Lecture #16 – blackboard scribble

1a) Processor utilization analysis (RM)

Task	C_i	D_i	T_i
τ_1	3	20	20
τ_2	10	30	30
τ_3	25	60	60

$D_i = T_i$

$$U = \frac{3}{20} + \frac{10}{30} + \frac{25}{60} = 0{,}15 + 0{,}333 + 0{,}417 = 0{,}90$$

$$U_{RM(3)} = n\left(2^{1/n} - 1\right) = \{n=3\} = 3\left(2^{1/3} - 1\right) \approx 0{,}78$$

$$U > U_{RM(3)} \Rightarrow \text{Test fails!}$$

Because the test is only sufficient the schedulability of the task set cannot be determined

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1 b) Response-time analysis (DM)

The final (converged) response times should be calculated for each task, regardless of whether the analysis fails or not!

	Task	C_i	D_i	T_i
H	τ_1	3	5	20
M	τ_2	10	25	30
L	τ_3	25	40	60

$R_1 = C_1 = 3 \leq D_1 = 5 \Rightarrow$ ok!

$R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1$ [Assume $R_2^0 = C_2 = 10$]

$R_2^1 = 10 + \left\lceil \frac{10}{20} \right\rceil \cdot 3 = 10 + 1 \cdot 3 = 13$

$R_2^2 = 10 + \left\lceil \frac{13}{20} \right\rceil \cdot 3 = 10 + 1 \cdot 3 = 13$

Convergence $\leq D_2 = 25 \Rightarrow$ ok!

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1b) (cont'd)

Task	C_i	D_i	T_i
τ_1	3	5	20
τ_2	10	25	30
τ_3	25	40	60

$$R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 \qquad [\text{Assume } R_3^0 = C_3 = 25]$$

$$R_3^1 = 25 + \left\lceil \frac{25}{20} \right\rceil \cdot 3 + \left\lceil \frac{25}{30} \right\rceil \cdot 10 = 25 + 2 \cdot 3 + 1 \cdot 10 = 41$$

$$R_3^2 = 25 + \left\lceil \frac{41}{20} \right\rceil \cdot 3 + \left\lceil \frac{41}{30} \right\rceil \cdot 10 = 25 + 3 \cdot 3 + 2 \cdot 10 = 54$$

$$R_3^3 = 25 + \left\lceil \frac{54}{20} \right\rceil \cdot 3 + \left\lceil \frac{54}{30} \right\rceil \cdot 10 = 25 + 3 \cdot 3 + 2 \cdot 10 = 54$$

Convergence

$> D_3 = 40$ FAIL!

Test for τ_3 fails as response time exceeds deadline.

Because the test is exact the task set is not schedulable.

Note that, although the analysis failed already at R_3^1, this problem asked for the final (converged) response time values.

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Task	C_i	D_i	T_i
τ_1	3	5	20
τ_2	10	25	30
τ_3	25	40	60

1c) Determine L_{max} (largest interval)

$$U = U_1 + U_2 + U_3 = 0,15 + 0,333 + 0,417 = 0,9$$

Since $U < 1$: $L_{max} = \min(L_{BRH}, L_{LCM})$

$$L_{BRH} = \max\left\{D_1, D_2, D_3, \frac{\sum_{i=1}^{3}(T_i - D_i)\cdot U_i}{1-U}\right\}$$

$(T_1 - D_1)\cdot U_1 = 15 \cdot 0,15 = 2,25$

$(T_2 - D_2)\cdot U_2 = 5 \cdot 0,333 = 1,667$

$(T_3 - D_3)\cdot U_3 = 20 \cdot 0,417 = 8,333$

$$L^* = \frac{\sum (T_i - D_i)U_i}{1-U} = \frac{2,25 + 1,667 + 8,333}{1-0,9} = \frac{12,25}{0,1} = 122,5 \leq 123$$

$$L_{BRH} = \max\{D_1, D_2, D_3, L^*\} = \max\{5, 25, 40, 123\} = 123$$

$$L_{LCM} = LCM\{T_1, T_2, T_3\} = LCM\{20, 30, 60\} = 60$$

$$L_{max} = \min(L_{BRH}, L_{LCM}) = \min(123, 60) = 60$$

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1 d) Control-point calculation for all tasks

$K_1 = \{5, 25, 45\}$ $K_2 = \{25, 55\}$ $K_3 = \{40\}$

$K = \{5, 25, 40, 45, 55\}$

Task	C_i	D_i	T_i
τ_1	3	5	20
τ_2	10	25	30
τ_3	25	40	60

1 e) Processor-demand analysis for the given task set.

Analysis should be performed for every control point, regardless of whether the analysis in another control point fails or not.

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L	$N_1^L \cdot C_1$	$N_2^L \cdot C_2$	$N_3^L \cdot C_3$	$C_p(0,L)$	$C_p(0,L) \leq L$?
5	$\left(\left\lfloor \frac{5-5}{20} \right\rfloor + 1\right) \cdot 3 = 3$	$\left(\left\lfloor \frac{5-25}{30} \right\rfloor + 1\right) \cdot 10 = 0$	$\left(\left\lfloor \frac{5-40}{60} \right\rfloor + 1\right) \cdot 25 = 0$	$3+0+0=3$	OK!
25	$\left(\left\lfloor \frac{25-5}{20} \right\rfloor + 1\right) \cdot 3 = 6$	$\left(\left\lfloor \frac{25-25}{30} \right\rfloor + 1\right) \cdot 10 = 10$	$\left(\left\lfloor \frac{25-40}{60} \right\rfloor + 1\right) \cdot 25 = 0$	$6+10+0=16$	OK!
40	$\left(\left\lfloor \frac{40-5}{20} \right\rfloor + 1\right) \cdot 3 = 6$	$\left(\left\lfloor \frac{40-25}{30} \right\rfloor + 1\right) \cdot 10 = 10$	$\left(\left\lfloor \frac{40-40}{60} \right\rfloor + 1\right) \cdot 25 = 25$	$6+10+25=41$	FAIL!
45	$\left(\left\lfloor \frac{45-5}{20} \right\rfloor + 1\right) \cdot 3 = 9$	$\left(\left\lfloor \frac{45-25}{30} \right\rfloor + 1\right) \cdot 10 = 10$	$\left(\left\lfloor \frac{45-40}{60} \right\rfloor + 1\right) \cdot 25 = 25$	$9+10+25=44$	OK!
55	$\left(\left\lfloor \frac{55-5}{20} \right\rfloor + 1\right) \cdot 3 = 9$	$\left(\left\lfloor \frac{55-25}{30} \right\rfloor + 1\right) \cdot 10 = 20$	$\left(\left\lfloor \frac{55-40}{60} \right\rfloor + 1\right) \cdot 25 = 25$	$9+20+25=54$	OK!

Remember that all control points need to be analyzed for this problem, despite the failure at t=40!

Test for L=40 fails as processor demand exceeds interval length.
Because the test is exact the task set is not schedulable.