Problem Set 8

1. Salience I

1(a)

$$\sigma(x,\bar{x}) = \frac{|x-\bar{x}|}{\bar{x}}$$

- **zero homogeneity**

  Let $\lambda > 0$

$$\sigma(\lambda x, \lambda\bar{x}) = \frac{|\lambda x - \lambda\bar{x}|}{\lambda\bar{x}} = \frac{|x-\bar{x}|}{\bar{x}} = \sigma(x,\bar{x})$$

- **ordering**

$$\underbrace{[x,y] \subsetneq [x',y']}_{\text{implicitly assumes } x\le y \quad x'\le y'} \implies \sigma(x,y) < \sigma(x',y')$$

$$\underbrace{[y,x] \subsetneq [y',x']}_{y\le x \quad y'\le x'} \implies \sigma(x,y) < \sigma(x',y')$$

$$\begin{cases} \sigma(x,y) = \dfrac{|x-y|}{y} = \dfrac{y-x}{y} = 1 - \dfrac{x}{y} < 1 - \dfrac{x'}{y'} = \dfrac{y'-x'}{y'} = \sigma(x',y') & x<y \\ \sigma(x,y) = \dfrac{|x-y|}{y} = \dfrac{x-y}{y} = \dfrac{x}{y} - 1 < \dfrac{x'}{y'} - 1 = \dfrac{x'-y'}{y'} = \sigma(x',y') & x>y \end{cases}$$

Interpretation

This function measures attribute salience as proportional difference from the average value

1(b)

Additional assumption: $x > \bar{x}, x' > \bar{x}'$

$$\begin{aligned} \frac{x'-\bar{x}'}{\bar{x}'} &> \frac{x-\bar{x}}{\bar{x}} \\ \frac{x'}{\bar{x}'} - 1 &> \frac{x}{\bar{x}} - 1 \\ \frac{x'}{\bar{x}'} &> \frac{x}{\bar{x}} \\ x' &> \frac{x}{\bar{x}} \times \bar{x}' \end{aligned}$$

$$[\bar{x}', \frac{x}{\bar{x}} \times \bar{x}'] \subsetneq [\bar{x}', x'] \implies \sigma(x',\bar{x}') > \sigma(\frac{x}{\bar{x}} \times \bar{x}', \bar{x}') = \sigma(x,\bar{x})$$

2. Salience II

$$\delta(c_1 - c_2) < q_1 - q_2 < \frac{c_1 - c_2}{\delta}$$

2(a)

$$\frac{q_1}{c_1} > \frac{q_2}{c_2}$$

$$j = \begin{cases} 1 & p_1 = \min\left\{\frac{q_1}{q_2}c_2, \frac{q_1 - q_2}{\delta} + c_2\right\} \\ 2 & p_2 = c_2 \end{cases}$$

Case 1

$$\frac{q_1}{q_2}c_2 \leq \frac{q_1 - q_2}{\delta} + c_2 \implies p_1 = \frac{q_1}{q_2}c_2$$

$$\frac{q_1}{p_1} = \frac{q_1}{\frac{q_1}{q_2}c_2} = \frac{q_2}{c_2} = \frac{q_2}{p_2}$$

$$\sigma(p_1, \bar{p}) = \sigma(q_1, \bar{q}) \implies p \text{ and } q \text{ are equally salient}$$

- If $u^S(q_1, \bar{p}_1) = q_1 - p_1 > u^S(q_2, p_2) = q_2 - p_2$, Firm 1 gets the entire market
- If $u^S(q_1, p_1) \leq u^S(q_2, p_2)$, apply "limit pricing" assumption

$$p_1 > c_1 \implies p_2 = c_2$$

If firm 1 reduces price by small ε to $p_1 - \varepsilon$

$$\frac{q_1}{p_1 - \varepsilon} = \frac{q_1}{\frac{q_1}{q_2}c_2 - \varepsilon} = \frac{q_2}{c_2 - \frac{q_2}{q_1}\varepsilon} > \frac{q_2}{c_2} = \frac{q_2}{p_2} \implies \text{quality is salient}$$

$$u^S(q_1, p_1 - \varepsilon) = q_1 - \delta(p_1 - \varepsilon) \geq q_1 - \delta(\frac{q_1 - q_2}{\delta} + c_2 - \varepsilon) = q_2 - \delta c_2 + \delta\varepsilon > q_2 - \delta c_2 = u^S(q_2, p_2)$$

Firm 1 can undercut firm 2 then firm 1 can get the entire market $d_1 = 1, d_2 = 0$

- Firm 2 does not deviate to $\tilde{p}_2 < p_2 = c_2$ because this is a weakly dominated strategy

  If $\tilde{p}_2 > p_2$, then $\frac{q_1}{p_1} > \frac{q_2}{\tilde{p}_2} \implies$ quality is salient

$$u^S(q_1, p_1) = q_1 - \delta p_1 \geq q_2 - \delta c_2 > q_2 - \delta\tilde{p}_2 = u^S(q_2, \tilde{p}_2)$$

