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Surface Waves Periodically Excited in a CO_2 Tube (*).

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Abstract. – Experimental results about surface waves that are periodically forced in a cylindrical tube filled with CO_2 at vapour pressure are reported. Relevant magnitudes were measured and understood by means of the general concepts of the theory of instabilities and surface waves.

Introduction. – In the last 20 years many works have been developed, both theoretical and experimental, in order to study pattern formation by hydrodynamical instabilities and their transition to the turbulence in many systems. Specific interest has been devoted to the appearance of spatial and temporal modes when a control parameter was varied from a threshold. Actually, the existence of several patterns and their competition are among the most interesting problems. Particular interest has been given to dynamical phenomena at fluid-fluid interfaces, when they are periodically excited [1, 2].

Among the theoretical approaches to this kind of problems, a review of a most interesting analysis was obtained in [3] and references therein. For interfaces periodically excited perpendicularly to gravity, a model was developed [4] where the only free parameter is a damping factor heuristically added. Depending on the frequencies, amplitudes and aspect ratio (considering circular geometries) a superposition of resonant perpendicular modes may appear, yielding chaotic regimes, for applied frequencies near to the natural resonance frequency. From an experimental point of view, there are not many results for these systems [1]. Their main purpose is to describe experiments in which all the phase diagram of the system has been measured and compared with the predictions of [4].

We report experimental results about interfacial waves periodically and horizontally forced which appear in a tube filled with CO_2 at vapour pressure (see experimental set-up) thus the experimental situation is similar to that cited above. The considered geometry, with very small aspect ratio, diminishes the class of solutions obtained in [4, 1] at linear or weakly non-linear regimes, depending on the applied frequencies. On the other hand, a wide range of

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frequencies was explored, far away from natural resonance, in order to observe regimes where either gravity or capillary effects are negligible or both are important. Parameters such as viscosity, density and surface tension, besides the fact of considering a closed basin, permit only the comparison with a few well-established results of general theories of interfacial waves [5]. Finally, the geometry, the physical properties of CO_2 (see experimental set-up) and applied forces imply very high adimensionalized wave numbers and almost-one-dimensional patterns for linear and weakly non-linear regimes.

Experimental set-up. – The experiments were performed using a Natterer tube, mounted horizontally on a plateholder built to minimize lateral vibrations and mechanic frictions, as is shown schematically in fig. 1. The Natterer tube is a CO_2-filled tube at vapour pressure and at room temperature (20 °C) (near the critical point: 32 °C). In our case, the geometry was a cylindrical tube of 1 cm diameter and 28 cm length. The CO_2 liquid phase filled just one half of the whole volume of the tube; therefore, in its horizontal position the maximum depth of the liquid phase was 0.5 cm. As the variations of temperature near the critical point produce appreciable variations in the physical properties, the experiments were done in a thermostatted chamber at constant temperature, controlled externally. For the present paper, the experiments were done at 20 °C with a stability better than 0.2 °C per hour. At 20 °C the main physical characteristics [6] are $\rho_l = 773\ \mathrm{kg/m^3}$, $\rho_g = 189\ \mathrm{kg/m^3}$, $\nu_l = 9.2\cdot 10^{-8}\ \mathrm{m/s}$, $\nu_g = 7.8\cdot 10^{-8}\ \mathrm{m/s}$, $\sigma_0 = 1.16\cdot 10^{-3}\ \mathrm{kg/s^2}$, where l and g subscripts mean liquid and gas phase and ρ, ν, σ are density, kinematical viscosity and surface tension.

A sinusoidal mechanic force was applied by a loudspeaker in the direction of the long axis of the tube. The driving force and amplitude were produced by a function generator connected to an amplifier with very low noise. The measurements of the driving amplitudes of the applied force were done by a microvoltmeter connected directly to the loudspeaker. We had verified that the mechanical displacements (about 2 mm) of the tube were proportional to the applied voltages. The driving frequencies lie between 5 and 110 Hz having a resolution of 10 mHz.

The deformations of the liquid-gas interface were detected by a schlieren method which allowed us to visualize spatial structure and to distinguish variations of the interface (compared to the rest position) of 10 μm. Also, a system of direct visualization by a video-camera CCD was used in order to study patterns and their evolution when the control parameters (frequencies and amplitudes) were changed. In all the cases we synchronized the signal to avoid the stroboscopic effect. For the working frequencies, extremities of the tube act as wavemakers, and so, for any mechanic excitation, waves are produced and afterwards are propagated to the middle of the tube.

