Transport properties of suspensions

Department of Materials
ETH Zürich

Goal of this chapter

- Apply what we have learned to colloidal dispersions
- Example of a heat transfer problem
- effective medium approach
- suspension drag and suspension viscosity
- Einsteins' transport theorems

1. Temperature fields around a single sphere
2. Effective medium theory
3. Stokes flow around a sphere
4. Einstein’s law for the viscosity of suspensions

APPLIED PHYSICS LETTERS VOLUME 83, NUMBER 14 6 OCTOBER 2003

Thermal conductivities of naked and monolayer protected metal nanoparticle based nanofluids: Manifestation of anomalous enhancement and chemical effects

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(Received 3 February 2003; accepted 30 June 2003)

Thermal conductivities of two kinds of Au nanoparticles were measured in water and toluene media. The water soluble particles, 10–20 nm in mean diameter, made with citrate stabilization showed thermal conductivity enhancement of 5%–21% in the temperature range of 30–60 °C at a loading of 0.000 26 (by volume). The effect was 7%–14% for Au particles stabilized with a monolayer of octadecanethiol even for a loading of 0.011%. Comparatively lower thermal conductivity enhancement was observed for larger diameter Ag particles for significantly higher loading. Effective enhancement of 9%, even at vanishing concentrations, points to additional factors in the thermal conductivity mechanism in nanofluids. Results also point to important chemical factors such as the need for direct contact of the metal surface with the solvent medium to improve enhancement.
 [DOI: 10.1063/1.1602578]

JOURNAL OF APPLIED PHYSICS **99**, 084308 (2006)

Thermal conductivity of nanoparticle suspensions

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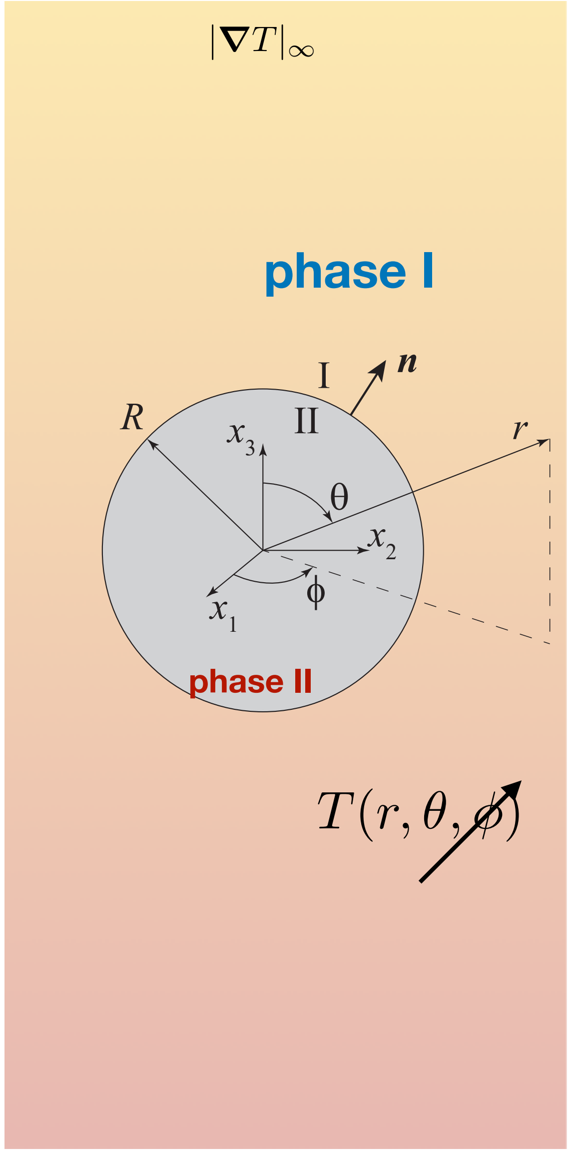
(Received 21 July 2005; accepted 24 February 2006; published online 26 April 2006)

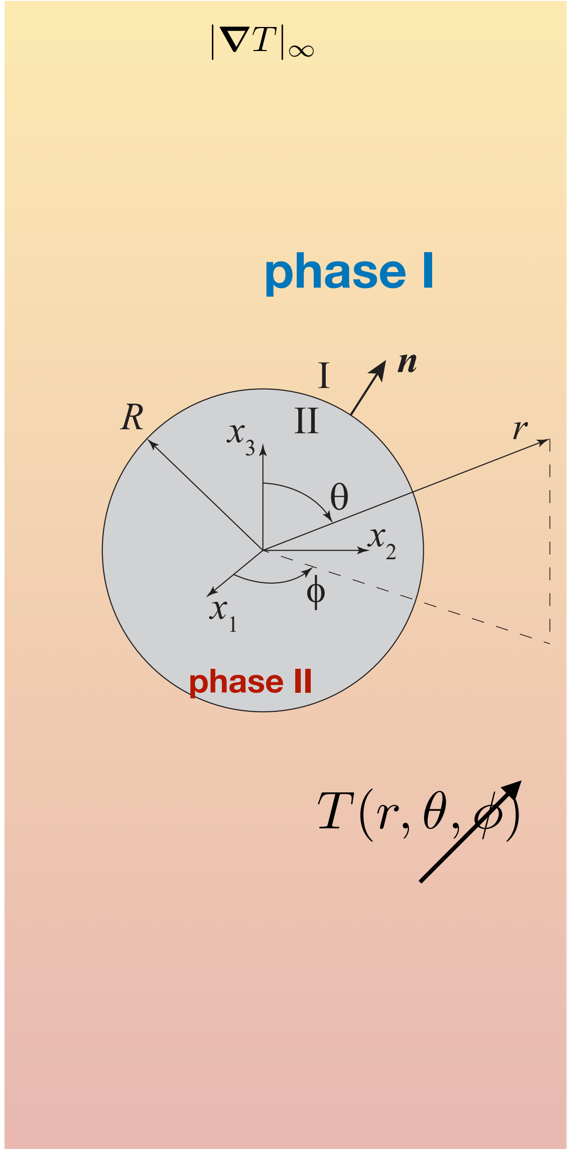
We describe an optical beam deflection technique for measurements of the thermal diffusivity of fluid mixtures and suspensions of nanoparticles with a precision of better than 1%. Our approach is tested using the thermal conductivity of ethanol-water mixtures; in nearly pure ethanol, the increase in thermal conductivity with water concentration is a factor of 2 larger than predicted by effective medium theory. Solutions of $C_{60}-C_{70}$ fullerenes in toluene and suspensions of alkanethiolate-protected Au nanoparticles were measured to maximum volume fractions of 0.6% and 0.35 vol %, respectively. We do not observe anomalous enhancements of the thermal conductivity that have been reported in previous studies of nanofluids; the largest increase in thermal conductivity we have observed is 1.3% ±0.8% for 4 nm diam Au particles suspended in ethanol. [DOI: 10.1063/1.2189933]

