ST3131 Regression Analysis

AY2016/2017 Semester 2

Predicting Bicycle Sharing Usage from Environmental and Social Factors

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Group 41

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Executive Summary

This report provides an analysis of several linear regression models that aim to predict the usage of bike sharing systems. Bike sharing systems is very significant as it plays an important role in traffic, environmental and health issues. This study develops a mathematical model to provide a more scientific explanation for the variation of total count of rental bikes and furthermore offers some suggestions to increase the popularity of these bike-sharing systems.

Section 1 describes the problem and objective in greater detail.

Section 2 explains the data collection and choice of variables. We chose 7 highly correlated variables instead of all 16 variables.

Section 3 describes the regression models fitted to the data. We used all regression and AIC methods to find out the first model. The first model has produced a good fit but it violated our normality assumption. Next, we transformed our first model. Since our data meets the condition for Box Cox transformation, we carried out Box Cox transformation and obtained the second model. We looked at the residual plot against the second model and looked at the residual plot against every predictor. Further transformation is required. In addition, a third model is produced (cnt^0.75 ~ weekday + weathersit + atemp2 + hum + windspeed) Finally, we run diagnostics on the final model. Residuals appeared normal, and several observations were found to be influential or high leverage.

Section 4 discusses interpretations of the models above. In general, all models confirmed the expected effects of the predictor variables, and the dominance of some effects over others. The final model presents a balance between predictive power and ease of interpretation.

Section 5 arrives at a conclusion, discusses policy implications, and delineates limitations of the study.

1. Description of the Problem

As countries around the world are developing, new and innovative ideas are emerging, making our daily lives more convenient with improved efficiency of our systems and technology. The bike-sharing system is one of the many examples. Bike-sharing systems are the new generation of traditional bike rentals which transforms the traditional bike renting system into an automatic process which makes borrowing a bike more convenient. This is done by allowing users to borrow and return bikes at any stations in the service region. Therefore, bike-sharing systems are commonly used for short-distance trips as a supplement for private motor vehicles as well as regular public transportation. In many countries, bike-sharing is used as a solution to the last-mile connectivity transportation solution. Apart from the fact that there are approximately more than 500 bike-sharing programs around the world, composing of over 500,000 bicycles, the use of bikes also help to alleviate traffic congestion and air pollution. This has led us to develop great interest for the topic.

The characteristics of data being generated by these systems make them attractive for the research and helps us get a sensing of the mobility in the city just by monitoring these data. Therefore, we want to explore the various factors that affects the usage of these systems and develop a mathematical model to provide a more scientific explanation for the variation of total count of rental bikes and furthermore offer some suggestions to increase the popularity of these bike-sharing systems.

2. Description of data

We obtained our data from UCI Machine Learning Repository (https://archive.ics.uci.edu/ml/datasets/Bike+Sharing+Dataset), which contains the hourly and daily count of rental bikes between 2011 and 2012 in the Capital bikeshare system with the corresponding weather and seasonal information. Out of a total of 16 attributes in the data set, we have chosen 7 variables that we are most interested in. They are:

1. **Weathersit**: The weather situation on the day.
   The weather situation is indicated by integers 1-4 in the following categories:
   - 1: Clear, Few clouds, Partly cloudy, Partly cloudy
   - 2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist
   - 3: Light Snow, Light Rain + Thunderstorm + Scattered clouds, Light Rain + Scattered clouds
   - 4: Heavy Rain + Ice Pellets + Thunderstorm + Mist, Snow + Fog

2. **Weekday**: Day of the week
   The days of the week are indicated by integers 0-6, where 0 is Sunday and 6 is Saturday.

3. **Temp**: Normalized Temperature in Degree Celsius.
   The values are derived via (t-t_min)/(t_max-t_min), t_min = -8, t_max = +39 (only in hourly scale).

4. **Atemp**: Normalized Feeling Temperature in Degree Celsius.
   What the temperature feels like. The values are derived via (t-t_min)/(t_max-t_min), t_min = -16, t_max = +50 (only in hourly scale)

5. **Hum**: Normalized Humidity.
   The values are divided to 100 (max)

6. **Windspeed**: Normalized wind speed.
   The values are divided to 67 (max)

7. **Cnt**: Count of total rental bikes (Response Variable)
   Count includes bike rentals by both casual and registered users.

The first 6 variables are factors that affect the hourly and daily count of rental bikes. With this, we aim to develop a regression model that helps us explain the total count of rental bikes.

3. Regression Analysis

The regression analysis is conducted in 3 parts:

3.1 Primary model
3.2 Transformation of variables
3.3 Work on Final model

The R code used in constructing the models are furnished in Appendix B, and the comprehensive outputs and plots are in Appendix C.

3.1 Primary model

Firstly, we consider all possible linear regression models of the six variables:

$$cnt = \beta 0 + \beta 1 weekday + \beta 2 weathersit + \beta 3 temp + \beta 4 atemp + \beta 5 hum + \beta 6 windspeed + \varepsilon$$

For each number of variables, the maximum value of R^2 and minimum value of C_p are obtained. The summarized R output is shown in the following table:

	Index of regressors	R^2	C_p
1	4	0.3982439	108.9298
2	2 4	0.4476672	42.2734
3	4 5 6	0.4631997	22.69634
4	2 4 5 6	0.4731351	10.89459
5	1 2 4 5 6	0.4781691	5.90161
6	1 2 3 4 5 6	0.4788182	7

From the value of R^2, models (5) and (6) have similar maximum values. Based on the value of C_p, model (5) and (6) are more adequate.

Next, we applied stepwise regression to verify the observation above based on Akaike Information Criterion (AIC). The null model is set as cnt ~ 1 and full model as cnt ~ weekday + weathersit + temp + atemp + hum + windspeed, with direction specified as “both”. The stepwise function terminates with model (5): cnt ~ atemp + weathersit + windspeed + hum + weekday with AIC = 10601.43.

Among the output of three selection methods, AIC and C_p returned model (5) while R^2 returned model (6). Since the R^2 values of the two models differ by little, variables in model (5) (atemp, weathersit, windspeed, hum, weekday) should be chosen to be consistent with the goal of simplicity.

Diagnostics

Kolmogorov-Smirnov test is run on the residuals of model (5) and the null hypothesis of a normal distribution is not rejected at 95% significance level with a p value of 0.09258. However, this value of p may imply a weak evidence to null hypothesis to a certain extent. This contradicts with our assumption of normal distributed residuals; therefore, we conduct transformation of response variable as well as predictors to normalise residuals.

3.2 Transformation of variables

In order to check whether any higher order terms cause significant effect, we used Box and Cox transformation on the response variable. The required conditions are checked:

a) Y is always positive with a minimum of 22
b) $Y_{max} / Y_{min} = 8714 / 22 > 10$

Box and Cox transformation is applied and yielded l as 0.75. Hence, we considered model (5A) which makes transformation using $cnt^{0.75}$ (cnt^0.75 ~ weekday + weathersit + atemp + hum + windspeed). Figure 1 shows a comparison of residual against fitted value plots and normal Q-Q plots of the original model (5) and model (5A).

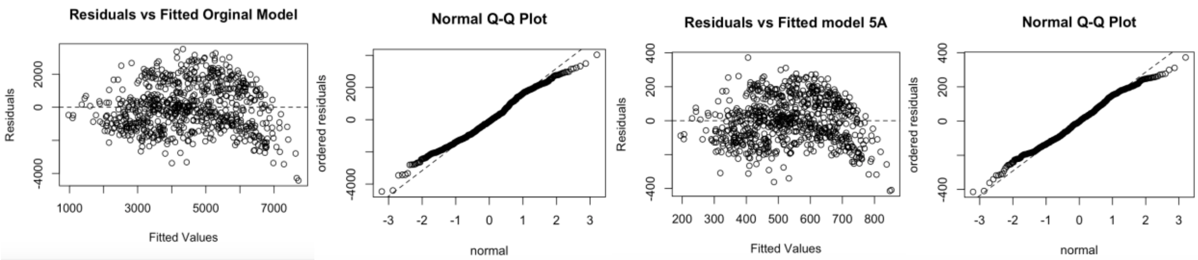

Figure 1

The scatter plot of did not yield any significant improvement in its pattern. Therefore, based on this model, we further tested residuals against each predictor. Among the three numerical variables, a quadratic pattern is observed with atemp in Figure 2.

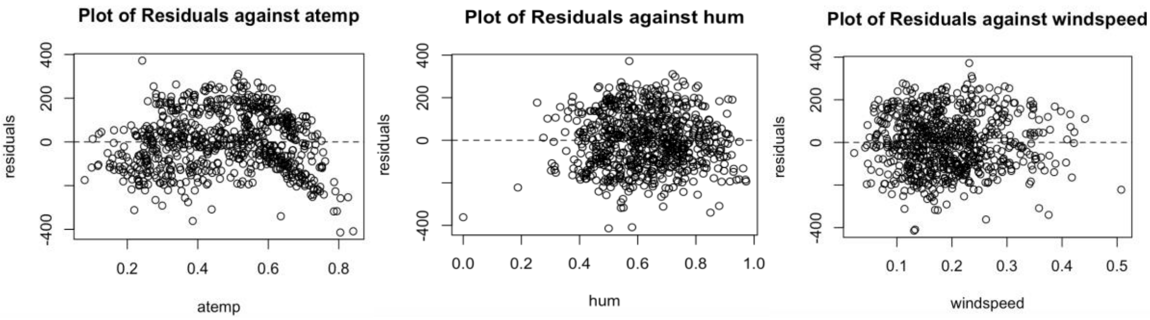

Figure 2

Hence, we fit a new model (5B):

$$cnt^{0.75} \sim weekday + weathersit + atemp^2 + hum + windspeed$$

The model (5B) yields the following summary:

```
Call:
lm(formula = cnt^0.75 ~ weekday + weathersit + atemp2 + hum +  windspeed)

Residuals:
    Min      1Q  Median      3Q     Max
-454.45 -103.01   -2.06  104.58  338.23

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  577.769     32.944  17.538  < 2e-16 ***
weekday        6.315      2.588   2.440   0.0149 *
weathersit   -56.342     12.458  -4.523 7.14e-06 ***
atemp2       653.422     35.167  18.580  < 2e-16 ***
hum         -115.185     48.794  -2.361   0.0185 *
windspeed   -373.830     71.257  -5.246 2.04e-07 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 139.6 on 725 degrees of freedom
Multiple R-squared:  0.4245,	Adjusted R-squared:  0.4206
F-statistic:   107 on 5 and 725 DF,  p-value: < 2.2e-16

> ks.test(res,"pnorm",mean(res),sd(res))
	One-sample Kolmogorov-Smirnov test
data:  res
D = 0.046762, p-value = 0.08177
```

alternative hypothesis: two-sided

Even though this model does not result in a more satisfactory R^2 and residual normality, they still suggest that the model is appropriate with normal distribution. Moreover, the normal Q-Q plots shows less curvature.

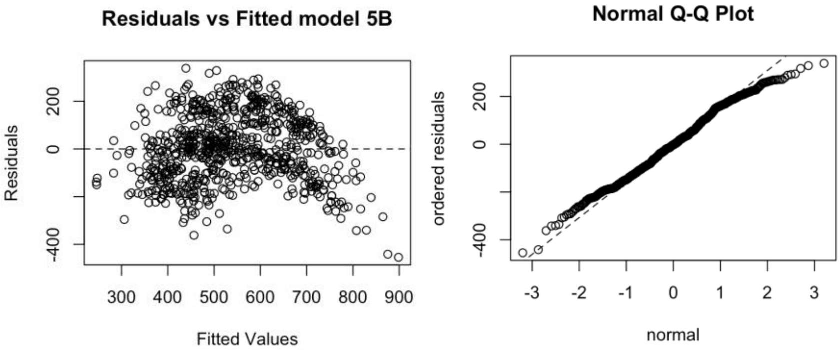


Figure 3(residual against fitted value plot and normal Q-Q plot for model (5B))

3.3 Work on Final Model

Test for serial correlation

We use the Durbin-Watson test to test for serial correlation and the test yields a p value of 0. This suggests that some of the variables are affected by others. Such correlation could be explained to a certain extend from the variable data, i.e. atemp is affected by hum and windspeed.