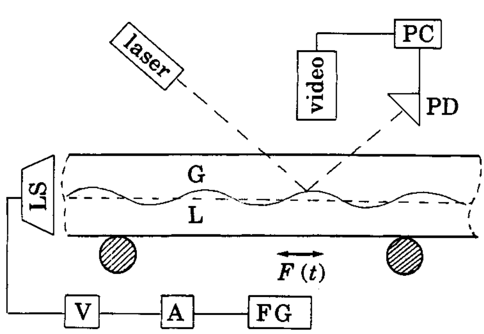

Fig. 1. – Schematic picture of experimental set-up. LS, A, V, PD, G, L mean loudspeaker, amplifier, voltmeter, photodetector, gas phase and liquid phase.

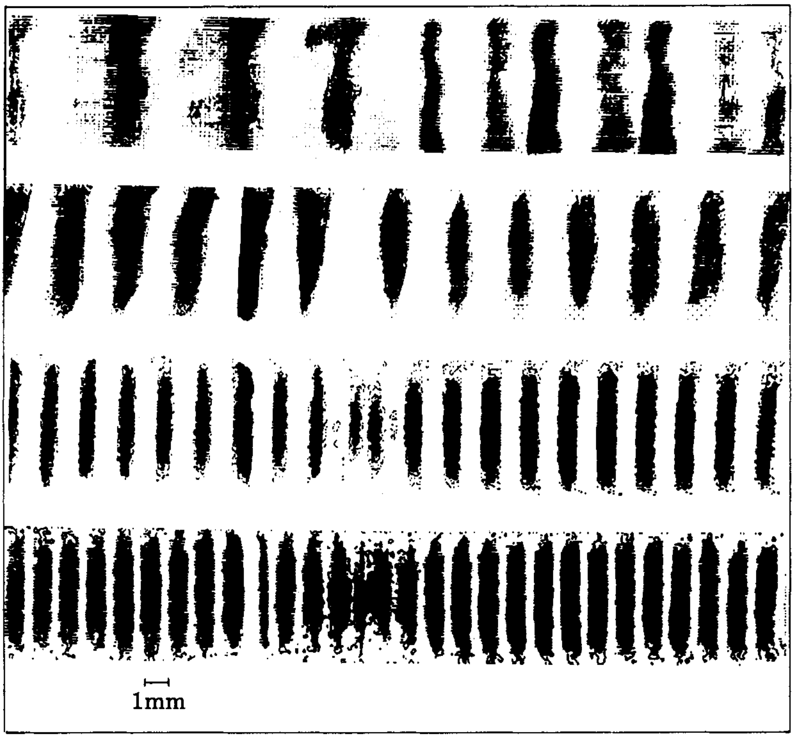

Fig. 2. – From top to bottom images with increasing driving frequencies (15, 30, 50, 80 Hz).

Due to the low viscosity, the effect of lateral walls is negligible, at least if the propagation of perturbations in the liquid phase is considered. Actually, the penetration length is 2% of the depth. Also, the surface tension is very small, and then, the meniscus effect can be neglected. However, the tube is submitted to periodic vibrations in a parallel direction to the lateral walls, so secular instabilities at the interface could be developed. Following the linear theory [7] and simple estimations of the involved parameters, such as applied amplitudes (~ 2 mm) and frequencies (≤ 110 Hz), viscosities, etc., always lead to Froude numbers sufficiently small satisfying the stability criterium.

As was said previously, for any amplitude of mechanical excitation waves are produced. However, only when this amplitude of excitation reaches a certain value, waves can be visualized, or detected by the methods cited above. These values are defined as a «threshold»: for each applied frequency, the amplitude of the driving force which gave us the first detected surface deformation was the «threshold» amplitude.

In fig. 2 typical patterns, as observed from above, are shown containing stripes dark and bright perpendicular to the longest axis of the tube which correspond to crests and troughs of the waves.

Results. – We measured «threshold» amplitudes for different exciting frequencies (fig. 3) and we found a minimum at 28 Hz distinguishing two regions. In the first one, on the left, «threshold» values increased when the driving frequency was increased. It can be observed that, qualitatively, the two regions corresponded, respectively, to gravity and capillary waves because the capillary wave number in this problem lies near the minimum. An easy

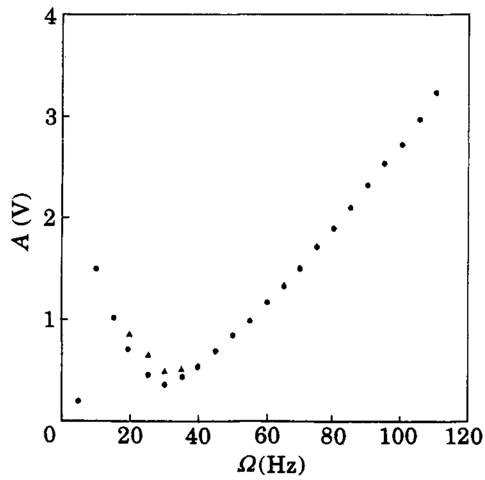

Fig. 3.

