Exam
Social Choice Theory
Spring 2016

Instructions:

- The duration of the exam is 90 minutes.
  Please answer all three problems.
  You can achieve up to 90 points.
- Please solve and answer all problems on the answer sheet that is distributed separately.
  Write your name and your student ID number on the answer sheet.
  Please have your student identity card ready.
- You must document the solution process and provide sufficient arguments for your solution.
- This question sheet has 4 pages (including cover page). Please check for completeness.
- You are not allowed to use any auxiliary equipment.
  Switch off all electronic devices.

Points:

- Problem 1 (Scoring Methods): 30 points
- Problem 2 (Manipulability): 30 points
- Problem 3 (Social Evaluation Functions): 30 points

Good Luck!

Problem 1: Scoring Methods (30 points)

Let $X = \{x, y, z\}$ be the set of alternatives. Consider the scoring method given by the vector $s = (s_1, s_2, s_3) = (a, b, c)$ of scores, where $a \geq b \geq c$. Hence each of the $n \geq 2$ voters assigns a points to the most-preferred alternative, b points to the second-most-preferred alternative, and c points to the least-preferred alternative (you can assume that all preferences are strict and treat $\mathscr{A} = \mathscr{P}^n$ as the universal domain).

(a) Apply this method to each of the following three profiles of preferences and determine the resulting social preferences. Make careful distinctions based on the values of a, b, c and on whether the inequalities in $a \geq b \geq c$ are strict or not. (4 points each)

(i)

	preferences
Voter 1	$x\ P_1\ y\ P_1\ z$
Voter 2	$x\ P_2\ z\ P_2\ y$
Voter 3	$y\ P_3\ z\ P_3\ x$

(ii)

	preferences
Voter 4	$x\ P_4\ y\ P_4\ z$
Voter 5	$y\ P_5\ z\ P_5\ x$
Voter 6	$z\ P_6\ x\ P_6\ y$

(iii)

	preferences
Voter 7	$x\ P_7\ y\ P_7\ z$
Voter 8	$z\ P_8\ y\ P_8\ x$

(b) Which of Arrow's axioms for SWFs are satisfied and which are violated by this scoring method? Again make careful distinctions based on the values and inequalities in the scoring vector (a, b, c). (10 points) 要仔细想想，每种等号不等号的时候都考虑一下

(c) Now suppose the three groups of voters given in $(i) - (iii)$ of part (a) of this problem form a joint group (with 8 members). What is the outcome under the scoring method for the entire group, assuming that $a > b > c$ and $(a + c)/2 = b$? (8 points)

Problem 2: Manipulability (30 points)

Let $X = \{x, y, z\}$ be the set of alternatives and $N = \{1, 2\}$ the set of voters. Assume that both voters have strict preferences, i.e. treat $\mathscr{A} = \mathscr{P}^2$ as the universal domain. The following table completely specifies an SCF on this domain. For instance, the field with row yxz and column xyz contains y, which means that alternative y is selected if voter 1 has preference yP_1xP_1z and voter 2 has preference xP_2yP_2z. The other fields are interpreted analogously.

$R_1 \backslash R_2$	xyz	xzy	yxz	yzx	zxy	zyx
xyz	x	x	y	y	z	z
xzy	x	x	z	z	z	z
yxz	y	z	y	y	z	z
yzx	y	z	y	y	z	z
zxy	z	z	z	z	z	z
zyx	z	z	z	z	z	z

(a) Which of Arrow's axioms for SCFs are satisfied and which are violated by this method? (10 points)

仔细看哦，别只往下看，yxz到xzy也是z maintains its position的变换

(b) Is this SCF surjective?
Which of Gibbard-Satterthwaite's axioms are satisfied and which are violated?
Identify all preference profiles at which this SCF is manipulable by some voter. (10 points)

(c) Now suppose we restrict the domain of the above SCF by excluding the preference xP_izP_iy for both voters $i = 1, 2$. Under this assumption, answer again all questions from part (b) of this problem. (10 points)

Problem 3: Social Evaluation Functions (30 points)

Let X be a set of alternatives. For any profile of utility functions $\mathbf{U} = (U_1, \ldots, U_n) \in \mathscr{U}^n$ and any alternative $x \in X$, define the weighted average of the largest and the smallest utility by

$$\mathrm{WA}(x, \mathbf{U}) = \gamma \max\{U_1(x), \ldots, U_n(x)\} + (1 - \gamma) \min\{U_1(x), \ldots, U_n(x)\},$$

where γ is a parameter with $0 < \gamma < 1$. Consider the SEF e^{WA} defined by

$$x \, e^{\mathrm{WA}}(\mathbf{U}) \, y \quad \Leftrightarrow \quad \mathrm{WA}(x, \mathbf{U}) \geq \mathrm{WA}(y, \mathbf{U}),$$

for all $x, y \in X$ and all $\mathbf{U} \in \mathscr{U}^n$.

(a) Determine $e^{\mathrm{WA}}(\mathbf{U})$ for the example given in the following table. Make a careful distinction based on the value of γ. (5 points)

$\mathbf{U}$	x	y	z
U_1	4	1	1
U_2	0	5	3

(b) With which of the 7 information structures introduced in class is the SEF e^{WA} consistent? When constructing counterexamples, you are allowed to use a specific value of γ, for example $\gamma = 1/2$. (25 points)