Practicum - Consultation: 03.03.2022, 09-11

Please upload all files to OLAT on time.

What	File name	When
Individual project	"01worksheet-Your-Name.zip"	07.03.2022 at 7 am.
Group solutions	"01worksheet-Group-Name.zip"	07.03.2022 at 7 am.
Group contribution	"02contribution-Group-Name.zip"	08.03.2022 at 22 pm.

Individual tasks
Your 01worksheet-Your-Name.zip file contains reproducible code necessary to generate your results and your report together with the resulting pdf-file.

Exercise 1 (Individual project (Part 1))

This individual exercise has 3 steps.

(a) Explain the main goal of the study by Baeten et al. (2013).

(b) Apply classical methods to analyse the data in Table 1.

(c) Compute the optimal sample size for a 1:1 design for comparison of 60% (Secukinumab) and 25% (Placebo) with power 80% and significance level 5%.

Table 1: ASAS20 responders at week 6: data provided explicitly and implicitly in Table 2 of Baeten et al. (2013).

Group	n	Responders x (%)
Secukinumab	23	14 (60.9%)
Placebo	6	1 (16.7%)

Report your results.

Exercise 2 (Individual task: elicitation of your personal opinion)

Consider the height of adult Swiss females. What is your personal opinion about the location (mean) and the spread (standard deviation) of this distribution (in cm)? Report the values of both the mean and the standard deviation.

Exercise 3 (Individual task: installation of programs)

Given the system you are working with, follow the instructions to download and install the following programs and packages:

1. R: `https://www.r-project.org/`
2. RStudio: `https://www.rstudio.com/`
3. JAGS: `http://mcmc-jags.sourceforge.net/`
4. R-packages: rjags, coda, bayesmeta

Please report: Were you able to successfully download and install the programs and packages? Did you face any difficulties?

Group tasks
Your 01worksheet-Group-Name.zip (one per group) file contains reproducible code necessary to generate your results and your report together with the resulting pdf-file, which can contain scans of your handwritten solutions.

Exercise 4 (Bayes theorem)

Prove the conditional Bayes theorem:

$$P[A|B,I] = \frac{P[B|A,I]P[A|I]}{P[B|I]}$$

Exercise 5 (Application of the Bayes theorem)

Consider a diagnostic test with sensitivity $P[T^+|D^+] = 0.96$ and specificity $P[T^-|D^-] = 0.97$. What is the probability that someone tested positive T^+ actually is healthy $P[D^-|T^+]$ given that the prevalence of the disease is $P[D^+] = 0.002$. Discuss your findings.

Exercise 6 (Monte Carlo: random sample vs the true distribution)

Let the random variable X follow the target $\mathrm{N}(\mu, \sigma^2)$ distribution with $\mu = 160$ and $\sigma = 20$. For set.seed(44566), use the `rnorm()` function in `R` and generate a Monte Carlo sample (*i.i.d* realisations of X) of size $M = 1000$.
Accomplish the following tasks given the MC sample for X:

1. Report the true values of the expectation (mean), standard deviation, variance, median, and (0.025, 0.5, 0.975) quantiles of X;
2. Plot the traceplot of the MC sample for X;
3. Plot the histogram of the MC sample for X with the overlayed true density curve;
4. Summarize the MC sample by computing sample mean, standard deviation, variance and (0.025, 0.5, 0.975) quantiles and compare them with the true values;
5. Estimate the probabilities that $P[X > 175]$ and $P[150 < X < 180]$ and compare both estimates with the true values obtained with the `pnorm()` function.

Exercise 7 (Bayes Factor)

Let $Y|\mu \sim \mathrm{N}(\mu, \kappa^{-1})$ with known variance κ^{-1}. For H_0, $\mu = \mu_0$. For H_1, we suppose that the parameter μ is known with prior distribution $\mu \sim \mathrm{N}(\nu, \lambda^{-1})$, with ν and λ fixed.

(a) Show that

$$f(y|H_1) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{\kappa\lambda}{\kappa+\lambda}}\exp\Big\{-\frac{\kappa\lambda}{2(\kappa+\lambda)}(y-\nu)^2\Big\}$$

(b) Determine analytically the Bayes factor $BF_{01}(y)$.

(c) Show that $BF_{01}(y)$ tends to ∞ as λ tends to 0.

(d) Compute the posterior probability of H_0, denoted by $P[H_0|y]$, if $P[H_0] = P[H_1] = 0.5$, $\mu_0 = 0$, $\kappa = 1$, $\nu = 2$, $\lambda = 1/2$, and $y = 1$. Interpret this result.

Exercise 8 (Calibration of p-values: `pCalibrate`)

Use the `twoby2Calibrate` function of `pCalibrate` to compute the minimum Bayes factor (BF) and p-values from Fisher's exact test applied to data provided in Table 1 of Exercise 1. To get the result for a confirmative test, assume that the alternative is simple. Format this BF with function `formatBF` (Held and Ott, 2018). Is BF increasing or decreasing the prior odds of no effect $P[H_0]$? Assume that the prior probability of no effect $P[H_0]$ is equal 50%. Compute the posterior probability $P[H_0|\text{data}]$ with function `BF2pp` and interpret the result.

Group contributions

Exercise 9 (Group contributions for the lecture on 10.03.2022)

Please prepare a group contribution, which your group will present (ca. 5 min) during the next lecture.

(2.1) History of the Normal distribution and its properties.

(2.2) History of the Binomial distribution and its properties.

(2.3) History of the Beta distribution and its properties.

(2.4) Procedures in `R` for computation of confidence intervals for binary data.

Make sure that the file 02contribution-Group-Name.zip (one per group) contains the pdf-file and the `R` code you want to present.

References

Baeten, D., X. Baraliakos, J. Braun, J. Sieper, P. Emery, D. van der Heijde, I. McInnes, J. van Laar, R. Landewé, P. Wordsworth, J. Wollenhaupt, H. Kellner, J. Paramarta, J. Wei, A. Brachat, S. Bek, D. Laurent, Y. Li, Y. Wang, A. Bertolino, S. Gsteiger, A. Wright, and W. Hueber (2013). Anti-interleukin-17A monoclonal antibody secukinumab in treatment of ankylosing spondylitis: a randomised, double-blind, placebo-controlled trial. *The Lancet 382*, 1705–1713.

Held, L. and M. Ott (2018). On p-values and Bayes Factors. *Annual Review of Statistics and Its Application 5*, 393–419.