Heat transfer of a single sphere.



- What will you solve for?
- Sketch out the solution
- What will the boundary condition do?
- How do we go from a single sphere to an effective macroscopic value

Heat transfer of a single sphere.



temperature equation

$$\rho \hat{c}_{\hat{v}} \left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} T \right) = \lambda \nabla^2 T - \frac{\rho \hat{c}_{\hat{v}} (\gamma - 1)}{\alpha_p} (\boldsymbol{\nabla} \cdot \boldsymbol{v})$$
$$+ \eta [\boldsymbol{\nabla} \boldsymbol{v} + (\boldsymbol{\nabla} \boldsymbol{v})^{\mathrm{T}}] : \boldsymbol{\nabla} \boldsymbol{v} + \left(\eta_{\mathrm{d}} - \frac{2}{3} \eta \right) (\boldsymbol{\nabla} \cdot \boldsymbol{v})^2$$

simplifies to Laplace equation

$$\nabla^2 T = 0$$

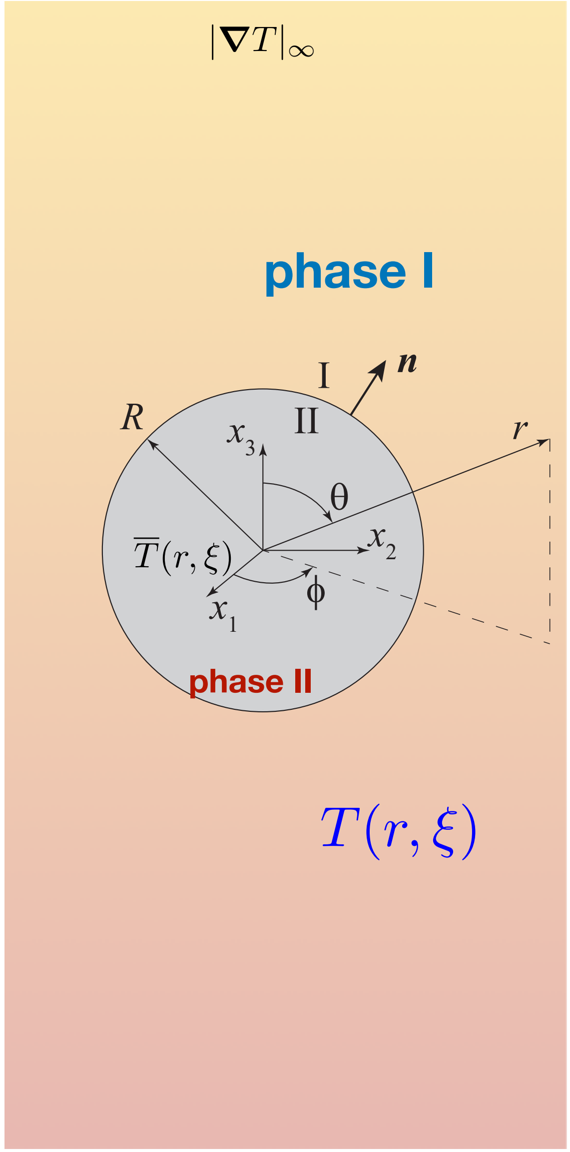
Symmetry and a useful coordinate transform

$$\xi = \cos \theta \qquad T(r, \xi)$$
$$d\xi = - \sin \theta \, d\theta \qquad \overline{T}(r, \xi)$$

Equation to be solved

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial \xi} \left[(1 - \xi^2) \frac{\partial T}{\partial \xi} \right] = 0$$

Heat transfer of a single sphere.



$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial \xi}\left[(1-\xi^2)\frac{\partial T}{\partial \xi}\right]=0$$

Separation of variables

$$T(r,\xi)=\Psi(r)\Phi(\xi)$$

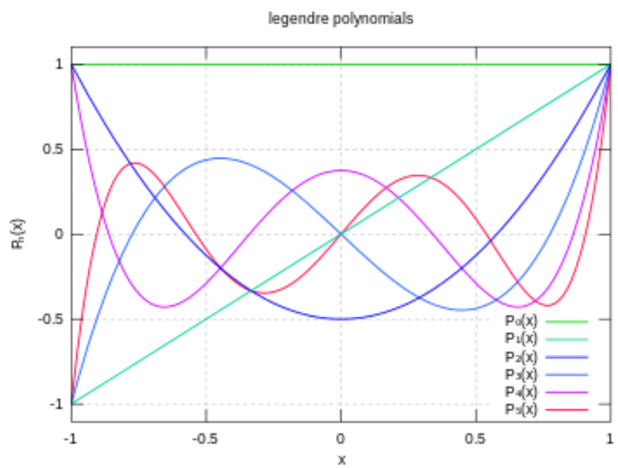
$$-\frac{1}{\Phi}\frac{d}{d\xi}\left[(1-\xi^2)\frac{d\Phi}{d\xi}\right]=\alpha^2=\frac{1}{\Psi}\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right)$$

