



Practicum - Consultation: 19.05.2022, 09-11

Please upload all files to OLAT on time.

| What | File name | When |
|--------------------|---------------------------------|----------------------|
| Individual project | "o6worksheet-Your-Name.zip" | 30.05.2022 at 7 am. |
| Group solutions | "o6worksheet-Group-Name.zip" | 30.05.2022 at 7 am. |
| Group contribution | "o7contribution-Group-Name.zip" | 31.05.2022 at 22 pm. |

Individual tasks

Your o6worksheet-Your-Name.zip file contains reproducible code necessary to generate your results and your report together with the resulting pdf-file.

Exercise 1 (Individual project (Part 6))

The individual project of worksheets 1–5 investigated the Bayesian analysis provided by Baeten et al. (2013). You have learned different methods that are relevant for Bayesian study design, prior elicitation, and Bayesian data analysis.

Please put together all pieces (text and R code) from all individual exercises. It is recommended to follow a time line for an evolving project as if you were a statistician coaching the whole study. Assume that the project must pass several stages:

1. study design and sample size computation
2. priori elicitation for placebo and treatment groups
3. data collection
4. data analysis
5. interpretation of the results

Explain your approach to a client.

Remark: You can also use the diagram to structure introduction, methods, results, and discussion sections in your report.

Exercise 2 (Individual task: reflection)

Reflect upon the whole course.



1. What have you learnt in this course?
2. What should be improved in this course and how?
3. What are the positive sides of this course?

Group tasks

Your `o6worksheet-Group-Name.zip` (one per group) file contains reproducible code necessary to generate your results and your report together with the resulting pdf-file, which can contain scans of your handwritten solutions. List the names of students who contributed to the solution of group tasks.

Exercise 3 (Bayesian meta-analysis with `bayesmeta`)

Use the description of the package `bayesmeta` provided by Röver (2020) to compute a Bayesian meta-analysis of $\log(OR)$ of treatment and placebo based on 8 historical studies to replicate the analysis shown in the script.

Total number of observations n_i in each study subject to placebo $i = 1, \dots, 8$:

```
pl_total ← (107, 44, 51, 39, 139, 20, 78, 35)
```

Number of cases x_i in each study subject to placebo $i = 1, \dots, 8$:

```
pl_case ← c(23, 12, 19, 9, 39, 6, 9, 10)
```

Total number of observations n_i in each study subject to a treatment $i = 1, \dots, 8$:

```
tr_total ← (208, 38, 150, 45, 138, 20, 201, 34)
```

Number of cases x_i in each study subject to a treatment $i = 1, \dots, 8$:

```
tr_case ← c(120, 18, 107, 26, 82, 16, 126, 23)
```

Note that these data extend the data used for the prior elicitation in the placebo group in the individual project Exercise 1 in Worksheet 5 by the treatment group (Adalimumab in studies 1 and 2, Etanercept in studies 3–6, Infliximab in studies 7 and 8). Explore and use plot functions provided by `bayesmeta` and report results obtained.

Exercise 4 (Bayesian meta-analysis with JAGS)

Run R code provided in the file `06worksheet_JAGSextension.R`. This model provides an alternative analysis of data considered in the Exercise 3 above. Discuss similarities and differences of the model provided in this R code and models that were used for Bayesian meta-analyses in the Exercise 3 above and in the individual project (Exercise 1 of Worksheet 5).



Exercise 5 (Moments of the Poisson-gamma distribution)

Let $Y \mid \lambda \sim \text{Po}(\lambda)$ with $\lambda \sim G(\alpha, \beta)$. Use the expressions for iterated expectation

$$\mathbb{E}(Y) = \mathbb{E}_\lambda[\mathbb{E}_Y(Y \mid \lambda)]$$

and variance (Held and Sabanés Bové, 2020, Section A.3.4)

$$\text{Var}(Y) = \text{Var}_\lambda[\mathbb{E}_Y(Y \mid \lambda)] + \mathbb{E}_\lambda[\text{Var}_Y(Y \mid \lambda)]$$

to derive both the expectation and the variance of the random variable Y .

Hints:

Poisson distribution $X \sim \text{Po}(\lambda)$: $\mathbb{E}(X) = \lambda$, $\text{Var}(X) = \lambda$

Gamma distribution $X \sim G(\alpha, \beta)$: $\mathbb{E}(X) = \alpha/\beta$, $\text{Var}(X) = \alpha/\beta^2$

Exercise 6 (Empirical Bayes)

Consider observed numbers of lip cancer cases per district for each of 56 districts in Scotland:

$y \leftarrow c(11, 5, 15, 9, 6, 9, 2, 3, 26, 39, 20, 31, 9, 16, 6, 16, 19, 17, 15, 11, 19, 7, 10, 0, 7, 7, 9, 2, 8, 8, 11, 6, 28, 4, 1, 1, 1, 8, 6, 3, 2, 1, 7, 10, 9, 11, 3, 11, 5, 8, 3, 7, 0, 8, 7, 13)$

Assume that these observations are *i.i.d.* realisations of the model $Y \mid \lambda \sim \text{Po}(\lambda)$ with $\lambda \sim G(\alpha, \beta)$. Apply and compare two different approaches to compute empirical Bayes estimates for each district:

- Numerical maximisation of the log-likelihood corresponding to the Poisson-gamma distribution as described by (Held and Sabanés Bové, 2020, p. 210) to obtain the marginal maximum likelihood estimator.
- Matching of moments based on the Exercise 5 above, which provides the marginal moment estimator.

Compare means and the lengths of equi-tailed 95%CrI obtained by both approaches. Report your results.

Group contributions

Exercise 7 (Group contributions for the lecture on 02.06.2022)



Please prepare a group contribution, which your group will present (ca. 5 min) during the next lecture.

Use the MATCH Uncertainty Elicitation Tool

(<http://optics.eee.nottingham.ac.uk/match/uncertainty.php>)

to elicit the prior for the height of adult Swiss females. You may try elicitation of different prior distributions. In any case, report the mean and the standard deviation of the elicited normal prior.

(7.1) Apply Roulette and report the results

(7.2) Apply Quartile and report the results

(7.3) Apply Hybrid and report the results

(7.4) Apply Tertile and report the results

Make sure that the file `07ontribution-Group-Name.zip` (one per group) contains the pdf-file and the R code you want to present.

References

- Baeten, D., X. Baraliakos, J. Braun, J. Sieper, P. Emery, D. van der Heijde, I. McInnes, J. van Laar, R. Landewé, P. Wordsworth, J. Wollenhaupt, H. Kellner, J. Paramarta, J. Wei, A. Brachat, S. Bek, D. Laurent, Y. Li, Y. Wang, A. Bertolino, S. Gsteiger, A. Wright, and W. Hueber (2013). Anti-interleukin-17A monoclonal antibody secukinumab in treatment of ankylosing spondylitis: a randomised, double-blind, placebo-controlled trial. *The Lancet* 382, 1705–1713.
- Held, L. and D. Sabanés Bové (2020). *Likelihood and Bayesian Inference: With Applications in Biology and Medicine*. Springer.
- Röver, C. (2020). Bayesian random-effects meta-analysis using the bayesmeta R package. *Journal of Statistical Software* 93(6), 1–51.