  Firm 2 still makes zero profit and has no incentive to set $\tilde{p}_2 > p_2$
- Firm 1 does not deviate to $\tilde{p}_1 < p_1$

$$\Pi_1(q_1, \tilde{p}_1, q_2, p_2) \leq \tilde{p}_1 - c_1 < p_1 - c_1 = \Pi_1(q_1, p_1, q_2, p_2)$$

- Firm 1 does not deviate to $\tilde{p}_1 > p_1$

$$\frac{q_1}{\tilde{p}_1} < \frac{q_1}{p_1} = \frac{q_2}{p_2} \implies \text{price is salient}$$

$$u^S(q_1, \tilde{p}_1) = \delta q_1 - \tilde{p}_1 < \delta q_1 - p_1 = \delta q_1 - \frac{q_1}{q_2}c_2$$

$$q_1 - q_2 < \frac{c_1 - c_2}{\delta}$$

$$\delta(q_1 - q_2) < c_1 - c_2 < \frac{q_1}{q_2} \times c_2 - c_2$$

$$\delta(q_1 - q_2) < c_2 \times (\frac{q_1}{q_2} - 1)$$

$$\delta q_1 - \frac{q_1}{q_2} \times c_2 < \delta q_2 - c_2$$

$$u^S(q_1, \tilde{p}_1) < u^S(q_2, p_2)$$

  Firm 2 gets the whole market $\Pi_1 = 0$

Case 2

$$\frac{q_1-q_2}{\delta}+c_2<\frac{q_1}{q_2}c_2 \implies p_1=\frac{q_1-q_2}{\delta}+c_2$$

$$\frac{q_1}{p_1}>\frac{q_1}{\frac{q_1}{q_2}c_2}=\frac{q_2}{c_2}=\frac{q_2}{p_2} \implies \text{quality is salient}$$

$$\begin{aligned} u^S(q_1,p_1) &= q_1-\delta p_1 \\ &= q_1-\delta\left[\frac{q_1-q_2}{\delta}+c_2\right] \\ &= q_2-\delta c_2 \\ &= u^S(q_2,p_2) \end{aligned}$$

If firm 1 switches to price $p_1-\varepsilon, \varepsilon>0$, then the quality is still salient and

$$\begin{aligned} u^S(q_1,p_1-\varepsilon) &= q_1-\delta p_1+\delta\varepsilon \\ &= q_1-\delta\left(\frac{q_1-q_2}{\delta}+c_2\right)+\delta\varepsilon \\ &= q_2-\delta c_2+\delta\varepsilon \\ &> q_2-\delta c_2=u^S(q_2,p_2) \end{aligned}$$

By "limit pricing", firm 1 gets the entire market

- Firm 2 never deviates
  - If $\tilde{p}_2<p_2=c_2$: this is weakly dominated strategy
  - If $\tilde{p}_2>p_2$, quality still salient and

    $$u^S(q_1,p_1)=u^S(q_2,p_2)=q_2-\delta p_2>q_2-\delta\tilde{p}_2=u^S(q_2,\tilde{p}_2)$$

    still $d_1=1, d_2=0$
- Firm 1 never deviates
  - If $\tilde{p}_1<p_1$:

    $$\Pi_1(\tilde{p}_1,q_1,p_2,q_2)=\tilde{p}_1-c_1<p_1-c_1=\Pi(p_1,q_1,p_2,q_2)$$

  - If $\tilde{p}_1>p_1$: Firm 2 gets the entire market independent of whether quality or price is salient

    $$\Pi_1=0$$

In conclusion

$$p_2=c_2, p_1=\min\left\{\frac{q_1}{q_2}c_2, \frac{q_1-q_2}{\delta}+c_2\right\} \text{ is an equilibrium}$$

2(b)

$$q_1-c_1>q_2-c_2$$

- If $\delta=1$, we have $p_2=c_2$ and $p_1=(q_1-q_2)+c_2$ in equilibrium, firm 1 gets the entire market

  $$\Pi_1^{\delta=1}=q_1-q_2+c_2-c_1$$

- In salient equilibrium

  $$\Pi_1^{\delta<1}\leq\frac{q_1}{q_2}c_2-c_1$$

$$
\begin{aligned}
\Pi_1^{\delta=1} &> \Pi_1^{\delta<1} \\
q_1 - q_2 + c_2 - c_1 &> \frac{q_1}{q_2} c_2 - c_1 \\
q_1 - q_2 &> \left(\frac{q_1}{q_2} - 1\right) c_2 \\
q_2 &> c_2
\end{aligned}
$$

Payoff from firm 1 is strictly higher in rational equilibrium than salience equilibrium iff $c_2 < q_2$

Intuition

Firm 1 wants to set high price if quality is salient but this high price would make price salient (against firm 1's advantage)

Firm 1 wants to set price low enough to keep quality salient, which reduces market power.