31

≤0.34)

6.4

題: $\hat{\theta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$, $\hat{\theta}_2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$

$E(x_i) = \mu$, $E(\bar{x}) = \mu$ $\boxed{\bar{x} = \frac{x_1 + x_2}{n}}$

$V(x_i) = \sigma^2 = E(x_i^2) - \mu^2$; $V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$

$$E(\hat{\theta}_1) = E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}\right] = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 - \sigma^2 + n\mu^2 - n\mu^2) = \frac{n-1}{n}\sigma^2 \neq \sigma^2$$

為偏誤估計量

$$E(\hat{\theta}_2) = E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}\right] = \frac{1}{n-1} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 - \sigma^2 + n\mu^2 - n\mu^2) = \sigma^2$$

為不偏誤估計量