Test for potential outliers

Table 1 summarizes the observations found to be potential outliers/influential observations based on several measures of leverage and influence on the final model.

	dffit	cov.r	cook.d	hat inf	
27	-0.252405	0.962	1.05e-02	0.00923	*
50	-0.258972	1.027	1.12e-02	0.03342	*
69	-0.781066	1.071	1.01e-01	0.10036	*
202	-0.281838	0.989	1.32e-02	0.01749	*
203	-0.415051	0.936	2.83e-02	0.01553	*
204	-0.409305	0.941	2.76e-02	0.01601	*
205	-0.294280	0.972	1.43e-02	0.01391	*
210	-0.265069	0.970	1.16e-02	0.01144	*
239	-0.364907	0.981	2.20e-02	0.02188	*
249	0.011914	1.029	2.37e-05	0.01999	*
293	0.064345	1.026	6.91e-04	0.01926	*
302	-0.135795	1.025	3.07e-03	0.02294	*
407	0.035052	1.027	2.05e-04	0.01913	*
421	-0.020783	1.027	7.21e-05	0.01853	*
478	-0.109287	1.026	1.99e-03	0.02165	*
560	0.109619	1.025	2.00e-03	0.02081	*
595	0.200355	0.966	6.64e-03	0.00670	*
657	0.144831	0.970	3.48e-03	0.00401	*
666	0.193585	0.969	6.21e-03	0.00662	*
668	-0.318006	0.992	1.68e-02	0.02145	*

Table 1

Without any strong suggestion of the cause of high influence/leverage, we to keep these observations, and retain them for further study.

4. Interpretation of Findings

In all models produced, our general expectations on the predictors' influences were confirmed:

There is a positive relationship when comparing cnt with atemp, weekday, whereas there is a negative relationship for the factors weathersit, hum, windspeed. Atemp proved to be the most significant variable, while hum had the smallest significance in predicting the value of cnt.

These translate to the following general observations:
1. The later into the week (Fridays and Saturdays), the more the number of bicycle rentals in a day.
2. The higher the feeling temperature, the more the number of bicycle rentals in a day.
3. The more adverse the weather situation, the less the number of bicycle rentals in a day.
4. The lower the humidity, the more the number of bicycle rentals in a day.
5. The lower the windspeed, the more the number of bicycle rentals in a day.
6. The variation of the actual temperature is insignificant in its effect on the number of bicycle rentals in a day.

In our final model, the relationship between the variables and cnt can be predicted using the following equation:

cnt^0.75 = 577.769 + 6.315weekday - 56.342weathersit +653.422atemp^2 - 115.185hum - 373.83windspeed + ε

5. Conclusion

This study aimed to examine the relationship between the number of bicycle-sharing rentals in a day (cnt) and environmental factors as well as social factors. These includes the weather situation (weathersit), the day of the week (weekday), Normalized Temperature (temp), normalized feeling temperature (atemp), normalized humidity (hum), as well as normalized wind speed (windspeed). Given data collected from 731 days, this study finds that there is a generally positive relationship when comparing cnt with atemp, weekday, whereas there is a generally negative relationship for the factors weathersit, hum, windspeed. During the initial test for an appropriate model, the variable temp was excluded from the model as it was deemed insignificant in predicting the value of cnt.

Although this study is based on a limited number of factors, its findings are useful in predicting an estimate of the number of bicycle rentals for each day given the above factors. The relationship between weather conditions and the number of rentals is very intuitive, since cities with bicycle-sharing services would also offer other modes of transport, such as buses, trains, or cars. If the weather is not ideal for outdoor activities, most users would switch to other modes of transport. The relationship between the day of the week and the number of rentals can also be explained by the fact that people go out for leisure towards the end of the week, where cycling is a less hurried mode of transport and is also considered a leisure activity to some. It may also be important to note that while it may seem logical for the actual temperature to be considered in predicting the number of rentals, our findings show otherwise. This could be because in some countries, the actual temperature may differ largely from the feeling temperature, which could be affected by the humidity and windspeed. Hence, it is far more accurate to predict the number of rentals based on the feeling temperature, which determines largely whether a user would rent a bicycle.

Limitations

Despite the rigorous procedures in the study, there are still some limitations. One weakness of the study is that the dataset is based on information from one bicycle-sharing company in Portugal. The factors studied are highly dependent on geographical location, since weather conditions are climate-based, and social habits of users on days of the week are also affected by the culture and norms of the area. Hence, it may not be accurate to extrapolate the results to estimate bicycle rental counts in other parts of the world. Another weakness is the number of factors involved. While the study uses key weather factors, there are other factors that might be significant in determining the number of rentals. For instance, road conditions might be important since riding a bicycle would be dangerous on slippery or wet grounds. This cannot be determined solely by the weather situation (weathersit) if the conditions were left by the weather situation on the day before. That said, our proposed model is still sufficient in giving a good gauge of the number of bicycle rentals based on the available factors. While other factors may be relevant, too many factors could result in an over-fitted model, which would not be ideal since it might show a false relationship between the factors and the number of rentals.

Appendices

A Dataset

	weekday	weathersit	temp	atemp	hum	windspeed	cnt
1	6	2	0.344167	0.363625	0.805833	0.160446	985
2	0	2	0.363478	0.353739	0.696087	0.248539	801
3	1	1	0.196364	0.189405	0.437273	0.248309	1349
4	2	1	0.2	0.212122	0.590435	0.160296	1562
5	3	1	0.226957	0.22927	0.436957	0.1869	1600
6	4	1	0.204348	0.233209	0.518261	0.089565	1606
7	5	2	0.196522	0.208839	0.498696	0.168726	1510
8	6	2	0.165	0.162254	0.535833	0.266804	959
9	0	1	0.138333	0.116175	0.434167	0.36195	822
10	1	1	0.150833	0.150888	0.482917	0.223267	1321
11	2	2	0.169091	0.191464	0.686364	0.122132	1263
12	3	1	0.172727	0.160473	0.599545	0.304627	1162
13	4	1	0.165	0.150883	0.470417	0.301	1406
14	5	1	0.16087	0.188413	0.537826	0.126548	1421
15	6	2	0.233333	0.248112	0.49875	0.157963	1248
16	0	1	0.231667	0.234217	0.48375	0.188433	1204
17	1	2	0.175833	0.176771	0.5375	0.194017	1000
18	2	2	0.216667	0.232333	0.861667	0.146775	683
19	3	2	0.292174	0.298422	0.741739	0.208317	1650
20	4	2	0.261667	0.25505	0.538333	0.195904	1927
21	5	1	0.1775	0.157833	0.457083	0.353242	1543
22	6	1	0.05913	0.07907	0.4	0.17197	981
23	0	1	0.096522	0.098839	0.436522	0.2466	986
24	1	1	0.097391	0.11793	0.491739	0.15833	1416
25	2	2	0.223478	0.234526	0.616957	0.129796	1985
26	3	3	0.2175	0.2036	0.8625	0.29385	506
27	4	1	0.195	0.2197	0.6875	0.113837	431
28	5	2	0.203478	0.223317	0.793043	0.1233	1167
29	6	1	0.196522	0.212126	0.651739	0.145365	1098
30	0	1	0.216522	0.250322	0.722174	0.073983	1096
31	1	2	0.180833	0.18625	0.60375	0.187192	1501
32	2	2	0.192174	0.23453	0.829565	0.053213	1360
33	3	2	0.26	0.254417	0.775417	0.264308	1526
34	4	1	0.186957	0.177878	0.437826	0.277752	1550
35	5	2	0.211304	0.228587	0.585217	0.127839	1708
36	6	2	0.233333	0.243058	0.929167	0.161079	1005
37	0	1	0.285833	0.291671	0.568333	0.1418	1623
38	1	1	0.271667	0.303658	0.738333	0.045408	1712
39	2	1	0.220833	0.198246	0.537917	0.36195	1530
40	3	2	0.134783	0.144283	0.494783	0.188839	1605
41	4	1	0.144348	0.149548	0.437391	0.221935	1538
42	5	1	0.189091	0.213509	0.506364	0.10855	1746
43	6	1	0.2225	0.232954	0.544167	0.203367	1472
44	0	1	0.316522	0.324113	0.457391	0.260883	1589
45	1	1	0.415	0.39835	0.375833	0.417908	1913
46	2	1	0.266087	0.254274	0.314348	0.291374	1815
47	3	1	0.318261	0.3162	0.423478	0.251791	2115
48	4	1	0.435833	0.428658	0.505	0.230104	2475
49	5	1	0.521667	0.511983	0.516667	0.264925	2927
50	6	1	0.399167	0.391404	0.187917	0.507463	1635
51	0	1	0.285217	0.27733	0.407826	0.223235	1812
52	1	2	0.303333	0.284075	0.605	0.307846	1107
53	2	1	0.182222	0.186033	0.577778	0.195683	1450
54	3	1	0.221739	0.245717	0.423043	0.094113	1917
55	4	2	0.295652	0.289191	0.697391	0.250496	1807
56	5	2	0.364348	0.350461	0.712174	0.346539	1461
57	6	1	0.2825	0.282192	0.537917	0.186571	1969
58	0	1	0.343478	0.351109	0.68	0.125248	2402
59	1	2	0.407273	0.400118	0.876364	0.289686	1446
60	2	1	0.266667	0.263879	0.535	0.216425	1851
61	3	1	0.335	0.320071	0.449583	0.307833	2134
62	4	1	0.198333	0.200133	0.318333	0.225754	1685

63	5	2	0.261667	0.255679	0.610417	0.203346	1944
64	6	2	0.384167	0.378779	0.789167	0.251871	2077
65	0	2	0.376522	0.366252	0.948261	0.343287	605
66	1	1	0.261739	0.238461	0.551304	0.341352	1872
67	2	1	0.2925	0.3024	0.420833	0.12065	2133
68	3	2	0.295833	0.286608	0.775417	0.22015	1891
69	4	3	0.389091	0.385668	0	0.261877	623
70	5	2	0.316522	0.305	0.649565	0.23297	1977
71	6	1	0.329167	0.32575	0.594583	0.220775	2132
72	0	1	0.384348	0.380091	0.527391	0.270604	2417
73	1	1	0.325217	0.332	0.496957	0.136926	2046
74	2	2	0.317391	0.318178	0.655652	0.184309	2056
75	3	2	0.365217	0.36693	0.776522	0.203117	2192
76	4	1	0.415	0.410333	0.602917	0.209579	2744
77	5	1	0.54	0.527009	0.525217	0.231017	3239
78	6	1	0.4725	0.466525	0.379167	0.368167	3117
79	0	1	0.3325	0.32575	0.47375	0.207721	2471
80	1	2	0.430435	0.409735	0.737391	0.288783	2077
81	2	1	0.441667	0.440642	0.624583	0.22575	2703
82	3	2	0.346957	0.337939	0.839565	0.234261	2121
83	4	2	0.285	0.270833	0.805833	0.243787	1865
84	5	1	0.264167	0.256312	0.495	0.230725	2210
85	6	1	0.265833	0.257571	0.394167	0.209571	2496
86	0	2	0.253043	0.250339	0.493913	0.1843	1693
87	1	1	0.264348	0.257574	0.302174	0.212204	2028
88	2	1	0.3025	0.292908	0.314167	0.226996	2425
89	3	2	0.3	0.29735	0.646667	0.172888	1536
90	4	3	0.268333	0.257575	0.918333	0.217646	1685
91	5	2	0.3	0.283454	0.68625	0.258708	2227
92	6	2	0.315	0.315637	0.65375	0.197146	2252
93	0	1	0.378333	0.378767	0.48	0.182213	3249
94	1	1	0.573333	0.542929	0.42625	0.385571	3115
95	2	2	0.414167	0.39835	0.642083	0.388067	1795
96	3	1	0.390833	0.387608	0.470833	0.263063	2808
97	4	1	0.4375	0.433696	0.602917	0.162312	3141
98	5	2	0.335833	0.324479	0.83625	0.226992	1471
99	6	2	0.3425	0.341529	0.8775	0.133083	2455
100	0	2	0.426667	0.426737	0.8575	0.146767	2895
101	1	2	0.595652	0.565217	0.716956	0.324474	3348
102	2	2	0.5025	0.493054	0.739167	0.274879	2034
103	3	2	0.4125	0.417283	0.819167	0.250617	2162
104	4	1	0.4675	0.462742	0.540417	0.1107	3267
105	5	1	0.446667	0.441913	0.67125	0.226375	3126
106	6	3	0.430833	0.425492	0.888333	0.340808	795
107	0	1	0.456667	0.445696	0.479583	0.303496	3744
108	1	1	0.5125	0.503146	0.5425	0.163567	3429
109	2	2	0.505833	0.489258	0.665833	0.157971	3204
110	3	1	0.595	0.564392	0.614167	0.241925	3944
111	4	1	0.459167	0.453892	0.407083	0.325258	4189
112	5	2	0.336667	0.321954	0.729583	0.219521	1683
113	6	2	0.46	0.450121	0.887917	0.230725	4036
114	0	2	0.581667	0.551763	0.810833	0.192175	4191
115	1	1	0.606667	0.5745	0.776667	0.185333	4073
116	2	1	0.631667	0.594083	0.729167	0.3265	4400
117	3	2	0.62	0.575142	0.835417	0.3122	3872
118	4	2	0.6175	0.578929	0.700833	0.320908	4058
119	5	1	0.51	0.497463	0.457083	0.240063	4595
120	6	1	0.4725	0.464021	0.503333	0.235075	5312
121	0	2	0.451667	0.448204	0.762083	0.106354	3351
122	1	2	0.549167	0.532833	0.73	0.183454	4401
123	2	2	0.616667	0.582079	0.697083	0.342667	4451
124	3	2	0.414167	0.40465	0.737083	0.328996	2633
125	4	1	0.459167	0.441917	0.444167	0.295392	4433
126	5	1	0.479167	0.474117	0.59	0.228246	4608
127	6	1	0.52	0.512621	0.54125	0.16045	4714
128	0	1	0.528333	0.518933	0.631667	0.074638	4333
129	1	1	0.5325	0.525246	0.58875	0.176	4362
130	2	1	0.5325	0.522721	0.489167	0.115671	4803
131	3	1	0.5425	0.5284	0.632917	0.120642	4182
132	4	1	0.535	0.523363	0.7475	0.189667	4864
133	5	2	0.5125	0.4943	0.863333	0.179725	4105
134	6	2	0.520833	0.500629	0.9225	0.13495	3409
135	0	2	0.5625	0.536	0.867083	0.152979	4553
136	1	1	0.5775	0.550512	0.787917	0.126871	3958

