Problem Set 13

1. (**Mechanism Design**)

   Suppose that three agents need to reach the collective decision to whom an **indivisible unit** of a single good should be allocated. Agent $i = 0$ currently owns the good ("seller"), while agents $i = 1, 2$ are potential future owners ("buyers"). An outcome thus is a vector

   $$x = ((k_0, k_1, k_2), (t_0, t_1, t, t_2)),$$

   where $k_i \in 0, 1$ indicates whether agent i owns the good, and $t_i \in \mathbb{R}$ are net monetary transfers to agent i. Note that $k_i = 1$ for exactly one agent.

   The set of feasible allocations, X, is defined by $\sum_{i=0}^{2} k_i = 1$ and $\sum_{i=0}^{2} t_i \leq 0$.

   All agents have a quailinear utility of the form

   $$u_i = \theta_i k_i + t_i$$

   We assume that there is no uncertainty regarding the valuation of the seller ($i = 0$), who always has $\theta_0 = 0$. That is, the seller derives her utility only from her monetary transfer. By contrast, the types of the two buyers are random and only private information, where $\Theta_1 = \Theta_2 = [0, 1]$, and each θ_i is iid uniformly distributed.

   (a) What does a buyer's type θ_i express?

   (b) Let $\theta = (\theta_1, \theta_2)$. Consider the following social choice function (SCF):

   $$f(\theta) = (k_1(\theta), k_2(\theta), t_1(\theta), t_2(\theta))$$

   $$k_1(\theta) = 1 \equiv \theta_1 \geq \theta_2$$

   $$k_2(\theta) = 1 \equiv \theta_2 > \theta_1$$

$$t_i(\theta) = -\theta_i k_i(\theta)$$

$$t_0(\theta) = t_1(\theta) + t_2(\theta)$$

Provide a verbal description of this SCF. Is it feasible? Is it ex-post efficient?

(c) Show that this SCF cannot be implemented in dominant strategies. (*Hint: Use the revelation theorem. That is: Show that this SCF cannot be implemented in dominant strategies by the direct revelation mechanism* (*where the buyers simply announce their types*).) Explain intuitively why truth-telling is not incentive-compatible here.

(d) Could you modify the transfers in the above SCF such that the modified SCF can be implemented in dominant strategies?