Global Poverty and Economic Development

Wenjie Tu

Fall Semester 2021

1 Adverse Selection

Setup:

- Projects need startup cost L
- Entrepreneurs (borrowers) vary in their unobservable type: risky or safe
  - Risky borrowers: invest in risky assets and obtain return $R' > L$ with probability p and zero return with probability $1-p$
  - Safe borrowers: invest in safe assets and always obtain return $R < R'$
  - No borrower's action/effort
- Only one potential borrower of each type and one lender who can issue only one single loan, L
  - If both borrowers apply, the lender randomly picks one (the lender cannot observe the borrow's type)

Solution:

- Maximum interest rate that borrowers will accept
  - Safe borrower: $i_s = \frac{R-L}{L}$
  - Risky borrower: $i_r = \frac{R'-L}{L}$ (she pays only if the project succeeds)
- If the lender offers a loan at interest rate i_s, both borrowers willy apply and the lender's expected profit is:

$$\pi_s = \frac{1}{2}L(1+i_s) + \frac{1}{2}Lp(1+i_s) - L$$

- If the lender offers a loan at interest rate i_r, only the risky borrower will apply and the lender's expected profit is:

$$\pi_r = pL(1+i_r) - L$$

- The lender will choose i_s if $\pi_s > \pi_r$

$$p < \frac{R}{2R'-R}$$

- Intuition
  - By raising the interest rate, only risky borrowers apply (adverse selection) $\rightarrow$ higher interest may reduce lender's profit
  - If the lender chooses i_s, there is **credit rationing**: demand exceeds supply at i_s, but the lender does not raise the price

2 Moral Hazard

Setup:

- An entrepreneur can invest in a project that leads to return R with probability e and 0 otherwise
- The entrepreneur chooses the effort level e:
  - Cost of effort: $c(e) = \frac{1}{2}ce^2$
- Opportunity cost of capital: ρ
- Opportunity cost of labor/time: u

Solution:

- If the entrepreneur can self-finance the project, her maximization problem is:

$$\max_{e} \quad eR + (1-e)0 - \frac{1}{2}ce^2 - \rho - u$$

- The optimal (First Best) level of effort is:

$$e^{FB} = \frac{R}{c}$$

  - assume an interior solution $e < 1$
- Now assume the entrepreneur cannot self-finance: she has illiquid wealth w that she can use as collateral for a loan
- The entrepreneur can get a loan from a lender:
  - He pays back interest r if the project succeeds
  - He pays collateral w if the project does not succeed (extreme case of limited liability: $w = 0$)
- Entrepreneur (borrower) payoff:

$$\pi^B = e(R-r) + (1-e)(-w) - \frac{1}{2}ce^2 - u$$

- Lender payoff:

$$\pi^L = er + (1-e)w - \rho$$

- If the two parts could contract on effort, they would choose the level that maximizes the joint surplus ($\pi^B + \pi^L$), which is again e^{FB}
- Now assume that the lender and the borrower cannot contract on effort
  - Notice that the lender observes the type of the borrower but he still cannot contract on the action of the borrower
- For a given interest, the borrower will choose the level of effort that maximizes π^B (*Incentive Compatibility Constraint*)

$$e^{SB} = \frac{R - r + w}{c}$$

  - If $w < r$, then $e^{SB} < e^{FB}$. Why?
- Assume perfect competition among lenders $\rightarrow$ Lender's expected profit must equal the cost of capital (*Zero Profit Condition*):

$$er + (1-e)w = \rho$$

- Blug the IC into the ZPC, we obtain
$$ce^2 - eR + (\rho - w) = 0$$

- The solution is the larger root:
$$e^*(w) = \frac{R + \sqrt{R^2 - 4c(\rho - w)}}{2c}$$
  - The lender is indifferent between two roots, but the borrower is better off with the larger root
  - e^* is increasing in w. If $w = \rho$, $e^* = e^{FB}$

- We can also solve for the equilibrium interest (i.e., loan$\times$(1+interest rate))
$$r^*(w) = w + \frac{R - \sqrt{R^2 - 4c(\rho - w)}}{2}$$