Bound solutions

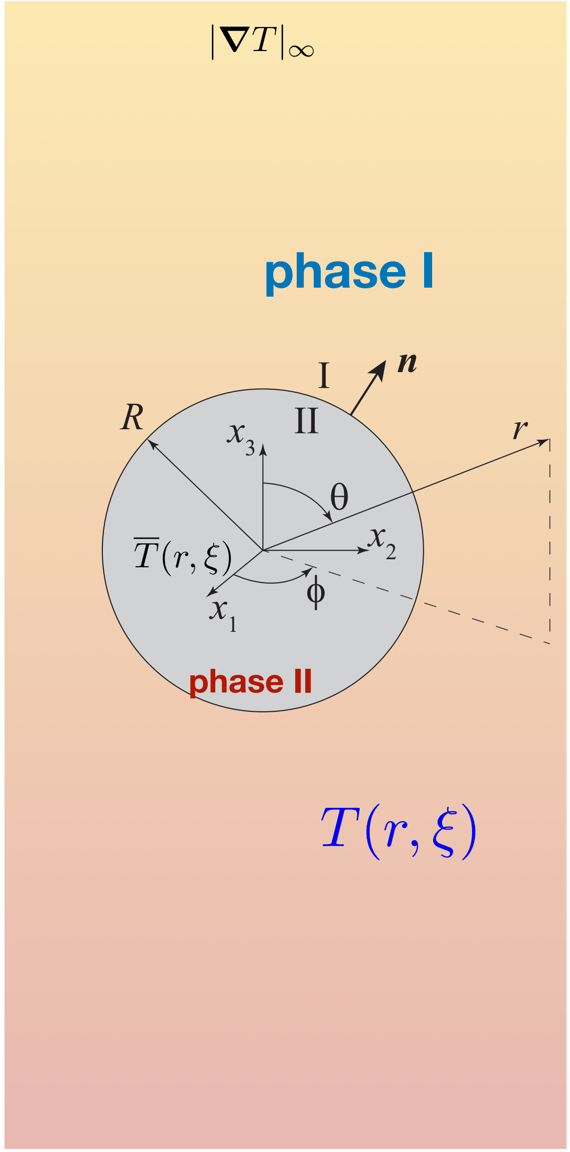
$$\Phi_n(\xi)=P_n(\xi)$$

$$\alpha^2 \to \alpha_n^2 = n(n+1) \text{ for } n=0,1,2...$$

Legendre polynomials



Heat transfer of a single sphere.



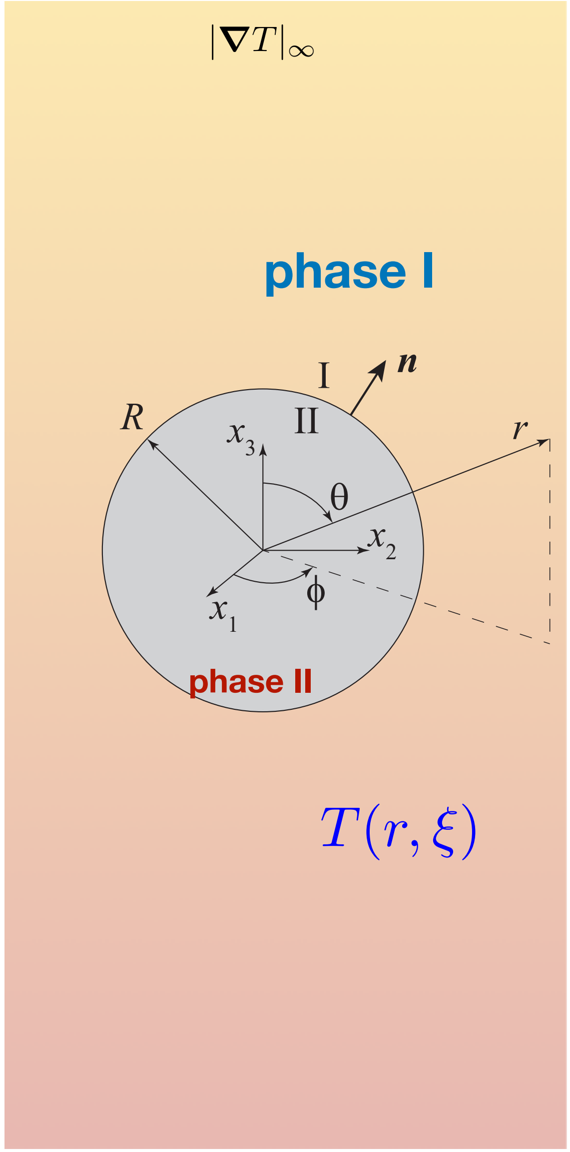
$$T(r,\xi) = \Psi(r)\Phi(\xi)$$

$$-\frac{1}{\Phi}\frac{d}{d\xi}\left[(1-\xi^2)\frac{d\Phi}{d\xi}\right] = \alpha^2 = \frac{1}{\Psi}\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right)$$

equidimensional - Euler differential equation

$$\Psi_n(r) = A_n r^n + B_n r^{-(n+1)}$$

Heat transfer of a single sphere.



temperature field in the medium

$$T(r,\xi) = \sum_{n=0}^{\infty} \left[A_n r^n + B_n r^{-(n+1)} \right] P_n(\xi)$$

