Problem Set 12

1. **(Asset Markets)** Consider the two person economy with John (J) and Tim (T) from the lecture with a single good ("crops") and uncertain endowments. There are two states. In state $s = 0$, which occurs with probability $\pi_0 \in (0, 1/2]$, John is endowed with 1 unit of crops, while Tim has nothing. In state $s = 1$ Tim is endowed with 1 unit of crops, while John has nothing. Thus the (state-dependent) endowments are

$$\omega_J = (1, 0) \qquad \omega_T = (0, 1)$$

There is a complete asset market with the following return structure

$$\begin{pmatrix} r_{01} \\ r_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} r_{02} \\ r_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha \end{pmatrix},$$

where $\alpha > 0$. Let $u_i(\cdot)$ be a strictly increasing and strictly concave Bernoulli utility function, representing the preferences of $i \in \{J, T\}$.

   (a) Let q_2 denote the (relative) price of the second asset. What are the opportunity costs of an additional unit of consumption in state $s = 0$? Explain the comparative-statics intuitively.

   (b) Derive the Radner equilibrium $(x_J^*, x_T^*; q_2^*)$.

   (c) How does q_2 depend on α? Explain!

2. (**Incomplete Markets**) Consider again the above two person economy. The asset market now has the following return structure

$$\begin{pmatrix} r_{01} \\ r_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} r_{02} \\ r_{12} \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

Let $u_i(x) = Ln(x)$ for $i \in \{J, T\}$. Show that a Radner Equilibrium does not exist. Why does this market fail?

3. (**Asset Markets with Aggregate Risk**)

Consider again the above two person economy. Suppose that the endowments now are

$$\omega_J = (1, 0) \qquad \omega_T = (0, 2)$$

There is a complete asset market with a full set of Arrow securities, where

$$\begin{pmatrix} r_{01} \\ r_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} r_{02} \\ r_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Suppose that both consumers have the same Bernoulli utility function $u(x) = Ln(x)$.

(a) Why is this economy said to feature "aggregate risk"?

(b) Characterize the Pareto set of this economy.

(c) Let q_2 denote the price of the second asset. Derive the Radner equilibrium $(x_J^*, x_T^*; q_2^*)$. Show that it is Pareto efficient. Why?

(d) Consider now the asset structure

$$\begin{pmatrix} r_{01} \\ r_{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} r_{02} \\ r_{12} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

What is the price q_2 of the second asset (normalizing $q_1 = 1$ as always)? (*Hint: Do not solve the model again. Instead: Think how the assets must be priced by relating the new return structure to the old one.*)