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Homogenized Moduli Evaluation of Periodic Materials with Finite-Volume Micromechanics and Abaqus

SIMULIA Regional User Meetings

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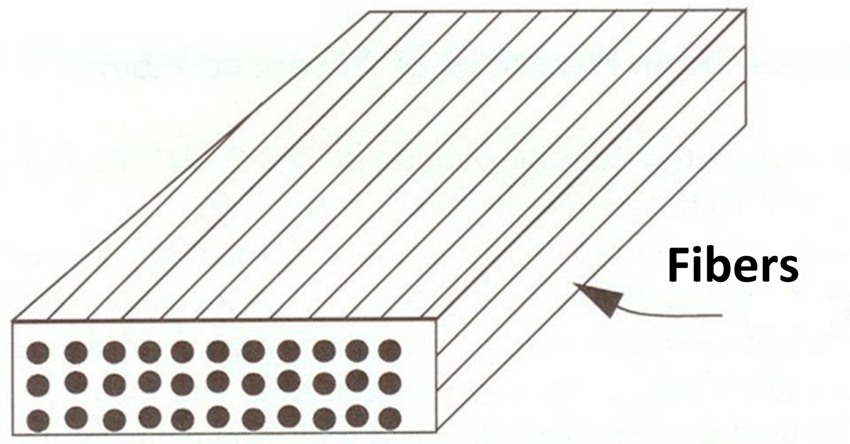
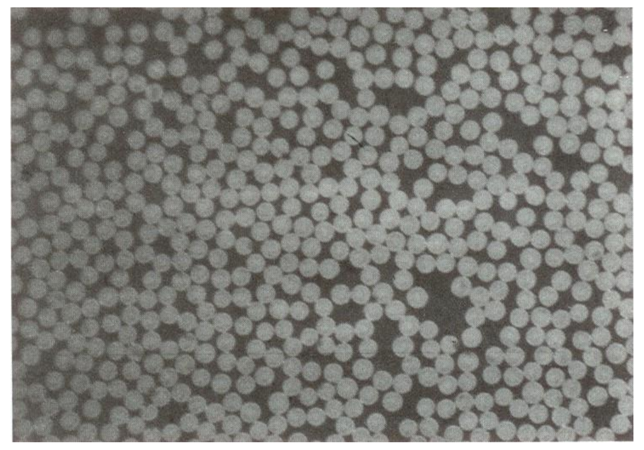
Introduction:

Q1: Properties of periodic materials without testing?

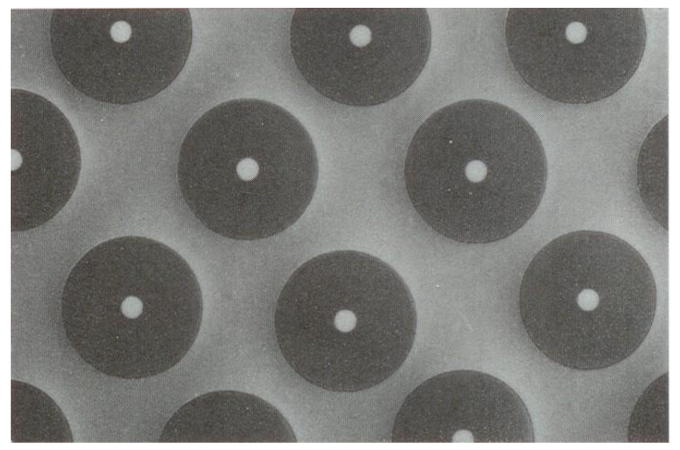
Q2: Local stress fields at constituents' level?

A: Micromechanics





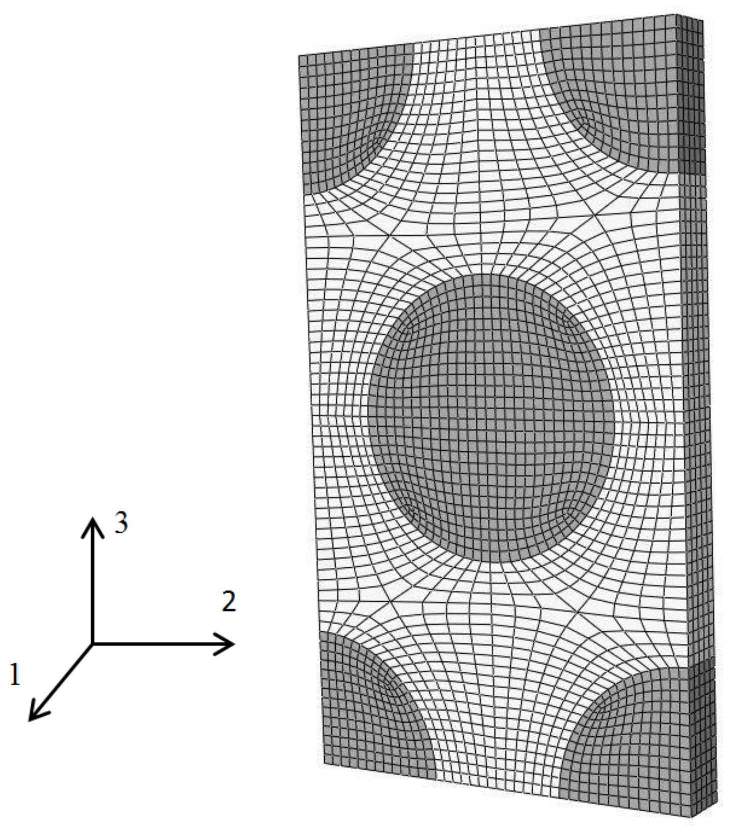
Graphite/Epoxy
Statistically homogeneous