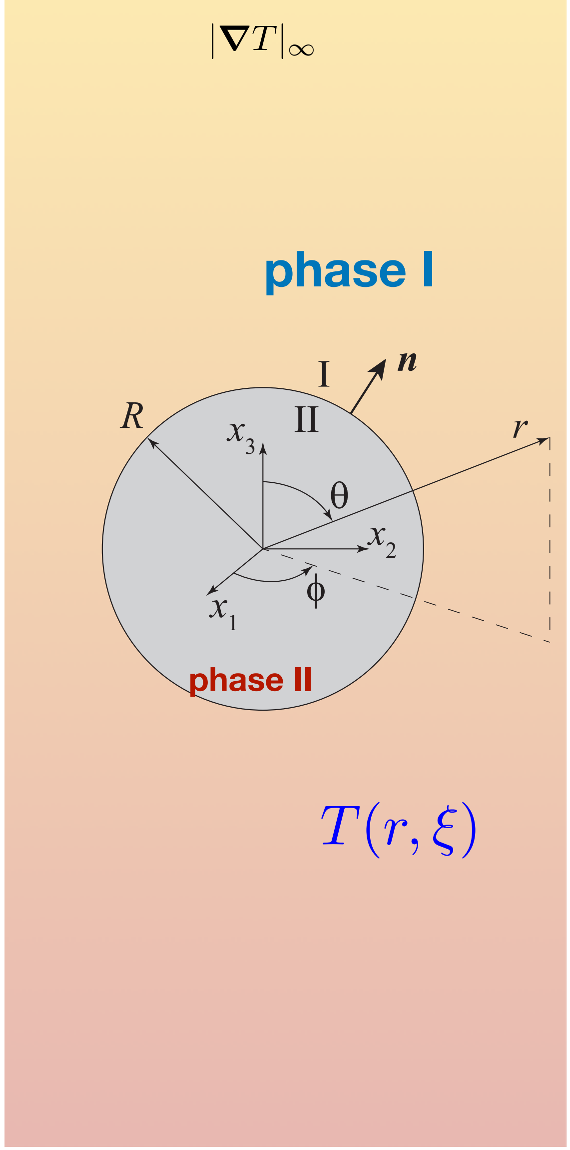
temperature within the sphere

$$\overline{T}(r,\xi) = \sum_{n=0}^{\infty} \overline{A}_n r^n P_n(\xi)$$

$$\overline{B}_n = 0$$

`Temperature needs to be finite`

Heat transfer of a single sphere.



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$$\overline{T}(r,\xi) = \sum_{n=0}^{\infty} \overline{A}_n r^n P_n(\xi)$$

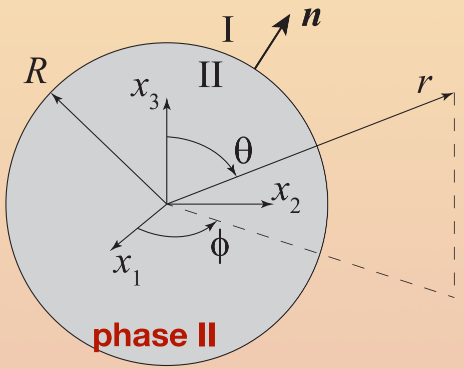
$$\overline{B}_n = 0$$

Temperature needs to be finite

Heat transfer of a single sphere.

$|\boldsymbol{\nabla} T|_\infty$

phase I



$T(r, \xi)$

BC1 : Far field conditions

$$r \rightarrow \infty \qquad \frac{\partial T}{\partial r}(\infty, \xi) = |\boldsymbol{\nabla} T|_\infty \xi$$

$$A_1 = |\boldsymbol{\nabla} T|_\infty \text{ and } A_n = 0 \text{ for } n \geq 2$$

BC2 : normal component fluxes

$$\boldsymbol{n} \cdot \boldsymbol{j}_q^{\mathrm{I}} = \boldsymbol{n} \cdot \boldsymbol{j}_q^{\mathrm{II}}$$

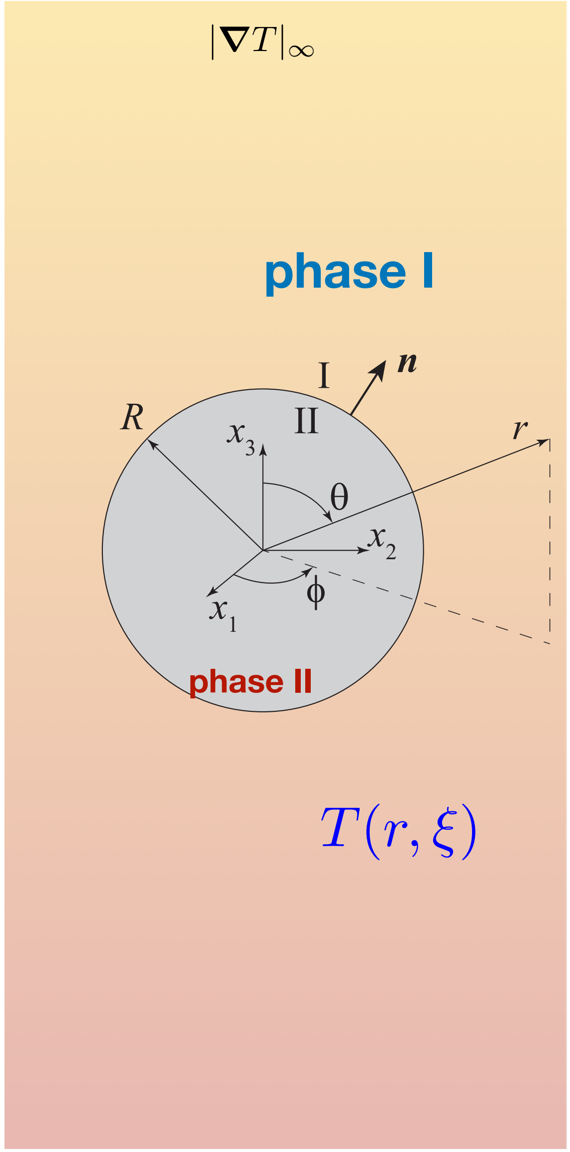
$$\frac{\partial T}{\partial r}(R, \xi) = \beta \frac{\partial \overline{T}}{\partial r}(R, \xi)$$

