

Lecture #12 – blackboard scribble

Calculation of response times:

$[\tau_1 \text{ has highest priority (DM)}]$

$$R_1 = C_1 = 4 \leq D_1 = 6 \Rightarrow \text{ok!}$$

	Task	C_i	D_i	T_i
H	τ_1	4	6	8
L	τ_2	3	14	16
M	τ_3	2	10	32

$[\tau_3 \text{ has medium priority (DM)}]$ Note: ceiling function

$$R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 \quad [\text{Assume } R_3^0 = C_3 = 2]$$

$$R_3^1 = 2 + \left\lceil \frac{2}{8} \right\rceil \cdot 4 = 2 + 1 \cdot 4 = 6$$

$$R_3^2 = 2 + \left\lceil \frac{6}{8} \right\rceil \cdot 4 = 2 + 1 \cdot 4 = 6$$

Convergence because

$$R_3^2 = R_3^1$$

$$\leq D_3 = 10 \Rightarrow \text{ok!}$$

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$[\tau_2 \text{ has lowest priority (DM)}]$

$$R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{R_2}{T_3} \right\rceil \cdot C_3 \quad [\text{Assume } R_2^0 = C_2 = 3]$$

$$R_2^1 = 3 + \left\lceil \frac{3}{8} \right\rceil \cdot 4 + \left\lceil \frac{3}{32} \right\rceil \cdot 2 = 3 + 1 \cdot 4 + 1 \cdot 2 = 9$$

$$R_2^2 = 3 + \left\lceil \frac{9}{8} \right\rceil \cdot 4 + \left\lceil \frac{9}{32} \right\rceil \cdot 2 = 3 + 2 \cdot 4 + 1 \cdot 2 = 13$$

$$R_2^3 = 3 + \left\lceil \frac{13}{8} \right\rceil \cdot 4 + \left\lceil \frac{13}{32} \right\rceil \cdot 2 = 3 + 2 \cdot 4 + 1 \cdot 2 = 13$$

Task	C_i	D_i	T_i
τ_1	4	6	8
τ_2	3	14	16
τ_3	2	10	32

H
L
M

Convergence

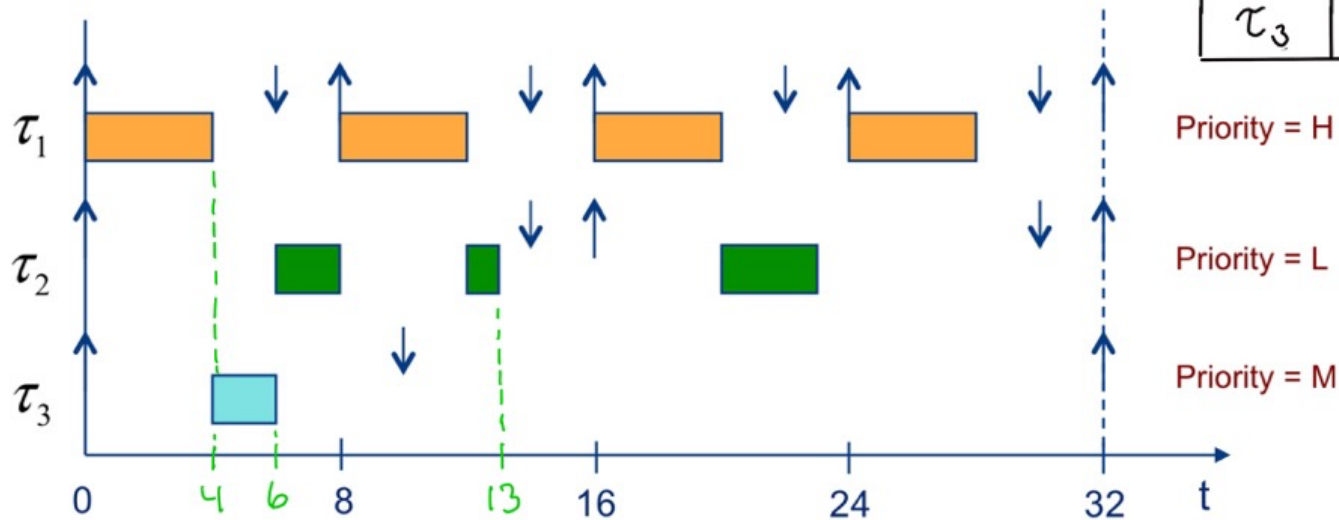
$\leq D_2 = 14 \Rightarrow \text{ok!}$

All deadlines are met!

