

Exercise #7 – blackboard scribble

Example 1:

The utilization of the task set is:

$$U = \sum_{i=1}^6 \frac{C_i}{T_i} = 0.2 + 0.4 + 0.4 + 0.1 + 0.4 + 0.07 = 1.97$$

The utilization bound for RMFF is {oh & Baker}

$$U_{RMFF} = m \left(2^{1/2} - 1 \right) = \{m=3\} = 3 \left(2^{1/2} - 1 \right) \approx 1.24$$

↑
of processors

Since $U > U_{RMFF}$ the test fails!

However, since the test is only sufficient failure does not imply non-schedulability.

We will use the RMFF algorithm to show schedulability.

Task	C _i	T _i	U _i
T ₁	2	10	0.2
T ₂	10	25	0.4
T ₃	12	30	0.4
T ₄	5	10	0.5
T ₅	8	20	0.4
T ₆	7	100	0.07

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Rate-Monotonic First-Fit (RMFF) {Dhall & Liu}

- Number the processors M_1, M_2, \dots, M_m
- Assign tasks in order of increasing periods
- For each task T_i , choose the lowest-indexed (previously-used) processor M_j such that T_i , together with all tasks that have already been assigned to M_j , can be feasibly scheduled according to Lin & Layland's test for RM

Task	C_i	T_i	U_i
T_1	2	10	0,2
T_2	10	25	0,4
T_3	12	30	0,4
T_4	5	10	0,5
T_5	8	20	0,9
T_6	7	100	0,07

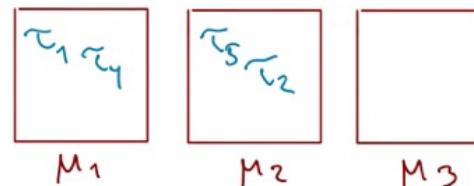
The three processors are numbered M_1, M_2 and M_3

The order of allocation (RM priority) is

$T_1, T_4, T_5, T_2, T_3, T_6$

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~~$\tau_1, \tau_4, \tau_5, \tau_2, \tau_3, \tau_6$~~



Task τ_1 can be allocated to M_1 since it's empty.

Task τ_4 can be allocated to M_1 since

$$U_1 + U_4 = 0,2 + 0,5 = 0,7 \leq U_{RM(1)} = 2(2^{1/2} - 1) \approx 0,82$$

Task τ_5 cannot be allocated to M_1 since

$$U_1 + U_4 + U_5 = 0,2 + 0,5 + 0,4 = 1,1 > 1$$

Task τ_5 can be allocated to M_2 since it is empty

Task τ_2 cannot be allocated to M_1 since

$$U_1 + U_4 + U_2 = 0,2 + 0,5 + 0,4 = 1,1 > 1$$

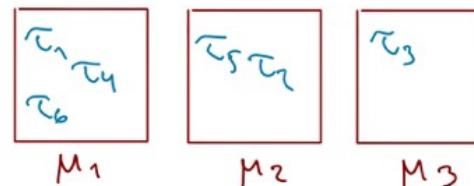
Task τ_2 can be allocated to M_2 since

$$U_5 + U_2 = 0,4 + 0,4 = 0,8 \leq U_{RM(2)} \approx 0,82$$

Task	C _i	T _i	U _i
τ_1	2	10	0,2
τ_2	10	25	0,4
τ_3	12	30	0,4
τ_4	5	10	0,5
τ_5	8	20	0,4
τ_6	7	100	0,07

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~~$\tau_1, \tau_4, \tau_5, \tau_2, \tau_3, \tau_6$~~



Task	C _i	T _i	U _i
τ_1	2	10	0,2
τ_2	10	25	0,4
τ_3	12	30	0,4
τ_4	5	10	0,5
τ_5	8	20	0,4
τ_6	7	100	0,07

Task τ_3 cannot be allocated to μ_1 since

$$U_1 + U_4 + U_3 = 0,2 + 0,5 + 0,4 = 1,1 > 1$$

Task τ_3 cannot be allocated to μ_2 since

$$U_5 + U_2 + U_3 = 0,4 + 0,9 + 0,4 = 1,2 > 1$$

Task τ_3 can be allocated to μ_3 since it is empty

Task τ_6 can be allocated to μ_1 since

$$U_1 + U_4 + U_6 = 0,2 + 0,5 + 0,07 = 0,77 \leq U_{RM}(3) = 3(2^{1/3} - 1) \approx 0,78$$

All tasks could successfully be assigned with RMFF, so the tasks are schedulable on 3 processors.