$$\beta = \bar{\lambda} / \lambda$$

BC: interfacial flux relations lead to

$$T(R, \xi) = \overline{T}(R, \xi) + R_{\mathrm{K}}^{\mathrm{s}} \lambda \frac{\partial T}{\partial r}(R, \xi)$$

$$R_{\mathrm{K}}^{\mathrm{s}} = R_{\mathrm{K}}^{\mathrm{Is}} + R_{\mathrm{K}}^{\mathrm{IIs}}$$



$$T^{\mathrm{I}} - T^{\mathrm{s}} = -R_{\mathrm{K}}^{\mathrm{Is}} \left[(\boldsymbol{v}^{\mathrm{I}} - \boldsymbol{v}^{\mathrm{s}}) h^{\mathrm{I}} + \boldsymbol{j}_q^{\mathrm{I}} \right] \cdot \boldsymbol{n}$$

$$T^{\mathrm{II}} - T^{\mathrm{s}} = R_{\mathrm{K}}^{\mathrm{IIs}} \left[(\boldsymbol{v}^{\mathrm{II}} - \boldsymbol{v}^{\mathrm{s}}) h^{\mathrm{II}} + \boldsymbol{j}_q^{\mathrm{II}} \right] \cdot \boldsymbol{n}$$

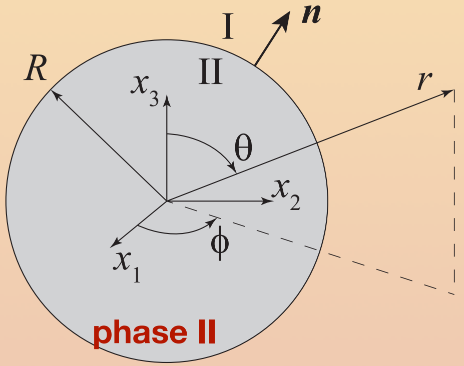
$$T(R,\xi) = \overline{T}(R,\xi) + R_{\mathrm{K}}^{\mathrm{s}} \lambda \frac{\partial T}{\partial r}(R,\xi)$$

$$R_{\mathrm{K}}^{\mathrm{s}} = R_{\mathrm{K}}^{\mathrm{Is}} + R_{\mathrm{K}}^{\mathrm{IIs}}$$

Heat transfer of a single sphere.

$|\nabla T|_\infty$

phase I



$T(r,\xi)$

BC1 : Far field conditions

$r \to \infty$

$$\frac{\partial T}{\partial r}(\infty,\xi) = |\nabla T|_\infty \xi$$

$A_1 = |\nabla T|_\infty$ and $A_n = 0$ for $n \geq 2$

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$\beta = \bar{\lambda}/\lambda$

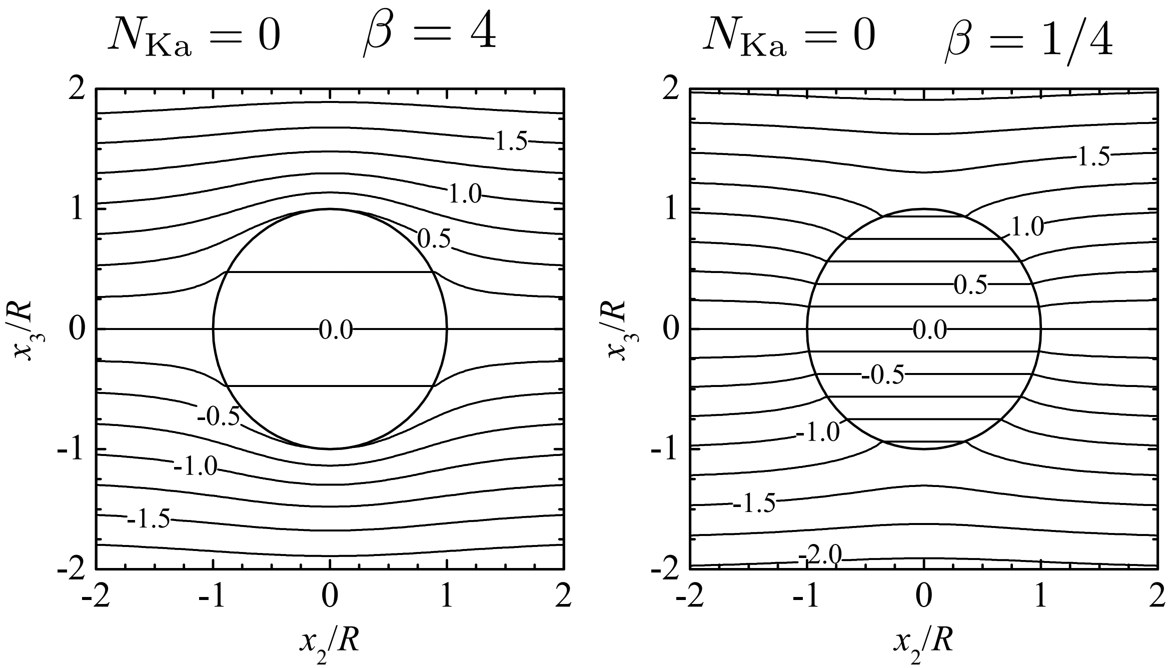
BC: interfacial flux relations lead to

$$T(R,\xi) = \overline{T}(R,\xi) + R_{\mathrm{K}}^{\mathrm{s}}\lambda\frac{\partial T}{\partial r}(R,\xi)$$

$R_{\mathrm{K}}^{\mathrm{s}} = R_{\mathrm{K}}^{\mathrm{Is}} + R_{\mathrm{K}}^{\mathrm{IIs}}$