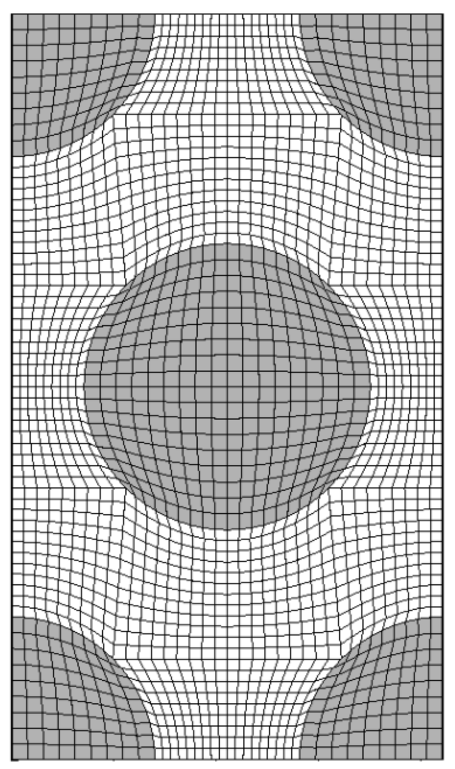
SiC/Titanium
Periodic (hexagonal)

Unidirectional composites are typically transversely isotropic → five elastic moduli: $E_A, E_T, G_A, G_T, \nu_A$

Finite Volume Direct Averaging Micromechanics (FVDAM) theory:



Abaqus (3D)



FVDAM(2D)

Basic building block of hexagonal unit cell with 40% volume fraction (Discretization is doubled for analysis)

- Displacement field : $u_i^{(q)}(\mathbf{x}, \mathbf{y}(\eta,\xi)) = \bar{\varepsilon}_{ij} x_j + u_i'^{(q)}(\eta,\xi)$
- Fluctuating displacement: $u_i'^{(q)} = W_{i(00)}^{(q)} + \eta W_{i(10)}^{(q)} + \xi W_{i(01)}^{(q)} + \frac{1}{2}(3\eta^2 - 1) W_{i(20)}^{(q)} + \frac{1}{2}(3\xi^2 - 1) W_{i(02)}^{(q)}$
- Global system of equations: $\mathbf{K}^{global}\hat{\mathbf{U}}' = \Delta\mathbf{C}\,\bar{\boldsymbol{\varepsilon}} + \boldsymbol{\Gamma} + \mathbf{G}$
- Homogenized Hooke's Law:

$$\bar{\sigma} = \frac{1}{\mathbf{V}}\int \boldsymbol{\sigma}(x)\,dV = \sum_{q=1}^{N_q} v^{(q)} \bar{\boldsymbol{\sigma}}^{(q)} = \mathbf{C}^* \bar{\boldsymbol{\varepsilon}} - (\bar{\sigma}^{th} + \bar{\sigma}^{pl})$$

Procedure to Compute Homogenized Properties in Abaqus:

$$\begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \\ \bar{\sigma}_4 \\ \bar{\sigma}_5 \\ \bar{\sigma}_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_1 \\ \bar{\varepsilon}_2 \\ \bar{\varepsilon}_3 \\ \bar{\gamma}_4 \\ \bar{\gamma}_5 \\ \bar{\gamma}_6 \end{Bmatrix}$$

Homogenized relation

Create 3D model → Automate the imposition of periodic boundary conditions with Python → Apply unit strain six times and analyze corresponding problem six times → Obtain effective stiffness matrix by averaging stress fields

