Solution to Warm Up Problemset

International Macroeconomics (Master)

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Exercise 1: Review of Basic Concepts

(a) To show, for small values of x:

$$\log(1+x) \approx x$$

and hence:

$$\log\left(\frac{x_{t+1}}{x_t}\right) \approx \frac{x_{t+1}-x_t}{x_t}$$

The first order Taylor approximation of $f(x)$ around the expansion point a is given by

$$f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a)$$

Using $f(x) = \log(1+x)$ and $a = 0$ as we are looking at values of x close to zero, we obtain

$$\begin{aligned} f(x) &= \log(1+x) \\ &\approx \log(1+0) + \left(\frac{1}{1+0}\right)\left(\frac{1}{1!}\right)(x-0) \\ &= \log(1) + x \\ &= 0 + x \\ &= x \end{aligned}$$

Approximately, the growth rate of a variable is equal to the difference of the log-levels of the variable.

Let G denote the (gross) growth rate of a variable x:

$$\begin{aligned} G &= \frac{x_{t+1}-x_t}{x_t} \\ &= \frac{x_{t+1}}{x_t} - 1 \end{aligned}$$

For small values of G:

$$\begin{aligned} G &\approx \log\left(1 + \frac{x_{t+1}-x_t}{x_t}\right) \\ &\approx \log\left(\frac{x_{t+1}}{x_t}\right) \\ &\approx \log(x_{t+1}) - \log(x_t) \end{aligned}$$

(b) Linear regression model:

$$\log(y) = \beta_1 + \beta_2 \log(x) + \epsilon$$

The coefficient β_2 denotes an elasticity and determines by how many percent y changes if x changes by 1 percent.

Example:
price elasticity of demand for a good

$$\begin{aligned} \text{elasticity} &= \frac{\%\text{ change in quantity}}{\%\text{ change in price}} \\ \eta &< 0 \end{aligned}$$

now, take

$$\begin{aligned} f(x) &= \ln(x) \\ f'(x) &= \frac{d\ln(x)}{dx} \\ &= \frac{1}{x} \\ d\ln(x) &= \frac{dx}{x} \end{aligned}$$

similarly:

$$d\ln(y) = \frac{dy}{y}$$

hence, β_2 is an elasticity:

$$\begin{aligned} \ln(y) &= \beta_1 + \beta_2 \ln(x) + \epsilon \\ \frac{d\ln(y)}{d\ln(x)} &= \beta_2 \\ \frac{\frac{dy}{y}}{\frac{dx}{x}} &= \beta_2 \end{aligned}$$

(c) Cobb-Douglas production function:

$F(K, L) = K^\alpha L^\beta$ where $\alpha + \beta = 1$.
Production function $F(\cdot)$, K stands for capital input and L is labor input.

$\alpha + \beta = 1$ implies constant returns to scale, that is

$$\lambda F(K, L) = F(\lambda K, \lambda L)$$

log-transformation of the Cobb-Douglas production function yields a linear relation:

$$\begin{aligned}
\ln F(K,L) &= \alpha \ln(K) + \beta \ln(L) \\
\frac{\partial \ln F(K,L)}{\partial \ln(K)} &= \frac{\frac{dF(K,L)}{F(K,L)}}{\frac{dK}{K}} \\
&= \alpha
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \ln F(K,L)}{\partial \ln(L)} &= \frac{\frac{dF(K,L)}{F(K,L)}}{\frac{dL}{L}} \\
&= \beta
\end{aligned}$$

Ceteris paribus, α is the relative change in output if K increases by 1%, and β is the relative change in output if L increases by 1%.

Exercise 2: Power Utility Function

(a) CRRA-utility, coefficient of relative risk aversion:

$$\begin{aligned}
U(C) &= \frac{C^{1-\gamma} - 1}{1-\gamma} \\
U'(C) &= C^{-\gamma} \\
U''(C) &= -\gamma C^{(-\gamma-1)}
\end{aligned}$$

- increasing and concave utility function
- declining marginal utility
- degree of concavity of the utility function measures the extent of risk aversion

coefficient of relative risk aversion

$$\begin{aligned}
RRA &= -\frac{U''(C)}{U'(C)} \cdot C \\
&= \gamma
\end{aligned}$$

Hence, the coefficient of relative risk aversion is constant for all levels of C.

(b) CRRA-utility with $\gamma = 1$:

$$U(C) = \frac{C^{1-\gamma} - 1}{1-\gamma}$$

- This function is not defined for $\gamma = 1$.
- What happens to $U(C)$ if $\gamma \to 1$?

