University of Zurich, Dept. of Economics
International Macroeconomics
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Exam Paper for Spring 2020 Lecture

`Answer all questions. Altogether you can reach up to 24 points. Note that unnecessarily lengthy expositions may be penalized. Using mathematics and formal exposition may facilitate answering even those questions that do not explicitly require calculations.`

`GOOD LUCK!`

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1. Consider a small open economy in which the representative agent maximizes

$$U_t = \boldsymbol{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s u(C_{t+s}) \right\}$$

with

$$u(C_{t+s}) = -\frac{1}{2}(h - C_{t+s})^2$$

(where h is a constant) subject to the intertemporal budget constraint

$$B_{t+1} = (1+r)B_t + NO_t - C_t$$

where $\lim_{s\to\infty} \frac{\boldsymbol{E}_t(B_{t+s})}{(1+r)^s} = 0$. The notation is familiar from the lecture:

B_t	Holdings of foreign bonds
r	World interest rate
C_t	Consumption
Y_t	Gross domestic product (GDP)
I_t	Gross Investment
G_t	Government consumption / expenditure
$NO_t = Y_t - I_t - G_t$	Net output / national cash-flow
$\boldsymbol{E}_t(.)$	Expectations operator
K_t	Capital stock
$I_t = K_{t+1} - K_t$	Investment

(12 POINTS TOTAL)

(a) Under the assumption that $\beta(1+r) = 1$, we have shown in the course that the current account balance, CA_t, is given by

$$CA_t = -\sum_{k=1}^{\infty} \frac{\boldsymbol{E}_t(\Delta NO_{t+k})}{(1+r)^k} \tag{1}$$

(You DO NOT have to derive this formula!). Assume that net output is given by the stochastic process

$$NO_t = 1.25NO_{t-1} - 0.25NO_{t-2} + \varepsilon_t$$

Calculate the response of the current account to a shock in NO_t, i.e. $\frac{\partial CA_t}{\partial \varepsilon_t}$. Determine its sign. Briefly discuss the economic intuition for your result. (2 POINTS).

(b) A researcher wants to test the present-value model of the current account empirically. To this end, she uses actual data to estimate a bivariate vector-autoregression (VAR) of the form

$$Z_t = \mathbf{A} Z_{t-1} + \epsilon_t$$

where $Z_t' = \left[\begin{array}{cc} \Delta NO_t, & CA_t \end{array}\right]$ is the vector of endogenous variables, ϵ_t a vector of residuals and the coefficient matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then she uses this VAR to compute a model-implied current account, $\widehat{CA}_t$, according to equation (1) and test the hypothesis $\widehat{CA}_t = CA_t$ for a given interest rate r. Show that this hypothesis implies that

$$a_{11} = a_{21} \text{ and } a_{12} = a_{22} - (1+r)$$

(3 POINTS).

(c) Now assume that output Y_t is produced according to a production function

$$Y_t = A_t F(K_t)$$

with the usual properties: $F'(K_t) > 0$, $\lim_{K \to 0} F'(K_t) = \infty$, and $F''(K_t) < 0$. Capital does not depreciate, so that investment is

$$I_t = \Delta K_{t+1} = K_{t+1} - K_t$$

Future productivity is risky and the productivity process A_t follows

$$\boldsymbol{E}_t(A_{t+1}) = \rho A_t$$

with $\rho = 1$, i.e. A_t follows a random walk.
GRAPHICALLY describe the adjustment over time of output, consumption, investment, and savings following a positive shock to A_t in period t (i.e. draw

the impulse responses. No calculations needed). In your answer, the impulse responses of output and consumption should appear in the same graph and those of savings and investment in a separate one.
(4 POINTS)

(d) In a seminal paper, Glick and Rogoff (JME 1995) ran the following regression of the current account on investment:

$$\Delta CA_t = constant + \gamma \Delta I_t + residual_t$$

What was the general pattern of sign and magnitude of the coefficient γ that Glick and Rogoff identified in the data for G7 countries? What would the model in (c) imply for the sign and magnitude of γ? Sketch one way in which you could change the model above in order to reconcile the model with the findings of Glick and Rogoff. Explain carefully!
(3 POINTS)

