Exam
Social Choice Theory
Fall 2012

Instructions:

- The duration of the exam is 90 minutes.
  Please answer **all three** problems.
  You can achieve up to 90 points.
- Please solve and answer all problems on the answer sheet that is distributed separately.
  Write your name and your student ID number on the answer sheet.
- This question sheet has 4 pages (including cover page). Please check for completeness.
- You are not allowed to use any auxiliary equipment (except for dictionaries).
  Switch off all electronic devices.
- Please have your student identity card ready.

Points:

- Problem 1 (Choice Functions): 30 points
- Problem 2 (Arrow's Theorem for SWFs): 30 points
- Problem 3 (Manipulability): 30 points

Good Luck!

Problem 1: Choice Functions (30 points)

Throughout this problem, $X = \{w, x, y, z\}$ is the set of alternatives for a single decision-maker.

(a) Give a formal and precise definition of the concept of a choice function on X.
How is a choice function interpreted? (10 points)

(b) The following table specifies three different choice functions C^i, $i = 1, 2, 3$, on X.
The table is read as follows. The first column lists all subsets $S \subseteq X$ with at least two elements. The second column describes which elements are chosen from each of these subsets according to choice function C^1. Columns three and four contain the analogous information for choice functions C^2 and C^3. Each C^i, $i = 1, 2, 3$, also satisfies $C^i(\{a\}) = \{a\}$ for each single alternative $a \in X$.

S	$C^1(S)$	$C^2(S)$	$C^3(S)$
$\{w, x\}$	$\{x\}$	$\{w\}$	$\{w\}$
$\{w, y\}$	$\{y\}$	$\{w, y\}$	$\{w\}$
$\{w, z\}$	$\{w\}$	$\{w\}$	$\{w\}$
$\{x, y\}$	$\{y\}$	$\{x\}$	$\{x, y\}$
$\{x, z\}$	$\{x\}$	$\{x\}$	$\{x\}$
$\{y, z\}$	$\{y\}$	$\{y\}$	$\{y\}$
$\{w, x, y\}$	$\{y\}$	$\{w, x\}$	$\{w\}$
$\{w, x, z\}$	$\{x\}$	$\{w, x\}$	$\{w\}$
$\{w, y, z\}$	$\{y\}$	$\{w, y\}$	$\{w\}$
$\{x, y, z\}$	$\{y\}$	$\{x\}$	$\{x, y\}$
$\{w, x, y, z\}$	$\{y\}$	$\{w, x\}$	$\{w\}$

Do these choice functions satisfy or violate the conditions α (contraction consistency) and/or β (expansion consistency)? (10 points)

(c) For each of the three choice functions C^i, $i = 1, 2, 3$, from part (b) of this problem, construct the base relation R_{C^i}. Are these relations transitive? (10 points)

Problem 2: Arrow's Theorem for SWFs (30 points)

(a) State Arrow's Impossibility Theorem for Social Welfare Functions (SWFs). Please give a precise definition of the underlying axioms. (10 points)

(b) Let $X = \{w, x, y, z\}$ be the set of alternatives, and assume all $n \geq 2$ voters have strict preferences, i.e. treat $\mathscr{A} = \mathscr{P}^n$ as the universal domain.

Consider the following method. Given the voters' preferences, first divide alternatives into two classes, those which are Pareto-efficient (X^E) and those which are Pareto-inefficient (X^I). Formally, for each $a \in X$, we have $a \in X^I$ if there exists $b \in X$ with bR_ia for all voters i and bP_ja for at least one voter j, and $a \in X^E$ otherwise. Society is now indifferent between all alternatives in X^E, and strictly prefers every alternative in X^E to every alternative in X^I. The social preference among alternatives in X^I coincides with the preference of voter 1 for these alternatives. 读题的时候看看仔细，这个和帕累托规则差不多，但是有区别的！

Apply this method to the following preference profile. (5 points)

	preferences
Voter 1	$x\ P\ z\ P\ y\ P\ w$
Voter 2	$z\ P\ y\ P\ w\ P\ x$
Voter 3	$z\ P\ w\ P\ x\ P\ y$

(c) For the method from part (b) of this problem, check which of Arrow's axioms for SWFs are satisfied and which are violated. When you think that an axiom is satisfied, give a brief argument. When you think an axiom is violated, give a counterexample. (8 points)

(d) Give an example of an SWF that satisfies all of Arrow's axioms except [P]. Explain your construction and arguments briefly. (7 points)

Problem 3: Manipulability (30 points)

Let $X = \{x, y, z\}$ be the set of alternatives and $N = \{1, 2\}$ the set of voters. Assume that both voters have strict preferences, i.e. treat $\mathscr{A} = \mathscr{P}^2$ as the universal domain. The following table completely specifies an SCF on this domain. For instance, the field with row yzx and column xyz contains x, which means that alternative x is selected if voter 1 has preference yP_1zP_1x and voter 2 has preference xP_2yP_2z. The other fields are interpreted analogously.

$R_1 \backslash R_2$	xyz	xzy	yzx	yxz	zxy	zyx
xyz	x	x	z	x	z	z
xzy	x	x	z	x	z	z
yzx	x	x	z	x	z	z
yxz	x	x	z	x	z	z
zxy	x	x	z	x	z	z
zyx	x	x	z	x	z	z

(a) Which of Arrow's axioms for SCFs are satisfied and which are violated by this method? Please explain your answer. (10 points)

(b) Which of Gibbard-Satterthwaite's axioms are satisfied and which are violated?
Discuss: Does this SCF contradict the Gibbard-Satterthwaite theorem? (10 points)

(c) Consider the linear reference order on X given by $x > y > z$. Suppose both voters have strict preferences which are single-peaked with respect to this order. Give an example of an SCF on this restricted domain which is surjective and satisfies axioms [$\bar{\text{D}}$] and [$\bar{\text{S}}$]. Please write down your example in table form, similar to above. (10 points)