Practicum - Consultation: 17.03.2022, 09-11

Please upload all files to OLAT on time.

What	File name	When
Individual project	"02worksheet-Your-Name.zip"	21.03.2022 at 7 am.
Group solutions	"02worksheet-Group-Name.zip"	21.03.2022 at 7 am.
Group contribution	"03contribution-Group-Name.zip"	22.03.2022 at 22 pm.

Individual tasks
Your 02worksheet-Your-Name.zip file contains reproducible code necessary to generate your results for both 2A and 2B parts and your report together with the resulting pdf-file.

Exercise 1 (Individual project (Part 2A))

Provide classical and Bayesian analyses for Secukinumab and placebo separately.

(a) Compute 95% confidence intervals (95%CI) for the true probability of response in the Secukinumab and in the placebo groups for the data provided in Table 1. Interpret both 95%CIs.

(b) Plot both Beta(0.5, 1) and the Beta(11, 32) priors and summarize them with prior mean, median, and the equi-tailed 95% interval.

(c) Consider the Secukinumab group in Table 1: plot the Beta(0.5, 1) prior, plot the posterior distribution of the response rate, and summarize the posterior distribution with posterior mean, median, and the equi-tailed 95% credible interval (95%CrI).

(d) Consider the placebo group in Table 1: plot the Beta(11, 32) prior, plot the posterior distribution of the response rate, and summarize the posterior distribution with posterior mean, median, and the equi-tailed 95%CrI.

(e) Interpret 95%CrIs in both Secukinumab and placebo groups.

Report your results.

Exercise 2 (Individual project (Part 2B))

Baeten et al. (2013) write in their supplementary material: "predictive distribution was approximated by a Beta density with matching mean and standard deviation". In this exercise

Table 1: ASAS20 responders at week 6: data and results provided explicitly and implicitly in Table 2 of Baeten et al. (2013). Priors: Beta(0.5, 1) in the Secukinumab group and Beta(11, 32) in the placebo group.

Group	n	Responders x (%)	Posterior response rate
Secukinumab	23	14 (60.9%)	59.2%
Placebo	6	1 (16.7%)	24.5%

you implement a function that computes α and β shape parameters of a Beta distribution with known mean and variance.

(a) Assume that sample mean and sample variance values of a Beta distribution are known. Derive analytically formulas that compute values of α and β shape parameters of the Beta distribution given its sample mean and sample variance (moments matching).

(b) Implement these formulas in a function in R. Input: sample mean and sample variance of a Beta distribution. Output: α and β shape parameters of the Beta distribution.

(c) Apply your function to mean= 0.255814 and variance= 0.004326663. What are the resulting values of α and β parameters of the Beta distribution?

Report your results.

Group tasks
Your 02worksheet-Group-Name.zip (one per group) file contains reproducible code necessary to generate your results and your report together with the resulting pdf-file, which can contain scans of your handwritten solutions.

Exercise 3 (Conjugate Bayes: analytical derivation)

Assume that $y_1, \ldots, y_n$ are realizations (observations) generated by *iid* random variables which follow a N(m, κ^{-1}) distribution. Moreover, assume that the prior of m follows a N(μ, λ^{-1}) distribution, where κ, μ and λ are fixed (known) constants. Derive analytically (with all constants!) the posterior distribution of m given that $y_1, \ldots, y_n$ have been observed.

Exercise 4 (Conjugate Bayesian analysis in practice)

Apply analytical formulas derived in Exercise 3 above to the vector of `Height` (cm) measurements 166, 168, 168, 177, 160, 170, 172, 159, 175, 164, 175, 167, 164 of 13 Swiss females. Assume that $y_1, \ldots, y_n$ are observations generated by $\mathrm{N}(m, \kappa^{-1})$ distribution with $\kappa = 1/900$. Moreover, assume a $\mathrm{N}(\mu, \lambda^{-1})$ prior for m with $\mu = 161$ and $\lambda = 1/70$.

(a) Apply descriptive graphs and summaries to the `Height` outcome. Compute a classical 95%CI for the true mean and interpret it.

(b) Plot the prior distribution of m, compute its expectation, standard deviation, median, and equi-tailed 95% interval. Estimate $P[m > 200]$ from the prior distribution.

(c) On the plot from task (b), plot the posterior distribution of $m|y_1, \ldots, y_n$. Moreover, compute the posterior mean, standard deviation, and median. In addition, compute the equi-tailed 95%CrI and interpret it.

(d) Estimate $P[m > 200|y_1, \ldots, y_n]$ under the posterior distribution.

(e) Discuss how much your opinion about moments of m and about $P[m > 200]$ have changed between the prior and the posterior.

Exercise 5 (Bayesian learning)

For an interim analysis after 12 subjects, 3 adverse events were observed. In the final study after seeing 64 subjects, 14 adverse events were seen.
Compare the results obtained for the following two Beta priors at each stage:

(a) $\alpha = \beta = 0.5$;

(b) $\alpha = 8$, $\beta = 24$.

How much evidence do you have for a response rate >0.4 before seeing any data, at the stage of an intermediate study and for the final study?
Plot priors and posteriors at each stage.
What are the mean, median, and the equi-tailed 95%CrI of the distribution at each stage?

Group contributions

Exercise 6 (Group contributions for the lecture on 24.03.2022)

Please prepare a group contribution, which your group will present (ca. 5 min) during the next lecture.

(3.1) History of the Monte Carlo method.

(3.2) Overview of random sample generators used for independent Monte Carlo sampling in R.

(3.3) Write a short program in R to estimate the area of the unit circle $x^2 + y^2 = 1$ by dropping randomly points (independent Monte Carlo sampling) on it.

(3.4) Review asymptotical results in the classical statistics.

Make sure that the file 03contribution-Group-Name.zip (one per group) contains the pdf-file and the R code you want to present.

References

Baeten, D., X. Baraliakos, J. Braun, J. Sieper, P. Emery, D. van der Heijde, I. McInnes, J. van Laar, R. Landewé, P. Wordsworth, J. Wollenhaupt, H. Kellner, J. Paramarta, J. Wei, A. Brachat, S. Bek, D. Laurent, Y. Li, Y. Wang, A. Bertolino, S. Gsteiger, A. Wright, and W. Hueber (2013). Anti-interleukin-17A monoclonal antibody secukinumab in treatment of ankylosing spondylitis: a randomised, double-blind, placebo-controlled trial. *The Lancet 382*, 1705–1713.