137	2	2	0.561667	0.538529	0.837917	0.277354	4123
138	3	2	0.55	0.527158	0.87	0.201492	3855
139	4	2	0.530833	0.510742	0.829583	0.108213	4575
140	5	1	0.536667	0.529042	0.719583	0.125013	4917
141	6	1	0.6025	0.571975	0.626667	0.12065	5805
142	0	1	0.604167	0.5745	0.749583	0.148008	4660
143	1	2	0.631667	0.590296	0.81	0.233842	4274
144	2	2	0.66	0.604813	0.740833	0.207092	4492
145	3	1	0.660833	0.615542	0.69625	0.154233	4978
146	4	1	0.708333	0.654688	0.6775	0.199642	4677
147	5	1	0.681667	0.637008	0.65375	0.240679	4679
148	6	1	0.655833	0.612379	0.729583	0.230092	4758
149	0	1	0.6675	0.61555	0.81875	0.213938	4788
150	1	1	0.733333	0.671092	0.685	0.131225	4098
151	2	1	0.775	0.725383	0.636667	0.111329	3982
152	3	2	0.764167	0.720967	0.677083	0.207092	3974
153	4	1	0.715	0.643942	0.305	0.292287	4968
154	5	1	0.62	0.587133	0.354167	0.253121	5312
155	6	1	0.635	0.594696	0.45625	0.123142	5342
156	0	2	0.648333	0.616804	0.6525	0.138692	4906
157	1	1	0.678333	0.621858	0.6	0.121896	4548
158	2	1	0.7075	0.65595	0.597917	0.187808	4833
159	3	1	0.775833	0.727279	0.622083	0.136817	4401
160	4	2	0.808333	0.757579	0.568333	0.149883	3915
161	5	1	0.755	0.703292	0.605	0.140554	4586
162	6	1	0.725	0.678038	0.654583	0.15485	4966
163	0	1	0.6925	0.643325	0.747917	0.163567	4460
164	1	1	0.635	0.601654	0.494583	0.30535	5020
165	2	1	0.604167	0.591546	0.507083	0.269283	4891
166	3	1	0.626667	0.587754	0.471667	0.167912	5180
167	4	2	0.628333	0.595346	0.688333	0.206471	3767
168	5	1	0.649167	0.600383	0.735833	0.143029	4844
169	6	1	0.696667	0.643954	0.670417	0.119408	5119
170	0	2	0.699167	0.645846	0.666667	0.102	4744
171	1	2	0.635	0.595346	0.74625	0.155475	4010
172	2	2	0.680833	0.637646	0.770417	0.171025	4835
173	3	1	0.733333	0.693829	0.7075	0.172262	4507
174	4	2	0.728333	0.693833	0.703333	0.238804	4790
175	5	1	0.724167	0.656583	0.573333	0.222025	4991
176	6	1	0.695	0.643313	0.483333	0.209571	5202
177	0	1	0.68	0.637629	0.513333	0.094533	5305
178	1	2	0.6825	0.637004	0.658333	0.107588	4708
179	2	1	0.744167	0.692558	0.634167	0.144283	4648
180	3	1	0.728333	0.654688	0.497917	0.261821	5225
181	4	1	0.696667	0.637008	0.434167	0.185312	5515
182	5	1	0.7225	0.652162	0.39625	0.102608	5362
183	6	1	0.738333	0.667308	0.444583	0.115062	5119
184	0	2	0.716667	0.668575	0.6825	0.228858	4649
185	1	2	0.726667	0.665417	0.637917	0.081479	6043
186	2	1	0.746667	0.696338	0.590417	0.126258	4665
187	3	1	0.72	0.685633	0.743333	0.149883	4629
188	4	1	0.75	0.686871	0.65125	0.1592	4592
189	5	2	0.709167	0.670483	0.757917	0.225129	4040
190	6	1	0.733333	0.664158	0.609167	0.167912	5336
191	0	1	0.7475	0.690025	0.578333	0.183471	4881
192	1	1	0.7625	0.729804	0.635833	0.282337	4086
193	2	1	0.794167	0.739275	0.559167	0.200254	4258
194	3	1	0.746667	0.689404	0.631667	0.146133	4342
195	4	1	0.680833	0.635104	0.47625	0.240667	5084
196	5	1	0.663333	0.624371	0.59125	0.182833	5538
197	6	1	0.686667	0.638263	0.585	0.208342	5923
198	0	1	0.719167	0.669833	0.604167	0.245033	5302
199	1	1	0.746667	0.703925	0.65125	0.215804	4458
200	2	1	0.776667	0.747479	0.650417	0.1306	4541
201	3	1	0.768333	0.74685	0.707083	0.113817	4332
202	4	2	0.815	0.826371	0.69125	0.222021	3784
203	5	1	0.848333	0.840896	0.580417	0.1331	3387
204	6	1	0.849167	0.804287	0.5	0.131221	3285
205	0	1	0.83	0.794829	0.550833	0.169171	3606
206	1	1	0.743333	0.720958	0.757083	0.090808	3840
207	2	1	0.771667	0.696979	0.540833	0.200258	4590
208	3	1	0.775	0.690667	0.402917	0.183463	4656
209	4	1	0.779167	0.7399	0.583333	0.178479	4390
210	5	1	0.838333	0.785967	0.5425	0.174138	3846

211	6	1	0.804167	0.728537	0.465833	0.168537	4475
212	0	1	0.805833	0.729796	0.480833	0.164813	4302
213	1	1	0.771667	0.703292	0.550833	0.156717	4266
214	2	1	0.783333	0.707071	0.49125	0.20585	4845
215	3	2	0.731667	0.679937	0.6575	0.135583	3574
216	4	2	0.71	0.664788	0.7575	0.19715	4576
217	5	1	0.710833	0.656567	0.630833	0.184696	4866
218	6	2	0.716667	0.676154	0.755	0.22825	4294
219	0	1	0.7425	0.715292	0.752917	0.201487	3785
220	1	1	0.765	0.703283	0.592083	0.192175	4326
221	2	1	0.775	0.724121	0.570417	0.151121	4602
222	3	1	0.766667	0.684983	0.424167	0.200258	4780
223	4	1	0.7175	0.651521	0.42375	0.164796	4792
224	5	1	0.708333	0.654042	0.415	0.125621	4905
225	6	2	0.685833	0.645858	0.729583	0.211454	4150
226	0	2	0.676667	0.624388	0.8175	0.222633	3820
227	1	1	0.665833	0.616167	0.712083	0.208954	4338
228	2	1	0.700833	0.645837	0.578333	0.236329	4725
229	3	1	0.723333	0.666671	0.575417	0.143667	4694
230	4	1	0.711667	0.662258	0.654583	0.233208	3805
231	5	2	0.685	0.633221	0.722917	0.139308	4153
232	6	1	0.6975	0.648996	0.674167	0.104467	5191
233	0	1	0.710833	0.675525	0.77	0.248754	3873
234	1	1	0.691667	0.638254	0.47	0.27675	4758
235	2	1	0.640833	0.606067	0.455417	0.146763	5895
236	3	1	0.673333	0.630692	0.605	0.253108	5130
237	4	2	0.684167	0.645854	0.771667	0.210833	3542
238	5	1	0.7	0.659733	0.76125	0.083963	4661
239	6	2	0.68	0.635556	0.85	0.375617	1115
240	0	1	0.707059	0.647959	0.561765	0.304659	4334
241	1	1	0.636667	0.607958	0.554583	0.159825	4634
242	2	1	0.639167	0.594704	0.548333	0.125008	5204
243	3	1	0.656667	0.611121	0.597917	0.083333	5058
244	4	1	0.655	0.614921	0.639167	0.141796	5115
245	5	2	0.643333	0.604808	0.727083	0.139929	4727
246	6	1	0.669167	0.633213	0.716667	0.185325	4484
247	0	1	0.709167	0.665429	0.742083	0.206467	4940
248	1	2	0.673333	0.625646	0.790417	0.212696	3351
249	2	3	0.54	0.5152	0.886957	0.343943	2710
250	3	3	0.599167	0.544229	0.917083	0.097021	1996
251	4	3	0.633913	0.555361	0.939565	0.192748	1842
252	5	2	0.65	0.578946	0.897917	0.124379	3544
253	6	1	0.66	0.607962	0.75375	0.153608	5345
254	0	1	0.653333	0.609229	0.71375	0.115054	5046
255	1	1	0.644348	0.60213	0.692174	0.088913	4713
256	2	1	0.650833	0.603554	0.7125	0.141804	4763
257	3	1	0.673333	0.6269	0.697083	0.1673	4785
258	4	2	0.5775	0.553671	0.709167	0.271146	3659
259	5	2	0.469167	0.461475	0.590417	0.164183	4760
260	6	2	0.491667	0.478512	0.718333	0.189675	4511
261	0	1	0.5075	0.490537	0.695	0.178483	4274
262	1	2	0.549167	0.529675	0.69	0.151742	4539
263	2	2	0.561667	0.532217	0.88125	0.134954	3641
264	3	2	0.595	0.550533	0.9	0.096404	4352
265	4	2	0.628333	0.554963	0.902083	0.128125	4795
266	5	2	0.609167	0.522125	0.9725	0.078367	2395
267	6	2	0.606667	0.564412	0.8625	0.078383	5423
268	0	2	0.634167	0.572637	0.845	0.050379	5010
269	1	2	0.649167	0.589042	0.848333	0.1107	4630
270	2	2	0.636667	0.574525	0.885417	0.118171	4120
271	3	2	0.635	0.575158	0.84875	0.148629	3907
272	4	1	0.616667	0.574512	0.699167	0.172883	4839
273	5	1	0.564167	0.544829	0.6475	0.206475	5202
274	6	2	0.41	0.412863	0.75375	0.292296	2429
275	0	2	0.356667	0.345317	0.791667	0.222013	2918
276	1	2	0.384167	0.392046	0.760833	0.083346	3570
277	2	1	0.484167	0.472858	0.71	0.205854	4456
278	3	1	0.538333	0.527138	0.647917	0.17725	4826
279	4	1	0.494167	0.480425	0.620833	0.134954	4765
280	5	1	0.510833	0.504404	0.684167	0.022392	4985
281	6	1	0.521667	0.513242	0.70125	0.045404	5409
282	0	1	0.540833	0.523983	0.7275	0.06345	5511
283	1	1	0.570833	0.542925	0.73375	0.042304	5117
284	2	2	0.566667	0.546096	0.80875	0.143042	4563