$\Rightarrow$ apply rule of L'Hôpital:

if

$$
\begin{aligned}
\lim_{x \to x_0} f(x) &= 0 \\
\lim_{x \to x_0} g(x) &= 0
\end{aligned}
$$

then use

$$
\lim_{x \to x_0} \left(\frac{f(x)}{g(x} \right) = \frac{f'(x)}{g'(x)}
$$

apply this rule:

$$
\begin{aligned}
\lim_{\gamma \to 1} \left(\frac{C^{1-\gamma} - 1}{1 - \gamma} \right) &= \lim_{\gamma \to 1} \left(\frac{f(\gamma)}{g(\gamma)} \right) \\
&= \lim_{\gamma \to 1} \left(\frac{f'(\gamma)}{g'(\gamma)} \right)
\end{aligned}
$$

here:

$$
\begin{aligned}
f(\gamma) &= C^{1-\gamma} - 1 \\
g(\gamma) &= 1 - \gamma
\end{aligned}
$$

$$
\begin{aligned}
f(\gamma) &= C^{1-\gamma} - 1 \\
&= \exp\left(\ln\left(C^{1-\gamma}\right)\right) - 1 \\
&= \exp\left((1-\gamma)\ln(C)\right) - 1 \\
f'(\gamma) &= \exp\left((1-\gamma)\ln(C)\right)(-\ln(C)) \\
&= C^{1-\gamma}(-\ln(C))
\end{aligned}
$$

$$
g'(\gamma) = -1
$$

$$
\begin{aligned}
\Rightarrow \lim_{\gamma \to 1} \left(\frac{f'(\gamma)}{g'(\gamma)} \right) &= \frac{-C^{1-\gamma} \ln(C)}{-1} \\
&= \ln(C)
\end{aligned}
$$

That is, for γ approaching 1, the power utlility function simplifies to $\ln(C)$.

Exercise 3: Balance of Payments

See for instance Krugman and Obstfeld (2008) pages 307–319.

(a) Balance of Payments, Current Account, Trade Balance

The **balance of payments** is a systematic summary of all international 'economic transactions' between residents of a country and non-residents during a specific time period (i.e., year or quarter).

The balance of payments compromises the current account (CA), the capital account, the financial account, (and net errors and omissions).

An 'economic transaction' in this context refers to private or public sector transactions on a transnational view. It can be:
- export or import of goods and services (trade)
- cross-boarder labor and investment income
- transfers (e.g. worker abroad transfers his wage to home country, development aid, repayment of debt)
- capital flows (e.g. resident of a country purchases stocks of a firm located abroad)

The balance of payments follows the principle of double-entry bookkeeping: each transaction is recorded twice. That is, at least one account is credited and at least one account is debited.[1]

The **current account (CA)** is the sum of the trade balance (TB), net factor income (interest and dividend payments), and net transfer payments (e.g. foreign aid).

The **trade balance (TB)** is the difference between a country's exports and imports of goods and services. That is, we can define the trade balance or net exports (NX) as

$$NX = X - IM.$$

If $NX > 0$, the country is likely to run a CA surplus and is a creditor to the rest of the world. Conversely, if $NX < 0$, the country is likely to run a CA deficit and is a debtor to the rest of the world. Net exports are a substantial component of total demand in open economies.

(b) Balance of Payments (BoP) scheme

Current Account
- trade balance: imports and exports of goods (merchandise) and services (e.g. legal assistance, shipping fees, tourist expenditures)
- net factor income: interests, dividends, profits, wages
- net transfer payments: e.g. foreign aid

Capital Account
The capital account incorporates other wealth transfers between countries not summarized in the financial account. These transfers correspond to non-produced, non-financial assets

[1] 'Credited' means that the country receives a payment and 'debited' means that the country makes a payment.

and non-market transactions, such as copyrights or patents.

Financial Account
The financial account measures the difference between the sale of financial assets to foreigners and the purchase of financial assets located abroad by domestic residents.

Net Errors and Omissions
This position accounts for statistical discrepancies.

The **BoP identity** is given by:
Current Account + Capital Account + Financial Account + $\epsilon = 0$

(c) Bookkeeping

(i) Take the viewpoint of Switzerland and convert US \$ into CHF. The transaction is worth 910,000 US \$. With an exchange rate of 0.91 \$/CHF this corresponds to

$$910,000\ \$ \cdot (0.91\ \$/\text{CHF})^{-1} = 1,000,000\ \text{CHF}.$$

Recall, receiving a payment means 'credited' (+) and making a payment means 'debited' (-).

> Current Account:
> delivery of the good → credit: 'export of goods' 1,000,000 CHF
>
> Financial Account:
> trade credit with maturity of one year → debit: 'trade credit' 1,000,000 CHF[2]

(ii) Swiss firm buys stocks of a German company worth 10,000,000 CHF.[3]

> Financial Account:
> purchase of stocks → debit: 'portfolio investment' 10,000,000 CHF[4]
>
> Financial Account:
> payment of stocks → credit: 'liability' 10,000,000 CHF[5]

(iii) Debt forgiveness of 5,000,000 CHF.

> Capital Account:
> forgiveness of foreign debt is a non-market transfer payment to foreigners → debit: 'debt forgiveness' 5,000,000 CHF
>
> Financial Account:
> reduction of claims on foreigners → debit: 'decline in claims' –5,000,000 CHF

References

Krugman, P. and M. Obstfeld (2008): *International Economics: Theory and Policy*, Addison-Wesley.

[2]This is a claim on foreigners and therefore capital export. It corresponds to a 'sale of a Swiss bank deposit' which can be interpreted as the import of an asset. That is why it pops up on the left–hand side of the financial account as a debit item.
[3]Again, think of it as an 'import of foreign assets'. Therefore it is a debit in the financial account.
[4]This is a claim on foreigners and thus a capital export.
[5]This is a claim of foreigners on domestic residents and thus a capital import.