$7 \pm 1.645\sqrt{\frac{15^2}{50}+\frac{}{50}}$

習9 標本数=6，標本平均数 $\bar{x}=\frac{15+18+9+13+17+14}{6}=14.33$

(1) 標準差 $S=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}=\sqrt{\frac{\Sigma x_i^2-n\bar{x}^2}{n-1}}=\sqrt{\frac{1284-6\times14.33^2}{5}}$

$=\sqrt{10.38}=3.22$

x_i	x_i^2
15	225
18	324
9	81
13	169
17	289
14	196
	1284

(2) $1-\alpha=0.9$ 自由度 $n-1=9$

$\frac{\alpha}{2}=\frac{0.1}{2}=0.05$

$\chi^2_{\frac{\alpha}{2}}(n-1)=\chi^2_{0.05}5=11.07$

$\chi^2_{1-\frac{\alpha}{2}}(n-1)=\chi^2_{0.95}5=1.15$

信頼区間公式

$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(\nu)}},\sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(\nu)}}\right)=\left(\sqrt{\frac{5\times10.38}{11.07}},\sqrt{\frac{5\times10.38}{1.15}}\right)=\left(\sqrt{\frac{51.9}{11.07}},\sqrt{\frac{51.9}{1.15}}\right)=(2.17,6.73)$

20. 題目：変異数不相等，常態分布 $n=9<30$ 小標本

$\sigma_1 \neq \sigma_2$

ν 自由度 $=\frac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1}\ \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2-1}}$

$\mu_1-\mu_2$ の $100(1-\alpha)\%$ 的信頼区間

$(\bar{x}-\bar{y})\pm t_{\frac{\alpha}{2}}(\nu)\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$

(1) 95% = 0.95 $1-\alpha = 0.95$ $\alpha = 0.05$ $\frac{\alpha}{2} = 0.025$

$s_1 = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$ $\bar{x}_1 = \frac{69}{9} = 7.67$

$= \sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n-1}} = \sqrt{\frac{1217 - 9 \times 7.67^2}{8}} = \sqrt{85.94} = 9.27$ ※

x_i^2 169 25 576 36 225 25 16 81 64 $\bar{y} = \frac{61}{9} = 6.78$

$s_2 = \sqrt{\frac{3993 - 9 \times 6.78^2}{8}} = \sqrt{\frac{3993 - 713.72}{8}} = \sqrt{447.41} = 21.15$ ※

y_i^2 361 1 256 961 676 400 1024 289 25

$$V = \frac{\left(\frac{9.27^2}{9} + \frac{21.15^2}{9}\right)^2}{\frac{\left(\frac{9.27^2}{9}\right)^2}{8} + \frac{\left(\frac{21.15^2}{9}\right)^2}{8}} = \frac{(9.55 + 49.70)^2}{\frac{(9.55)^2}{8} + \frac{(49.70)^2}{8}} = \frac{3510.56}{320.16} = 10.97 = 11$$ ※

$(7.67 - 6.78) \pm t_{0.025}(11)\sqrt{59.25} = 0.89 \pm t_{0.025}(11)\,7.7$

$= 0.89 \pm 2.201 \times (7.7)$

$= 0.89 \pm 16.95$

(1) $(-16.06, 17.94)$ ※

(2) 標準差 90% 信賴區間 $V = n-1$ $1-\alpha = 0.9$ $\alpha = 0.1$ $\frac{\alpha}{2} = 0.05$

$$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(V)}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}(V)}}\right) = \left(\sqrt{\frac{8 \times 9.27^2}{\chi^2_{0.05}(8)}}, \frac{8 \times 9.27^2}{\chi^2_{0.95}(8)}\right)$$

$$= \left(\sqrt{\frac{687.46}{15.51}}, \sqrt{\frac{687.46}{2.73}}\right) = (6.66, 15.87)$$

(3) ※ 變異數比的信賴區間

$$\frac{\sigma_1^2}{\sigma_2^2}, \left(\frac{S_1^2}{S_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(V_1, V_2)}, \frac{S_1^2}{S_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}}(V_1, V_2)}\right) \quad F_{0.95}(8,8) = \frac{1}{F_{0.05}(8,8)}$$

$V_1, V_2 = (n_1 - 1, n_2 - 1)$

$$\left(\frac{9.27^2}{21.15^2} \times \frac{1}{F_{0.05}(8,8)}, \frac{9.27^2}{21.15^2} \times \frac{1}{F_{0.95}(8,8)}\right) = \left(\frac{85.93}{447.32} \times \frac{1}{3.44}, \frac{85.93}{447.32} \times \frac{1}{0.29}\right)$$

$= (0.06, 0.66)$