2. Consider a world economy which consists of two countries, Home and Foreign, and exists for two periods. The representative agent in the Home and Foreign country has known first–period income Y_1 and $Y_1^\star$, respectively. In addition, both countries exhibit a zero initial net foreign asset position ($B_1 = B_1^\star = 0$). On date 2, $\mathcal{S} \geq 2$ different states of nature $s \in \{1, \ldots, \mathcal{S}\}$ are possible, implying that second–period output level at home $Y_2(s)$ and abroad $Y_2^\star(s)$ is uncertain at date 1. Each state of nature may occur with probability $\pi(s) > 0$. Asset markets are complete, i.e. for each state of the world there is a state contingent Arrow–Debreu security available for trade. The price of an Arrow–Debreu security which pays one unit of consumption if state s occurs and nothing in all other states is given by $\widetilde{p}(s) = \frac{p(s)}{1+r}$, where $p(s)$ is the state price and r is the return on a risk–free bond. The representative household in the Home country maximizes lifetime expected utility

$$u(C_1) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2(s)),$$

subject to his/her period resource constraints. By analogy, the representative household in the Foreign country maximizes lifetime expected utility

$$u(C_1^\star) + \beta \sum_{s=1}^{\mathcal{S}} \pi(s) u(C_2^\star(s)),$$

subject to his/her period resource constraints. Home and Foreign consumers share the same preferences, which are described by a standard CRRA utility function:

$$u(C) = \frac{C^{1-\rho} - 1}{1 - \rho}.$$

(12 POINTS TOTAL)

(a) Assume that $u(C) = \log(C)$ and $\mathcal{S} = 2$. The current account at date one is given by

$$CA_1 = \frac{\beta}{1+\beta} Y_1 - \frac{1}{1+\beta} \left[\frac{p(1)Y_2(1) + p(2)Y_2(2)}{1+r} \right].$$

(You DO NOT have to derive this formula!).

1. Consider the following situation:
$\beta = 0.9$, $\pi(1) = 2/3$, $\pi(2) = 1/3$, $Y_1 = Y_1^* = 2$, $Y_2(1) = 1$, $Y_2^*(1) = 3$, $Y_2(2) = 5/2$, $Y_2^*(2) = 3/2$
Is the Home country running a current account surplus or deficit? Explain why. (2 POINTS)
2. Now assume instead that probabilities are $\pi(1) = 1/4$ and $\pi(2) = 3/4$. Does the sign of the Home country's current account change? Explain why or why not. (2 POINTS)

(b) Our model implies

$$\frac{C_2(s)}{C_1} = \frac{C_2^\star(s)}{C_1^\star} = \frac{Y_2^W(s)}{Y_1^W} \qquad \forall s \in \{1, 2, \ldots, \mathcal{S}\}. \tag{2}$$

(You DO NOT have to derive this formula!). What does equation (2) imply for the correlation between a country's output growth and its consumption growth? Use this implication to derive a regression-based test of complete risk sharing and explicitly state the null hypothesis of this test. How could you use the results from this regression to measure the extent of risk sharing? Compare the typical findings of this test when it is run on data from the regions of an individual country to the results obtained when the regression is run using data from a cross-section of countries. Briefly interpret these findings.
(4 POINTS)

(c) Now consider a home investor who can invest into a domestic bond yielding interest rate i and a foreign-currency denominated bond yielding interest rate i^*. In the lecture we have shown that the first-order conditions for this investor are given by

$$-u'(C_1) + \beta \boldsymbol{E}_1\{u'(C_2)(1+i)\} = 0$$
$$-u'(C_1)S_1 + \beta \boldsymbol{E}_1\{u'(C_2)S_2(1+i^*)\} = 0$$

where S_t is the spot exchange rate expressed in units of home currency per foreign currency.

1. Show formally that uncovered interest parity will hold in this model if $C_2(s) = \boldsymbol{E}_1(C_2)$ for each state s of the world. (2 POINTS)
2. Now assume that in a regression of nominal exchange rate changes on interest rate differentials of the form

$$\log\left(\frac{S_{t+1}}{S_t}\right) = constant + \alpha(i_t - i_t^*) + residual_{t+1} \tag{3}$$

a researcher finds estimates for α that are significantly lower than unity. Which stochastic properties does the exchange rate have to fulfill so that the pattern found by the researcher in regression (3) can be explained by the theoretical model? Interpret these properties carefully. (2 POINTS)