

NUMERICAL OPTIMIZATION: ASSIGNMENT 3

DEADLINE: the lab on 2024.03.25 (the lab on 2024.03.18 is canceled due to the rectorial election debate)

In this assignment, we are going to do least-squares optimization.

1. **1 point** Write a function which solves polynomial least-squares fitting, ie. for $n \in \mathbb{N}$, vectors $x, y \in \mathbb{R}^M$, find a polynomial p^* of degree at most n which minimizes the cost function

$$\sum_{i=1}^M (p^*(x_i) - y_i)^2.$$

2. **0.5 points** Modify the function from the previous task to factor in a weight function w , ie. to minimize the expression

$$\sum_{i=1}^M w(x_i)(p^*(x_i) - y_i)^2.$$

Test it for $x_i \in [-1, 1]$ and weight functions $w(t) = 1 - t^2$, $w(t) = t + 1$.

3. **0.5 points** Generate 100 points (x_i, y_i) with x_i being from the uniform distribution over $[0, 4]$ and $y_i = 0.5x_i^4 - 7.1x_i^3 - 4.2x_i^2 + 2$. Plot the function. Use the function from the first task to find optimal fittings of degrees 1, 2, 3, 4, 5, 6, 7. What do you observe?
4. **1 point** Generate 100 points (x_i, y_i) with x_i being from the uniform distribution over $[0, 4]$ and $y_i = 0.5x_i^4 - 7.1x_i^3 - 4.2x_i^2 + 2 + u_i$, where u_i is from the normal distribution $N(0, 1)$.
 - (a) Try and fit a polynomial of degree 99. What is the result?
 - (b) Experiment with lasso and ridge regressions (`from sklearn.linear_model import Lasso, Ridge`), ie. regression with L1 and L2 regularization, respectively. Experiment with different values of the parameter *alpha*.
5. **2 points** Find a perfect place for a society of rail enthusiasts meeting. In the text files, you'll find the distances between certain Polish cities via rail and the number of members of the society in each city. People can travel to the meeting place only using the specified rail connections. The cost function is thus the sum of the squared distances between the city of origin and the meeting place for each member. Consider two variants:
 - (a) the meeting can take place only in one of the cities;
 - (b) the meeting can take place at any location along the rail connection.