Exercise #7 – blackboard scribble

Example 1:

The utilization of the task set is:

$$U = \sum_{\forall i} \frac{C_i}{T_i} = 0{,}2 + 0{,}4 + 0{,}4 + 0{,}5 + 0{,}4 + 0{,}07 = 1{,}97$$

The utilization bound for RMFF is {Oh & Baker}

$$U_{RMFF} = \underset{\substack{\uparrow \\ \text{\# of processors}}}{m}\left(2^{1/2} - 1\right) = \{m = 3\} = 3\left(2^{1/2} - 1\right) \approx 1{,}24$$

Since $U > U_{RMFF}$ the test fails!

However, since the test is only sufficient failure does not imply non-schedulability.

We will use the RMFF algorithm to show schedulability.

Task	C_i	T_i	U_i
τ_1	2	10	0,2
τ_2	10	25	0,4
τ_3	12	30	0,4
τ_4	5	10	0,5
τ_5	8	20	0,4
τ_6	7	100	0,07

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Rate-Monotonic First-Fit (RMFF) {Dhall & Liu}

- Number the processors $M_1, M_2, \ldots M_m$
- Assign tasks in order of increasing periods
- For each task τ_i, choose the lowest-indexed (previously-used) processor M_j such that τ_i, together with all tasks that have already been assigned to M_j, can be feasibly scheduled according to Liu & Layland's test for RM

Task	C_i	T_i	U_i
τ_1	2	10	0,2
τ_2	10	25	0,4
τ_3	12	30	0,4
τ_4	5	10	0,5
τ_5	8	20	0,4
τ_6	7	100	0,07

The three processors are numbered M_1, M_2 and M_3

The order of allocation (RM priority) is

$$\tau_1, \tau_4, \tau_5, \tau_2, \tau_3, \tau_6$$

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$\cancel{\tau_1}, \cancel{\tau_4}, \cancel{\tau_5}, \cancel{\tau_2}, \tau_3, \tau_6$

M_1	M_2	M_3
τ_1 τ_4	τ_5 τ_2	

Task	C_i	T_i	U_i
τ_1	2	10	0,2
τ_2	10	25	0,4
τ_3	12	30	0,4
τ_4	5	10	0,5
τ_5	8	20	0,4
τ_6	7	100	0,07

Task τ_1 can be allocated to M_1 since it is empty.

Task τ_4 can be allocated to M_1 since

$$U_1 + U_4 = 0{,}2 + 0{,}5 = 0{,}7 \leq U_{RM(2)} = 2(2^{1/2} - 1) \approx 0{,}82$$

Task τ_5 cannot be allocated to M_1 since

$$U_1 + U_4 + U_5 = 0{,}2 + 0{,}5 + 0{,}4 = 1{,}1 > 1$$

Task τ_5 can be allocated to M_2 since it is empty

Task τ_2 cannot be allocated to M_1 since

$$U_1 + U_4 + U_2 = 0{,}2 + 0{,}5 + 0{,}4 = 1{,}1 > 1$$

Task τ_2 can be allocated to M_2 since

$$U_5 + U_2 = 0{,}4 + 0{,}4 = 0{,}8 \leq U_{RM(2)} \approx 0{,}82$$

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$\cancel{\tau_1}, \cancel{\tau_4}, \cancel{\tau_5}, \cancel{\tau_2}, \cancel{\tau_3}, \cancel{\tau_6}$

μ_1: τ_1 τ_4 τ_6

μ_2: τ_5 τ_2

μ_3: τ_3

Task	C_i	T_i	U_i
τ_1	2	10	0,2
τ_2	10	25	0,4
τ_3	12	30	0,4
τ_4	5	10	0,5
τ_5	8	20	0,4
τ_6	7	100	0,07

Task τ_3 cannot be allocated to μ_1 since

$U_1 + U_4 + U_3 = 0{,}2 + 0{,}5 + 0{,}4 = 1{,}1 > 1$

Task τ_3 cannot be allocated to μ_2 since

$U_5 + U_2 + U_3 = 0{,}4 + 0{,}4 + 0{,}4 = 1{,}2 > 1$

Task τ_3 can be allocated to μ_3 since it is empty

Task τ_6 can be allocated to μ_1 since

$U_1 + U_4 + U_6 = 0{,}2 + 0{,}5 + 0{,}07 = 0{,}77 \leq U_{RM}(3) = 3(2^{1/3} - 1) \approx 0{,}78$

All tasks could successfully be assigned with RMFF, so the tasks are schedulable on 3 processors.

