Mock Exam
Social Choice Theory
Fall 2012

几个找反例的思路

1. 可测/可比：从无差异出发，看看经过什么线性变换之后会不会变得有差异了
2. IIA：每个选项不是equally/symmetrically treated的，比如博达投票法则之类的，想办法调调无关顺序使得整个票数变动。
Score-based method很可能不是IIA的，因为每个选项给分多半不一样
3. P/M：试着找找non-responsive的地方，比如不管偏好怎么样，我这个地方的社会选择都是这个选项。典型就是2/3-多数投票，0-2/3的地方不管我票是多是少都不改变选项的

Instructions:

- The duration of the exam is 90 minutes.
  Please answer **all three** problems.
  You can achieve up to 90 points.
- Please solve and answer all problems on the answer sheet that is distributed separately.
  Write your name and your student ID number on the answer sheet.
- This question sheet has 4 pages (including cover page). Please check for completeness.
- You are not allowed to use any auxiliary equipment (except for dictionaries).
  Switch off all electronic devices.
- Please have your student identity card ready.

Points:

- Problem 1 (Preferences): 30 points
- Problem 2 (Arrow's Theorem for SCFs): 30 points
- Problem 3 (Manipulability): 30 points

Good Luck!

90分钟90分，大概1分钟一分，分的多少就是按难度和时间给的

Problem 1: Preferences (30 points)

Throughout this problem, $X = \{w, x, y, z\}$ is the set of alternatives for a single decision-maker.

(a) Define formally the concept of a preference on X. Please give a precise definition of all concepts and terms that you are using. (10 points) 今年考试不会考这种定义题的，因为开卷

(b) Each of the following three tables specifies a different binary relation R^i, $i = 1, 2, 3$, on X. To understand how these tables must be read, consider the first (leftmost) table, which defines relation R^1. The checkmark in the field with row x and column w, for instance, means that $(x, w) \in R^1$, or equivalently xR^1w. The fact that there is no checkmark in the field with row w and column x means that $(w, x) \notin R^1$, or equivalently $\neg wR^1x$. The other fields and tables are interpreted analogously.

有些证明就是枚举，考试的时候如果全部列出来很长，你就随便取一个检查一下，然后说你检查了全部。或者就把整个偏好写出来

R^1	w	x	y	z
w	✓			✓
x	✓	✓		✓
y	✓	✓	✓	✓
z				✓

R^2	w	x	y	z
w	✓	✓	✓	✓
x	✓	✓	✓	✓
y			✓	
z		✓	✓	✓

R^3	w	x	y	z
w	✓	✓	✓	✓
x		✓	✓	✓
y		✓	✓	✓
z				✓

Which of these binary relations are a preference on X, and which are not?
Which of them are a strict (i.e. antisymmetric) preference? (10 points)

(c) For each of the three binary relations R^i, $i = 1, 2, 3$, from part (b) of this problem, derive the set of best elements of $S = \{w, y, z\}$, i.e. the choice set $C(S, R^i)$. (10 points)

Problem 2: Arrow's Theorem for SCFs (30 points)

(a) State Arrow's Impossibility Theorem for Social Choice Functions (SCFs). Please give a precise definition of the underlying axioms. (10 points)

(b) Let $X = \{w, x, y, z\}$ be the set of alternatives, and assume all $n \geq 2$ voters have strict preferences, i.e. treat $\mathscr{A} = \mathscr{P}^n$ as the universal domain.

Consider the following voting method. Each voter gives 1 vote to each of his/her *two* top ranked alternatives. These votes are added for each alternative, and the alternative with the largest number of votes is selected as the winner. If there are ties, i.e. two or more alternatives obtain the same largest number of votes, then the one which comes first in the alphabet is selected among them.

Apply this method to the following preference profile. (5 points)

#	preferences
4	$w\ P\ x\ P\ y\ P\ z$
3	$x\ P\ y\ P\ z\ P\ w$
5	$y\ P\ z\ P\ w\ P\ x$
1	$z\ P\ w\ P\ x\ P\ y$
1	$x\ P\ w\ P\ z\ P\ y$

(c) For the method from part (b) of this problem, check which of Arrow's axioms for SCFs are satisfied and which are violated. When you think that an axiom is satisfied, give a brief argument. When you think that an axiom is violated, give a counterexample. (8 points)

(d) Give an example of an SCF that satisfies all of Arrow's axioms except [$\bar{\mathrm{P}}$]. Explain your construction and arguments briefly. (7 points)

Problem 3: Manipulability (30 points)

Let $X = \{x, y, z\}$ be the set of alternatives and $N = \{1, 2\}$ the set of voters. Assume that both voters have strict preferences, i.e. treat $\mathscr{A} = \mathscr{P}^2$ as the universal domain. The following table completely specifies an SCF on this domain. For instance, the field with row yxz and column xyz contains y, which means that alternative y is selected if voter 1 has preference yP_1xP_1z and voter 2 has preference xP_2yP_2z. The other fields are interpreted analogously.

$R_1 \backslash R_2$	xyz	xzy	yxz	yzx	zxy	zyx
xyz	x	x	y	y	z	z
xzy	x	x	y	z	z	z
yxz	y	y	y	y	z	z
yzx	y	z	y	y	z	z
zxy	z	z	z	z	z	z
zyx	z	z	z	z	z	z

(a) Which of Arrow's axioms for SCFs are satisfied and which are violated by this method? Please explain your answer. (10 points)

(b) Is this SCF surjective?
Which of Gibbard-Satterthwaite's axioms are satisfied and which are violated?
Identify all preference profiles at which this SCF is manipulable by some voter. (10 points)

(c) Consider the linear reference order on X given by $x > y > z$.
Which of the strict preferences are single-peaked with respect to this order?
Now suppose we restrict the domain of the above SCF to these single-peaked preferences.
Is it still surjective? Check again Gibbard-Satterthwaite's axioms. (10 points)