Mid-Term Exercise

1

$$
\begin{gathered}
U_t = \boldsymbol{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s u(C_{t+s}) \right\} \\
\text{with} \quad u(C_{t+s}) = -\frac{1}{2}(h - C_{t+s})^2 \\
\text{IBC:} \quad B_{t+1} = (1+r)B_t + NO_t - C_t \\
\lim_{k\to\infty} \frac{B_{t+k}}{(1+r)^k} = 0
\end{gathered}
$$

1(a)

Under the assumption that $\beta(1+r) = 1$,

$$
CA_t = -\sum_{k=1}^{\infty} \frac{\boldsymbol{E}_t(\Delta NO_{t+k})}{(1+r)^k}
$$

VAR:

$$
\underbrace{\begin{bmatrix} \Delta NO_{t+1} \\ CA_{t+1} \end{bmatrix}}_{\boldsymbol{Z}_{t+1}} = \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \Delta NO_t \\ CA_t \end{bmatrix}}_{\boldsymbol{Z}_t} + \underbrace{\begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \end{bmatrix}}_{\boldsymbol{u}_{t+1}}
$$

$$
\boldsymbol{Z}_{t+1} = \boldsymbol{A}\boldsymbol{Z}_t + \boldsymbol{u}_{t+1}
$$

$$
\begin{cases} \Delta NO_{t+1} = a_{11}\Delta NO_t + a_{12}CA_t + u_{1,t+1} \\ CA_{t+1} = a_{21}\Delta NO_t + a_{22}CA_t + u_{2,t+1} \end{cases}
$$

Note: using time series data of CA & NO, we can estimate $\boldsymbol{A}$

Using the first row of VAR:

$$
\Delta NO_{t+1} = [1 \quad 0]\boldsymbol{A}\boldsymbol{Z}_t + u_{1,t+1}
$$

Iterate forward k period and take expectation

$$
E_t[\Delta NO_{t+k}] = [1 \quad 0]\boldsymbol{A}^k \boldsymbol{Z}_t
$$

$$
\begin{aligned}
\widehat{CA}_t &= -\sum_{k=1}^{\infty} \frac{\boldsymbol{E}_t(\Delta NO_{t+k})}{(1+r)^k} \\
&= -[1 \quad 0] \sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k \boldsymbol{A}^k \boldsymbol{Z}_t \\
&\frac{1}{1+r}|\boldsymbol{A}| < 1 \quad \text{and} \quad \boldsymbol{A} \text{ is invertible} \\
&= -\left[\frac{1}{1+r} \quad 0\right] \boldsymbol{A} \left[\boldsymbol{I}_2 - \left(\frac{1}{1+r}\right)\boldsymbol{A}\right]^{-1} \boldsymbol{Z}_t \\
&= \boldsymbol{B}\boldsymbol{Z}_t
\end{aligned}
$$

Using the second row of VAR:

$$
CA_{t+1} = [0 \quad 1]\boldsymbol{A}\boldsymbol{Z}_t + u_{2,t+1}
$$

$$CA_t = [0 \quad 1]\boldsymbol{Z}_t$$

Wald test:

$$-\left[\frac{1}{1+r} \quad 0\right]\boldsymbol{A}\left[\boldsymbol{I}_2 - \left(\frac{1}{1+r}\right)\boldsymbol{A}\right]^{-1} = [0 \quad 1]$$

Why may it be important to include the current account in the forecasting equation for ΔNO_{t-k}?

Present value relation itself is a powerful statement: the current account is a sufficient statistics for people's expectation of future outputs. So, if we include past values of CA into the info-set in our output forecast equation, we may actually do very well without having to proxy the infinite complexity of people's info set.

1(b)

$$\Delta NO_t = \lambda \Delta P_t + (1-\lambda)\Delta T_t \quad \text{where } 0 \le \lambda \le 1$$

$$\begin{cases} \Delta P_t = \alpha \Delta P_{t-1} + \eta_t & \alpha \in (0,1) \\ \Delta T_t = (\rho - 1)T_t + \nu_t & \rho \in (0,1) \end{cases}$$

$$\frac{\partial E_t[\Delta P_{t+k}]}{\partial \eta_t} = \alpha^k$$

$$\frac{\partial E_t[\Delta T_{t+k}]}{\partial \nu_t} = \frac{\partial E_t[T_{t+k} - T_{t+k-1}]}{\partial \nu_t} = \rho^k - \rho^{k-1} = (\rho - 1)\rho^{k-1}$$

$$\begin{aligned} CA_t &= -\sum_{k=1}^{\infty} \frac{\boldsymbol{E}_t(\Delta NO_{t+k})}{(1+r)^k} \\ &= -\sum_{k=1}^{\infty} \frac{\lambda E_t[\Delta P_{t+k}] + (1-\lambda)E_t[\Delta T_{t+k}]}{(1+r)^k} \end{aligned}$$

$$\begin{aligned} \frac{\partial CA_t}{\partial \eta_t} &= -\sum_{k=1}^{\infty} \frac{1}{(1+r)^k}\left(\lambda \underbrace{\frac{\partial E_t[\Delta P_{t+k}]}{\partial \eta_t}}_{\alpha^k} + (1-\lambda)\underbrace{\frac{\partial E_t[\Delta T_{t+k}]}{\partial \eta_t}}_{0}\right) \\ &= -\lambda \sum_{k=1}^{\infty}\left(\frac{\alpha}{1+r}\right)^k \\ &= -\frac{\lambda\alpha}{1+r-\alpha} \quad \text{Negative} \end{aligned}$$

Consistent with the permanent income hypothesis.

$$\begin{aligned} \frac{\partial CA_t}{\partial \nu_t} &= -\sum_{k=1}^{\infty} \frac{1}{(1+r)^k}\left(\lambda \underbrace{\frac{\partial E_t[\Delta P_{t+k}]}{\partial \nu_t}}_{0} + (1-\lambda)\underbrace{\frac{\partial E_t[\Delta T_{t+k}]}{\partial \nu_t}}_{(\rho-1)\rho^{k-1}}\right) \\ &= -\frac{(1-\lambda)(\rho-1)}{\rho}\sum_{k=1}^{\infty}\left(\frac{\rho}{1+r}\right)^k \\ &= \frac{(1-\lambda)(1-\rho)}{1+r-\rho} \quad \text{Positive} \end{aligned}$$

1(c)

We can parametrize this using $\lambda^E > \lambda^D$

Emerging economies will have a higher countercyclical CA.

$$\begin{cases} \dfrac{\partial CA^E}{\partial \eta_t} < \dfrac{\partial CA^D}{\partial \eta_t} \\ \dfrac{\partial CA^E}{\partial \nu_t} < \dfrac{\partial CA^D}{\partial \nu} \end{cases}$$

Or

$$\begin{cases} \alpha^E > \alpha^D \\ \rho^E > \rho^D \end{cases}$$

CA is more countercyclical if shock has larger permanent component.

Note:

- Developed economies will have a procyclical CA (larger transitory component).
  - Positive shocks lead to an increase in consumption that is less than 1 to 1 and the CA is increasing.
- Emerging market economies will have a higher countercyclical CA (larger permanent component).
  - Positive shocks today induce consumption to increase by more than 1 to 1 such that the country runs a CA deficit.