Problemset 10

International Macroeconomics (Master)

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Exercise 1: Market Completeness and Risk Sharing

Consider a world economy which consists of two countries, Home and Foreign, and exists for two periods. The representative agent in the Home and Foreign country has known first–period income Y_1 and $Y_1^\star$, respectively. In addition, both countries exhibit a zero initial net foreign asset position ($B_1 = B_1^\star = 0$). On date 2, $\mathcal{S} \geq 2$ different states of nature $s \in \{1, \ldots, \mathcal{S}\}$ are possible, implying that second–period output level at home $Y_2(s)$ and abroad $Y_2^\star(s)$ is uncertain at date 1. Each state of nature may occur with probability $\pi(s) > 0$.

Asset markets are complete, i.e. for each state of the world there is a state contingent Arrow–Debreu security available for trade. The price of an Arrow–Debreu security which pays one unit of consumption if state s occurs and nothing in all other states is given by $\widetilde{p}(s) = \frac{p(s)}{1+r}$, where $p(s)$ is the state price and r is the return on a risk–free bond. The representative household in the Home country maximizes lifetime expected utility

$$u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s)),$$

subject to his/her period resource constraints. By analogy, the representative household in the Foreign country maximizes lifetime expected utility

$$u(C_1^\star) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2^\star(s)),$$

subject to his/her period resource constraints.

Home and Foreign consumers share the same preferences, which are described by a standard CRRA utility function:

$$u(C) = \frac{C^{1-\rho} - 1}{1 - \rho}.$$

(a) Solve the optimization problem of the representative household in the Home country.

(i) Set up the representative household's optimization problem.

(ii) Show that the intertemporal Euler Equations are given by

$$\beta \pi(s) \left(\frac{C_1}{C_2(s)} \right)^\rho = \frac{p(s)}{1+r} \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \tag{1}$$

(iii) Provide a brief economic intuition for (1).

(b) Equilibrium in the world market at every point in time and all possible states requires that

$$\begin{aligned} C_1 + C_1^\star &= Y_1 + Y_1^\star \equiv Y_1^W \\ C_2(s) + C_2^\star(s) &= Y_2(s) + Y_2^\star(s) \equiv Y_2^W(s) \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \end{aligned}$$

Show that for any two states s and s' we must have

$$\left(\frac{Y_2^W(s)}{Y_2^W(s')}\right)^{-\rho} \frac{\pi(s)}{\pi(s')} = \frac{p(s)}{p(s')}. \tag{2}$$

What is the intuition behind equation (2)?

(c) Show that our model implies

$$\frac{C_2(s)}{C_1} = \frac{C_2^\star(s)}{C_1^\star} = \frac{Y_2^W(s)}{Y_1^W} \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \tag{3}$$

(d) What does condition (3) imply for the international co-movement of consumption? Why are actually observed patterns of consumption co-movement puzzling?

(e) Assume now that $u(C) = \log(C)$ and $\mathcal{S} = 2$. Show that the current account at date one is given by

$$CA_1 = \frac{\beta}{1+\beta} Y_1 - \frac{1}{1+\beta}\left[\frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right].$$

(f) Let $p(s)^A$ and r^A denote autarky state prices and interest rate, respectively. Show that the current account can be written as

$$CA_1 = \frac{Y_2(1)}{1+\beta}\left[\frac{p(1)^A}{1+r^A} - \frac{p(1)}{1+r}\right] + \frac{Y_2(2)}{1+\beta}\left[\frac{p(2)^A}{1+r^A} - \frac{p(2)}{1+r}\right].$$

What is the economic interpretation for this current account equation?

(g) Show that the country's gross purchases of the individual Arrow–Debreu securities satisfy

$$\begin{aligned} \frac{p(1)}{1+r} B_2(1) &= \pi_1 CA_1 + \frac{p(2)\pi_2 Y_2(1)}{1+r}\left[\frac{p(1)^A}{p(2)^A} - \frac{p(1)}{p(2)}\right] \\ \frac{p(2)}{1+r} B_2(2) &= \pi_2 CA_1 - \frac{p(2)\pi_2 Y_2(1)}{1+r}\left[\frac{p(1)^A}{p(2)^A} - \frac{p(1)}{p(2)}\right]. \end{aligned}$$

HINT: Show first that optimal consumption at date 2 in state s is given by

$$\frac{p(s)}{1+r} C_2(s) = \frac{\pi_s \beta}{1+\beta}\left[Y_1 + \frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r}\right]$$

Provide an intuition for the optimal portfolio structure in a situation in which $CA = 0$.