Problem Set 4

1. Let $X = (-\infty, \infty) \times \mathbb{R}_+^{L-1}$ be the choice set, and consider quasilinear and strictly quasiconcave preference relation on X, represented by a differentiable utility function. Normalize the price of the numeraire such that $p_1 = 1$.

   (a) Characterize the Walrasian demand for non-numeraire commodities by means of first-order conditions.

   (b) Describe the wealth effects for non-numeraire commodities.

2. We say that a Walrasian demand correspondence $x(p, w)$ satisfies the **Weak Axiom of Revealed Preferences (WARP)** if for any two different price-wealth situations (p, w) and (p', w') the following property holds:

$$p \cdot x(p', w') \leq w \;\wedge\; x(p', w') \neq x(p, w) \implies p' \cdot x(p, w) > w'$$

   (a) Explain verbally why this condition is equivalent to the definition of WARP from the lecture.

   (b) Suppose that $x(p, w)$ is a Walrasian demand function derived from solving a UMP. Show that $x(p, w)$ must verify WARP.

3. Let $X = \mathbb{R}_+^2$, and consider the two utility functions $u_1 = \min\{x_1, 2x_2\}$, $u_2 = \min\{\sqrt{x_1}, x_2\}$. Let $p > 0$ and $w > 0$.

   (a) Depict the indifference map for these two utility functions in the (x_1, x_2)-plane. Interpret.

   (b) Do u_1 or u_2 represent homothetic preferences?

   (c) Derive the Walrasian demand function $x(p, w)$ implied by u_1.