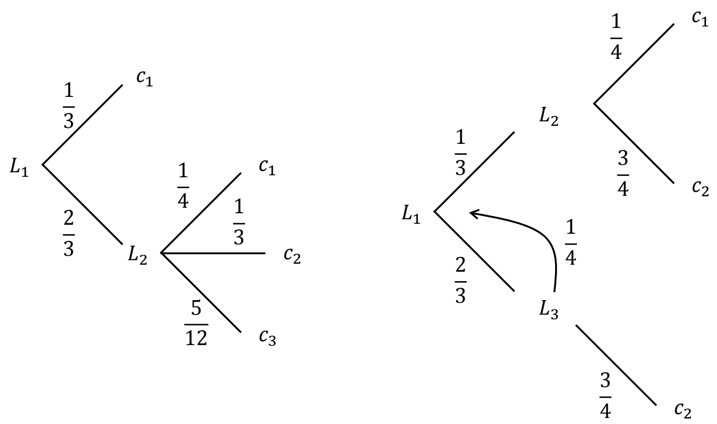
Problem Set 5

1. Let $c_1, ..., c_n$ denote final outcomes. Derive the simple lotteries induced by the following two compound lotteries.



2. Let $n = 4$ and let $U(L) = \sum p_l u_l$ be a vNM utility function.
   (a) Suppose that $u_1 - u_2 > u_3 - u_4$. Show that $L_1 = (\frac{1}{2}, 0, 0\frac{1}{2}) \succ (0, \frac{1}{2}, \frac{1}{2}, 0) = L_2$.
   (b) Is vNM utility an ordinal concept?

3. Let $U : \mathcal{L} \to \mathbb{R}$ be a vNM utility function representing $\succeq$. Show that $\tilde{U}(L) = \beta U(L) + \gamma$ also is a vNM utility function representing $\succeq$ whenever $\beta > 0$ and $\gamma \in \mathbb{R}$.

4. Show that if the preference $\succeq$ over $\mathfrak{L}$ satisfies the independence axiom, then for any $L, L', L'' \in \mathfrak{L}$ and any $\alpha \in (0, 1)$, we have

$$L \sim L' \iff \alpha L + (1 - \alpha)L'' \sim \alpha L' + (1 - \alpha)L''.$$

5. Let $M = [0, 1]$ be the money domain, and consider a strictly risk averse expected utility decision maker with Bernoulli utility $u(x) = \sqrt{x}$.

   (a) Let L be a lottery that gives $x = 2/3$ with certainty. What is its corresponding distribution function F_L? What is $U(F_L)$?

   (b) Consider the lottery L with support M and uniform density $f_L(x) = 1$. Derive $U(F_L)$.

   (c) Derive the certainty equivalent $c(F_L, \sqrt{\cdot})$ of the last lottery.