Temperature field : isotherms

$$\frac{(T - T_0)}{(R|\boldsymbol{\nabla} T|_\infty)}$$



$$\beta = \bar{\lambda}/\lambda$$

$$N_{Ka} = \frac{R_K^s \lambda}{R}$$

$$\frac{\overline{T}(r,\xi) - T_0}{R|\boldsymbol{\nabla} T|_\infty} = \frac{3}{2 + \beta(1 + 2N_{\mathrm{Ka}})}\left(\frac{r}{R}\right)\xi$$

$$\frac{T(r,\xi) - T_0}{R|\boldsymbol{\nabla} T|_\infty} = \left(\frac{r}{R}\right)\xi + \frac{1 - \beta(1 - N_{\mathrm{Ka}})}{2 + \beta(1 + 2N_{\mathrm{Ka}})}\left(\frac{R}{r}\right)^2\xi$$

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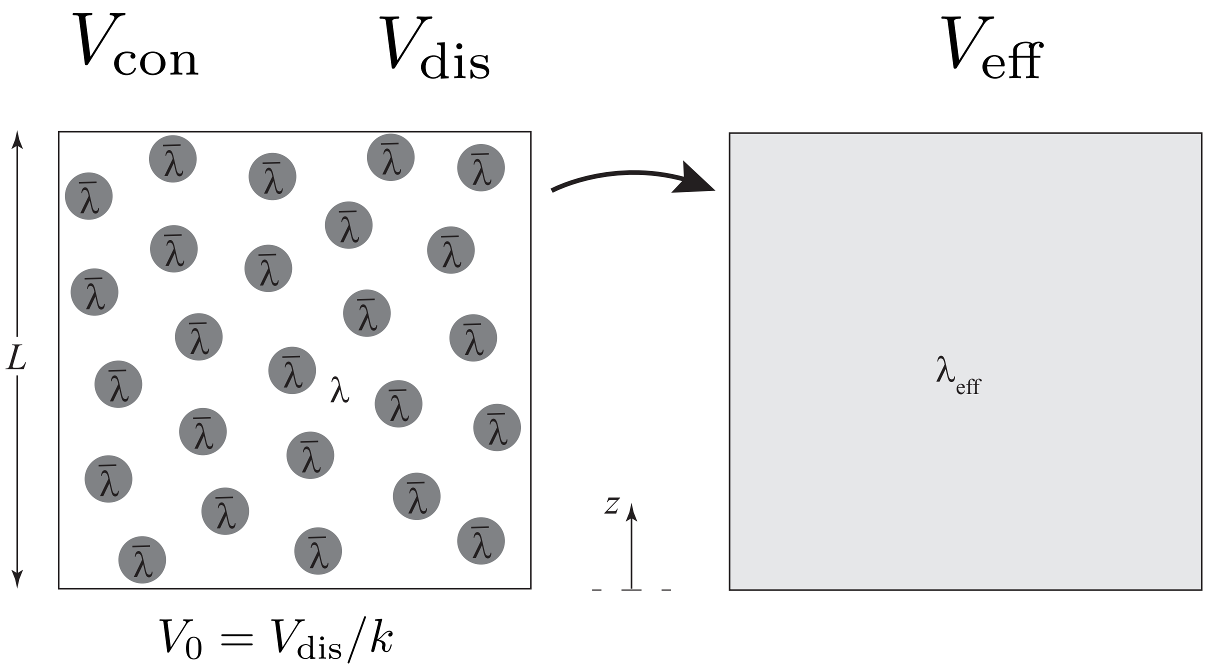


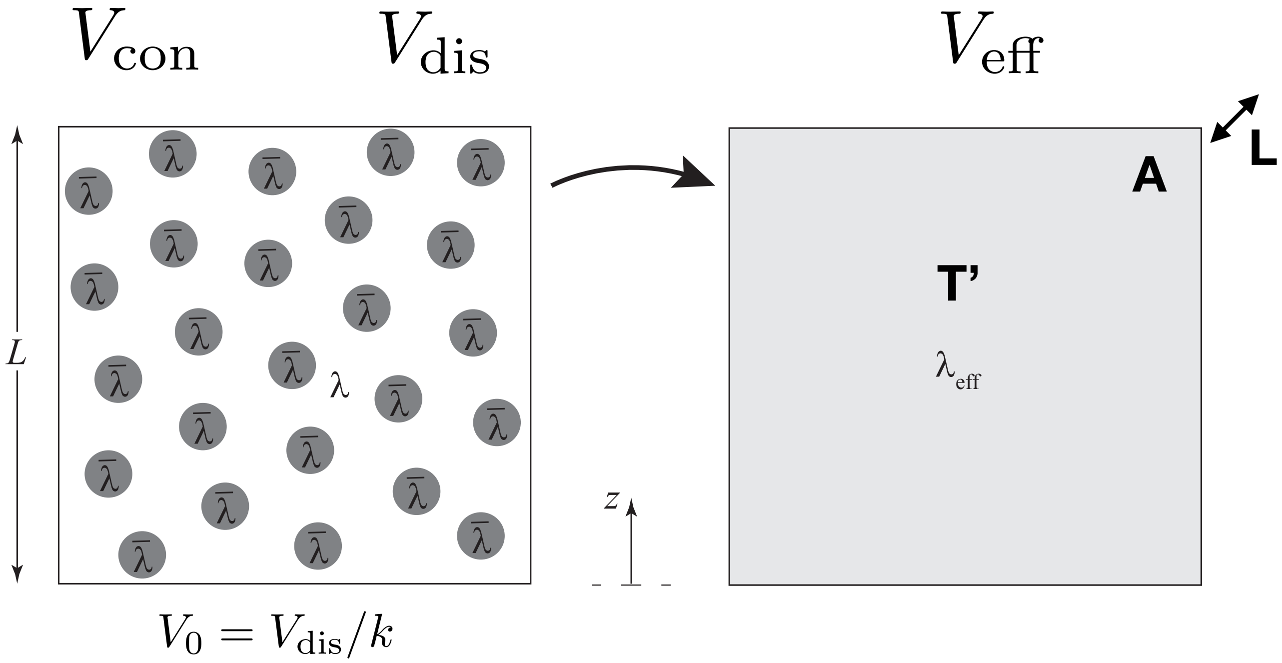
$$V_{\text{eff}}^{1/3} << x$$

$$\phi = kV_0/V_{\text{eff}} \ll 1$$

Volume fraction

Effective medium theory : volume average of a quantity

$$\langle(\ldots)\rangle = \frac{1}{V_{\text{eff}}}\int_{V_{\text{eff}}}(\ldots)dV$$