- It can be shown that, for $w < \rho$, $\frac{\partial r^*(w)}{\partial w} < 0$
  - Richer borrowers get the loan at a lower interest rate and in equilibrium they will be more successful in their projects
  - If w is very low, it may be impossible to satisfy the lenders' ZPC while also ensuring the borrower's utility is above u $\rightarrow$ poor borrowers do not receive the loan

Example:

Question 1: Agents can undertake a project at cost of 1. The project has outcome y if it succeeds and 0 otherwise. The probability of success is equal to the amount of effort e the agent exerts (or probability $= 1$ if $e > 1$). The cost of effort is $\frac{1}{2}ce^2$

1. What is the first best effort choice? In this and next questions, assume $c \geq y$.
$$\max_e \quad ey + (1-e)0 - \frac{1}{2}ce^2 - 1 \implies e^{FB} = \max\left\{\frac{y}{c}, 1\right\}$$

2. Suppose the agent cannot self-finance the project, but she has to borrow from a bank at (gross) interest rate $r > 1$. Assume the agent has limited liability. Write down the borrower's problem. What is the level of effort chosen by the borrower? How does it compare to the first best? Why?
$$\max_e \quad e(y-r) + (1-e)0 - \frac{1}{2}ce^2 - 1 \implies e^{SB} = \frac{y-r}{c} < e^{FB}$$

3. Now suppose two borrowers i, j (with same c) are in a group lending scheme: if agent i succeeds but agent j fails, agent i pays a cost k to the lender (assume $k < c$). Suppose the two borrowers choose independently their level of effort, taking as given the choice of the other borrower. What is the (symmetric) level of effort the borrowers choose?
$$\max_{e_i} \quad e_i[(y-r) - (1-e_j)k] + (1-e_i)[0 + (1-e_j)0] - \frac{1}{2}ce^2 - 1$$
$$e_i = \frac{y - r - k(1-e_j)}{c} \overset{e_i = e_j = e^{GL}}{\implies} e^{GL} = \frac{y-r-k}{c-k}$$

Question 2: An entrepreneur can invest k in a project and obtain $F(k)$. He was own wealth $w < k$ and need to borrow the rest at interest rate r. When the time to repay the loan comes, the entrepreneur can run away by paying a cost η per unit of capital. In other words, the lender cannot enforce repayment.

1. When will the borrower choose to default? Therefore, what is the maximum amount a lender will lend?
$$F(k) - r(k-w) < F(k) - \eta k \implies r(k-w) > \eta k$$

2. What is the relationship between the amount invested and wealth?

$$r(k - w) = \eta k \implies k = \frac{rw}{r - \eta} \implies \frac{\partial k}{\partial w} = \frac{r}{r - \eta} < 1$$

Now suppose that the borrower's cost of defaulting is zero unless the lender bears a monitoring cost ϕ (in which case the cost of defaulting is again η per unit of k). Also, suppose that the cost of capital is ρ per unit. The equilibrium in the lending market is driven by a zero profit condition for the lender that equates the profits lender makes on loan to the cost of capital.

3. What is zero profit condition for the lender?

$$r(k - w) - \rho(k - w) - \phi = 0$$

4. What is the maximum loan amount a borrower can get? (hint: equate the lender zero profit condition and the incentive constraint for the borrower)

$$k - w = \frac{\phi - \eta w}{\eta - \rho}$$

5. What is the interest rate when the credit constraint binds? How does the interest rate compare to the cost of capital ρ? How does this comparison depend on the monitoring cost ϕ?

$$r = \rho + \frac{\eta - \rho}{\phi - \eta w}\phi$$

3 Quasi-Hyperbolic Discounting vs. EU

$$U^t(c_t, c_{t+1}, \cdots, c_T) = \delta^{t-1}u(c_t) + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-1}u(c_\tau)$$

$$\delta_{t,s} = \begin{cases} 1, & \text{if } t = s \\ \beta\delta^{t-s}, & \text{if } t > s \end{cases}$$

