Problem Set 2

1. **(WARP and Demand)** Let $x(p, w)$ be a Walrasian demand function. Consider the following property of $x(p, w)$:

   Let (p, w) and (p', w') be two price-wealth situations for which

   $$(p \cdot x(p', w') \leq w \ \textit{and} \ x(p', w') \neq x(p, w)) \implies p' \cdot x(p, w) > w'. \quad (1)$$

   (a) State a verbal interpretation of condition (1).

   (b) Suppose that $x(p, w)$ verifies condition (1) for any two price-wealth situations (p, w) and (p', w'). Show that the corresponding choice structure satisfies WARP.

   (c) Draw a picture in $X = \mathbb{R}^2_+$ that illustrates: If condition (1) is violated, then $x(p, w)$ also violates WARP.

2. **(Non-continuous preferences)** Let $X = \mathbb{R}^2_+$, and define $x \succeq y$ if either $x_1 > y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$. This preference relation is called "lexicographic" (why the name?).

   (a) Illustrate that $\succeq$ is rational, strongly monotone and strictly convex.

   (b) Can you find a utility function that represents these $\succeq$?

   (c) Show that the lexicographic preference relation fails to be continuous.

3. (**Preferences and Utility**) Let the choice set be $X = \mathbb{R}^L_+$, where

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_L \end{pmatrix} \in \mathbb{R}^L_+$$

denotes a choice alternative. Let a preference relation be defined by

$$x \succeq y \quad \Leftrightarrow \quad x \geq y,$$

where $x \geq y$ holds iff $x_l \geq y_l$, $\forall l = 1, ..., L$.

(a) Can these preferences be represented by a utility function?

(b) Depict the indifference set $I(x) = \{y \in X : y \sim x\}$ for the case where $L = 2$.

4. **(Preferences and Utility)** Consider the utility function $u(x, y) = 3Ln(x) + 2Ln(y)$ defined on $X = \mathbb{R}^2_{++}$.

(a) Are the underlying preferences (strictly) convex? Continuous? Monotonic? Locally non-satiated?

(b) Does this utility function represent a homothetic preference relation?

5. **(Demand and Utility)** Consider a Walrasian demand function $x(p, w)$ where

$$x_l(p, w) = \frac{w}{\sum_{i=1}^{L} p_i}$$

for $p, w > 0$ and any $l = 1, ..., L$.

(a) Does $x(p, w)$ verify zero-homogeneity and Walras' Law?

(b) Derive the matrix of price effects $D_p x(p, w)$ for this demand.

(c) Does this demand function feature any gross substitutes?

(d) Let $L = 2$, such that

$$x_1(p, w) = x_2(p, w) = \frac{w}{p_1 + p_2}.$$

Can you find a utility function that yields this Walrasian demand as a solution to the UMP?

(Hint: Try to think how the indifference curves must look like for such a demand function to emerge as the solution of an UMP.)