285	3	3	0.543333	0.517717	0.90625	0.24815	2416
286	4	2	0.589167	0.551804	0.896667	0.141787	2913
287	5	2	0.550833	0.529675	0.71625	0.223883	3644
288	6	1	0.506667	0.498725	0.483333	0.258083	5217
289	0	1	0.511667	0.503154	0.486667	0.281717	5041
290	1	1	0.534167	0.510725	0.579583	0.175379	4570
291	2	2	0.5325	0.522721	0.701667	0.110087	4748
292	3	3	0.541739	0.513848	0.895217	0.243339	2424
293	4	1	0.475833	0.466525	0.63625	0.422275	4195
294	5	1	0.4275	0.423596	0.574167	0.221396	4304
295	6	1	0.4225	0.425492	0.629167	0.092667	4308
296	0	1	0.421667	0.422333	0.74125	0.099513	4381
297	1	1	0.463333	0.457067	0.772083	0.118792	4187
298	2	1	0.471667	0.463375	0.622917	0.166658	4687
299	3	2	0.484167	0.472846	0.720417	0.148642	3894
300	4	2	0.47	0.457046	0.812917	0.197763	2659
301	5	2	0.330833	0.318812	0.585833	0.229479	3747
302	6	3	0.254167	0.227913	0.8825	0.351371	627
303	0	1	0.319167	0.321329	0.62375	0.176617	3331
304	1	1	0.34	0.356063	0.703333	0.10635	3669
305	2	1	0.400833	0.397088	0.68375	0.135571	4068
306	3	1	0.3775	0.390133	0.71875	0.082092	4186
307	4	1	0.408333	0.405921	0.702083	0.136817	3974
308	5	2	0.403333	0.403392	0.6225	0.271779	4046
309	6	1	0.326667	0.323854	0.519167	0.189062	3926
310	0	1	0.348333	0.362358	0.734583	0.092054	3649
311	1	1	0.395	0.400871	0.75875	0.057225	4035
312	2	1	0.408333	0.412246	0.721667	0.069038	4205
313	3	1	0.4	0.409079	0.758333	0.062196	4109
314	4	2	0.38	0.373721	0.813333	0.189067	2933
315	5	1	0.324167	0.306817	0.44625	0.314675	3368
316	6	1	0.356667	0.357942	0.552917	0.212062	4067
317	0	1	0.440833	0.43055	0.458333	0.281721	3717
318	1	1	0.53	0.524612	0.587083	0.306596	4486
319	2	2	0.53	0.507579	0.68875	0.199633	4195
320	3	3	0.456667	0.451988	0.93	0.136829	1817
321	4	2	0.341667	0.323221	0.575833	0.305362	3053
322	5	1	0.274167	0.272721	0.41	0.168533	3392
323	6	1	0.329167	0.324483	0.502083	0.224496	3663
324	0	2	0.463333	0.457058	0.684583	0.18595	3520
325	1	3	0.4475	0.445062	0.91	0.138054	2765
326	2	3	0.416667	0.421696	0.9625	0.118792	1607
327	3	2	0.440833	0.430537	0.757917	0.335825	2566
328	4	1	0.373333	0.372471	0.549167	0.167304	1495
329	5	1	0.375	0.380671	0.64375	0.098896	2792
330	6	1	0.375833	0.385087	0.681667	0.068421	3068
331	0	1	0.459167	0.4558	0.698333	0.208954	3071
332	1	1	0.503478	0.490122	0.743043	0.142122	3867
333	2	2	0.458333	0.451375	0.830833	0.258092	2914
334	3	1	0.325	0.311221	0.613333	0.271158	3613
335	4	1	0.3125	0.305554	0.524583	0.220158	3727
336	5	1	0.314167	0.331433	0.625833	0.100754	3940
337	6	1	0.299167	0.310604	0.612917	0.095783	3614
338	0	1	0.330833	0.3491	0.775833	0.083958	3485
339	1	2	0.385833	0.393925	0.827083	0.062208	3811
340	2	3	0.4625	0.4564	0.949583	0.232583	2594
341	3	3	0.41	0.400246	0.970417	0.266175	705
342	4	1	0.265833	0.256938	0.58	0.240058	3322
343	5	1	0.290833	0.317542	0.695833	0.082717	3620
344	6	1	0.275	0.266412	0.5075	0.233221	3190
345	0	1	0.220833	0.253154	0.49	0.066542	2743
346	1	1	0.238333	0.270196	0.670833	0.06345	3310
347	2	1	0.2825	0.301138	0.59	0.14055	3523
348	3	2	0.3175	0.338362	0.66375	0.060958	3740
349	4	2	0.4225	0.412237	0.634167	0.268042	3709
350	5	2	0.375	0.359825	0.500417	0.260575	3577
351	6	2	0.258333	0.249371	0.560833	0.243167	2739
352	0	1	0.238333	0.245579	0.58625	0.169779	2431
353	1	1	0.276667	0.280933	0.6375	0.172896	3403
354	2	2	0.385833	0.396454	0.595417	0.061571	3750
355	3	2	0.428333	0.428017	0.858333	0.2214	2660
356	4	2	0.423333	0.426121	0.7575	0.047275	3068
357	5	1	0.373333	0.377513	0.68625	0.274246	2209
358	6	1	0.3025	0.299242	0.5425	0.190304	1011

359	0	1	0.274783	0.279961	0.681304	0.155091	754
360	1	1	0.321739	0.315535	0.506957	0.239465	1317
361	2	2	0.325	0.327633	0.7625	0.18845	1162
362	3	1	0.29913	0.279974	0.503913	0.293961	2302
363	4	1	0.248333	0.263892	0.574167	0.119412	2423
364	5	1	0.311667	0.318812	0.636667	0.134337	2999
365	6	1	0.41	0.414121	0.615833	0.220154	2485
366	0	1	0.37	0.375621	0.6925	0.192167	2294
367	1	1	0.273043	0.252304	0.381304	0.329665	1951
368	2	1	0.15	0.126275	0.44125	0.365671	2236
369	3	2	0.1075	0.119337	0.414583	0.1847	2368
370	4	1	0.265833	0.278412	0.524167	0.129987	3272
371	5	1	0.334167	0.340267	0.542083	0.167908	4098
372	6	1	0.393333	0.390779	0.531667	0.174758	4521
373	0	1	0.3375	0.340258	0.465	0.191542	3425
374	1	2	0.224167	0.247479	0.701667	0.0989	2376
375	2	1	0.308696	0.318826	0.646522	0.187552	3598
376	3	2	0.274167	0.282821	0.8475	0.131221	2177
377	4	2	0.3825	0.381938	0.802917	0.180967	4097
378	5	1	0.274167	0.249362	0.5075	0.378108	3214
379	6	1	0.18	0.183087	0.4575	0.187183	2493
380	0	1	0.166667	0.161625	0.419167	0.251258	2311
381	1	1	0.19	0.190663	0.5225	0.231358	2298
382	2	2	0.373043	0.364278	0.716087	0.34913	2935
383	3	1	0.303333	0.275254	0.443333	0.415429	3376
384	4	1	0.19	0.190038	0.4975	0.220158	3292
385	5	2	0.2175	0.220958	0.45	0.20275	3163
386	6	2	0.173333	0.174875	0.83125	0.222642	1301
387	0	2	0.1625	0.16225	0.79625	0.199638	1977
388	1	2	0.218333	0.243058	0.91125	0.110708	2432
389	2	1	0.3425	0.349108	0.835833	0.123767	4339
390	3	1	0.294167	0.294821	0.64375	0.161071	4270
391	4	2	0.341667	0.35605	0.769583	0.073396	4075
392	5	2	0.425	0.415383	0.74125	0.342667	3456
393	6	1	0.315833	0.326379	0.543333	0.210829	4023
394	0	1	0.2825	0.272721	0.31125	0.24005	3243
395	1	1	0.269167	0.262625	0.400833	0.215792	3624
396	2	1	0.39	0.381317	0.416667	0.261817	4509
397	3	1	0.469167	0.466538	0.507917	0.189067	4579
398	4	2	0.399167	0.398971	0.672917	0.187187	3761
399	5	1	0.313333	0.309346	0.526667	0.178496	4151
400	6	2	0.264167	0.272725	0.779583	0.121896	2832
401	0	2	0.265833	0.264521	0.687917	0.175996	2947
402	1	1	0.282609	0.296426	0.622174	0.1538	3784
403	2	1	0.354167	0.361104	0.49625	0.147379	4375
404	3	2	0.256667	0.266421	0.722917	0.133721	2802
405	4	1	0.265	0.261988	0.562083	0.194037	3830
406	5	2	0.280833	0.293558	0.54	0.116929	3831
407	6	3	0.224167	0.210867	0.73125	0.289796	2169
408	0	1	0.1275	0.101658	0.464583	0.409212	1529
409	1	1	0.2225	0.227913	0.41125	0.167283	3422
410	2	2	0.319167	0.333946	0.50875	0.141179	3922
411	3	1	0.348333	0.351629	0.53125	0.1816	4169
412	4	2	0.316667	0.330162	0.752917	0.091425	3005
413	5	1	0.343333	0.351629	0.634583	0.205846	4154
414	6	1	0.346667	0.355425	0.534583	0.190929	4318
415	0	2	0.28	0.265788	0.515833	0.253112	2689
416	1	1	0.28	0.273391	0.507826	0.229083	3129
417	2	1	0.287826	0.295113	0.594348	0.205717	3777
418	3	1	0.395833	0.392667	0.567917	0.234471	4773
419	4	1	0.454167	0.444446	0.554583	0.190913	5062
420	5	2	0.4075	0.410971	0.7375	0.237567	3487
421	6	1	0.290833	0.255675	0.395833	0.421642	2732
422	0	1	0.279167	0.268308	0.41	0.205229	3389
423	1	1	0.366667	0.357954	0.490833	0.268033	4322
424	2	1	0.359167	0.353525	0.395833	0.193417	4363
425	3	2	0.344348	0.34847	0.804783	0.179117	1834
426	4	1	0.485833	0.475371	0.615417	0.226987	4990
427	5	2	0.353333	0.359842	0.657083	0.144904	3194
428	6	2	0.414167	0.413492	0.62125	0.161079	4066
429	0	1	0.325833	0.303021	0.403333	0.334571	3423
430	1	1	0.243333	0.241171	0.50625	0.228858	3333
431	2	1	0.258333	0.255042	0.456667	0.200875	3956
432	3	1	0.404167	0.3851	0.513333	0.345779	4916

