Problem Set 13

1. Mechanism Design

- Agent $i = 0$ currently owns the good (**seller**)
- Agent $i = 1, 2$ are potential future owners (**buyers**)

$$\text{an outcome vector } x = ((k_0, k_1, k_2), (t_0, t_1, t_2))$$
$$\text{where} \begin{cases} k_i \in 0, 1 & \text{indicates whether agent } i \text{ owns the good} \\ t_i \in \mathbb{R} & \text{net monetary transfers to agent } i \end{cases}$$

$$\text{The set of feasible allocations } X \text{ is defined by} \begin{cases} \sum_{i=1}^{2} k_i = 1 \\ \sum_{i=0}^{2} t_i \leq 0 \end{cases}$$

$$\text{quasilinear utility:} \quad u_i = \theta_i k_i + t_i$$

$$\begin{cases} \theta_0 = 0 \\ \Theta_1 = \Theta_2 \sim [0, 1] \end{cases}$$

1(a)

$$u_i(k_i, t_i, \theta_i) = \theta_i k_i + t_i$$

For buyers $i = 1, 2$

$$\begin{cases} u_i(0, \tilde{t}_i, \theta_i) = \tilde{t}_i \\ u_i(1, \tilde{t}_i - \theta_i, \theta_i) = \theta_i + (\tilde{t} - \theta_i) = \tilde{t}_i \end{cases}$$

θ_i is how much buyer i is willing to pay for the good. θ is the valuation.

1(b)

$$\theta = (\theta_1, \theta_2)$$

SCF:

$$f(\theta) = (k_1(\theta), k_2(\theta), t_1(\theta), t_2(\theta))$$
$$\begin{cases} k_1(\theta) = 1, & \theta_1 \geq \theta_2 \\ k_2(\theta) = 1, & \theta_1 < \theta_2 \end{cases}$$
$$t_i(\theta) = -\theta_i k_i(\theta)$$
$$t_0(\theta) = -(t_1(\theta) + t_2(\theta))$$

Description:

- Buyer with higher valuation is allocated the good and pays his/her valuation to the seller.
- Ties are broken in favor of buyer 1 .

Feasibility:

$$k_0(\theta) = 0 \quad \forall \theta$$

$$\begin{cases} k_0(\theta) + k_1(\theta) + k_2(\theta) = 0 + 1 + 0 = 1 & \text{if } \theta_1 \geq \theta_2 \\ k_0(\theta) + k_1(\theta) + k_2(\theta) = 0 + 0 + 1 = 1 & \text{if } \theta_1 < \theta_2 \end{cases}$$

$$\text{Transfers:} \quad \sum_{i=0}^{2} t_i(\theta) = t_0(\theta) + t_1(\theta) + t_2(\theta) = 0$$

The social choice function is feasible.

Ex-Post Efficiency:

Total utility by implementing SCF f :

$$U = \sum_{i=1}^{2} u_i = \theta_0 k_0(\theta) + t_0(\theta) + \theta_1 k_1(\theta) + t_1(\theta) + \theta_2 k_2(\theta) + t_2(\theta)$$