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Example 2:

Task	C_i	T_i	U_i
τ_1	1	7	0,14
τ_2	4	19	0,21
τ_3	9	20	0,45
τ_4	11	22	0,5

The utilization of the task set is

$$U = \sum_{\forall i} \frac{C_i}{T_i} = 0{,}14 + 0{,}21 + 0{,}45 + 0{,}5 = 1{,}30$$

The utilization bound for RM-US is

$$U_{RM\text{-}US} = m^2/(3m-2) = \{m=3\} = 9/7 \approx 1{,}29$$

Since $U > U_{RM\text{-}US}$ the test fails!

However, since the test is only sufficient failure does not imply non-schedulability. Instead, we will show schedulability using response-time analysis for global scheduling.

$$R_i = C_i + 1/m \sum_{\forall j \in hp(i)} \left(\left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j + C_j \right)$$

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$m = 3$ processors

	Task	C_i	T_i	U_i
M	τ_1	1	7	0,14
L	τ_2	4	19	0,21
H	τ_3	9	20	0,45
H	τ_4	11	22	0,5

Calculate utilization-separation (US) bound:

$$m/(3m-2) = 3/3\cdot3-2 = 3/7 \approx 0{,}43$$

Derive task priorities:

- Based on the RM-US bound (0,43) tasks τ_3 and τ_4 are considered "heavy" tasks, and are assigned highest priority.
- Task τ_1 has highest RM priority
- Task τ_2 has lowest RM priority

Since we have 3 processors, tasks τ_3, τ_4 and τ_1 are trivially schedulable ($C_i < T_i$) on one processor each

However, for task τ_2 we need to calculate the response time!

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$$R_i^{n+1} = C_i + \frac{1}{m} \sum_{\forall j \in hp(i)} \left(\left\lceil \frac{R_i^n}{T_j} \right\rceil C_j + C_j \right)$$

Task	C_i	T_i	U_i
τ_1	1	7	0,14
τ_2	4	19	0,21
τ_3	9	20	0,45
τ_4	11	22	0,5

$$R_2^0 = C_2 = 4$$

$$R_2^1 = C_2 + \frac{1}{m}\left(\left(\left\lceil \frac{R_2^0}{T_3} \right\rceil C_3 + C_3 \right) + \left(\left\lceil \frac{R_2^0}{T_4} \right\rceil \cdot C_4 + C_4 \right) + \left(\left\lceil \frac{R_2^0}{T_1} \right\rceil \cdot C_1 + C_1 \right) \right) =$$

$$4 + \frac{1}{3}\left(\left(\overset{1}{\left\lceil \frac{4}{20} \right\rceil} \cdot 9 + 9 \right) + \left(\overset{1}{\left\lceil \frac{4}{22} \right\rceil} \cdot 11 + 11 \right) + \left(\overset{1}{\left\lceil \frac{4}{7} \right\rceil} \cdot 1 + 1 \right) \right) = 4 + \frac{1}{3}(18 + 22 + 2) = 4 + \frac{42}{3} = 18$$

$$R_2^2 = 4 + \frac{1}{3}\left(\left(\overset{1}{\left\lceil \frac{18}{20} \right\rceil} \cdot 9 + 9 \right) + \left(\overset{1}{\left\lceil \frac{18}{22} \right\rceil} \cdot 11 + 11 \right) + \left(\overset{3}{\left\lceil \frac{18}{7} \right\rceil} \cdot 1 + 1 \right) \right) = 4 + \frac{1}{3}(18 + 22 + 4) = 4 + \frac{44}{3} = 18{,}67$$

$$R_2^3 = 4 + \frac{1}{3}\left(\left(\left\lceil \frac{18{,}67}{20} \right\rceil \cdot 9 + 9 \right) + \left(\left\lceil \frac{18{,}67}{22} \right\rceil \cdot 11 + 11 \right) + \left(\left\lceil \frac{18{,}67}{7} \right\rceil \cdot 1 + 1 \right) \right) = 4 + \frac{1}{3}(18 + 22 + 4) = 4 + \frac{44}{3} = 18{,}67$$

Since $R_2^3 \leq T_2 = 19$ task τ_2 is also schedalable