**University of Zurich, Dept. of Economics**
International Macroeconomics
**Prof. Dr. Mathias Hoffmann**
**Final problem set FS2021**

`Note: this problem set reviews material previously covered in the lecture and/or the tutorial. It is provided for self-study purposes only and will not be discussed in class.`

1. Consider a small open economy in which the representative agent maximizes

$$U_t = \boldsymbol{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s u(C_{t+s}) \right\}$$

with

$$u(C_{t+s}) = -\frac{1}{2}(h - C_{t+s})^2$$

subject to the intertemporal budget constraint

$$B_{t+1} = (1+r)B_t + NO_t - C_t$$

and $\lim_{k\to\infty} \frac{B_{t+k}}{(1+r)^k} = 0$ .

The notation is familiar from the lecture:

| | |
|---|---|
| $B_t$ | Holdings of foreign bonds |
| $r$ | World interest rate |
| $C_t$ | Consumption |
| $Y_t$ | Gross Domestic Product (GDP) |
| $I_t$ | Gross Investment |
| $NO_t = Y_t - I_t$ | Net Output / National Cash-Flow |
| $\boldsymbol{E}_t(\cdot)$ | Expectations Operator |
| $K_t$ | Capital stock |
| $I_t = K_{t+1} - K_t$ | Investment |

(a) Under the assumption that $\beta(1+r) = 1$, we have shown in the course that the current account balance, $CA_t$, is given by

$$CA_t = -\sum_{k=1}^{\infty} \frac{\boldsymbol{E}_t(\Delta NO_{t+k})}{(1+r)^k} \tag{1}$$

(You DO NOT have to derive this formula!). Carefully explain an empirical test of equation (1). In your answer give special attention to why it is important to include the current account in the forecasting equation for $\Delta NO_{t+k}$.

(b) Now assume that net output is composed of permanent ($P_t$) and transitory ($T_t$) components so that

$$\Delta NO_t = \lambda \Delta P_t + (1 - \lambda)\Delta T_t$$

where $0 \leq \lambda \leq 1$ and

$$\Delta P_t = \alpha \Delta P_{t-1} + \eta_t$$
$$\Delta T_t = (\rho - 1)T_{t-1} + \nu_t$$

with $\alpha, \rho$ between zero and one and where $\eta_t$ and $\nu_t$ are time $t$ shocks. Use equation (1) to show that the response of the current account in period $t$ ...

1. ... to a shock in $\Delta P_t$ equals $-\frac{\lambda\alpha}{1+r-\alpha}$
2. ... to a shock in $\Delta T_t$ equals $\frac{(1-\lambda)(1-\rho)}{1+r-\rho}$

(c) In the simplest static Keynesian models of undergraduate macroeconomics texts, higher current income may be associated with a current account deficit as imports rise. Thus, the current account can be countercyclical. Indeed, early critics of the intertemporal approach to the current account (as embodied by the model described above) argued that a major empirical flaw of the approach is its inability to yield countercyclical current accounts. Are they right? Explain carefully using the results from exercise (b) above.

2. Consider a static production economy model with traded and non–traded goods. Production technology in both sectors is described by $Y_T = A_T L_T^\alpha$ and $Y_N = A_N L_N^\alpha$ with $\alpha \in (0,1)$. Labor is perfectly mobile across sectors. Aggregate labor is fixed and normalized to one such that labor market equilibirum requires $L_T + L_N = 1$. The production possibilities frontier (PPF) is given by

$$Y_T = A_T \left(1 - \left(\frac{Y_N}{A_N}\right)^{\frac{1}{\alpha}}\right)^\alpha .$$

The representative household derives utility $U(C)$ from a CES consumption bundle $C$ composed of tradable ($C_T$) and non–tradable ($C_N$) goods:

$$C(C_N, C_T) = \left[\gamma^{\frac{1}{\theta}} C_N^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} .$$

The country has net foreign bond holdings of $B_0$, which bear interest at the world rate $r$. The budget restriction is

$$CA = rB_0 + Y_T + \frac{P_N}{P_T} Y_N - \frac{P_N}{P_T} C_N - C_T,$$

where $CA$ is the current account balance and $\frac{P_N}{P_T}$ is the relative price of non–tradable goods.

(a) Show that optimal production requires that

$$\frac{P_N}{P_T} = \frac{A_T}{A_N} \left(\frac{L_N}{L_T}\right)^{1-\alpha} .$$

Provide a brief intuition for this optimality condition.