433	4	1	0.5275	0.524604	0.5675	0.441563	5382
434	5	2	0.410833	0.397083	0.407083	0.4148	4569
435	6	1	0.2875	0.277767	0.350417	0.22575	4118
436	0	1	0.361739	0.35967	0.476957	0.222587	4911
437	1	1	0.466667	0.459592	0.489167	0.207713	5298
438	2	1	0.565	0.542929	0.6175	0.23695	5847
439	3	1	0.5725	0.548617	0.507083	0.115062	6312
440	4	1	0.5575	0.532825	0.579583	0.149883	6192
441	5	2	0.435833	0.436229	0.842083	0.113192	4378
442	6	2	0.514167	0.505046	0.755833	0.110704	7836
443	0	2	0.4725	0.464	0.81	0.126883	5892
444	1	1	0.545	0.532821	0.72875	0.162317	6153
445	2	1	0.560833	0.538533	0.807917	0.121271	6093
446	3	2	0.531667	0.513258	0.82125	0.089558	6230
447	4	1	0.554167	0.531567	0.83125	0.117562	6871
448	5	2	0.601667	0.570067	0.694167	0.1163	8362
449	6	2	0.5025	0.486733	0.885417	0.192783	3372
450	0	2	0.4375	0.437488	0.880833	0.220775	4996
451	1	1	0.445833	0.43875	0.477917	0.386821	5558
452	2	1	0.323333	0.315654	0.29	0.187192	5102
453	3	1	0.484167	0.47095	0.48125	0.291671	5698
454	4	1	0.494167	0.482304	0.439167	0.31965	6133
455	5	2	0.37	0.375621	0.580833	0.138067	5459
456	6	2	0.424167	0.421708	0.738333	0.250617	6235
457	0	2	0.425833	0.417287	0.67625	0.172267	6041
458	1	1	0.433913	0.427513	0.504348	0.312139	5936
459	2	1	0.466667	0.461483	0.396667	0.100133	6772
460	3	1	0.541667	0.53345	0.469583	0.180975	6436
461	4	1	0.435	0.431163	0.374167	0.219529	6457
462	5	1	0.403333	0.390767	0.377083	0.300388	6460
463	6	1	0.4375	0.426129	0.254167	0.274871	6857
464	0	1	0.5	0.492425	0.275833	0.232596	5169
465	1	1	0.489167	0.476638	0.3175	0.358196	5585
466	2	1	0.446667	0.436233	0.435	0.249375	5918
467	3	1	0.348696	0.337274	0.469565	0.295274	4862
468	4	1	0.3975	0.387604	0.46625	0.290429	5409
469	5	1	0.4425	0.431808	0.408333	0.155471	6398
470	6	1	0.495	0.487996	0.502917	0.190917	7460
471	0	1	0.606667	0.573875	0.507917	0.225129	7132
472	1	1	0.664167	0.614925	0.561667	0.284829	6370
473	2	1	0.608333	0.598487	0.390417	0.273629	6691
474	3	2	0.463333	0.457038	0.569167	0.167912	4367
475	4	1	0.498333	0.493046	0.6125	0.065929	6565
476	5	1	0.526667	0.515775	0.694583	0.149871	7290
477	6	1	0.57	0.542921	0.682917	0.283587	6624
478	0	3	0.396667	0.389504	0.835417	0.344546	1027
479	1	2	0.321667	0.301125	0.766667	0.303496	3214
480	2	1	0.413333	0.405283	0.454167	0.249383	5633
481	3	1	0.476667	0.470317	0.427917	0.118792	6196
482	4	2	0.498333	0.483583	0.756667	0.176625	5026
483	5	1	0.4575	0.452637	0.400833	0.347633	6233
484	6	2	0.376667	0.377504	0.489583	0.129975	4220
485	0	1	0.458333	0.450121	0.587083	0.116908	6304
486	1	2	0.464167	0.457696	0.57	0.171638	5572
487	2	2	0.613333	0.577021	0.659583	0.156096	5740
488	3	1	0.564167	0.537896	0.797083	0.138058	6169
489	4	2	0.56	0.537242	0.768333	0.133696	6421
490	5	1	0.6275	0.590917	0.735417	0.162938	6296
491	6	2	0.621667	0.584608	0.756667	0.152992	6883
492	0	2	0.5625	0.546737	0.74	0.149879	6359
493	1	2	0.5375	0.527142	0.664167	0.230721	6273
494	2	2	0.581667	0.557471	0.685833	0.296029	5728
495	3	2	0.575	0.553025	0.744167	0.216412	4717
496	4	1	0.505833	0.491783	0.552083	0.314063	6572
497	5	1	0.533333	0.520833	0.360417	0.236937	7030
498	6	1	0.564167	0.544817	0.480417	0.123133	7429
499	0	1	0.6125	0.585238	0.57625	0.225117	6118
500	1	2	0.573333	0.5499	0.789583	0.212692	2843
501	2	2	0.611667	0.576404	0.794583	0.147392	5115
502	3	1	0.636667	0.595975	0.697917	0.122512	7424
503	4	1	0.593333	0.572613	0.52	0.229475	7384
504	5	1	0.564167	0.551121	0.523333	0.136817	7639
505	6	1	0.6	0.566908	0.45625	0.083975	8294
506	0	1	0.620833	0.583967	0.530417	0.254367	7129

507	1	2	0.598333	0.565667	0.81125	0.233204	4359
508	2	2	0.615	0.580825	0.765833	0.118167	6073
509	3	2	0.621667	0.584612	0.774583	0.102	5260
510	4	1	0.655	0.6067	0.716667	0.172896	6770
511	5	1	0.68	0.627529	0.747083	0.14055	6734
512	6	1	0.6925	0.642696	0.7325	0.198992	6536
513	0	1	0.69	0.641425	0.697083	0.215171	6591
514	1	1	0.7125	0.6793	0.67625	0.196521	6043
515	2	1	0.7225	0.672992	0.684583	0.2954	5743
516	3	2	0.656667	0.611129	0.67	0.134329	6855
517	4	1	0.68	0.631329	0.492917	0.195279	7338
518	5	2	0.654167	0.607962	0.755417	0.237563	4127
519	6	1	0.583333	0.566288	0.549167	0.186562	8120
520	0	1	0.6025	0.575133	0.493333	0.184087	7641
521	1	1	0.5975	0.578283	0.487083	0.284833	6998
522	2	2	0.540833	0.525892	0.613333	0.209575	7001
523	3	1	0.554167	0.542292	0.61125	0.077125	7055
524	4	1	0.6025	0.569442	0.567083	0.15735	7494
525	5	1	0.649167	0.597862	0.467917	0.175383	7736
526	6	1	0.710833	0.648367	0.437083	0.144287	7498
527	0	1	0.726667	0.663517	0.538333	0.133721	6598
528	1	2	0.720833	0.659721	0.587917	0.207713	6664
529	2	2	0.653333	0.597875	0.833333	0.214546	4972
530	3	1	0.655833	0.611117	0.582083	0.343279	7421
531	4	1	0.648333	0.624383	0.569583	0.253733	7363
532	5	1	0.639167	0.599754	0.589583	0.176617	7665
533	6	1	0.631667	0.594708	0.504167	0.166667	7702
534	0	1	0.5925	0.571975	0.59875	0.144904	6978
535	1	2	0.568333	0.544842	0.777917	0.174746	5099
536	2	1	0.688333	0.654692	0.69	0.148017	6825
537	3	1	0.7825	0.720975	0.592083	0.113812	6211
538	4	1	0.805833	0.752542	0.567917	0.118787	5905
539	5	1	0.7775	0.724121	0.57375	0.182842	5823
540	6	1	0.731667	0.652792	0.534583	0.179721	7458
541	0	1	0.743333	0.674254	0.479167	0.145525	6891
542	1	1	0.715833	0.654042	0.504167	0.300383	6779
543	2	1	0.630833	0.594704	0.373333	0.347642	7442
544	3	1	0.6975	0.640792	0.36	0.271775	7335
545	4	1	0.749167	0.675512	0.4225	0.17165	6879
546	5	1	0.834167	0.786613	0.48875	0.165417	5463
547	6	1	0.765	0.687508	0.60125	0.161071	5687
548	0	1	0.815833	0.750629	0.51875	0.168529	5531
549	1	1	0.781667	0.702038	0.447083	0.195267	6227
550	2	1	0.780833	0.70265	0.492083	0.126237	6660
551	3	1	0.789167	0.732337	0.53875	0.13495	7403
552	4	1	0.8275	0.761367	0.457917	0.194029	6241
553	5	1	0.828333	0.752533	0.450833	0.146142	6207
554	6	1	0.861667	0.804913	0.492083	0.163554	4840
555	0	1	0.8225	0.790396	0.57375	0.125629	4672
556	1	2	0.710833	0.654054	0.683333	0.180975	6569
557	2	2	0.720833	0.664796	0.6675	0.151737	6290
558	3	1	0.716667	0.650271	0.633333	0.151733	7264
559	4	1	0.715833	0.654683	0.529583	0.146775	7446
560	5	2	0.731667	0.667933	0.485833	0.08085	7499
561	6	2	0.703333	0.666042	0.699167	0.143679	6969
562	0	1	0.745833	0.705196	0.717917	0.166667	6031
563	1	1	0.763333	0.724125	0.645	0.164187	6830
564	2	1	0.818333	0.755683	0.505833	0.114429	6786
565	3	1	0.793333	0.745583	0.577083	0.137442	5713
566	4	1	0.77	0.714642	0.600417	0.165429	6591
567	5	2	0.665833	0.613025	0.844167	0.208967	5870
568	6	3	0.595833	0.549912	0.865417	0.2133	4459
569	0	2	0.6675	0.623125	0.7625	0.093921	7410
570	1	1	0.741667	0.690017	0.694167	0.138683	6966
571	2	1	0.750833	0.70645	0.655	0.211454	7592
572	3	1	0.724167	0.654054	0.45	0.1648	8173
573	4	1	0.776667	0.739263	0.596667	0.284813	6861
574	5	1	0.781667	0.734217	0.594583	0.152992	6904
575	6	1	0.755833	0.697604	0.613333	0.15735	6685
576	0	1	0.721667	0.667933	0.62375	0.170396	6597
577	1	1	0.730833	0.684987	0.66875	0.153617	7105
578	2	1	0.713333	0.662896	0.704167	0.165425	7216
579	3	1	0.7175	0.667308	0.6775	0.141179	7580
580	4	1	0.7525	0.707088	0.659583	0.129354	7261

581	5	2	0.765833	0.722867	0.6425	0.215792	7175
582	6	1	0.793333	0.751267	0.613333	0.257458	6824
583	0	1	0.769167	0.731079	0.6525	0.290421	5464
584	1	2	0.7525	0.710246	0.654167	0.129354	7013
585	2	2	0.735833	0.697621	0.70375	0.116908	7273
586	3	2	0.75	0.707717	0.672917	0.1107	7534
587	4	1	0.755833	0.699508	0.620417	0.1561	7286
588	5	2	0.715833	0.667942	0.715833	0.238813	5786
589	6	2	0.6925	0.638267	0.732917	0.206479	6299
590	0	1	0.700833	0.644579	0.530417	0.122512	6544
591	1	1	0.720833	0.662254	0.545417	0.136212	6883
592	2	1	0.726667	0.676779	0.686667	0.169158	6784
593	3	1	0.706667	0.654037	0.619583	0.169771	7347
594	4	1	0.719167	0.654688	0.519167	0.141796	7605
595	5	1	0.723333	0.2424	0.570833	0.231354	7148
596	6	1	0.678333	0.618071	0.603333	0.177867	7865
597	0	2	0.635833	0.603554	0.711667	0.08645	4549
598	1	2	0.635833	0.595967	0.734167	0.129979	6530
599	2	1	0.649167	0.601025	0.67375	0.072771	7006
600	3	1	0.6675	0.621854	0.677083	0.070283	7375
601	4	1	0.695833	0.637008	0.635833	0.084596	7765
602	5	2	0.7025	0.6471	0.615	0.072146	7582
603	6	2	0.661667	0.618696	0.712917	0.244408	6053
604	0	2	0.653333	0.595996	0.845833	0.228858	5255
605	1	1	0.703333	0.654688	0.730417	0.128733	6917
606	2	1	0.728333	0.66605	0.62	0.190925	7040
607	3	1	0.685	0.635733	0.552083	0.112562	7697
608	4	1	0.706667	0.652779	0.590417	0.077117	7713
609	5	1	0.764167	0.6894	0.5875	0.168533	7350
610	6	2	0.753333	0.702654	0.638333	0.113187	6140
611	0	2	0.696667	0.649	0.815	0.064071	5810
612	1	1	0.7075	0.661629	0.790833	0.151121	6034
613	2	1	0.725833	0.686888	0.755	0.236321	6864
614	3	1	0.736667	0.708983	0.74125	0.187808	7112
615	4	2	0.696667	0.655329	0.810417	0.142421	6203
616	5	1	0.703333	0.657204	0.73625	0.171646	7504
617	6	2	0.659167	0.611121	0.799167	0.281104	5976
618	0	1	0.61	0.578925	0.5475	0.224496	8227
619	1	1	0.583333	0.565654	0.50375	0.258713	7525
620	2	1	0.5775	0.554292	0.52	0.092054	7767
621	3	1	0.599167	0.570075	0.577083	0.131846	7870
622	4	1	0.6125	0.579558	0.637083	0.082721	7804
623	5	1	0.633333	0.594083	0.6725	0.103863	8009
624	6	1	0.608333	0.585867	0.501667	0.247521	8714
625	0	1	0.58	0.563125	0.57	0.090183	7333
626	1	2	0.580833	0.55305	0.734583	0.151742	6869
627	2	2	0.623333	0.565067	0.8725	0.357587	4073
628	3	1	0.5525	0.540404	0.536667	0.215175	7591
629	4	1	0.546667	0.532192	0.618333	0.118167	7720
630	5	1	0.599167	0.571971	0.66875	0.154229	8167
631	6	1	0.65	0.610488	0.646667	0.283583	8395
632	0	1	0.529167	0.518933	0.467083	0.223258	7907
633	1	1	0.514167	0.502513	0.492917	0.142404	7436
634	2	1	0.55	0.544179	0.57	0.236321	7538
635	3	1	0.635	0.596613	0.630833	0.2444	7733
636	4	2	0.65	0.607975	0.690833	0.134342	7393
637	5	2	0.619167	0.585863	0.69	0.164179	7415
638	6	1	0.5425	0.530296	0.542917	0.227604	8555
639	0	1	0.526667	0.517663	0.583333	0.134958	6889
640	1	2	0.520833	0.512	0.649167	0.090804	6778
641	2	3	0.590833	0.542333	0.871667	0.104475	4639
642	3	2	0.6575	0.599133	0.79375	0.066546	7572
643	4	2	0.6575	0.607975	0.722917	0.117546	7328
644	5	1	0.615	0.580187	0.6275	0.10635	8156
645	6	1	0.554167	0.538521	0.664167	0.268025	7965
646	0	2	0.415833	0.419813	0.708333	0.141162	3510
647	1	2	0.383333	0.387608	0.709583	0.189679	5478
648	2	2	0.446667	0.438112	0.761667	0.1903	6392
649	3	1	0.514167	0.503142	0.630833	0.187821	7691
650	4	1	0.435	0.431167	0.463333	0.181596	7570
651	5	1	0.4375	0.433071	0.539167	0.235092	7282
652	6	1	0.393333	0.391396	0.494583	0.146142	7109
653	0	1	0.521667	0.508204	0.640417	0.278612	6639
654	1	2	0.561667	0.53915	0.7075	0.296037	5875