Flow chart of computing homogenized properties in Abaqus

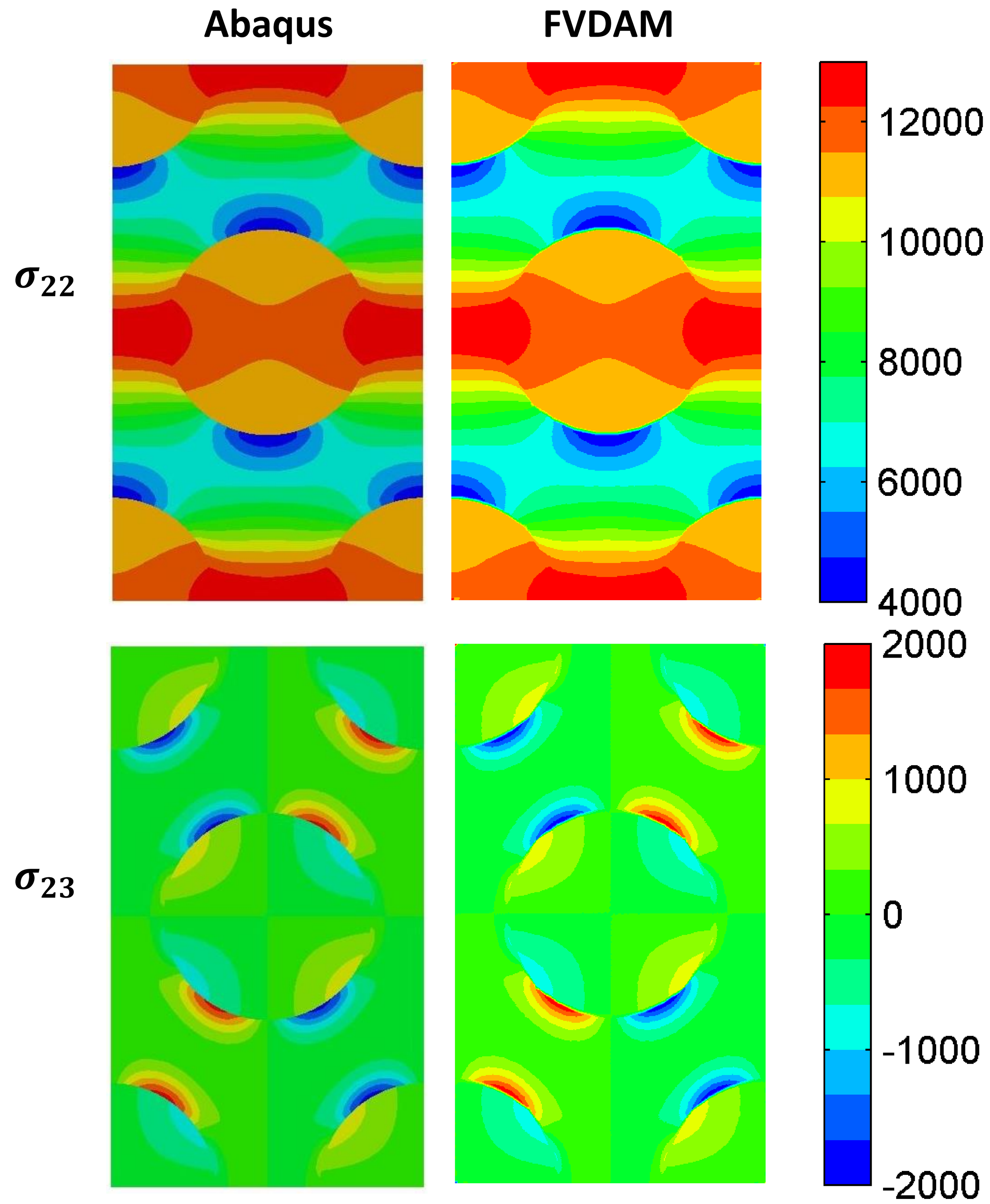
Results: Homogenized Properties

	E (GPa)	G (GPa)	ν
Glass fiber	80	33.3	0.2
Epoxy	3.35	1.24	0.35

	E_A(GPa)	E_T(GPa)	G_T(GPa)	G_A(GPa)	ν_A	Subvolumes /Elements	Computational time (s)
FVDAM	34.02	7.31	2.51	2.70	0.28	9,028	17.7
Abaqus	34.02	7.31	2.51	2.70	0.28	10,3488	606.6

FVDAM: Windows 7 64bit OS with Intel(R) Core(TM) i7-2760QM CPU @2.40GHz, 8GM RAM
Abaqus: Windows 7 64bit OS with Intel(R) Core(TM) i7-2820QM CPU@2.3GHz, 16GB RAM
Note: FVDAM uses generalized plane strain analysis to generate the entire set of properties while a 3D mesh is used in Abaqus - hence the difference in execution times.

Selected stress fields with imposition of only unit $\bar{\varepsilon}_{22}$:



Conclusions:

1) FVDAM's accuracy and efficiency in computing homogenized properties and local stress fields are verified with Abaqus.
2) Python scripting capability in Abaqus makes it possible to automate the imposition of periodic boundary conditions via coupling equations.

References:

1) H. Khatam, M.J. Pindera, Parametric Finite-Volume Micromechanics of Periodic Materials with Elastoplastic Phases, Int J Plasticity, 2009,25:1386–1411
2) Y. Yang et al., Z. Pan, M.J. Pindera, Capturing the Multiscale Effects in the Response of Coated Woven Fabrics (submitted).

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