Taking Linear Logic Apart

Wen Kokke
University of Edinburgh
Edinburgh, Scotland
wen.kokke@ed.ac.uk

Structural Rules.

$$\frac{}{x{\leftrightarrow}y \vdash x : A, y : A^{\perp}}\ \textsc{Ax} \qquad \frac{P \vdash \Gamma, x : A \qquad Q \vdash \Delta, x : A^{\perp}}{(\nu x)(P \mid Q) \vdash \Gamma, \Delta}\ \textsc{Cut}$$

Logical Rules.

$$\frac{P \vdash \Gamma, y : A \qquad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}\ (\otimes) \qquad \frac{P \vdash \Gamma, y : A, x : B}{x(y).P \vdash \Gamma, x : A \parr B}\ (\parr)$$

$$\frac{}{x[].0 \vdash x : \mathbf{1}}\ (\mathbf{1}) \qquad \frac{P \vdash \Gamma}{x().P \vdash \Gamma, x : \perp}\ (\perp)$$

$$\frac{P \vdash \Gamma, x : A}{x \triangleleft \texttt{inl}.P \vdash \Gamma, x : A \oplus B}\ (\oplus_1) \qquad \frac{P \vdash \Gamma, x : B}{x \triangleleft \texttt{inr}.P \vdash \Gamma, x : A \oplus B}\ (\oplus_2)$$

$$\frac{P \vdash \Gamma, x : A \qquad Q \vdash \Gamma, x : B}{x \triangleright \{\texttt{inl} : P; \texttt{inr} : Q\} \vdash \Gamma, x : A \,\&\, B}\ (\&)$$

$$(\text{no rule for } \mathbf{0}) \qquad \frac{}{x \triangleright \{\} \vdash \Gamma, x : \top}\ (\top)$$

Figure 1: Classical Processes (CP)

Theorem 1. *If $P \vdash \Gamma$ in CP, then $P \vdash \Gamma$ in HCP.*

Theorem 2. *If $P \vdash \Gamma$ in HCP, then $P \vdash \Gamma$ in CP.*

Definition 1.
$$\begin{aligned} \parr\cdot &= \perp \\ \parr(x_1 : A_1, \dots, x_n : A_n) &= A_1 \parr \cdots \parr A_n \end{aligned}$$

Lemma 1. *If $\vdash \Gamma$ in HCP, then $\vdash \parr\Gamma$ in HCP.*

Definition 2.
$$\begin{aligned} \bigotimes\varnothing &= \mathbf{1} \\ \bigotimes(\Gamma_1 \mid \dots \mid \Gamma_n) &= \parr\Gamma_1 \otimes \dots \otimes \parr\Gamma_n \end{aligned}$$

Theorem 3. *If $\vdash \mathcal{G}$ in HCP, then $\vdash \bigotimes\mathcal{G}$ in HCP.*

Corollary 1 (Conservative Extension)**.** *If $\vdash \mathcal{G}$ in HCP, then $\vdash \bigotimes\mathcal{G}$ in CP.*

Theorem 4. *If $P \vdash \mathcal{G}$ in HCP, then $P \vdash \mathcal{G}$ in HTA.*

Theorem 5. *If $\vdash \mathcal{G}$ in HTA, then $\vdash \mathcal{G}$ in HCP.*

References

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Structural Rules.

$$\frac{}{x{\leftrightarrow}y \vdash x : A, y : A^{\perp}}\ \textsc{Ax} \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, x : A \qquad Q \vdash \mathcal{H} \mid \Delta, x : A^{\perp}}{(\nu x)(P \mid Q) \vdash \mathcal{G} \mid \mathcal{H} \mid \Gamma, \Delta}\ \textsc{Cut}$$

$$\frac{P \vdash \mathcal{G} \qquad Q \vdash \mathcal{H}}{P \mid Q \vdash \mathcal{G} \mid \mathcal{H}}\ \textsc{H-Mix} \qquad \frac{}{0 \vdash \varnothing}\ \textsc{H-Halt}$$

Logical Rules.

