Problem Set 7

1. (Bayesian Updating)

Consider a rational, risk-neutral entrepreneur, who must decide whether to implement a business idea. The success of the business plan is uncertain. If he decides to implement the plan, he must pay a non-refundable fixed cost $V > 0$. If implemented and the idea is a success (S), the entrepreneur earns a prize $X > V$, if the project fails (F), he earns nothing.

The entrepreneur can acquire a signal τ indicating whether the project is successful ($\tau = +$) or not ($\tau = -$). The signal structure verifies $P(+|S) = \theta$ and $P(-|F) = \theta$. Acquiring a signal costs $s > 0$.

Suppose that the entrepreneur can acquire at most two signals. The entrepreneur can decide sequentially as follows:

- Acquire the first signal? If no: Decide whether to implement or not.
- If yes: Observe the outcome of the signal. Then decide whether to acquire a second signal. If no: Decide whether to implement or not.
- If yes: Observe the outcome of the second signal and then decide whether to implement or not.

In the following, let $X = 2F$, $\theta = 4/5$ and $P(S) = 1/2$ for the prior probability that the project is a success.

(a) Suppose that the entrepreneur is a perfect Bayesian updater. Find the optimal decision strategy.

Hint: First derive the posteriors for all possible circumstances, and then use these to determine the optimal choice in each possible situation by backward induction.

(b) Suppose now that the entrepreneur suffers from underinference. Specifically, if p denotes his current prior about success, he updates according to

$$P(S|+) = \frac{1}{1 + \left(\frac{1-\theta}{\theta}\right)^c \left(\frac{1-p}{p}\right)}$$

after observing a positive signal. What is the optimal choice of a biased entrepreneur if $c = 0$ (maximal underinference)?

2. (**Optimal Information Acquisition**)

Consider the following investment model:

- You must decide in which of two projects $j = 0, 1$ to invest.
- In the end, exactly one project will be successful, and you earn a utility of 1 if you invested into the successful project. (Denote by ω_0 and ω_1 that project $j = 0$ or $j = 1$, respectively, is successful).
- However, project $j = 0$ is more prestigious, and you suffer from a reputation damage if you invest into this project and it fails. Specifically, you earn a utility of $-c$, where $c \in [0, 1)$, if you invest into $j = 0$ and this project fails. By contrast, you earn utility 0 if you invest into $j = 1$ and this project fails.
- Prior to making your investment, you can conduct an investigation (i.e. read business reports, analysts' views etc.).
- By conducting an investigation, you acquire a (Bayes-consistent) simple signal structure with state-space $S = \{S_0, S_1\}$ about which company is likely to be the successful one.
- The prior belief that $j = 0$ is successful is $\pi_0 = 1/2$.

(a) Calculate the ex ante expected utility assuming a separating signal structure (i.e., the investor chooses to invest in $j = 0$ after seeing signal S_0, and in $j = 1$ otherwise).

(b) Identify the set where separating signals yield a higher utility than deciding according to the prior in a $(P(S_0|\omega_0), P(S_1|\omega_1))$-plane.

(c) Consider a separating signal structure, and suppose that you have one marginal unit of attention at your disposal. You can either invest this unit of attention to increase $P(S_0|\omega_0)$ or $P(S_1|\omega_1)$ by a marginal unit. How should you use your attention?