Assignment 1

(Please return the solutions before March 31st 2021)

1. ***[10% of the total grade].*** In the 6 dice game, you bet 1 CHF in every game that has the following payoff structure after 6 independent fair dice are rolled at the same time

Outcome	Payoff
If 1 face shows 6	You win 1 CHF
If 2 faces show 6	You win 1.5 CHF
If 3 faces show 6	You win 2 CHF
If 4 faces show 6	You win 2.5 CHF
If 5 faces show 6	You win 3 CHF
If 6 faces show 6	You win 3.5 CHF

Find a solution to know whether it is a good idea to play this game. Please provide the (analytical) intuition behind the solution to this problem as well as a computer code that leads to the solution of this problem.

2. ***[30% of the total grade].*** Write your own code for the Metropolis algorithm introduced in lecture 1 to find out the bias of a coin given by the hidden parameter θ. In this problem assume that a coin is flipped $N = 20$ times and you obtain $z = 14$ heads. Assume that you have no prior information about the bias of the coin, and therefore assume a beta prior distribution with parameters $a = 1$ and $b = 1$. Your goal is to find the posterior distribution of θ given the observed data. For this case, recall that in the Metropolis algorithm, the move of the proposed new sample θ is given by

$$\min\left(1, \frac{\text{Bernoulli}(z, N|\theta_{\text{pro}})\text{beta}(\theta_{\text{pro}}|a, b)}{\text{Bernoulli}(z, N|\theta_{\text{cur}})\text{beta}(\theta_{\text{cur}}|a, b)}\right)$$

Where θ_{cur} is the current position of theta and θ_{pro} is the proposed move. Assume that the proposed moves are given by random draws from a normal distribution with parameters $N(\mu = \theta_{\text{cur}}, \sigma = 0.2)$ (in case the random draw is $\theta_{\text{pro}} > 1$ set the proposed value to 1, and in case the random draw is $\theta_{\text{pro}} < 0$ set the proposed value to 0). Generate three chains, each starting at $\theta_{\text{cur}} = 0.1$, $\theta_{\text{cur}} = 0.9$ and $\theta_{\text{cur}} = 0.5$, respectively. For each chain, generate 5,000 samples using the first 2,500 samples as burn-in and use the last 2,500 samples for inference. Based on this information, please complete the following steps:

a. Write a computer code that generates the three chains based on the Metropolis algorithm
b. Plot the three chains in different ways (e.g. as overlapping line plots or density plots) to study whether the three chains converged to the same posterior distribution
c. Concatenate the three chains and study whether you can conclude that the coin is fair (i.e. heads and tails are generated with equal probability)
d. Given the same data (a coin is flipped $N = 20$ times and you obtain $z = 14$ heads) use JAGS to model the problem. Compare the results obtained between your Metropolis algorithm and those obtained with JAGS.

3. ***[20% of the total grade].*** We introduced how one can use JAGS to estimate both the mean μ and SD σ when it is assumed that the data are distributed following a normal distribution. Based on the code "OneGroupNorm.r" provided with the materials of Lecture 4, please complete the following steps

    a. Use the inferred mean μ to indicate whether you can conclude that the expected value of the IQ test is greater than 100
    b. The *cohen's d* is defined as the standardized expected mean (or difference of means when one compared two distributions) which provides an indication of the *effect size* strength (see table below). Use the posterior distributions for the mean and SD obtained based on data provided in "OneGroupNorm.r" to derive a new *effect size* distribution given by $d = \mu/\sigma$

        i. Discuss whether the effect size is larger than 0 (please add the 95% HDI).
        ii. Based on this result, what can you conclude of the effect size in light of the effect size metric (see table below and do a little research about *cohen's d*)

Effect size	***d***	**Reference**
Very small	0.01	Sawilowsky, 2009
Small	0.20	Cohen, 1988
Medium	0.50	Cohen, 1988
Large	0.80	Cohen, 1988
Very large	1.20	Sawilowsky, 2009
Huge	2.0	Sawilowsky, 2009

4. ***[40% of the total grade].*** Place the file "dataFit.RData" in your working directory and load it in R using the command: `load("dataFit.RData")`. Once you load the file, you will find two variables: x and y. Based on this data, please complete the following steps:
   a. Plot the data to get an impression of the possible relationship between x and y.
   b. Fit the data based on three different models
      i. Model 1: The relationship between the two variables is given by a straight line y = a + bx
      ii. Model 2: The relationship between the two variables is given by a quadratic function: y = a + bx + cx^2
      iii. Model 3: Fit a "null" model where the relationship between the two variables is constant: y = a
   c. Plot the real data and overlap the predictions of the three models
   d. Evaluate which of the three models provides a better fit to the data after penalizing for model complexity using the DIC as metric. Briefly describe and discuss the results.