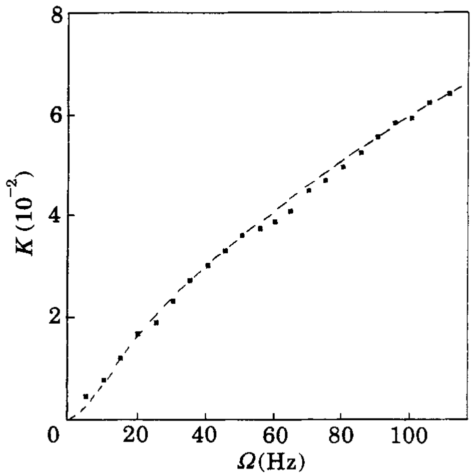

Fig. 4.

Fig. 3. – «Threshold» amplitudes for various driving frequencies. The distinguished regions around the minimum can be seen.

Fig. 4. – Dimensionless wave numbers for different applied frequencies at «threshold» values. The dashed curve is the fit to the model [8].

calculation gives for the adimensionalized capillary wave number (*i.e.* $k_c = 2L/\lambda_c$) [8]

$$k_c = \frac{1}{\pi}\left[\frac{(\rho_l - \rho_g)gL^2}{\sigma_0}\right]^{1/2} \approx 167,$$

where L is the length of the tube. Hysteresis can be observed around the minimum with regard to increasing and decreasing driving amplitude.

We also measured wave numbers for various frequencies applied at the «threshold» comparing these experimental results with the predictions by a linear model [8] (fig. 4). Wave numbers increase with the applied frequency because, for greater frequencies, the wavemakers produce waves at a greater rate.

If we suddenly increased the exciting amplitude from 0 to the «threshold», there would exist a delay time before any pattern could be observed (fig. 5). The delay time is expected to be the characteristic time for the transient (proportional to k^{-2}, where k is the adimensionalized wave number); however, at the capillary region the time delay values are greater than expected, because it is necessary to take into account the time interval between waves that are created at the extremities of the tube and arrive at the centre of the tube. In fact, the first interaction of the wave train created at both extremities corresponds to a destructive interference.

We also measured whether propagating patterns exist or not, taking a one-dimensional image throughout time (fig. 6). The slope of the stripes gave us the propagation velocities. The existence of propagating patterns for frequencies higher than a cut-off frequency corresponding to the capillary wave number calculated above was stated. Propagating patterns went in both directions and fig. 7 shows the absolute values of their velocities.

We could determine the existence of modulation amplitudes, near the «threshold».

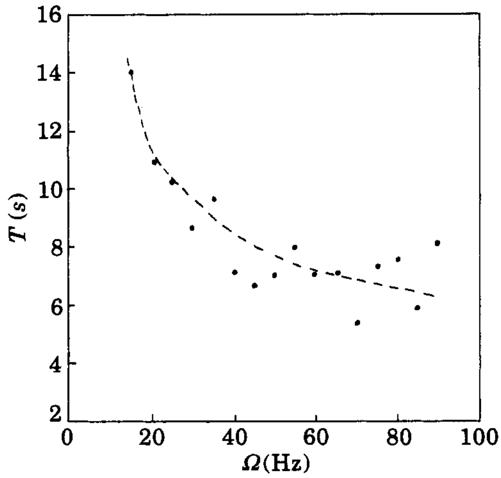

Fig. 5.

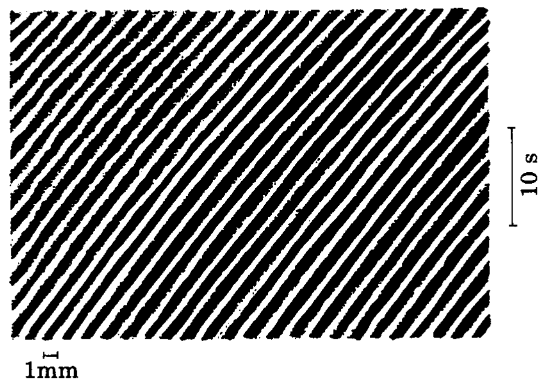

Fig. 6.