655	2	1	0.468333	0.460846	0.558333	0.182221	7534
656	3	1	0.455833	0.450108	0.692917	0.101371	7461
657	4	2	0.5225	0.512625	0.728333	0.236937	7509
658	5	2	0.563333	0.537896	0.815	0.134954	5424
659	6	1	0.484167	0.472842	0.572917	0.117537	8090
660	0	1	0.464167	0.456429	0.51	0.166054	6824
661	1	1	0.4875	0.482942	0.568333	0.081483	7058
662	2	1	0.544167	0.530304	0.641667	0.094546	7466
663	3	1	0.5875	0.558721	0.63625	0.072779	7693
664	4	2	0.55	0.529688	0.800417	0.124375	7359
665	5	2	0.545833	0.52275	0.807083	0.132467	7444
666	6	2	0.53	0.515133	0.72	0.235692	7852
667	0	2	0.4775	0.467771	0.694583	0.398008	4459
668	1	3	0.44	0.4394	0.88	0.3582	22
669	2	2	0.318182	0.309909	0.825455	0.213009	1096
670	3	2	0.3575	0.3611	0.666667	0.166667	5566
671	4	2	0.365833	0.369942	0.581667	0.157346	5986
672	5	1	0.355	0.356042	0.522083	0.266175	5847
673	6	2	0.343333	0.323846	0.49125	0.270529	5138
674	0	1	0.325833	0.329538	0.532917	0.179108	5107
675	1	1	0.319167	0.308075	0.494167	0.236325	5259
676	2	1	0.280833	0.281567	0.567083	0.173513	5686
677	3	2	0.295833	0.274621	0.5475	0.304108	5035
678	4	1	0.352174	0.341891	0.333478	0.347835	5315
679	5	1	0.361667	0.355413	0.540833	0.214558	5992
680	6	1	0.389167	0.393937	0.645417	0.057846	6536
681	0	1	0.420833	0.421713	0.659167	0.1275	6852
682	1	1	0.485	0.475383	0.741667	0.173517	6269
683	2	2	0.343333	0.323225	0.662917	0.342046	4094
684	3	1	0.289167	0.281563	0.552083	0.199625	5495
685	4	2	0.321667	0.324492	0.620417	0.152987	5445
686	5	1	0.345	0.347204	0.524583	0.171025	5698
687	6	1	0.325	0.326383	0.545417	0.179729	5629
688	0	1	0.3425	0.337746	0.692917	0.227612	4669
689	1	2	0.380833	0.375621	0.623333	0.235067	5499
690	2	2	0.374167	0.380667	0.685	0.082725	5634
691	3	1	0.353333	0.364892	0.61375	0.103246	5146
692	4	1	0.34	0.350371	0.580417	0.052871	2425
693	5	1	0.368333	0.378779	0.56875	0.148021	3910
694	6	1	0.278333	0.248742	0.404583	0.376871	2277
695	0	1	0.245833	0.257583	0.468333	0.1505	2424
696	1	1	0.313333	0.339004	0.535417	0.04665	5087
697	2	2	0.291667	0.281558	0.786667	0.237562	3959
698	3	1	0.296667	0.289762	0.50625	0.210821	5260
699	4	1	0.28087	0.298422	0.555652	0.115522	5323
700	5	1	0.298333	0.323867	0.649583	0.058471	5668
701	6	2	0.298333	0.316904	0.806667	0.059704	5191
702	0	2	0.3475	0.359208	0.823333	0.124379	4649
703	1	1	0.4525	0.455796	0.7675	0.082721	6234
704	2	1	0.475833	0.469054	0.73375	0.174129	6606
705	3	1	0.438333	0.428012	0.485	0.324021	5729
706	4	1	0.255833	0.258204	0.50875	0.174754	5375
707	5	2	0.320833	0.321958	0.764167	0.1306	5008
708	6	2	0.381667	0.389508	0.91125	0.101379	5582
709	0	2	0.384167	0.390146	0.905417	0.157975	3228
710	1	2	0.435833	0.435575	0.925	0.190308	5170
711	2	2	0.353333	0.338363	0.596667	0.296037	5501
712	3	2	0.2975	0.297338	0.538333	0.162937	5319
713	4	1	0.295833	0.294188	0.485833	0.174129	5532
714	5	1	0.281667	0.294192	0.642917	0.131229	5611
715	6	1	0.324167	0.338383	0.650417	0.10635	5047
716	0	2	0.3625	0.369938	0.83875	0.100742	3786
717	1	2	0.393333	0.4015	0.907083	0.098258	4585
718	2	1	0.410833	0.409708	0.66625	0.221404	5557
719	3	1	0.3325	0.342162	0.625417	0.184092	5267
720	4	2	0.33	0.335217	0.667917	0.132463	4128
721	5	2	0.326667	0.301767	0.556667	0.374383	3623
722	6	1	0.265833	0.236113	0.44125	0.407346	1749
723	0	1	0.245833	0.259471	0.515417	0.133083	1787
724	1	2	0.231304	0.2589	0.791304	0.07723	920
725	2	2	0.291304	0.294465	0.734783	0.168726	1013
726	3	3	0.243333	0.220333	0.823333	0.316546	441
727	4	2	0.254167	0.226642	0.652917	0.350133	2114
728	5	2	0.253333	0.255046	0.59	0.155471	3095

729	6	2	0.253333	0.2424	0.752917	0.124383	1341
730	0	1	0.255833	0.2317	0.483333	0.350754	1796
731	1	2	0.215833	0.223487	0.5775	0.154846	2729

B Selected R Codes

Reading Data from CSV

```
d1=read.table("/Users/chenxingru/Desktop/Bike-Sharing-Dataset/day.csv",sep=",",header=TRUE)
attach(d1)
bikecnt=d1[,c("workingday","weathersit","temp","atemp","hum","windspeed","cnt")]
attach(bikecnt)
```

Obtaining All Regressions

```
library(leaps)
modelT=lm(cnt~weekday+weathersit+temp+atemp+hum+windspeed)
anova(modelT)
summary(modelT)
x=model.matrix(modelT)[,-1]
allrsq=leaps(x,cnt,method="r2")
names(allrsq)
allrsq$which
allrsq$r2
##select the max r^2
for (i in 2:7){
  maxr2=max(allrsq$r2[allrsq$size==i])
  whichmodel=allrsq$which[allrsq$r2==maxr2,]
  namemodel=names(whichmodel)[whichmodel==T]
  cat(namemodel,"\n",maxr2,"\n")
}
```

Stepwise Regression, Cp

```
allCp=leaps(x,cnt,method="Cp")
for (i in 2:7){
  minCp=min(allCp$Cp[allCp$size==i])
  whichmodel=allCp$which[allCp$Cp==minCp,]
  namemodel=names(whichmodel)[whichmodel==T]
  cat(namemodel,"\n",minCp,"\n")
}
```

Stepwise Regression, AIC

```
nullmodel<-lm(cnt~1)
fullmodel<-lm(cnt~.)
step(nullmodel, scope=list(upper=fullmodel, lower=nullmodel), direction="both", k=2, test='F')
```

Box Cox Transformation

```
###check the condition
max(bikecnt[,"cnt"])
min(bikecnt[,"cnt"])
##8714/22 > 10, so we can use box cox method

library(MASS)
modelNew=lm(cnt~weekday+weathersit+atemp+hum+windspeed)
modelbc=boxcox(modelNew,lambda = seq(-1,5,0.01))
modelbc$x[modelbc$y==max(modelbc$y)]
## ans : 0.75
```

Variance Stabilizing Transformation

```
plot(modelNew)
summary(modelNew)
modeltrans=lm((cnt**.75)~.)
plot(modeltrans)
#Test significance of transformed model
anova(modeltrans)
summary(modeltrans)
```

Check and Compare Transformation Models

```
modelTrans=lm(cnt^0.75~weekday+weathersit+atemp+hum+windspeed)
summary(modeltTrans)
modelTrans2=lm(cnt^2~weekday+weathersit+atemp+hum+windspeed)
summary(modeltTrans)
summary(modelNew)
```

Residual Plots

```
###residual plots modelTrans
modelTrans=lm(cnt^0.75~weekday+weathersit+atemp+hum+windspeed)
res2=modelTrans$residuals
fv2=modelTrans$fitted.values
plot(fv2,res2,xlab="Fitted Values",ylab = "Residuals",main = "Plot of Residuals against the Fitted values")
abline(h=0,lty=2)
```

```
### Result is not very good, we look at the residual plot against every predictors

####residual plots on predictors atemp
plot(atemp,res2,xlab="atemp",ylab="residuals",main = "Plot of Residuals against atemp")
abline(h=0,lty=2)
###There is a quadratic trend

####residual plots on predictors hum
plot(hum,res2,xlab="hum",ylab="residuals",main = "Plot of Residuals against hum")
abline(h=0,lty=2)

### residual plots on predictor windspeed
plot(windspeed,res2,xlab="windspeed",ylab="residuals",main = "Plot of Residuals against windspeed")
abline(h=0,lty=2)
```

Model Transformation: cnt to cnt^0.75 and atemp to atemp^2

```
atemp2 = atemp^2
modelNTrans=lm(cnt^0.75~weekday+weathersit+atemp2+hum+windspeed) ##final model
res=modelNTrans$residuals
fv=modelNTrans$fitted.values
plot(fv,res,xlab="Fitted Values",ylab = "Residuals",main = "Plot of Residuals against the Fitted
values")
abline(h=0,lty=2)
summary(modelNTrans)
```

Test for Independence (Durbin Watson Test)

```
library(car)
durbinWatsonTest(modelNTrans)
```

Normality Test (KS Test)

```
ks.test(res,"pnorm",mean(res),sd(res)) ### final model
cnt^0.75~weekday+weathersit+atemp^2+hum+windspeed
ks.test(res1,"pnorm",mean(res1),sd(res1))  ### original model
cnt~weekday+weathersit+temp+atemp+hum+windspeed
```

Normality Test (QQ Plot)

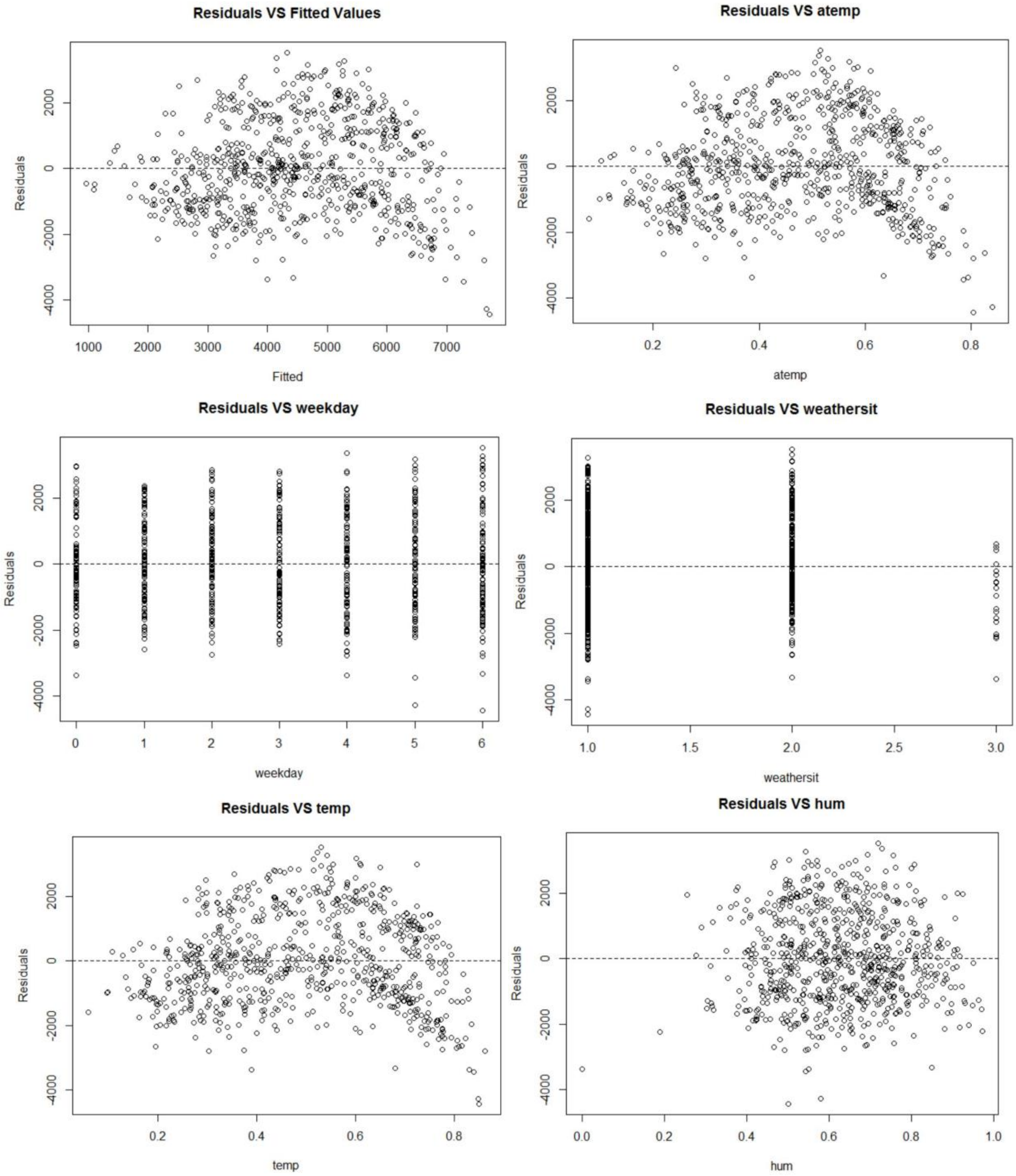
```
qqnorm(res,xlab="normal",ylab="ordered residuals")
qqline(res,lty=2)
```