- $\beta = 1$: standard exponential discounting
- $\beta < 1$: present bias
- The time-inconsistency here comes from comparing future periods; the discounting between today and tomorrow, and between one month from now vs. two months from now, are discounted differently
- We hit all future periods with an extra β

Example:

Suppose $\beta = 0.9$ and $\delta = 1$

1. Choose between \$99 in $t = 1$ and \$100 in $t = 2$
2. Choose between \$99 in $t = 3$ and in \$100 $t = 4$

In $t = 1$:

$$\begin{aligned} U^1 &= \delta^0 u(c_1) = 1 \times 99 = 99 \\ U^1 &= \beta\delta^1 u(c_2) = 0.9 \times 1 \times 100 = 90 \\ U^1 &= \beta\delta^2 u(c_3) = 0.9 \times 1 \times 99 = 89.1 \\ U^1 &= \beta\delta^3 u(c_4) = 0.9 \times 1 \times 100 = 90 \end{aligned}$$

In $t = 3$:

$$U^3 = \delta^0 u(c_3) = 1 \times 99 = 99$$
$$U^3 = \beta\delta^1 u(c_4) = 0.9 \times 1 \times 100 = 90$$

4 Self-Control

- There are three periods
- Income $= Y_1$ (no other income sources in other periods)
- There are matching contributions: M times the amount saved by the start of $t = 3$
- $t = 1, 2$, agent must make an allocation decision between savings and consumption
- The consumer has quasi-hyperbolic preferences with $\delta = 1$ for simplicity and $\beta \in (0, 1]$
- Assume sophistication: agent knows his future β and there is no uncertainty
- Utility is given by an instantaneous utility function $u(c_t)$ which is increasing and concave

$$u'(\cdot) > 0 \text{ and } u''(\cdot) < 0$$

- Agent's maximization problem is as follows:
  - In $t = 1$:
  $$\max \quad U_1(c_1, c_2, c_3) \equiv u(c_1) + \beta[u(c_2) + u(c_3)]$$
  - In $t = 2$:
  $$\max \quad U_2(c_2, c_3) \equiv u(c_2) + \beta u(c_3)$$

No commitment savings

Solve recursively:

- In $t = 3$, the agent consumes whatever is left
- In $t = 2$, solve the following maximization problem:

$$\max_{c_2} \quad u(c_2) + \beta u\left((Y_1 - c_1 - c_2)(1 + M)\right)$$

$$u'(c_2) = \beta(1 + M)u'\left((Y_1 - c_1 - c_2)(1 + M)\right)$$

- In $t = 1$, the agent takes the $t = 2$ constraint as given and solves:

$$\begin{aligned} \max_{c_1} \quad & u(c_1) + \beta[u(c_2) + u(c_3)] \\ \text{s.t.} \quad & c_3 = (Y_1 - c_1 - c_2)(1 + M) \\ & u'(c_2) = \beta(1 + M)u'(c_3) \\ & c_1, c_2, c_3 \geq 0 \end{aligned}$$

  - Defining $Y_2 \equiv Y_1 - c_1$

$$\begin{aligned} u'(c_1) &= \beta\left[u'(c_2)\frac{\mathrm{d}c_2}{\mathrm{d}Y_2} + u'(c_3)\frac{\mathrm{d}c_3}{\mathrm{d}Y_2}\right] \\ u'(c_2) &= \beta(1 + M)u'(c_3) \\ c_3 &= (Y_1 - c_1 - c_2)(1 + M) \end{aligned}$$

– Euler equation:
$$u'(c_1) = \left[\beta \frac{\mathrm{d}c_2}{\mathrm{d}Y_2} + \left(1 - \frac{\mathrm{d}c_2}{\mathrm{d}Y_2}\right)\right] u'(c_2)$$