$$\frac{P \vdash \mathcal{G} \mid \Gamma, y : A \qquad Q \vdash \mathcal{H} \mid \Delta, x : B}{x[y].(P \mid Q) \vdash \mathcal{G} \mid \mathcal{H} \mid \Gamma, \Delta, x : A \otimes B}\ (\otimes) \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, y : A, x : B}{x(y).P \vdash \mathcal{G} \mid \Gamma, x : A \parr B}\ (\parr)$$

$$\frac{}{x[].0 \vdash x : \mathbf{1}}\ (\mathbf{1}) \qquad \frac{P \vdash \mathcal{G} \mid \Gamma}{x().P \vdash \mathcal{G} \mid \Gamma, x : \perp}\ (\perp)$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : A}{x \triangleleft \mathtt{inl}.P \vdash \mathcal{G} \mid \Gamma, x : A \oplus B}\ (\oplus_1) \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, x : B}{x \triangleleft \mathtt{inr}.P \vdash \mathcal{G} \mid \Gamma, x : A \oplus B}\ (\oplus_2)$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : A \qquad Q \vdash \mathcal{G} \mid \Gamma, x : B}{x \triangleright \{\mathtt{inl} : P; \mathtt{inr} : Q\} \vdash \mathcal{G} \mid \Gamma, x : A \,\&\, B}\ (\&)$$

$$\text{(no rule for } \mathbf{0}\text{)} \qquad \frac{}{x \triangleright \{\} \vdash \Gamma, x : \top}\ (\top)$$

Figure 2: Hypersequent Classical Processes (HCP)

Structural Rules.

$$\frac{}{x{\leftrightarrow}y \vdash x : A, y : A^{\perp}}\ \textsc{Ax} \qquad \frac{P \vdash \mathcal{G} \mid x : A, \Gamma \mid x : A^{\perp}, \Delta}{(\nu x)P \vdash \mathcal{G} \mid \Gamma, \Delta}\ \textsc{H-Cycle}$$

$$\frac{P \vdash \mathcal{G} \qquad Q \vdash \mathcal{H}}{P \mid Q \vdash \mathcal{G} \mid \mathcal{H}}\ \textsc{H-Mix} \qquad \frac{}{0 \vdash \varnothing}\ \textsc{H-Halt}$$

Logical Rules.

$$\frac{P \vdash \mathcal{G} \mid \Gamma, y : A \mid \Delta, x : B}{x[y].P \vdash \mathcal{G} \mid \Gamma, \Delta, x : A \otimes B}\ \otimes \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, y : A, x : B}{x(y).P \vdash \mathcal{G} \mid \Gamma, x : A \parr B}\ (\parr)$$

$$\frac{P \vdash \mathcal{G}}{x[].P \vdash \mathcal{G} \mid x : \mathbf{1}}\ \mathbf{1} \qquad \frac{P \vdash \mathcal{G} \mid \Gamma}{x().P \vdash \mathcal{G} \mid \Gamma, x : \perp}\ (\perp)$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : A}{x \triangleleft \mathtt{inl}.P \vdash \mathcal{G} \mid \Gamma, x : A \oplus B}\ (\oplus_1) \qquad \frac{P \vdash \mathcal{G} \mid \Gamma, x : B}{x \triangleleft \mathtt{inr}.P \vdash \mathcal{G} \mid \Gamma, x : A \oplus B}\ (\oplus_2)$$

$$\frac{P \vdash \mathcal{G} \mid \Gamma, x : A \qquad Q \vdash \mathcal{G} \mid \Gamma, x : B}{x \triangleright \{\mathtt{inl} : P; \mathtt{inr} : Q\} \vdash \mathcal{G} \mid \Gamma, x : A \,\&\, B}\ (\&)$$

$$\text{(no rule for } \mathbf{0}\text{)} \qquad \frac{}{x \triangleright \{\} \vdash \Gamma, x : \top}\ (\top)$$

(Where each logical rule has the side condition that $x \notin \mathcal{G}$.)

Figure 3: Hypersequent Classical Processes *Taken Apart* (HTA)