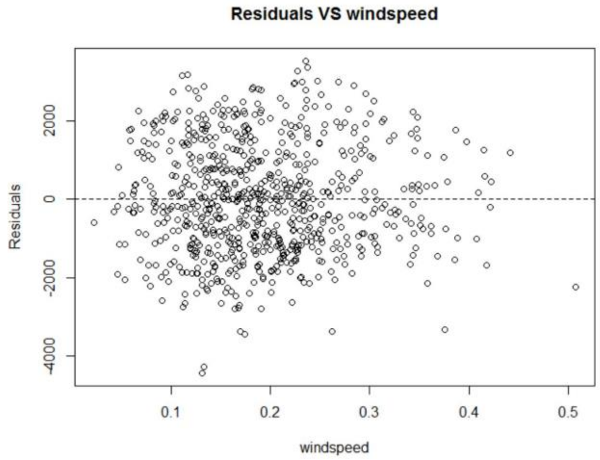
Test for Outliers

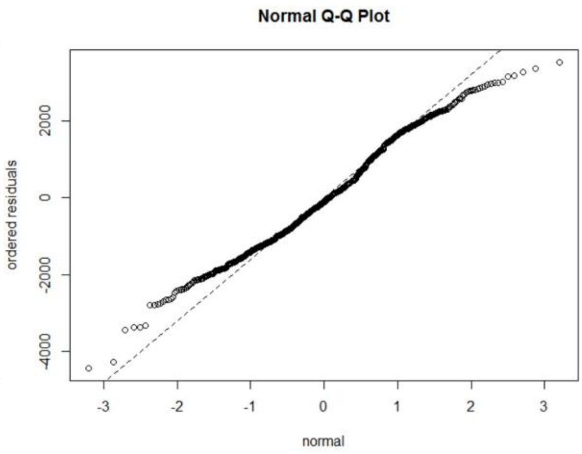
```
library(car)
influence.measures(modelNTrans)
rstudent(modelNTrans)
```

C Residual Plots for Four Primary Models

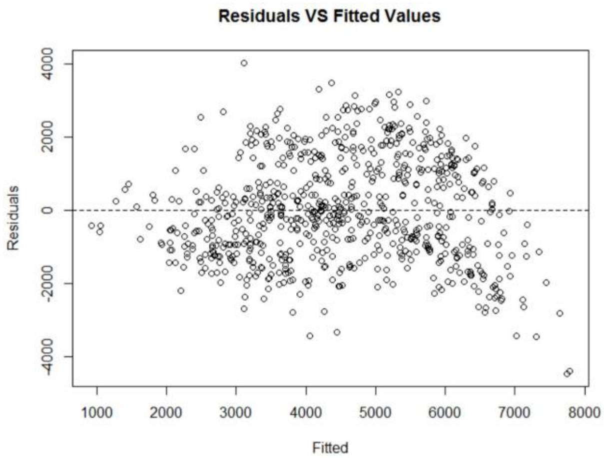
Model 1: $cnt = \beta_0 + \beta_1 weekday + \beta_2 weathersit + \beta_3 temp + \beta_4 atemp + \beta_5 hum + \beta_6 windspeed + \varepsilon$

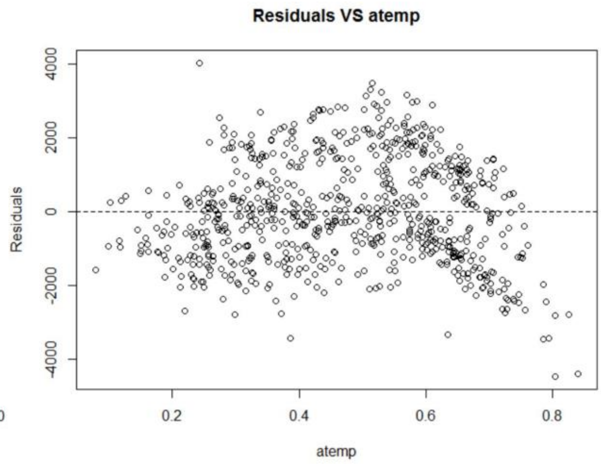


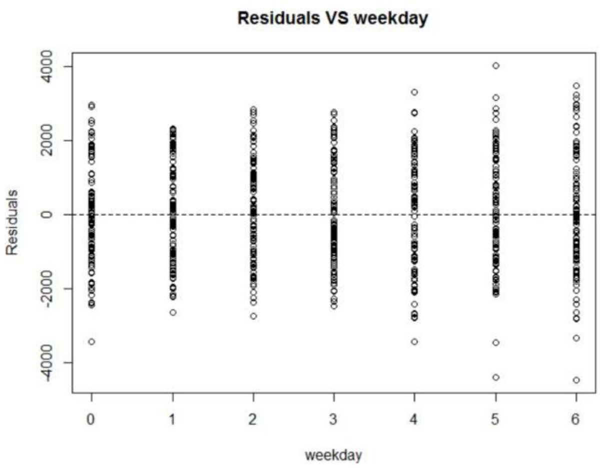


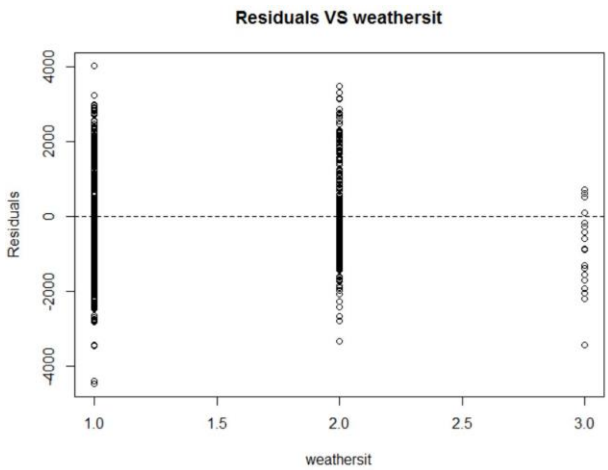


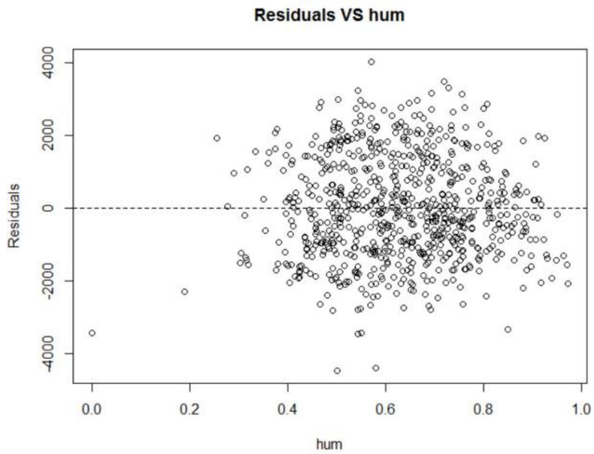
Model 2: $cnt = \beta_0 + \beta_1 weekday + \beta_2 weathersit + \beta_4 atemp + \beta_5 hum + \beta_6 windspeed + \varepsilon$

