

NUMERICAL OPTIMIZATION

Sheet 6: Review so far

Note: I strongly recommend you to try to solve all of these exercises with minimal use of the internet and online tools. The role of the exercises is for you to practice how ready are you for a practical test.

EXERCISE ONE Let's make some chocolate! As chocolate is in demand throughout the year, it is important to plan the production properly.

The predicted demand according to which we want to plan production is $d_i > 0$ for the i -th month in tons.

Create a linear program to express how much chocolate is to be produced in each month in the following year, so that we always meet demand while spending as little as possible on production.

A change in production volume of 1 ton between the following months costs 1500 PLN (due to redundancies or recruitment etc.) and storing 1 ton of chocolate costs 600PLN (counting from one month to the next).

Don't count the production of the chocolate itself, as it is paid for out of sales. Also assume that the chocolate does not spoil and you should not have any left at the end of December.

Hint: Define s_i as the surplus of chocolate in the i th month, and set $s_0 = s_{12} = 0$.

EXERCISE TWO Consider the function $f(x, y) = 2x^2 - 4xy + y^4 + 5y^2 - 10y$. Is this a convex function?

EXERCISE THREE

Consider again the function $f(x, y) = 2x^2 - 4xy + y^4 + 5y^2 - 10y$. Perform one step of the gradient descent method with backtracking via the Armijo condition, starting at the initial point $(0, 0)$. For the backtracking, start with the initial step length $\alpha = 1$ and use the parameters $c = 0.1$ (sufficient decrease parameter of Armijo) and $\rho = 0.1$ (contraction multiplier that is used to contract α).

EXERCISE FOUR Is the following object a convex set?

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + 2y^2 - 8y \geq 0, x^2 - (y - 2)^2 \leq 20.\}$$

EXERCISE FIVE Consider the following object (notice that the inequality is different than in the last exercise):

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + 2y^2 - 8y \leq 0.\}$$

Is the object an ellipsoid? How should we check this for a general quadratic object?