Mock-Exam-Questions
Advanced Microeconomics II
Spring 2021

Student ID: Name:

Question 1 (30 Minutes)

Consider a strictly risk averse decision-maker (DM) with Bernoulli utility function $u(m)$ over money m, where $u' > 0$ and $u'' < 0$. There are two financial assets.

- Asset 1 has a unit price $p_1 > 0$, and always pays 1 unit of money per asset held.
- Asset 2 has a unit price $p_2 > 0$, and pays $z_2 > 0$ units of money per asset held (i.e., if $z_2 = 2.3$, each unit of asset 2 pays 2.3 unit of money). The return z_2 is distributed with a density function $f(z_2)$, such that $\int z_2 f(z_2) dz_2 = 1$.

The DM has wealth $w > 0$ which he can spend on the two assets.

1. Denote by $q_1 \geq 0$ and $q_2 \geq 0$ the amounts of asset 1 and 2, respectively. State the budget set for this DM, and depict it in an appropriate diagram.
2. Prove that whenever $p_1 > p_2$, then the DM has a strictly positive demand for asset 2 (you may assume that the DM always wants to use all his wealth to acquire assets).

Question 2 (35 Minutes)

Consider an exchange economy with uncertainty. There are two states, denoted as $s = 0, 1$, two consumers and one physical good. Consumer $i = 1$ has a state-independent, strictly increasing and strictly concave Bernoulli utility function $u_1(x_{s1})$, where $x_{si} \geq 0$ denotes the consumption of the good if state $s \in \{0, 1\}$ has occurred. Consumer 1 behaves as an expected utility maximizer. Consumer $i = 2$ has a linear Bernoulli utility function given by $u_2(z) = z$. The endowments are state-dependent, where

$$\begin{pmatrix} \omega_{01} \\ \\ \omega_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} \omega_{02} \\ \\ \omega_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ \\ 1 \end{pmatrix}.$$

Both consumers believe that state $s = 0$ occurs with probability $\pi \in (0, 1)$.

a) What can be said about the risk attitude of consumers $i = 1$ and $i = 2$ (short explanation required)?

b) There are two assets $k = 1, 2$ with the return structure

$$r_1 = \begin{pmatrix} 1 \\ \\ 0 \end{pmatrix} \qquad r_2 = \begin{pmatrix} 1 \\ \\ 1 \end{pmatrix}$$

Let q denote the price of the second asset. Derive the (unique) Radner equilibrium (asset market equilibrium)

$$(x_{01}^*, x_{11}^*, x_{02}^*, x_{12}^*; q)\,.$$

(*Hint: You can assume that this equilibrium is interior, meaning that* $x_{si} > 0$ *for* $s = 0, 1$ *and* $i = 1, 2$.)

c) In what sense does the equilibrium feature an "efficient allocation of the risk"?

Question 3 (30 Minutes)

Let the choice set be $X = \{(x_1, x_2) \in \mathbb{R}^2_+ : x_1 > 1, x_2 < 2\}$. Consider a preference relation $\succeq$ on X represented by the utility function

$$U(x_1, x_2) = (x_1 - 1)(2 - x_2)^{-2}$$

Let $(p_1, p_2) \in \mathbb{R}^2_{++}$ denote the price vector, and normalize income to $w = 1$.

a) Prove or disprove the following claim: The preferences represented by $U(x_1, x_2)$ verify local non-satiation on X. (3P)

b) Derive the Walrasian Demand Function

$$x(p_1, p_2) = \begin{pmatrix} x_1(p_1, p_2) \\ \\ x_2(p_1, p_2) \end{pmatrix},$$

assuming prices are such that an interior solution $x(p_1, p_2) \in X$ results. That is, you can assume that $x_1^* > 1$ and $x_2^* \in (0, 2)$ when solving the Lagrange.

c) Use your solution x_1^*, x_2^* from b) to identify the set of prices $(p_1, p_2) \in \mathbb{R}^2_{++}$ for which indeed $x(p_1, p_2) \in X$. Depict this set of prices in a (p_1, p_2)-diagram.

d) The Law of Demand states that the demand decreases in the "own" price, that is:

$$\frac{\partial x_j(p_1, p_2)}{\partial p_j} > 0$$

for $j \in \{1, 2\}$. Is the Law of Demand satisfied for all prices for which $x(p_1, p_2) \in X$?

Question 4 (40 Minutes)

Consider an economy with a single consumer and two firms, which are owned by the consumer. There are three goods in the economy, two consumption goods x_1, x_2 and labor L. The consumer owns one unit of labor, and can supply any infinitesimal quantity of L as an input to any firm. Each firm uses labor to produce its consumption goods. The respective production functions are

$$q_1 = 2\sqrt{L_1}$$

$$q_2 = \sqrt{L_2}$$

where $L_1, L_2 \geq 0$ denote the amount of labor used in the respective production process. The consumer's utility function is $u(x_1, x_2) = x_1 x_2$. Consumer income w consists of labor and capital income (firm profits).

Normalize the price of labor (wage) to 1, and denote by p_1, p_2 the Walrasian prices of the consumption goods.

a) Explain in words why both prices p_1, p_2 must be strictly positive in any Walrasian equilibrium.

b) Explain in words why this consumer will supply all her labor to the market in any Walrasian equilibrium.

c) Derive the (unique) Walrasian price vector (p_1^*, p_2^*) and the Walrasian equilibrium allocation $(x_1^*, x_2^*, L_1^*, L_2^*)$ of this economy (that is: assume that the consumer and the firms are independent entities that maximize their well-being taking prices as given).

d) Provide an intuitive explanation why $p_1^* < p_2^*$.