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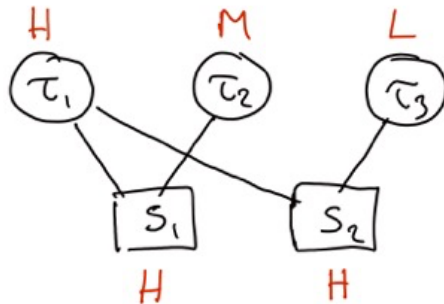
As we saw in the beginning of the lecture the resulting schedule looks like this:

Task	C_i	D_i	T_i
τ_1	4	6	8
τ_2	3	14	16
τ_3	2	10	32



Consequently, the analysis calculates worst-case response times that correspond exactly to the response times of the first instance of each task.

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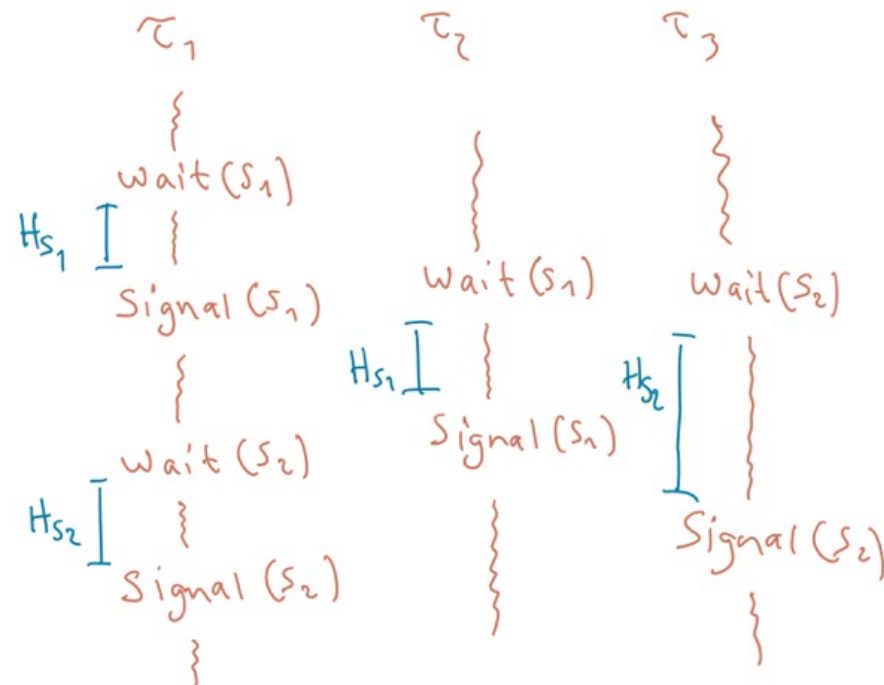


Task	C_i	D_i	T_i	H_{s1}	H_{s2}
τ_1	2	4	5	1	1
τ_2	3	12	12	1	—
τ_3	8	24	25	—	2

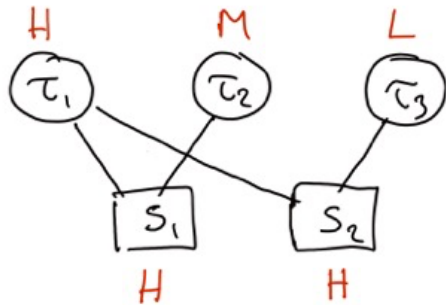
a) Ceiling priorities

$$S_1: \max\{H, M\} = H$$

$$S_2: \max\{H, L\} = H$$



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Task	C_i	D_i	T_i	H_{s1}	H_{s2}
τ_1	2	4	5	1	1
τ_2	3	12	12	1	-
τ_3	8	24	25	-	2

b) Blocking factors:

