Problem Set 6

1. (**EU Theory**) Let $u(\cdot)$ be a strictly increasing and concave Bernoulli utility function for a person with vNM utility $U(\cdot)$. Show that for any lottery F_L[1]

$$\bar{X}_L = \int x dF_L(x) \geq c(F_L, u)$$

2. (**State-Space-model**) Consider the state-space model with two uncertain future states $s = 1, 2$, with respective probability π_1, π_2, where $\pi_1 + \pi_2 = 1$. There are two perfectly divisible assets $k = 1, 2$, and each unit of each asset costs 1. Asset $k = 1$ pays one unit of wealth in both states, while asset $k = 2$ pays a random dividend of r units of wealth in state 2, where r is uniformly distributed over the real interval $[1, 2]$. Asset $k = 2$ pays nothing in state 1.

   An agent has current wealth $w > 0$ and wants to decide on the quantities $x_1, x_2 \geq 0$ of assets to acquire (selling is not possible). The agent has state-dependent Bernoulli utility functions u_1, u_2 over her future wealth positions, where $u_1(z) = Log(z)$ and $u_2(z) = 2u_1(z)$. Derive the parameter range of π_1, π_2 for which the agent acquires a non-zero amount of asset $k = 2$.

[1] You can assume that the corresponding integrals exist and are finite.

3. (**Prospect Theory**) Consider a decision maker (DM) with Bernoulli utility function
$$u(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ -\sqrt{-x}, & x < 0 \end{cases} .$$
Further, the initial wealth position of the DM is $w \in \mathbb{R}$. Consider the fair gamble, where the DM wins or loses 10 with equal probability.

   (a) Depict this utility function. Is it consistent with Prospect Theory?

   (b) Let $w = 0$. Show that the DM is indifferent about accepting the gamble if $w = 0$.

   (c) Suppose that $w \in (-10, 0)$. Show that the DM always chooses the gamble.

   (d) Let $w = 0$. The DM is in a Casino and can play the above gamble repeatedly. He decides sequentially about whether to gamble again or not given his current wealth. Suppose that he always chooses to gamble if his current wealth is zero (actually, he is indifferent). Moreover, there is a time constraint (the Casino closes), such that he can play maximally 4 gambles. Describe how the DM will gamble for any possible contingency given the above utility function. Calculate the expected outcome of this strategy by finding the various outcome states the DM can have.

   (e) Suppose that somebody offered the above gamble as a one-shot lottery over the final outcome states. Would our decision maker accept this gamble?

4. (**Rabin's Critique**) Suppose we know that Johnny is a risk-averse expected-utility maximizer, and that he will always turn down the 50-50 gamble of losing \$10 or gaining \$11. Consider now another gamble that with a 50 percent chance of losing \$100 and a 50 percent chance of winning some

amount \$$Y$. Make a guess: What is the biggest Y such that Jonny would prefer to turn down the gamble?