Problem Set 1

1. (**Preference Relations**) Prove Proposition 1.1 from the lecture (on the properties of $\succ$ and $\sim$).

2. (**Preference Relations**) Suppose that Jim strictly prefers being stone-cold sober over being drunk. However, Jim's preferences over pints of beer verify

$$x + 1 \succ x$$

for any number of pints $x \geq 0$. Suppose that Jim's preferences over $X = \mathbb{N}$ are complete. Are they rational?

3. (**Indifference sets**) Let X be a non-empty set, and $\succeq$ a rational preference relation on X. For any $x \in X$, the indifference set of x is defined as

$$I(x) \equiv \{y \in X : y \sim x\}.$$

   (a) Prove that different indifference sets cannot intersect.

   (b) Give an example of a complete preference relation on $X = \{a, b, c\}$ where different indifference sets intersect.

4. (**WARP**) Let $X = \{x, y, z\}$ and

$$\mathscr{B}_1 = (\{x, y\}, \{x, y, z\}) \quad \mathscr{B}_2 = (\{x, y\}, \{x, z\}, \{y, z\}).$$

Consider the two choice structures $(\mathscr{B}_1, C_1(\cdot))$,$(\mathscr{B}_2, C_2(\cdot))$ separately in the following two questions, where

$$C_1(\{x, y\}) = \{x, y\}, \quad C_1(\{x, y, z\}) = \{y, z\}$$

$$C_2(\{x, y\}) = x, \quad C_2(\{x, z\}) = z, \quad C_2(\{y, z\}) = y$$

(a) Do they satisfy WARP?

(b) Can they be rationalized by a rational preference relation?

5. (**Contour Sets**) Illustrate graphically that convex $\succeq$ can fail LNS, and that $\succeq$ with LNS or even monotonicity need not be convex.

6. (**Utility and Preferences**) Suppose that $X = \mathbb{R}^2_+$, and let $\succeq$ be a rational preference relation represented by the utility function $u(x, y) = xy$.

(a) Which utility functions represent the same preferences? i) $u_1(x, y) = 10(Ln(x) + Ln(y))$, ii) $u_2(x, y) = e^{xy}$, iii) $u(x, y) = \sqrt{2xy} - 10$

(b) Illustrate the indifference set, the upper contour set and the lower contour set for the bundles $(x_1, y_1) > (0, 0)$ and $(x_2, y_2) = (0, 0)$.

(c) Are these preferences (strongly) monotone? Are they (strictly) convex?