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Example 2:

The utilization of the task set is

$$U = \sum_{i=1}^m \frac{C_i}{T_i} = 0,14 + 0,21 + 0,45 + 0,5 = 1,30$$

The utilization bound for RM-VS is

$$U_{RM-VS} = \frac{m^2}{(3m-2)} = \{m=3\} = 9/7 \approx 1,29$$

Since $U > U_{RM-VS}$ the test fails!

However, since the test is only sufficient failure does not imply non-schedulability. Instead, we will show schedulability using response-time analysis for global scheduling.

$$R_i = C_i + \frac{1}{m} \sum_{T_j \in \text{exp}(i)} \left(\left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j + C_j \right)$$

Task	C _i	T _i	U _i
T ₁	1	7	0,14
T ₂	4	19	0,21
T ₃	9	20	0,45
T ₄	11	22	0,5

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$m = 3$ processors

Calculate utilization-round-separation (URS) bound:

$$m/(3m-2) = 3/(3 \cdot 3 - 2) = 3/7 \approx 0.43$$

Derive task priorities:

- Based on the RM-URS bound (0.43) tasks $\tilde{\tau}_3$ and $\tilde{\tau}_4$ are considered "heavy" tasks, and are assigned highest priority.
- Task $\tilde{\tau}_1$ has highest RM priority
- Task $\tilde{\tau}_2$ has lowest RM priority

Since we have 3 processors, tasks $\tilde{\tau}_3$, $\tilde{\tau}_4$ and $\tilde{\tau}_1$ are trivially schedulable ($C_i < T_i$) on one processor each

However, for task $\tilde{\tau}_2$ we need to calculate the response time!

Task	C_i	T_i	U_i
M	$\tilde{\tau}_1$	7	0.14
L	$\tilde{\tau}_2$	19	0.21
H	$\tilde{\tau}_3$	20	0.45
H	$\tilde{\tau}_4$	22	0.5

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$$R_i^{n+1} = C_i + \frac{1}{m} \sum_{\forall j \in h_p(i)} \left(\left[\frac{R_j^n}{T_j} \right] C_j + c_j \right)$$

$$R_2^{\circ} = C_2 = 4$$

Task	C_i	T_i	U_i
T_1	1	7	0,14
T_2	4	19	0,21
T_3	9	20	0,45
T_4	11	22	0,5

$$R_2^1 = C_2 + \frac{1}{m} \left(\left(\left[\frac{R_2^{\circ}}{T_3} \right] C_3 + C_3 \right) + \left(\left[\frac{R_2^{\circ}}{T_4} \right] C_4 + C_4 \right) + \left(\left[\frac{R_2^{\circ}}{T_1} \right] C_1 + C_1 \right) \right) = \\ 4 + \frac{1}{3} \left(\left(\left[\frac{4}{20} \right] \cdot 9 + 9 \right) + \left(\left[\frac{4}{22} \right] \cdot 11 + 11 \right) + \left(\left[\frac{4}{7} \right] \cdot 1 + 1 \right) \right) = 4 + \frac{1}{3} (18 + 22 + 2) = 4 + \frac{42}{3} = 18$$

$$R_2^2 = 4 + \frac{1}{3} \left(\left(\left[\frac{18}{20} \right] \cdot 9 + 9 \right) + \left(\left[\frac{18}{22} \right] \cdot 11 + 11 \right) + \left(\left[\frac{18}{7} \right] \cdot 1 + 1 \right) \right) = 4 + \frac{1}{3} (18 + 22 + 4) = 4 + \frac{44}{3} = 18,67$$

$$R_2^3 = 4 + \frac{1}{3} \left(\left(\left[\frac{18,67}{20} \right] \cdot 9 + 9 \right) + \left(\left[\frac{18,67}{22} \right] \cdot 11 + 11 \right) + \left(\left[\frac{18,67}{7} \right] \cdot 1 + 1 \right) \right) = 4 + \frac{1}{3} (18 + 22 + 4) = 4 + \frac{44}{3} = 18,67$$

Since $R_2^3 \leq T_2 = 19$ task T_2 is also schedutable