

Lab 2

January 30, 2018

Convolution and LTI System

INSTRUCTIONS:

All lab submissions include a written report and source code in the form of an m-file. The report contains all plots, images, and figures specified within the lab. All figures should be labeled appropriately. Answers to questions given in the lab document should be answered in the written report. *The written report must be in PDF format.* Submissions are done electronically through [Compass 2g](#).

1 1D Convolution for an Image

Report Item: In this problem, read and use the image **lena.gif** for the following operations. Use the `conv()` function to perform convolution. Before using it, read the documentation on `conv()`

(a) Use `imread()` to read the **lena.gif** image in Matlab and display it using the **imshow** function.

(b) Consider the 1D impulse response $h[n] = \frac{1}{6}\{1, 1, 1, 1, 1, 1\}$. Use it to perform 1D convolution along each row of the **lena.gif** image and display the resulting image. Comment on the result.

(c) Similarly, use the above impulse response to perform 1D convolution along each column of the image and display the resulting image. Compare and comment on this image with the one obtained in part (b).

(d) Lastly, use the above $h[n]$ to perform 1D convolution along each column of the resulting image obtained from part (b) and display the resulting image. How does it compare with the above two resulting images?

2 Zero-Input and Zero-State Responses

Report Item: Compute and plot the response of the following systems:

$$y_1[n] = \frac{n}{n+3}y_1[n-1] + x[n], \quad y_1[-1] = 1$$

$$y_2[n] = 0.7y_2[n-1] + x[n], \quad y_2[-1] = 1$$

- (a) to the input $x[n] = \delta[n]$, for $0 \leq n \leq 20$. Comment on the result.
- (b) to the input $x[n] = \delta[n-4]$, for $0 \leq n \leq 20$. Comment on the result.

3 Bitcoin Analysis

Report Item: Load the **bitcoin.mat** that contains the price of a bitcoin and execute the operations below. Please make sure you are only using the **conv** to verify your results

- (a) Write a Matlab script to compute the two following moving averages

$$y_1[n] = \frac{1}{51} \sum_{k=0}^{50} x[n-k]$$

$$y_2[n] = \frac{1}{51} \sum_{k=-25}^{25} x[n-k]$$

In this case, $y_1[n]$ will start with an index $n = 50$, and $y_2[n]$ will start with an index $n = 25$. Use the **conv** to verify your results.

- (b) Plot the sequences $x[n]$, $y_1[n]$ and $y_2[n]$ for $0 \leq n \leq N-1$ on the same plot and comment on the results. Use function **plot** and represent each sequence with different colors. Let $N-1$ denote the last sample index where system y_1 and y_2 can be implemented.

4 Analytical Expression and Finite Length Representation of a Convolution

Report Item: Let $x[n] = h[n] = (0.95)^n u[n]$ and $y[n] = x[n] * h[n]$

(a) Determine $y[n]$ analytically and plot using Matlab. In the report, typeset the math (in whatever method you choose) or include a picture. Both are valid.

(b) Take the first 50 samples of $x[n]$ and $h[n]$. Compute and plot $y[n]$ using the **conv** function.

(c) Use the **conv** function to determine and plot the first 99 samples of $y[n]$

(d) Which of the outputs in part (b) and (c) come close to that in (a)? Explain.

5 Stable or Unstable?

Report Item: Write a Matlab function to compute and plot the output of the discrete-time system

$$y[n] = 5y[n-1] + x[n], \quad y[-1] = 0$$

for $x[n] = u[n]$, $0 \leq n \leq 1000$.

Based on the results, can you make a statement regarding the stability of the system?

6 Convolution Sum

Report Item: The sum $A_x := \sum_n x[n]$ can be thought of as a measure of the "area" under a sequence $x[n]$.

(a) With the convolution sum defined as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \quad \text{for } -\infty < n < \infty$$

show that $A_y = A_x A_h$

(b) Given the sequences

$$x = \sin(2\pi \cdot 0.01 \cdot (0:100)) + 0.05 \cdot \text{randn}(1,101); \quad h = \text{ones}(1,5);$$

Compute $y[n] = h[n] * x[n]$. Check whether $A_y = A_x A_h$ and plot $x[n]$ and $y[n]$ on the same graph.

(c) Normalize $h[n]$ so that $A_h = 1$ and repeat part (b)

(d) If $A_h = 1$, then $A_y = A_x$. Use this result to explain the difference between the plots obtained in parts (b) and (c).

7 Edge Detector for an Image

Report Item: The second derivative operation $y = \frac{d^2x}{dt^2}$ is approximated by the difference equation

$$y[n] = x[n+1] - 2x[n] + x[n-1]$$

which is a noncausal LTI system and is used as an edge detector in image processing.

(a) Determine the impulse response of this edge detector.

(b) Read the image **lena.gif** image in Matlab and process it row-by-row using the above impulse response. Display the resulting image and comment on its appearance.

(c) Now process the **lena.gif** image column-by-column using the same impulse response. Display the resulting image and comment on its appearance.