Problemset 5

International Macroeconomics (Master)

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Exercise 1: Present-Value Model of the Current Account

A representative household maximizes his time–separable lifetime utility,

$$U_t = \sum_{s=0}^{\infty} \beta^s u\left(C_{t+s}\right)$$

subject to the period budget constraint

$$B_{t+1} = (1+r)\, B_t + NO_t - C_t.$$

(a) Derive the present value form of the intertemporal budget constraint.

(b) Show that with $\beta = \frac{1}{1+r}$, the representative household consumes the *annuity value* of his total discounted net wealth,

$$C_t = \frac{r}{1+r}\left[(1+r)\, B_t + \sum_{s=0}^{\infty} \frac{NO_{t+s}}{(1+r)^s}\right].$$

(c) Show that the current account is the present value of future changes in net output, i.e. $CA_t = -\sum_{s=1}^{\infty} \frac{\Delta NO_{t+s}}{(1+r)^s}$.

(i) Using the current account identity $CA_t = B_{t+1} - B_t$ and the intertemporal budget constraint, show that the following relationship holds true:

$$CA_t = -\frac{r}{1+r}\left[\sum_{s=0}^{\infty} \frac{NO_{t+s} - NO_t}{(1+r)^s}\right].$$

(ii) Show that the current account equals the present discounted value of future net output changes, i.e.,

$$CA_t = -\sum_{s=1}^{\infty} \frac{\Delta NO_{t+s}}{(1+r)^s}.$$

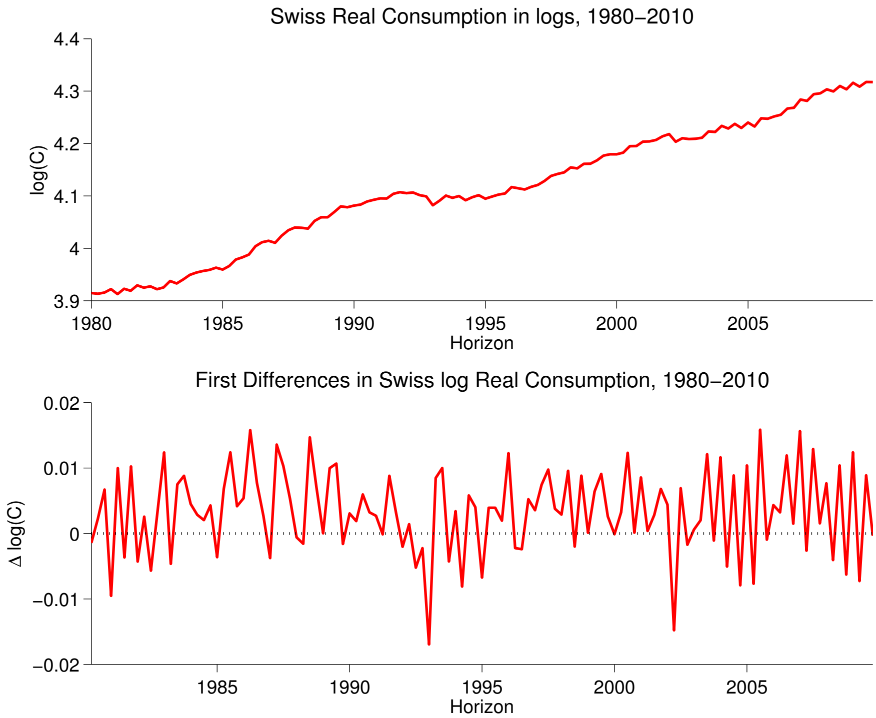
Exercise 2: Isoelastic Utility

Consider the following isoelastic utility function,

$$u(C_t) = \begin{cases} \frac{C_t^{1-\rho}-1}{1-\rho} & \text{for} \quad \rho > 0 \\ \log(C_t) & \text{for} \quad \rho = 1 \end{cases}$$

(a) Assume that gross consumption growth $\frac{C_{t+1}}{C_t}$ follows a log–normally distributed process.[1] Show that the Euler equation $\beta E_t\left[\frac{u'(C_{t+1})}{u'(C_t)}\right] = \frac{1}{1+r}$ with $\beta = \frac{1}{1+r}$ implies that $\log(C_{t+1})$ follows a random walk with a constant drift.

(b) The figure below portrays Swiss real log consumption and first differences of real log consumption. In light of this graph, is the result derived in part (a) consistent with what we observe in the data?



[1] If X is a normally distributed random variable with mean μ_X and variance σ_X^2, then $\exp(X)$ is log–normal with mean $E[\exp(X)] = \exp\left(\mu_X + \frac{1}{2}\sigma_X^2\right)$.

Exercise 3 (optional): Intertemporal Approach to the Current Account

Similar to Exercise 1, in the *stochastic* intertemporal infinite–horizon model of the current account with quadratic utility function and $\beta = \frac{1}{1+r}$, the current account equals the present discounted value of expected future net output changes, i.e.

$$CA_t = -\sum_{s=1}^{\infty} \frac{E_t\left[\Delta NO_{t+s}\right]}{(1+r)^s}. \tag{1}$$

(a) Show that $CA_{t+1} - \Delta NO_{t+1} - (1+r)CA_t \equiv \eta_{t+1}$ is the (negative) prediction error in consumption.

(b) Show that equation (1) holds if and only if the variable $CA_{t+1} - \Delta NO_{t+1} - (1+r)CA_t$ is statistically uncorrelated with date t (or earlier) variables.