Exam
Social Choice Theory
Spring 2017

Instructions:

- The duration of the exam is 90 minutes.
  Please answer all three problems.
  You can achieve up to 90 points.
- Please solve and answer all problems on the answer sheet that is distributed separately.
  Write your name and your student ID number on the answer sheet.
  Please have your student identity card ready.
- You must document the solution process and provide sufficient arguments for your solution.
- This question sheet has 4 pages (including cover page). Please check for completeness.
- You are not allowed to use any auxiliary equipment.
  Switch off all electronic devices.

Points:

- Problem 1 (Individual Choice Functions): 30 points
- Problem 2 (Manipulability): 30 points
- Problem 3 (Social Evaluation Functions): 30 points

Good Luck!

Problem 1: Individual Choice Functions (30 points)

Let $X = \{x, y, z\}$ be a set of alternatives. The following table specifies five different choice functions C^i, $i = 1, \ldots, 5$, on X. The table is read as follows. The first column lists all subsets $S \subseteq X$ with at least two elements. The second column describes which elements are chosen from each of these subsets according to choice function C^1. Columns three to six contain the analogous information for choice functions C^2 to C^5.

S	$C^1(S)$	$C^2(S)$	$C^3(S)$	$C^4(S)$	$C^5(S)$
$\{x,y,z\}$	$\{y\}$	$\{x\}$	$\{x,z\}$	$\{z\}$	$\{x\}$
$\{x,y\}$	$\{y\}$	$\{x\}$	$\{x\}$	$\{y\}$	$\{x,y\}$
$\{x,z\}$	$\{z\}$	$\{z\}$	$\{x,z\}$	$\{z\}$	$\{z\}$
$\{y,z\}$	$\{y\}$	$\{y\}$	$\{z\}$	$\{y,z\}$	$\{z\}$

(a) Do these choice functions satisfy or violate the conditions α (contraction consistency) and/or β (expansion consistency)? (10 points)

(b) For each of the choice functions, construct the base relation. (10 points)

(c) Are the base relations derived in part (b) of this problem transitive? Do they rationalize the respective choice function? (10 points)

考点在于什么是base relation：一定是完备和反身的，然后choice func.里有什么就加什么

还有什么是传递性：xRy, yRz要推出xRz，所以只要xRz在base relation里就行了，至于会不会还有zRx无所谓的

Problem 2: Manipulability (30 points)

Let $X = \{x, y\}$ be the set of alternatives and $N = \{1, 2, 3\}$ the set of voters. In this setting, the following tables completely specify a social choice function (SCF) on the domain $\mathscr{A} = \mathscr{R}^3$. Voter 1's possible preferences are listed in the rows of the tables. Voter 2's possible preferences are listed in the columns of the tables. Voter 3's possible preferences correspond to the three different tables. For instance, if voter 1 strictly prefers x over y, voter 2 strictly prefers y over x, and voter 3 is indifferent between x and y, then the field with row xP_1y and column yP_2x in the middle table shows that alternative x is selected.

xP_3y:

	xP_2y	xI_2y	yP_2x
xP_1y	x	x	x
xI_1y	x	x	x
yP_1x	x	x	y

xI_3y:

	xP_2y	xI_2y	yP_2x
xP_1y	x	x	x
xI_1y	x	y	y
yP_1x	y	y	y

yP_3x:

	xP_2y	xI_2y	yP_2x
xP_1y	x	y	y
xI_1y	y	y	y
yP_1x	y	y	y

(a) Which of Arrow's axioms for SCFs are satisfied and which are violated by this method? (15 points)

(b) Is this SCF surjective? Which of Gibbard-Satterthwaite's axioms are satisfied and which are violated by this method? (9 points)

(c) Explain the relation between your findings in part (b) of this problem and the Gibbard-Satterthwaite impossibility result. (6 points)

Problem 3: Social Evaluation Functions (30 points)

Let X be a set of alternatives. For any profile of utility functions $\mathbf{U} = (U_1, \ldots, U_n) \in \mathscr{U}^n$ and any alternative $x \in X$, first define the average utility by

$$\bar{U}(x, \mathbf{U}) = \frac{1}{n} \sum_{i=1}^{n} U_i(x).$$

Then define the mean absolute deviation of utilities by

$$\mathrm{MAD}(x, \mathbf{U}) = \frac{1}{n} \sum_{i=1}^{n} \left| U_i(x) - \bar{U}(x, \mathbf{U}) \right|,$$

where $\left| U_i(x) - \bar{U}(x, \mathbf{U}) \right|$ denotes the absolute value of the number $U_i(x) - \bar{U}(x, \mathbf{U})$, i.e., omitting the minus if negative. Consider the social evaluation function (SEF) e^{MAD} defined by

$$x \, e^{\mathrm{MAD}}(\mathbf{U}) \, y \quad \Leftrightarrow \quad \mathrm{MAD}(x, \mathbf{U}) \leq \mathrm{MAD}(y, \mathbf{U}),$$

for all $x, y \in X$ and all $\mathbf{U} \in \mathscr{U}^n$.

(a) Determine $e^{\mathrm{MAD}}(\mathbf{U})$ for the example given in the following table. (5 points)

$\mathbf{U}$	x	y	z
U_1	2	1	9
U_2	3	1	7
U_3	4	1	5

(b) With which of the seven information structures introduced in class is e^{MAD} consistent? (25 points)