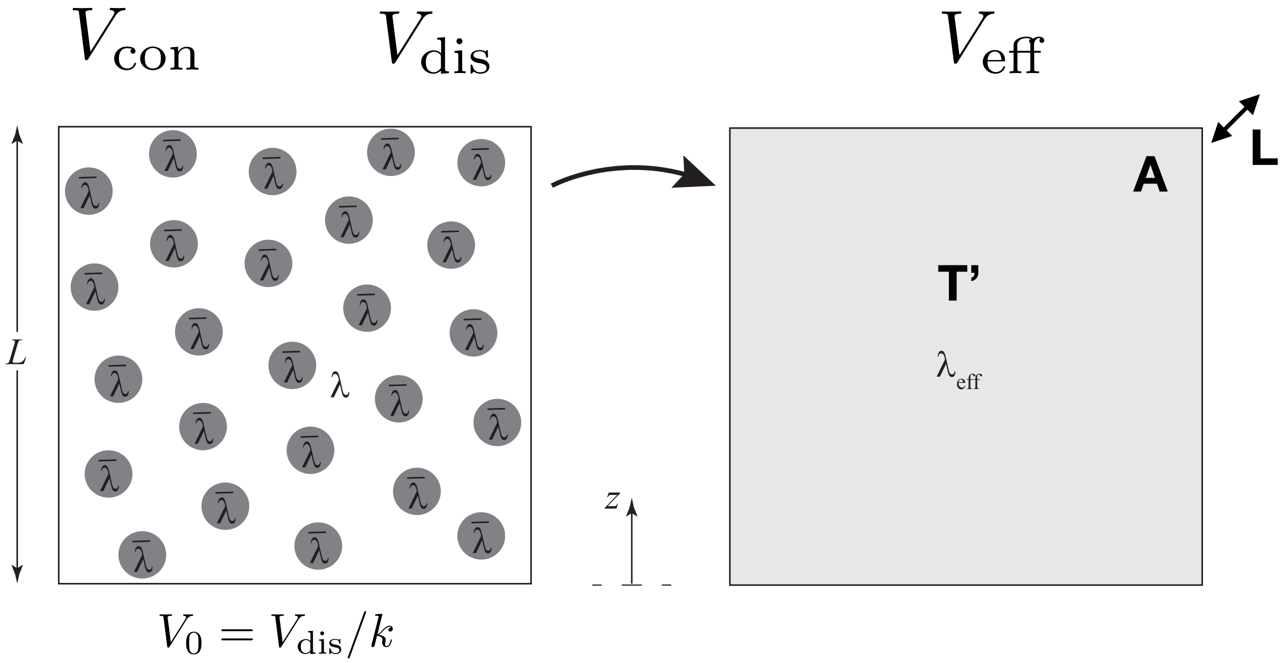
$$\Delta T = \int_L |\boldsymbol{\nabla} T'| dz$$

$$\Delta T / L = \langle |\boldsymbol{\nabla} T'| \rangle$$

$$\dot{Q} = \int_A |\boldsymbol{j}_q| dA$$

$$\dot{Q} / A = \langle |\boldsymbol{j}_q| \rangle$$

volume-averaged quantities



$$\dot{Q}/A \propto \Delta T/L$$

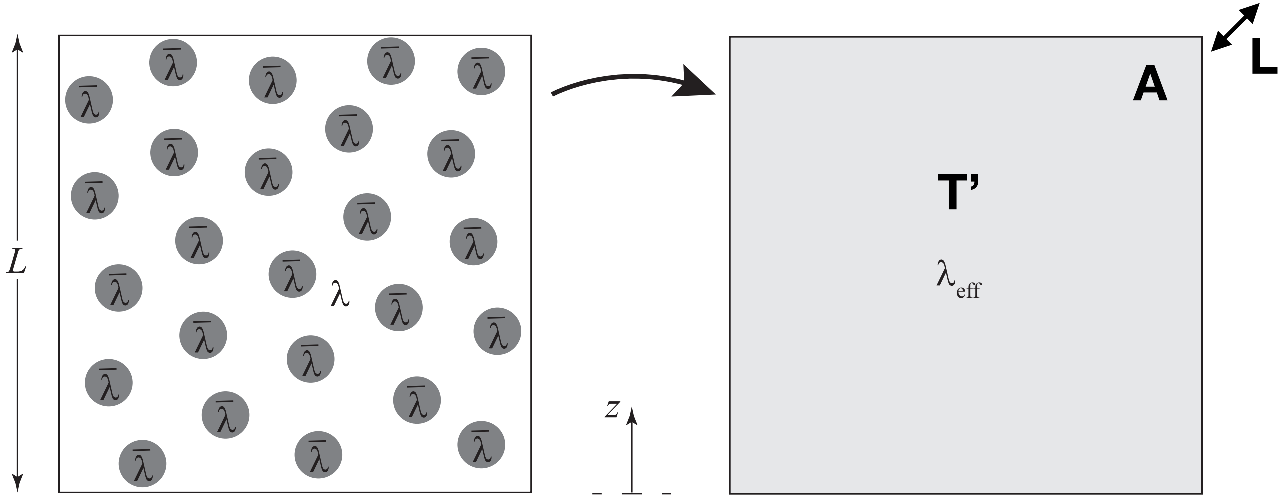
$$T(r,\xi) \sim r^{-2}$$

$$\langle \boldsymbol{j}_q \rangle = -\lambda_{\text{eff}} \langle \boldsymbol{\nabla} T' \rangle$$

$$\langle \boldsymbol{j}_q \rangle = -\lambda \langle \boldsymbol{\nabla} T' \rangle - \frac{\bar{\lambda} - \lambda}{V_{\text{eff}}} \int_{V_{\text{dis}}} \boldsymbol{\nabla} \overline{T} dV$$