Commitment savings

- In $t = 1$, the agent would like to set $u'(c_2) = (1 + M)u'(c_3)$
- When the agent has self-control problems, he is unable to ensure this pattern of consumption, as in $t = 2$ he would prefer to set $u'(c_2) = \beta(1 + M)u'(c_3)$, which is more than he would like to in $t = 1$
- The agent solves the problem as a $t = 1$ maximization for all periods, which gives the following set of equations for the solution

$$\begin{aligned} u'(c_1) &= \beta u'(c_2) \\ u'(c_2) &= (1 + M)u'(c_3) \\ c_3 &= (Y_2 - c_2)(1 + M) \end{aligned}$$

  – If $\beta = 1$, commitment savings has no effect
  – If $\beta = 0$, no savings
  – If $\beta \in (0, 1)$, two opposing effects on the impact of commitment on savings
    * Without commitment, $t = 2$ self will deviate further from optimal consumption in $t = 1$. The impact on savings of having a commitment device is larger for increased present bias.
    * However, $t = 1$ self also has a decreasing β, therefore less of a desire to allocate consumption to later periods.

5 Self-Enforcing Contracts

5.1 Product Quality Problem

- A firm can choose to produce a good with two qualities
  – The cost of high quality: c_1
  – The cost of low quality: c_0
- A buyer is willing to buy p_1 for a high-quality product and p_0 for a low-quality product
  – We assume $c_0 = p_0 = 0$ and $p_1 > c_1 > 0$
  – The buyer cannot observe the quality of the good before buying, but only after buying
- A one-period game
  – High-quality production will not be an equilibrium
    * Whatever price the buyer is willing to pay, the seller's will give him a low-quality product
    * Anticipating this, the buyer will only be willing to pay p_0
    * In a static game, the only (Nash) equilibrium is a low quality one
- A finitely repeated game
  – In T, the seller will cheat because she has no reputation to maintain
    * The buyer knows this and offers p_0
  – In $T - 1$, both parts know that they will not "cooperate" in T
    * If the buyer offered p_1 in $T - 1$, the seller would cheat because she has not incentives to have a reputation in T
  – By backward induction, the price is always p_0 and the quality is always low

- An infinitely repeated game
  - Assume that the buyer and the seller interact over an infinite amount of periods
    - The seller has a discount factor: $\delta < 1$
  - Consider the following buyer's strategy
    - The buyer pays p_1 in the first period
    - In subsequent periods: he pays p_1 if the seller offers high-quality product, otherwise pays p_0
  - Cooperative equilibrium

$$p_1 - c_1 + \sum_{t=1}^{\infty} \delta^t (p_1 - c_1) \geq p_1 - 0 + \sum_{t=1}^{\infty} \delta^t \times 0$$

    - For given p_1, c_1, the cooperative equilibrium is sustainable if δ is above a certain threshold (*Folk Theorem*)

- Building brands
  - If firms produce the low-quality good and sell it at a high price, they get punished and exit the market, and re-enter with a different name
    - Suppose there is a fixed cost of entry A (e.g. adverting to create a recognizable name)
    - Assume $p_1 = 1$
    - Consider a firm that has already paid the entry cost, and it can decide whether to produce
      - high-quality good, the flow of discounted profits is: $\frac{1-c_1}{1-\delta}$
      - low-quality good, the flow of discounted profits is: $1 + \delta(-A + \frac{1-c_1}{1-\delta})$
    - The firm will produce high-quality goods if

$$\frac{1-c_1}{1-\delta} > 1 + \delta(-A + \frac{1-c_1}{1-\delta})$$

      - $A > \frac{c_1}{\delta}$: the cost of re-entering is too high relative to the benefit from cheating the consumer this period
      - We also need $A < \frac{1-c_1}{1-\delta}$ to induce entry in the first place

5.2 Loan Default Problem

- Consider a case where an entrepreneur borrows L at interest rate i to finance a project that gives return $R > (1+i)L$
- Without a loan, he gets (exogenous) utility v
- Consider an infinitely repeated game (discount factor δ)
- A cooperative equilibrium where the borrower does not default is sustainable only if:

$$R - (1+i)L + \frac{\delta}{1-\delta}[R - (1+i)L] \geq R + \frac{\delta}{1-\delta} v$$

$$\frac{\delta}{1-\delta}[R - (1+i)L - v] \geq (1+i)L$$