Lecture #15 – blackboard scribble

RTA $\quad R_i^{k+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^k}{T_j} \right\rceil \cdot C_j \quad \leq D_i$

Consider the decision problem
That is, we terminate the iterations if a deadline is missed

Assume that $D_i \leq T_i$ (constrained-deadline tasks)
and that all $O_i = 0$ (synchronous tasks)

1) Show that RTA is a number problem

$0_i < C_i \leq D_i \leq T_i \leq \max_{\forall i} T_i$ Largest number
not restricted by n (problem size)

2) Show that RTA has tractable time complexity

Number of iterations per task $\leq D_i \leq T_i \leq \max_{\forall i} T_i$ (largest number)

Total number of iterations ($\forall$ tasks) $\leq n \cdot \max_{\forall i} T_i$

pseudo-polynomial time function of problem size n and largest number $\max_{\forall i} T_i$

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PDA

Assume that $U < 1$, $D_i \leq T_i$ (constrained-deadline tasks)

and that all $O_i = 0$ (synchronous tasks)

1) Show that PDA is a number problem

$O_i < C_i \leq D_i \leq T_i \leq \max_{\forall i} T_i$ Largest number

not restricted by n (problem size)

2) Show that PDA has tractable time complexity

a constant

largest number

Number of control points $\leq L^{max}_{BRH} = \max\left(1, \frac{U}{1-U}\right) \cdot \max_{\forall i} T_i$

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Total number of iterations ($\forall$ tasks) $\leq n \cdot \max\left(1, \frac{U}{1-U}\right) \cdot \max_{\forall i} T_i$

pseudo-polynomial time function of problem size n and largest number $\max_{\forall i} T_i$

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PDA $$L_{BRH} = \max(D_1, D_2, \dots, D_n, L^*), \text{ where } L^* = \frac{\sum_{\forall i} (T_i - D_i) \cdot U_i}{1-U}$$

Note that:

$$\frac{\sum_{\forall i} (T_i - D_i) U_i}{1-U} \leq \frac{\max_{\forall i}(T_i - D_i) \cdot \sum_{\forall i} U_i}{1-U} \leq \frac{\max_{\forall i}(T_i) \cdot \sum_{\forall i} U_i}{1-U} = \frac{U \cdot \max_{\forall i}(T_i)}{1-U}$$

Hence:

$$\max(D_1, D_2, \dots, D_n, L^*) \leq \max\left(\max_{\forall i}(D_i), \frac{U \cdot \max_{\forall i}(T_i)}{1-U}\right) \leq \max\left(\max_{\forall i}(T_i), \frac{U \cdot \max_{\forall i}(T_i)}{1-U}\right)$$

We can now define L_{BRH}^{max}, a useful upper bound of L_{BRH}:

$$\boxed{L_{BRH}^{max}} = \max\left(\max_{\forall i}(T_i), \frac{U \cdot \max_{\forall i}(T_i)}{1-U}\right) = \max\left(1, \frac{U}{1-U}\right) \cdot \max_{\forall i} T_i$$

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