Solution to Problemset 5
International Macroeconomics (Master)

Prof. Dr. Hoffmann
Chair of International Trade and Finance
University of Zurich

Spring Semester 2021

Exercise 1: Intertemporal Approach to the Current Account

(c) (ii) From part (i) we know that

$$CA_t = -\frac{r}{1+r}\left[\sum_{s=0}^{\infty}\frac{NO_{t+s}-NO_t}{(1+r)^s}\right].$$

Now, consider

$$\sum_{s=0}^{\infty}\frac{NO_{t+s}-NO_t}{(1+r)^s} = \frac{NO_t-NO_t}{(1+r)^0}+\frac{NO_{t+1}-NO_t}{(1+r)^1}+ \\ + \underbrace{\frac{NO_{t+2}-NO_t}{(1+r)^2}}_{\text{expand by } \frac{-NO_{t+1}+NO_{t+1}}{(1+r)^2}} + \dots$$

$$= \frac{\Delta NO_{t+1}}{(1+r)}+\frac{\Delta NO_{t+2}+\Delta NO_{t+1}}{(1+r)^2}+\frac{\Delta NO_{t+3}+\Delta NO_{t+2}+\Delta NO_{t+1}}{(1+r)^3}+\dots$$

Rewrite this as

$$\begin{aligned}
&= \frac{\Delta NO_{t+1}}{(1+r)}+\frac{\Delta NO_{t+1}}{(1+r)^2}+\frac{\Delta NO_{t+1}}{(1+r)^3}+\dots \\
&+ \frac{\Delta NO_{t+2}}{(1+r)^2}+\frac{\Delta NO_{t+2}}{(1+r)^3}+\frac{\Delta NO_{t+2}}{(1+r)^4}+\dots \\
&+ \frac{\Delta NO_{t+3}}{(1+r)^3}+\frac{\Delta NO_{t+3}}{(1+r)^4}+\frac{\Delta NO_{t+3}}{(1+r)^5}+\dots \\
&+ \dots
\end{aligned}$$

and then as

$$
\begin{aligned}
&= \frac{\Delta NO_{t+1}}{(1+r)}\left(1+\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\ldots\right) \\
&\quad + \frac{\Delta NO_{t+2}}{(1+r)^2}\left(1+\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\ldots\right) \\
&\quad + \frac{\Delta NO_{t+3}}{(1+r)^3}\left(1+\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\ldots\right) \\
&\quad + \ldots \\
&= \frac{1+r}{r}\left[\frac{\Delta NO_{t+1}}{(1+r)}+\frac{\Delta NO_{t+2}}{(1+r)^2}+\frac{\Delta NO_{t+3}}{(1+r)^3}+\ldots\right]
\end{aligned}
$$

Hence, for the CA we get

$$
CA_t = -\frac{r}{1+r}\frac{1+r}{r}\left[\frac{\Delta NO_{t+1}}{(1+r)}+\frac{\Delta NO_{t+2}}{(1+r)^2}+\frac{\Delta NO_{t+3}}{(1+r)^3}+\ldots\right]
$$

$$
\Leftrightarrow \qquad CA_t = -\sum_{s=1}^{\infty}\frac{\Delta NO_{t+s}}{(1+r)^s}.
$$

This is a version of the fundamental current account equation. The current account is the negative present value of all future changes in net output. That is, the CA incorporates all future changes in income. Thus, CA surpluses and deficits over multiple periods can indeed be optimal. They arise because households maximize lifetime utility.