Problem Set 10

1. Consider a two-consumer exchange economy with two consumption goods, and utility functions ($\alpha \in (0,1)$)

$$u^1(x_{11}, x_{21}) = x_{11}^{\alpha}, x_{21}^{1-\alpha},$$

$$u^2(x_{12}, x_{22}) = x_{12}^{\beta}, x_{22}^{1-\beta}.$$

The endowment is $(\omega_1, \omega_2) = (1, 1)$. Let $\alpha = 1/3$ and $\beta = 2/3$. Use a first-order approach to characterize all Pareto optimal allocations (the Pareto set) in the Edgeworth Box.

2. Consider a competitive economy consisting of one firm and one consumer. There are three goods, indexed by $i = 1, 2, 3$. Let $\omega = (\omega_1\ \omega_2\ \omega_3) = (0\ 1\ 0)$ be the initial endowment of the consumer. The consumer's utility function is given by $u = x_1 - \alpha x_3$, where $\alpha > 0$ and x_i is her consumption of good i. The firm only can use good 2 as input and if an amount of z_2 of good 2 is used, *simultaneously* both good 1 and 3 are produced according to the following technology ($z_2 \geq 0$):

$$y_1 \leq \sqrt{z_2}, \quad y_3 = \beta z_2$$

where $\beta \geq 0$. Note that good 3 cannot be disposed ("unproduced").

   (a) State an interpretation of the goods in this economy.

   (b) Characterize the Pareto set of this economy.

   (c) Suppose that good 3 cannot be priced but its supply always equals to "demand", i.e., $x_3 = y_3$. Show that a unique Price Equilibrium exists.[1] Identify the parametric conditions under which the PE is or is not PO.

[1] In General Equilibrium Theory "unique" means unique up to relative prices.

(d) Suppose now that good 3 indeed can be priced and traded. For simplicity let $\alpha = \beta = 1$, and show that the WE is a PO.

3. Consider a competitive economy with a single firm, a single consumer, and two goods: labour and food. The firm produces food x from labour according to the production function $x = f(e) = e$, where e is employment. Let w be the wage rate, and normalize the price of food to one. The consumer owns the firm. The consumer is endowed with one unit of labour, and has utility $U = Ln(x) + Ln(1-e)$. Now suppose that a small tax $\tau \in (0,1)$ on each unit of food sold is introduced to the economy. The revenues from the tax, $T = \tau e$, are distributed lump-sump to the consumer, such that his budget constraint is $x = we + T + \Pi$, where Π are firm profits.

   (a) How do w, e and x depend on τ in the Walrasian equilibrium?

   (b) How does welfare depend on τ? Explain!

4. Consider a pure exchange economy with $L = 2$ goods and $I = 2$ consumers. Let the choice sets be $X_i = \mathbb{R}^2_+$. Consumers have utility functions

$$u_1(x_{11}, x_{21}) = x_{11}^{\alpha} x_{21}^{1-\alpha}, \quad \alpha \in (0,1)$$
$$u_2(x_{12}, x_{22}) = x_{12}^{\beta} x_{22}^{1-\beta}, \quad \beta \in (0,1).$$

The total endowment of the two goods are $\omega_1 = \omega_2 = 1$.

A function $W : \mathbb{R}^2 \to \mathbb{R}$, $W = W(u_1, u_2)$ is called a (Bergson-Samuelson) **social welfare function** (see MWG, p. 117ff). This function normatively expresses a society's judgment about consumer utilities.

   (a) Consider the social planer problem of finding a feasible allocation $(x^*_{11}, ..., x^*_{22}) \in \mathbb{R}^4_+$ that maximizes the following social welfare function:

$$W = Ln(u_1) + Ln(u_2)$$

Derive the optimal planer allocation.

(b) In what follows, let $\alpha = \beta$. Suppose that the two consumers can freely trade their endowments in a market. The endowments are

$$\begin{pmatrix} \omega_{11} \\ \omega_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \qquad \begin{pmatrix} \omega_{12} \\ \omega_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}.$$

Normalize $p_1 = 1$ and find the unique Walrasian equilibrium.

(c) Suppose that the government desires to implement the social optimum from (a) as a market equilibrium by means of a redistributive tax T_1, T_2 on the income w_1, w_2 of the consumers; hence $T_1 + T_2 = 0$. What tax system (T_1, T_2) implements this social optimum?