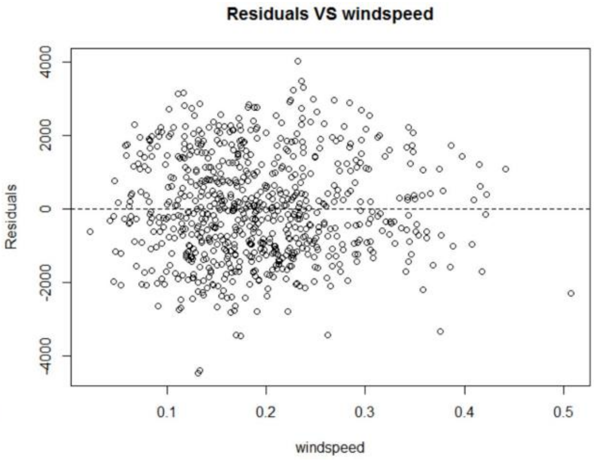


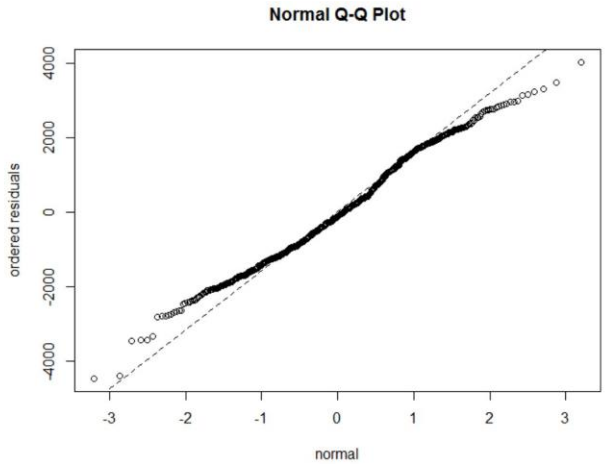




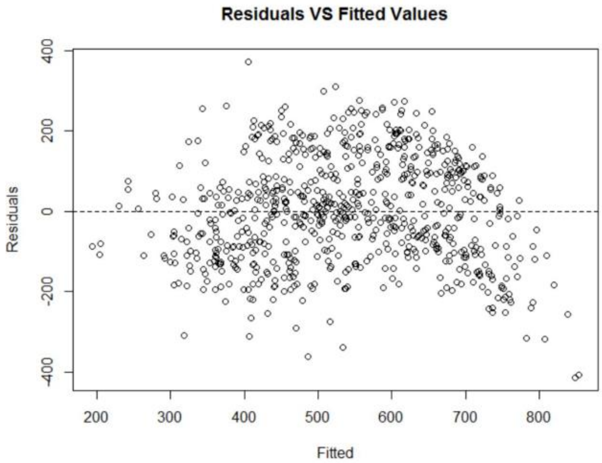


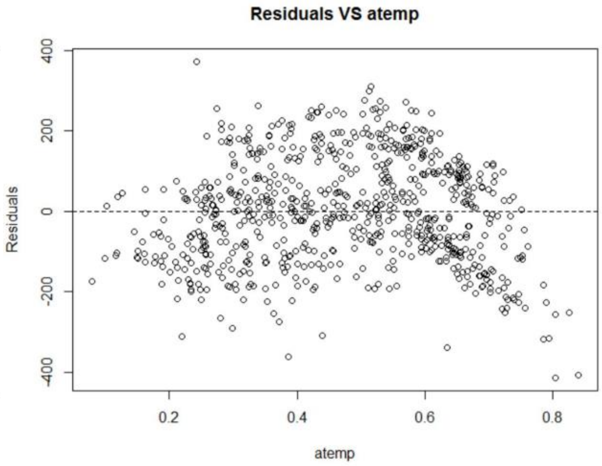


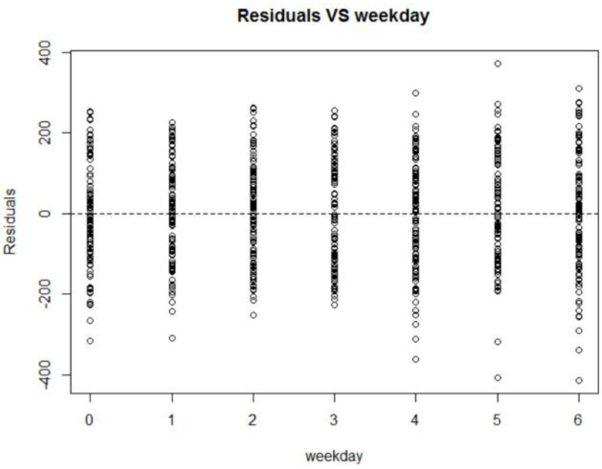


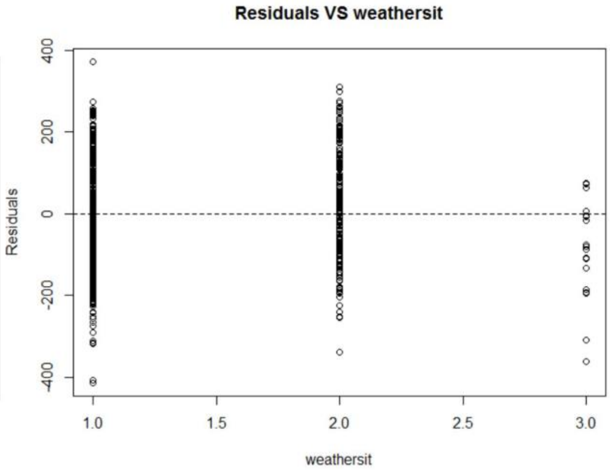


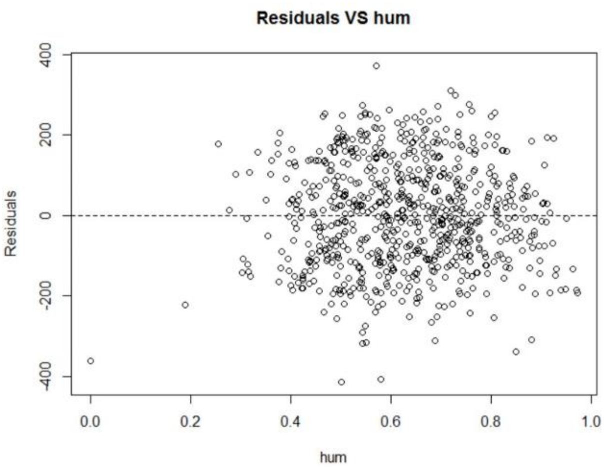
Model 3: $cnt^{0.75} = \beta_0 + \beta_1 weekday + \beta_2 weathersit + \beta_4 atemp + \beta_5 hum + \beta_6 windspeed + \varepsilon$

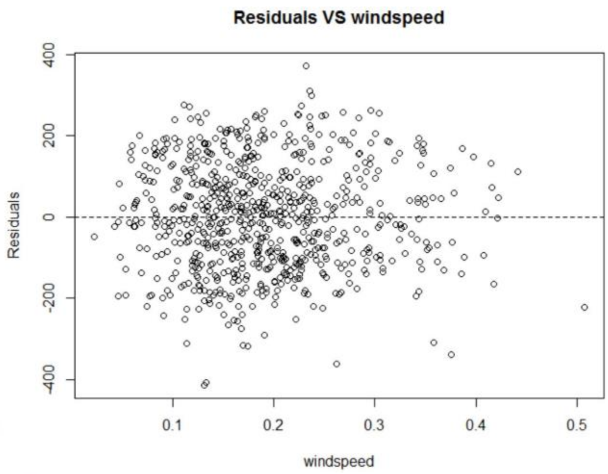

Residuals VS Fitted Values
Residuals
Fitted
-400 -200 0 200 400
200 300 400 500 600 700 800

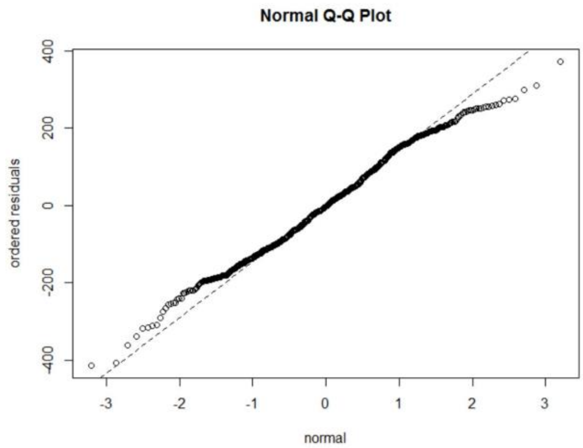

Residuals VS atemp
Residuals
atemp
-400 -200 0 200 400
0.2 0.4 0.6 0.8


Residuals VS weekday
Residuals
weekday
-400 -200 0 200 400
0 1 2 3 4 5 6

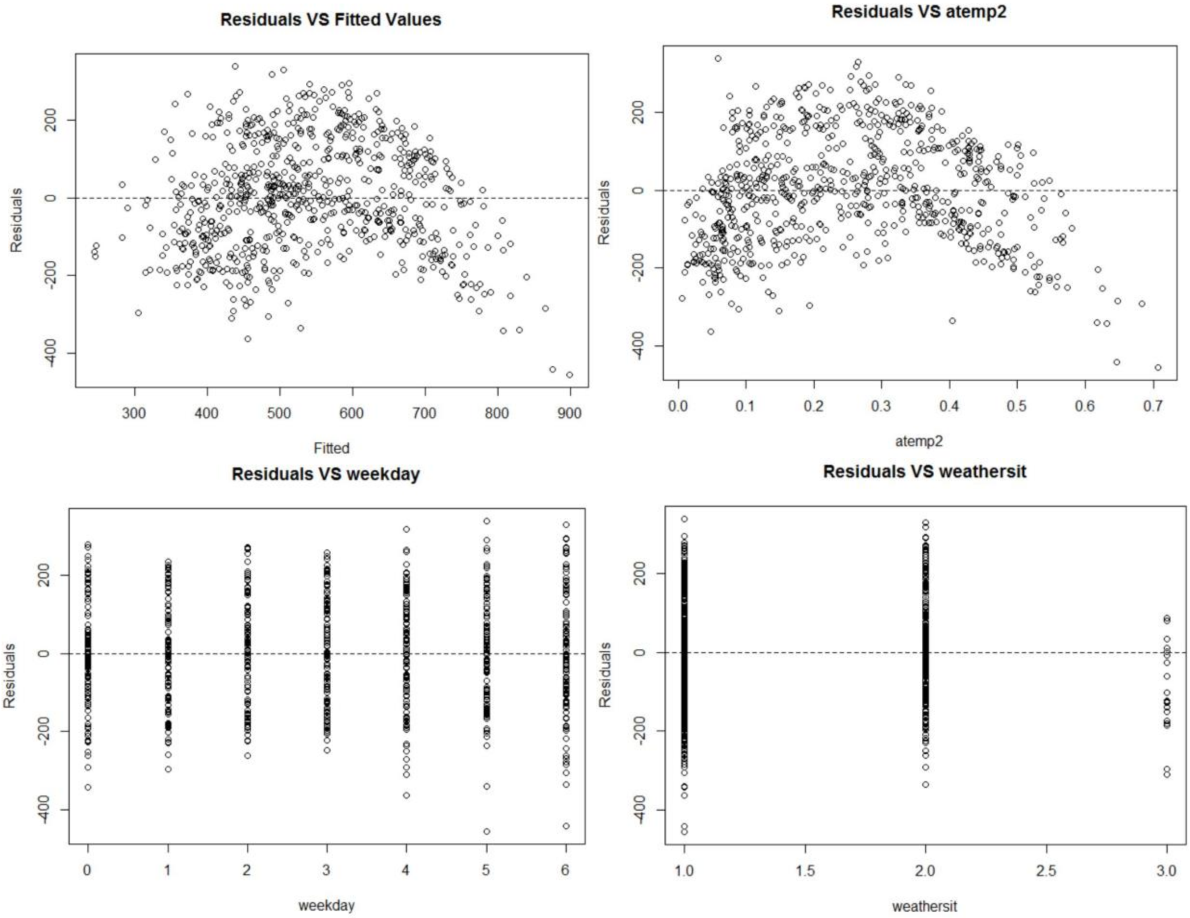

Residuals VS weathersit
Residuals
weathersit
-400 -200 0 200 400
1.0 1.5 2.0 2.5 3.0

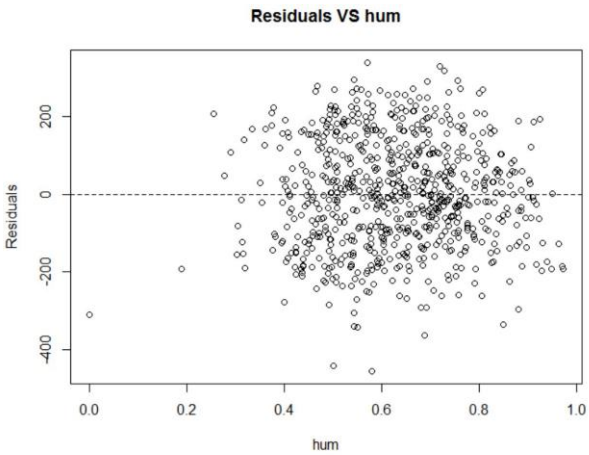

Residuals VS hum
Residuals
hum
400
200
0
-200
-400
0.0
0.2
0.4
0.6
0.8
1.0

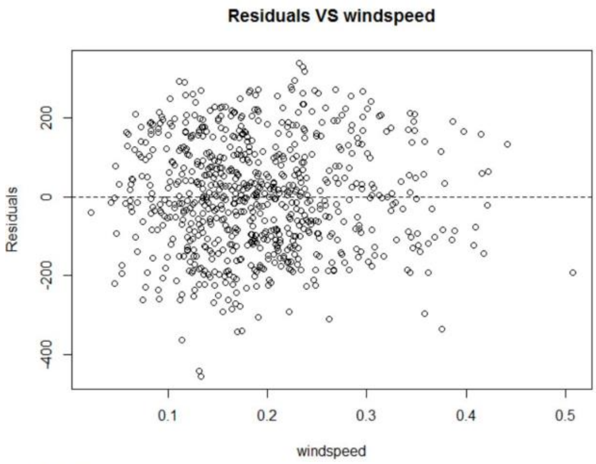

Residuals VS windspeed
Residuals
windspeed
400
200
0
-200
-400
0.1
0.2
0.3
0.4
0.5

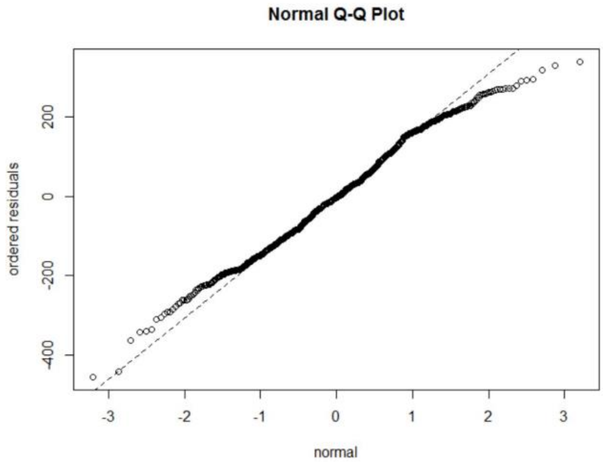

Normal Q-Q Plot
ordered residuals
normal
400
200
0
-200
-400
-3
-2
-1
0
1
2
3

Model 4: $cnt^{0.75} = \beta_0 + \beta_1 weekday + \beta_2 weathersit + \beta_4 atemp^2 + \beta_5 hum + \beta_6 windspeed + \varepsilon$




Residuals VS hum
Residuals
hum
200
0
-200
-400
0.0
0.2
0.4
0.6
0.8
1.0


Residuals VS windspeed
Residuals
windspeed
200
0
-200
-400
0.1
0.2
0.3
0.4
0.5


Normal Q-Q Plot
ordered residuals
normal
200
0
-200
-400
-3
-2
-1
0
1
2
3