Lab 4
February 27, 2018
Z to Fourier: The Tale of Two Transforms

INSTRUCTIONS:
All lab submissions include a written report and source code in the form of an m-file. The report contains all plots, images, and figures specified within the lab. All figures should be labeled appropriately. Answers to questions given in the lab document should be answered in the written report. ***The written report must be in PDF format***. Submissions are done electronically through Compass 2G. This lab will be due March 13, 2018 at 4:59 PM Central Time.

1 An Introduction

In this lab, we will draw connections between the system response in lab 3 and the notion of frequency response. This lab will also make clear the connection between the z-transform and the Discrete-Time Fourier Transform. Lastly, the lab will end with an application problem dealing with Nyquist sampling theorem. In the next lab, we'll extend these concepts to some cool audio applications.

2 Tutorial Problems for Frequency Response

In this section, you will be asked to look at some starter-kit codes and run them. You will then answer a few question about these functions for the report.

Report Item 0:
(1.) Open **starter_kit1.m**. Read through the lines in this file. This file demonstrates the use of the **freqz** function. Please answer in your lab report what **freqz** does, what the arguments of the function represent, and what are the outputs of the function.

(2) Modify the file and plot the magnitude and phase response $H(z) = 1 - z^{-1}$

(3) Open **starter_kit2.m**. Read through the lines in this file. This file demonstrates a way you can plot the magnitude and frequency given $H(z)$. Run this file and examine the plots. Comment on the difference from this approach versus the previous approach **starter_kit1.m**.

(4) Modify the file and plot the magnitude and phase response $H(z) = 1 - z^{-1}$

(5) Look up the angle() and phase () function in Matlab. What is the difference between these two functions in MATLAB when plotting the phase of the frequency response or the DTFT.

3 Frequency Response

In this section, we will be looking at frequency responses of various systems.

Report Item 1:
Suppose now we are given the following sequence $h[n] = 1, -2, 3, -4, 0, 4, -3, 2, -1$. Now, let's perform the following tasks.
(1) By hand,compute the Z-transform of $h[n]$, note that the first entry starts at the 0^{th} index.

(2) In MATLAB,evaluate this result in (1), $H(z)$ with $z = e^{j\omega}$. Let ω be an equally spaced vector of 1000 points between $-\pi$ and π.

(3) Plot the result in (2) (Magnitude and Phase). Note this is the DTFT of sequence $h[n]$.

(4) Verify the result in (3) using freqz() by plotting the magnitude and phase of the output of freqz()

Report Item 2:
Consider the system $y[n] = 0.5y[n-1] + 0.5x[n-1]$ with frequency response $H_d(\omega)$. In addition, recall the following theorem regarding the system response to sinusoids:

If $H_d(\omega) = |H_d(\omega)|e^{j\angle H_d(\omega)}$, then the system response to sinusoid $x[n] = cos(\omega_0 n)$ is:

$y[n] = |H_d(\omega_0)|cos(\omega_0 n + \angle H_d(\omega_0))$

Note that this is when $h[n]$, the impulse response, is *real*.

(1) Hand calculate $y_1[n]$ and $y_2[n]$ for the input $x_1[n] = 3cos(\pi n/2)$ and $x_2[n] = 3sin(\pi n/4)$. First calculate $H_d(\omega)$ and include it in your hand calculations

(2) Provide plots of $y_1[n]$ and $y_2[n]$ for the two input signals $x_1[n]$ and $x_2[n]$. Do this for n= 0 to 30. Do this in a two panel plot.

(3) Using $H(z)$ from the system, obtain the b, a coeffiencts from $H(z) = \frac{B(z}{A(z)}$. Use the filter function to calculate $y_1[n]$ and $y_2[n]$ for the input $x_1[n] = 3cos(\pi n/2)$ and $x_2[n] = 3sin(\pi n/4)$. Do this for n = 0 to 30. Plot $y_1[n]$ and $y_2[n]$.

(4) Use freqz() to plot the frequency response of the system response $H_d(\omega)$ with 8 equally spaced points between 0 and π

(5) Compare the output from (2) and (3). Are they different?

4 An FM Channel Problem

In communications, a channel is defined as a transmission media like a wire or even free space. In this problem, we will model a channel as the following.

Report Item 3:
A communications channel is modeled as the following difference equation $y[n] = 0.8y[n-1] + 0.2x[n]$. The channel is excited by

$$x[n] = cos(\frac{\pi B}{F_s N} n^2)$$

where $B = 10Hz$,$F_s = 100Hz$,$\tau = N/F_s = 10s$

In this problem, it will be useful to use the relationship $\omega = \Omega * T_s$ where $T_s = \frac{1}{F_s}$

(1) Plot the DTFT of system described by the above difference equation. Do this in MATLAB for the portion of the spectrum between DC and $10Hz$. Make your spacing for ω a 1000 length vector.

(2) Plot $x[n]$ for the first $10s$
(3) Use the filter () function to calculate the response of the system to the input $x[n]$. Plot the output response and overlay the frequency response in (1) on top of it. Does the resulting plot make sense?

5 Tweet-Tweet (No, Not A Problem About Twitter)

One waveform that we use a lot in signal processing is called the chirp. It's a signal whose frequency ramps up in time. The chirp signal has a wide use of applications from RADAR to WiFi. We will simply listen to the chirp to better our understanding of the Nyquist Rate.

Report Item 4:
Consider the continuous time signal :

$$x(t) = sin(\Omega_o t + \frac{1}{2}\beta t^2)$$

Taking the derivative of the argument of sin(), we find the instantaneous frequency to be

$$\Omega_{inst} = \Omega_o + \beta t$$

Suppose we have sampling rate $\Omega_{samp} = 2\pi(8192) rad/s$.
(1) For the parameters $\Omega_o = 2\pi(3000) rad/s$ and $\beta = 2000 rad^2/s$, plot $x[n]$, the sampled version of $x(t)$ above. Do this for $1s$. This can be done setting a variable t to be equal to $0 : \frac{1}{F_s} : 1$ and subsituting it into $x(t)$ above to form $x[n]$, the samples of the sequence to be plotted.

(2) Use the sound function and listen to x. Use the documentation if you don't know the sound function. If the documentation is still confusing, ask one of the two TAs in the room. Write down what you hear.

(3) Determine analytically at which time (in seconds) and the sample index when $x[n]$ is aliasing.
(4) Verify this by plotting out $x[n]$ to $7s$ passed the aliasing point. Also listen to the signal using sound(). Can you hear the aliasing?