

Method:

1. Initialize: $\hat{\mathbf{y}}_0 = \widehat{\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_0 \end{bmatrix}} = \mathbf{0}_{2n \times 1}$, $R_0 = \text{Var}[\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_0 \end{bmatrix}] = \begin{bmatrix} P_0 & 0 \\ 0 & 0 \end{bmatrix}_{2n \times 2n}$.

2. Propagate: $\hat{\mathbf{y}}_t = F_t \hat{\mathbf{y}}_{t-1}$, $R_t = F_t R_{t-1} F_t^T + G_t Q_t G_t^T$, $0 < t \leq \ell$,

$$\text{where } F_t = \begin{bmatrix} A_t & B_t L_t \\ K_t H_t A_t & A_t + B_t L_t - K_t H_t A_t \end{bmatrix}, G_t = \begin{bmatrix} V_t & 0 \\ K_t H_t V_t & K_t W_t \end{bmatrix}, \text{ and } Q_t = \begin{bmatrix} M_t & 0 \\ 0 & N_t \end{bmatrix}.$$

3. Truncate:

(a) Determine the set of all obstacles in the workspace that potentially intersect the distribution $\mathcal{N}[\mathbf{p}_t, \Sigma_t]$, where the vector $\mathbf{p}_t \in \mathbb{R}^s$ comprises of the corresponding spatial dimensions of the state vector $\mathbf{y}_t = \hat{\mathbf{y}}_t + \begin{bmatrix} \mathbf{x}_t^* \\ \mathbf{x}_t^* \end{bmatrix}$, and Σ_t is the marginal distribution over the spatial dimensions of the distribution $\mathcal{N}[\hat{\mathbf{y}}_t, R_t]$.

(b) Each obstacle in the potentially intersecting set, $\mathcal{O}_i, i = 0, \dots, k$, defines a feasible half-space defined by the tuple (\mathbf{a}_i, b_i) , where $\mathbf{a}_i \in \mathbb{R}^s$ is a unit vector normal to the obstacle defined such that $\mathbf{a}_i \mathbf{p}_t \leq b_i$.

(c) Truncate the joint distribution $\mathcal{N}[\mathbf{y}_t, R_t]$ against the hyperplane $(\tilde{\mathbf{a}}_i, b_i)$, where $\tilde{\mathbf{a}}_i = [\mathbf{a}_i, 0, \dots, 0]^T$ is a hyperplane in \mathbb{R}^{2n} that is degenerate in all dimensions except the spatial dimensions.

Applying an affine transformation $\mathbf{y}'_t = \tilde{\mathbf{a}}_i^T \mathbf{y}_t$ transforms the distribution $\mathcal{N}[\mathbf{y}_t, R_t]$ to a one-dimensional Gaussian distribution $\mathcal{N}[\tilde{\mathbf{a}}_i^T \mathbf{y}_t, \tilde{\mathbf{a}}_i^T R_t \tilde{\mathbf{a}}_i]$ along an axis that is normal to the hyperplane $(\tilde{\mathbf{a}}_i, b_i)$. The problem now simplifies to truncating the one-dimensional Gaussian distribution at a specified upper bound given by b_i , which is well-known from standard literature. Let μ and σ^2 be the truncated mean and variance of the transformed distribution.

The truncated mean of the original distribution are then given by: $\tilde{\mathbf{y}}_t = \mathbf{y}_t + \sum_{i=0}^k \frac{R_t \tilde{\mathbf{a}}_i}{\tilde{\mathbf{a}}_i^T R_t \tilde{\mathbf{a}}_i} (\mu - \tilde{\mathbf{a}}_i^T \mathbf{y}_t)$ and

the truncated variance is given by: $\tilde{R}_t = R_t + \sum_{i=0}^k \frac{R_t \tilde{\mathbf{a}}_i}{\tilde{\mathbf{a}}_i^T R_t \tilde{\mathbf{a}}_i} (\sigma^2 - \tilde{\mathbf{a}}_i^T \mathbf{y}_t \tilde{\mathbf{a}}_i) \frac{\tilde{\mathbf{a}}_i^T R_t \tilde{\mathbf{a}}_i}{\tilde{\mathbf{a}}_i^T R_t \tilde{\mathbf{a}}_i}$. The term $\frac{R_t \tilde{\mathbf{a}}_i}{\tilde{\mathbf{a}}_i^T R_t \tilde{\mathbf{a}}_i}$ gives the multiplicative *gain* (similar to the gain term in the Kalman filter measurement update equations) that scales the correction to be applied to the mean \mathbf{y}_t and the variance R_t of the original distribution.

(d) The mean of the joint distribution is now given by: $\hat{\mathbf{y}}_t = (\tilde{\mathbf{y}}_t - \begin{bmatrix} \mathbf{x}_t^* \\ \mathbf{x}_t^* \end{bmatrix})$ and the variance given by: $R_t = \tilde{R}_t$.

4. Repeat steps (2) and (3) till the end of the plan.