(b) As you know, optimal consumption behavior requires that

$$\frac{C_T}{C_N} = \frac{1-\gamma}{\gamma} \left(\frac{P_N}{P_T}\right)^\theta .$$

(You DO NOT have to derive this formula!)
Show that the current account to output ratio $\frac{CA}{Y}$ can be derived as

$$\frac{CA}{Y} = \frac{rB_0}{Y} + \frac{1 - \frac{1-\gamma}{\gamma}\left(\frac{P_N}{P_T}\right)^\theta \frac{Y_N}{Y_T}}{1 + \frac{P_N}{P_T}\frac{Y_N}{Y_T}},$$

where $Y$ denotes GDP expressed in units of tradable goods.

(c) In his treatise 'The Economic Consequences of the Peace', John Maynard Keynes argued that following its victory in World War I France would import inflation due to the income generated from foreign reparation claims (in particular on Germany) as stipulated in the Versailles Peace Treaty.

i. Use a $(C_T, C_N)$–diagram to provide a graphical illustration of Keynes' argument. You can assume an endowment economy with exogenous initial levels of $Y_N$ and $Y_T$.

ii. Bertil Ohlin put forward a critique of Keynes, arguing that Keynes' reasoning overestimated the required real appreciation for France, because it neglected the supply-side adjustment of the economy. Illustrate Ohlin's argument by comparing the production economy described above to an equivalent endowment economy as in exercise (c) i.

3. Consider a world economy which consists of two countries, Home and Foreign, and exists for two periods. The representative agent in the Home and Foreign country has known first–period income $Y_1$ and $Y_1^\star$, respectively. In addition, both countries exhibit a zero initial net foreign asset position ($B_1 = B_1^\star = 0$). On date 2, $\mathcal{S} \geq 2$ different states of nature $s \in \{1, \ldots, \mathcal{S}\}$ are possible, implying that second–period output level at home $Y_2(s)$ and abroad $Y_2^\star(s)$ is uncertain at date 1. Each state of nature may occur with probability $\pi(s) > 0$. Asset markets are complete, i.e. for each state of the world there is a state contingent Arrow–Debreu security available for trade. The price of an Arrow–Debreu security which pays one unit of consumption if state $s$ occurs and nothing in all other states is given by $\widetilde{p}(s) = \frac{p(s)}{1+r}$, where $p(s)$ is the state price and $r$ is the return on a risk–free bond. The representative household in the Home country maximizes lifetime expected utility

$$u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s)),$$

subject to his/her period resource constraints. By analogy, the representative household in the Foreign country maximizes lifetime expected utility

$$u(C_1^\star) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2^\star(s)),$$

subject to his/her period resource constraints. Home and Foreign consumers share the same preferences, which are described by a standard CRRA utility function:

$$u(C) = \frac{C^{1-\rho} - 1}{1-\rho}.$$

(a) Solve the optimization problem of the representative household in the Home country. Set up the representative household's optimization problem and show that the intertemporal Euler Equations are given by

$$\beta\pi(s) \left(\frac{C_1}{C_2(s)}\right)^{\rho} = \frac{p(s)}{1+r} \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \tag{2}$$

Provide a brief economic intuition for (2).

(b) Equilibrium in the world market at every point in time and all possible states requires that

$$\begin{aligned} C_1 + C_1^\star &= Y_1 + Y_1^\star \equiv Y_1^W \\ C_2(s) + C_2^\star(s) &= Y_2(s) + Y_2^\star(s) \equiv Y_2^W(s) \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \end{aligned}$$

Show that for any two states $s$ and $s'$ we must have

$$\left(\frac{Y_2^W(s)}{Y_2^W(s')}\right)^{-\rho} \frac{\pi(s)}{\pi(s')} = \frac{p(s)}{p(s')}. \tag{3}$$

Carefully explain the intuition behind equation (3).

(c) Show that our model implies

$$\frac{C_2(s)}{C_1} = \frac{C_2^\star(s)}{C_1^\star} = \frac{Y_2^W(s)}{Y_1^W} \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \tag{4}$$

(d) Use equation (4) to derive a regression-based test of complete risk sharing and explicitly state the null hypothesis of this test. Compare the typical findings of this test when it is run on data from the regions of an individual country to the results obtained when the regression is run using data from a cross-section of countries. Briefly interpret these findings.

(e) Assume now that $u(C) = \log(C)$ and $\mathcal{S} = 2$. Show that the current account at date one is given by

$$CA_1 = \frac{\beta}{1+\beta}Y_1 - \frac{1}{1+\beta}\left[\frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right].$$

Interpret this expression carefully. Specifically, compare it to equation (1) in Exercise 1.