Problemset 7

International Macroeconomics (Master)

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Spring Semester 2021

Exercise 1: Investment in the PVMCA

A representative agent maximizes his time–separable expected lifetime utility,

$$U_t = E_t\left[\sum_{s=0}^{\infty} \beta^s u\left(C_{t+s}\right)\right]$$

subject to the flow budget constraint

$$C_t = (1+r)\, B_t - B_{t+1} + Y_t - I_t - G_t.$$

Output is produced according to

$$Y_t = A_t F(K_t)$$

and, assuming that there is no depreciation, the change in the stock of capital is investment

$$I_t = \Delta K_{t+1} = K_{t+1} - K_t.$$

(a) By solving the consumer's optimization problem with $\beta(1+r) = 1$, show that the expected rate of return of capital is given by

$$E_t\left[A_{t+1}F'(K_{t+1})\right] = r - \operatorname{cov}\left(\frac{u'(C_{t+1})}{u'(C_t)}, A_{t+1}F'(K_{t+1})\right).$$

What interpretation can be given to the covariance term in the equation above?

(b) Assume that productivity follows a random walk process

$$A_{t+1} = A_t + \epsilon_{t+1}$$

$$E_t[A_{t+1}] = A_t.$$

(i) Explain why a change in the productivity parameter A, which comes as a surprise by the random walk assumption, generates a response of future output that is predictable.

(ii) In the framework of the present value model of the current account, the distinction between global and country-specific shocks is key in understanding the dynamics of current accounts. In what way?

(iii) Glick and Rogoff (JME 1995)[1] derive the following estimable equations for the change in the current account and investment (whereby they start from a production function with adjustment costs)

$$\Delta CA_t = (R-1)CA_{t-1} + \gamma_1 I_{t-1} + \gamma_2 \Delta A_t^c + \gamma_3 \Delta A_t^w$$

$$\Delta I_t = (\beta_1 - 1)I_{t-1} + \beta_2 \Delta A_t^c + \beta_3 \Delta A_t^w$$

where ΔA^c and ΔA^w denote a country-specific and a global productivity shock respectively.

What estimates for γ_2, γ_3, β_2 and β_3 do you expect to find under the PVMCA with permanent productivity shocks?

(c) In the PVMCA with a quadratic utility function and $\beta(1+r) = 1$, consumption equals

$$C_t = \frac{r}{1+r}\left((1+r)B_t + E_t\left[\sum_{s=0}^{\infty} \frac{NO_{t+s}}{(1+r)^s}\right]\right)$$

(i) Assume that $\Delta B_t = CA_{t-1} = 0$ to derive the following equation for the change in consumption

$$\Delta C_t = \frac{r}{1+r}\left(\Delta A_t F(K_t) - I_t + \frac{1}{r}\left(A_t F(K_t + I_t) - A_{t-1}F(K_t)\right)\right)$$

(ii) Use a Taylor approximation of the production function to show that

$$\Delta C_t = \Delta Y_t + \frac{I_t}{1+r}\left(-r + A_t F'(K_t)\right).$$

What can you say about the relative size of ΔY_t and ΔC_t? Given $\Delta A_t^c > 0$, what does this imply for the relative size of ΔCA_t and ΔI_t?

[1] Reuven Glick, Kenneth Rogoff, Global versus country-specific productivity shocks and the current account, Journal of Monetary Economics, Volume 35, Issue 1, February 1995, Pages 159-192.