Fig. 5. – Apparition time for patterns at «threshold» amplitudes. The dashed curve is the fit according to the idea exposed on the text.

Fig. 6. – One-dimensional spatio-temporal digital-image acquisition showing pattern propagation at 60 Hz. Time increasing from top to bottom.

Unfortunately, we are only in a position to say that the modulation amplitudes' frequencies were smaller than 100 MHz.

Discussion. – Due to the moving boundary conditions, the interface develops all the modes allowed by the symmetry conditions, although for each driving frequency there obviously exists a set of resonant modes which prevails. These sets are discretized because the experiments were performed in a closed basin. From here on we shall label these sets as «spatial resonance curves». Spatial resonance curves, which depend on the applied frequency, have absolute maxima. Those maxima of spatial resonance curves vary strongly

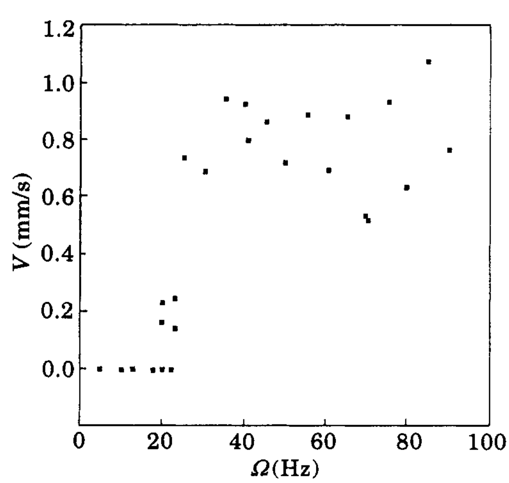

Fig. 7. – Pattern propagation velocity. For gravity waves there do not exist propagative patterns.

among neighbouring applied frequencies giving relative maxima and minima [8]. Moreover, our experiments lie far away from the natural resonance frequency and the system will get high wave number values. This justifies considering continuous spatial resonance curves.

For the region of capillary waves the variation of «threshold» with frequency increases because the amount of energy necessary to create a wave of a given height is, mainly, due to surface tension effects. Then, if the measured wave number increases, so does the interface curvature and also the amount of energy to deform the interface.

The experimental results at the gravity wave region corresponding to fig. 3 must be interpreted in a different way. As a matter of fact, in the linear model [8] it is obtained that the distance between relative maxima decreases when the applied frequency decreases. So, even for very small fluctuations of the driving frequency, there will be destructive interaction between the correspondent neighbouring maxima, being this effect more important for lower frequencies. A consequence would be that, to achieve a «threshold» as defined above, it is necessary to give an amount of energy greater than expected (fig. 3). At the capillary region this kind of arguments obviously becomes less important and then the surface tension effects cited above prevail. The hysteresis around the minimum of fig. 3 is just a non-linear effect of the resonance forwarded by kinematical considerations of neighbouring modes [9]. Actually, it lies between the resonances with the first subharmonic mode (Wilton ripples) and the second harmonic (internal resonances) being it a maximum hysteresis at the resonance with very near modes at the capillary wave number. In fact, the description given above permits to explain the experimental results of fig. 5.

The agreement of the capillary-gravity frequency with a cut-off frequency for the propagating patterns suggests the link between them [8].

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REFERENCES

[1] Nobili M. *et al.*, *Europhys. Lett.*, **7** (1988) 587; Funakoshi M. and Inoue S., *J. Fluid Mech.*, **192** (1988) 219.
[2] Douady S. and Fauve S., *Europhys. Lett.*, **6** (1988) 221; Fauve S. *et al.*, *Phys. Rev. Lett.*, **68** (1992) 3160.
[3] Miles J. W. and Henderson D. M., *Annu. Rev. Fluid Mech.*, **22** (1990) 143.
[4] Miles J. W., *J. Fluid Mech.*, **149** (1984) 15.
[5] Segel L. A., *Mathematics applied to Continuum Mechanics* (Dover Publications, Inc.) 1987, part C.
[6] Weast R. C. (Editor), *CRC Handbook of Chemistry and Physics*(CRC Press, Inc.) 1983.
[7] Yih C. S., *Fluid Mechanics* (McGraw-Hill, Inc.) 1969, pp. 516-524.
[8] González-Viñas W. and Salán J., *A model for surface wave periodically excited in a* CO_2 *tube*, submitted to *Phys. Fluids*.
[9] Hammack J. L. and Henderson D. M., *Annu. Rev. Fluid Mech.*, **25** (1993) 55.