Problemset 4

International Macroeconomics (Master)

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Exercise 1:   Production Economy

Consider a two–period economy with concave production function

$$Y_t = A_t K_t^{\alpha}, \qquad \text{with} \quad 0 < \alpha < 1,$$

where Y_t denotes the output level of the numeraire good, K_t is capital stock and A_t is the level of factor productivity in period t. The dynamic evolution of capital is given by

$$K_{t+1} = K_t + I_t,$$

i.e. there is no capital depreciation. The period budget constraint of the representative household is

$$C_t + I_t + B_{t+1} = Y_t + B_t(1 + r).$$

(a) Determine the intertemporal budget constraint of representative household.

Consider now the case of a **closed economy**.

(b) Derive formally the intertemporal Production Possibilities Frontier (PPF) and illustrate it in a graphical way. For which values of α is the PPF strictly concave?

(c) For a given interest rate r, determine the profit–maximizing levels of capital stock K_2 and output Y_2.

(d) Calculate the profit–maximizing investment profile I_1 as a function of r and show that it is downward sloping.

(e) Lifetime utility is given by

$$U = \log(C_1) + \beta \log(C_2).$$

Derive optimal intertemporal consumption (C_1 and C_2) and illustrate your result in the diagram from part (b).

(f) Calculate the savings schedule S_1 as a function of the interest rate r and show that it is upward sloping.

(g) How does the equilibrium interest rate react to changes in productivity?

  (i) How does I_1 react to changes in A_1? Interpret your result.

  (ii) How does S_1 react to changes in A_1? Interpret your result.

  (iii) Illustrate the adjustment of the equilibrium interest rate in an investment–saving diagram.

Exercise 2:   Open Economy with Production: Two-Country Model

Consider two open economies with production when there are only two periods. Lifetime utility in the domestic and foreign country (denoted by $*$) are given by

$$U(C_1, C_2) = u(C_1) + \beta u(C_2) \qquad\qquad U(C_1^*, C_2^*) = u(C_1^*) + \beta u(C_2^*)$$

where $u(C_t) = \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ are the same isoelastic utility functions. The production function for the domestic and foreign country are given by

$$Y_t = A_t F(K_t) = A_t K_t^{\alpha} \qquad\qquad Y_t^* = A_t^* F(K_t^*) = A_t^* K_t^{*\alpha}$$

with $\alpha \in (0,1)$. Capital accumulation is given by

$$K_{t+1} = K_t + I_t \qquad\qquad K_{t+1}^* = K_t^* + I_t^*$$

i.e., there is no capital depreciation. Suppose there are no initial holdings of foreign assets and the initial capital stocks are given by K_1 and K_1^*. Finally, the period budget constraint of the domestic and foreign representative household are

$$C_t + I_t + B_{t+1} = Y_t + B_t(1 + r) \qquad C_t^* + I_t^* + B_{t+1}^* = Y_t^* + B_t^*(1 + r).$$

Domestic and foreign aggregate saving as $S_t = Y_t - C_t$ and as $S_t^* = Y_t^* - C_t^*$ respectively.

(a) Discuss the solution to the optimal investment decisions in the open economy and show it graphically both for the domestic and the foreign economy.

(b) Derive the slope of the savings schedule analytically and show that optimal savings are not affected by the impact of the interest rate on the investment decision.

(c) Combine your findings from (b) with the figure in (a) and determine the equilibrium in the open economy.