Since both semaphores have highest priority ceiling (H) task τ_1 and τ_2 may be blocked by a task with lower priority regardless of which semaphore that lower-priority task uses.

$$B_1 = \max\{1, 2\} = 2 \begin{cases} \tau_2 \text{ may use semaphore } S_1 \text{ or} \\ \tau_3 \text{ may use semaphore } S_2 \end{cases}$$

$$B_2 = 2 \begin{cases} \tau_3 \text{ may use semaphore } S_2 \\ \text{NOTE: } \tau_2 \text{ may be blocked although it does not use } S_2 \end{cases}$$

$$B_3 = 0 \quad \leftarrow \text{NOTE: lowest-priority task can never be blocked}$$

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c) Calculate response times!

[τ_1 has highest DM priority]

$$R_1 = C_1 + B_1 = 2 + 2 = 4 \leq D_1 = 4 \Rightarrow \text{ok! (but barely)}$$

Task	C_i	D_i	T_i	H_{s1}	H_{s2}
τ_1	2	4	5	1	1
τ_2	3	12	12	1	—
τ_3	8	24	25	—	2

[τ_2 has medium DM priority]

$$R_2 = C_2 + B_2 + \left\lceil \frac{R_2}{T_1} \right\rceil \cdot C_1 \quad [\text{Assume } R_2^0 = C_2 = 3]$$

$$R_2^1 = 3 + 2 + \left\lceil \frac{3}{5} \right\rceil \cdot 2 = 3 + 2 + 1 \cdot 2 = 7$$

$$R_2^2 = 3 + 2 + \left\lceil \frac{7}{5} \right\rceil \cdot 2 = 3 + 2 + 2 \cdot 2 = 9$$

$$R_2^3 = 3 + 2 + \left\lceil \frac{9}{5} \right\rceil \cdot 2 = 3 + 2 + 2 \cdot 2 = 9 \quad \left. \begin{array}{l} \text{Convergence} \\ \leq D_2 = 12 \Rightarrow \text{ok!} \end{array} \right\}$$

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$[\tau_3 \text{ has lowest priority}]$

$$R_3 = C_3 + \left\lceil \frac{R_3}{T_2} \right\rceil C_2 + \left\lceil \frac{R_3}{T_1} \right\rceil C_1 \quad \left[\text{Assume } R_3^0 = C_3 = 8 \right]$$

Task	C_i	D_i	T_i	$H_{\delta 1}$	$H_{\delta 2}$
τ_1	2	4	5	1	1
τ_2	3	12	12	1	—
τ_3	8	24	25	—	2

$$R_3^1 = 8 + \left\lceil \frac{8}{12} \right\rceil \cdot 3 + \left\lceil \frac{8}{5} \right\rceil \cdot 2 = 8 + 1 \cdot 3 + 2 \cdot 2 = 15$$

$$R_3^2 = 8 + \left\lceil \frac{15}{12} \right\rceil \cdot 3 + \left\lceil \frac{15}{5} \right\rceil \cdot 2 = 8 + 2 \cdot 3 + 3 \cdot 2 = 20$$

$$R_3^3 = 8 + \left\lceil \frac{20}{12} \right\rceil \cdot 3 + \left\lceil \frac{20}{5} \right\rceil \cdot 2 = 8 + 2 \cdot 3 + 4 \cdot 2 = 22$$

$$R_3^4 = 8 + \left\lceil \frac{22}{12} \right\rceil \cdot 3 + \left\lceil \frac{22}{5} \right\rceil \cdot 2 = 8 + 2 \cdot 3 + 5 \cdot 2 = 24$$

$$R_3^5 = 8 + \left\lceil \frac{24}{12} \right\rceil \cdot 3 + \left\lceil \frac{24}{5} \right\rceil \cdot 2 = 8 + 2 \cdot 3 + 5 \cdot 2 = 24$$

Convergence

$\leq D_3 = 24 \Rightarrow \text{ok!}$

(but barely)

All deadlines are met!