Problem Set 11

1. **(Partial Equilibrium)**

   Consider the partial equilibrium model of the lecture. There is a single consumer with utility $u = m + Ln(x)$. The consumer owns all firms and has an endowment $\omega = 1$ of the numeraire. N firms produce good x with the same technology $q_j = \sqrt{z_j}$, where $z_j \geq 0$ is the amount of the numeraire good used by firm j in the production. Let p denote the price of the consumption good. Find the competitive equilibrium $(p, x^*, m^*, (q_1^*, ..., q_N^*), (z_1^*, ..., z_N^*))$ of this economy.

2. **(Partial Equilibrium: Pareto Set)**

   Consider the partial equilibrium model of the lecture. There are $I = 2$ consumers with utility $u_i = m_i + \phi_i(x)$, where $\phi_i' > 0$ and $\phi_i'' < 0$. $J = 1$ firm uses the numeraire good to produce output y_j with the cost function $c_j(y_j) \geq 0$, where $c_j' c_j'' > 0$. Hence $C_j(y_j)$ corresponds to the quantity of input use by firm j to produce $y_j \geq 0$ units of output. The aggregate endowment of the production good (numeriate) is $\omega > 0$.

   (a) Characterize the Pareto optima of this economy by the first-order approach, assuming an interior solution (meaning that $x_i, y_j > 0$ for every i, j). Compare these conditions to the one derived by the lecture.

   (b) Let $W : \mathbb{R}^2 \to \mathbb{R}$, $W = W(u_1, u_2)$ be a strictly increasing and strictly concave social welfare function (SWF). Suppose that society wants to find a feasible allocation that maximizes this SWF. Show that this allocation must select a particular Pareto optimum (you can restrict attention to interior optima).

3. **(Partial Equilibrium: Unconditional Basic Income)**

Consider again the partial equilibrium model. There are $I > 1$ consumers with utility $u_i = m_i + \alpha_i Ln(x_i)$, where $\alpha_i > 0$ is a parameter. Each consumer holds one unit of the numeraire ($\omega_i = 1$). There is a single firm with cost function $C(y) = y^2$. Let $p > 0$ denote the price of the consumption good.

Society decides to introduce an "unconditional basic income" for all consumers, which is financed by a sales tax levied on the firm in the market for the consumption good. That is, if the firm raises a revenue py, the total tax income is $T = \tau py$, $\tau \in [0, 1]$. The tax income is equally redistributed to consumers, such that each consumer receives an income T/I.

(a) Derive the Walrasian equilibrium.

(b) Let $x_i(\tau)$ denote equilibrium consumption of consumer i, and $X(\tau) = \sum_i x_i(\tau)$ is aggregate consumption. How does relative *consumption inequality*, i.e., the dispersion of consumption shares $s_i^*(\tau) = x_i(\tau)/X(\tau)$ depend on τ? How does absolute consumption inequality, i.e., differences of the form $x_i(\tau) - x_{i'}(\tau)$, depend on τ?

(c) Equilibrium income is $w_i(\tau) = 1 + \theta_i \Pi + \frac{T}{I}$, where $\theta_i \in [0, 1]$ denotes i's claim to firm profits. How does w_i depend on τ? Does income inequality decrease? Do all consumers benefit from the unconditional basic income?

(d) How does total equilibrium welfare depend on τ? Explain!