V_{con} V_{dis}

V_{con} V_{dis} V_{eff}



$$V_0 = V_{\mathrm{dis}}/k$$

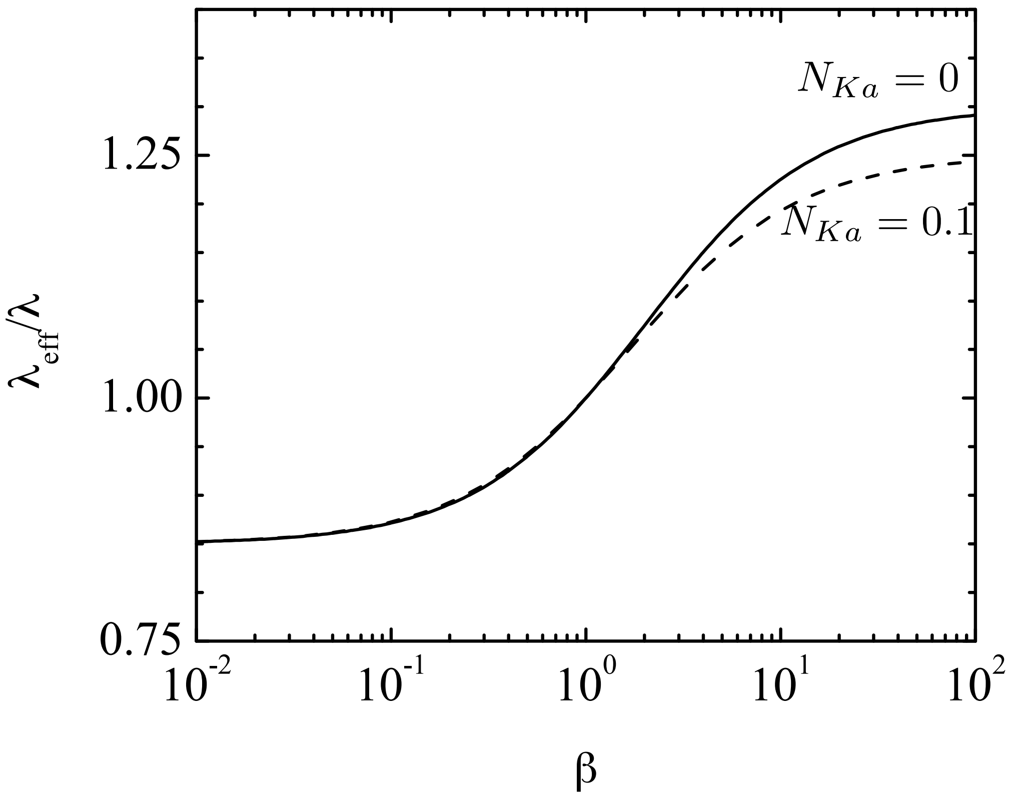
$$\langle \boldsymbol{j}_q \rangle = -\lambda \langle \boldsymbol{\nabla} T' \rangle - \frac{\overline{\lambda} - \lambda}{V_{\mathrm{eff}}} \int_{V_{\mathrm{dis}}} \boldsymbol{\nabla} \overline{T} dV$$

$$\frac{\overline{T}(r,\xi) - T_0}{R|\boldsymbol{\nabla} T|_\infty} = \frac{3}{2 + \beta(1 + 2N_{\mathrm{Ka}})} \left(\frac{r}{R}\right) \xi$$

$$\frac{1}{V_{\mathrm{eff}}} \int_{V_{\mathrm{dis}}} \boldsymbol{\nabla} \overline{T} dV = \frac{k}{V_{\mathrm{eff}}} \int_{V_0} \boldsymbol{\nabla} \overline{T} dV = \phi \frac{3}{2 + \beta(1 + 2N_{\mathrm{Ka}})} |\boldsymbol{\nabla} T|_\infty \boldsymbol{\delta}_3$$

$$\phi = kV_0/V_{\mathrm{eff}} \ll 1$$ DILUTE!

$$\frac{\lambda_{\text{eff}}}{\lambda} = 1 + \frac{3(\beta - 1)}{2 + \beta(1 + 2N_{\text{Ka}})}\phi$$



$$\beta = \bar{\lambda}/\lambda$$

$$N_{Ka} = \frac{R_K^s \lambda}{R}$$

$$\frac{\lambda_{\text{eff}}}{\lambda} = \begin{cases} 1 + 3\phi & \text{for} \quad \beta \to \infty, \\ 1 - \frac{3}{2}\phi & \text{for} \quad \beta \to 0. \end{cases}$$

Assignment 1 : Ok so now how about nanoparticle based fluids?

Warrier, P., Yuan, Y., Beck, M. P., & Teja, A. S. (2010). Heat transfer in nanoparticle suspensions: modeling the thermal conductivity of nanofluids. AIChE journal, 56(12), 3243-3256

Assignment 2. What's the role of the Kapitza resistance in polymer composites?

Ma, T., Zhao, Y., Ruan, K., Liu, X., Zhang, J., Guo, Y., ... & Gu, J. (2019). Highly thermal conductivities, excellent mechanical robustness and flexibility, and outstanding thermal stabilities of aramid nanofiber composite papers with nacre-mimetic layered structures. ACS Applied Materials & Interfaces, 12(1), 1677-1686.

Ruan, K., Shi, X., Guo, Y., & Gu, J. (2020). Interfacial thermal resistance in thermally conductive polymer composites: a review. Composites Communications, 22, 100518

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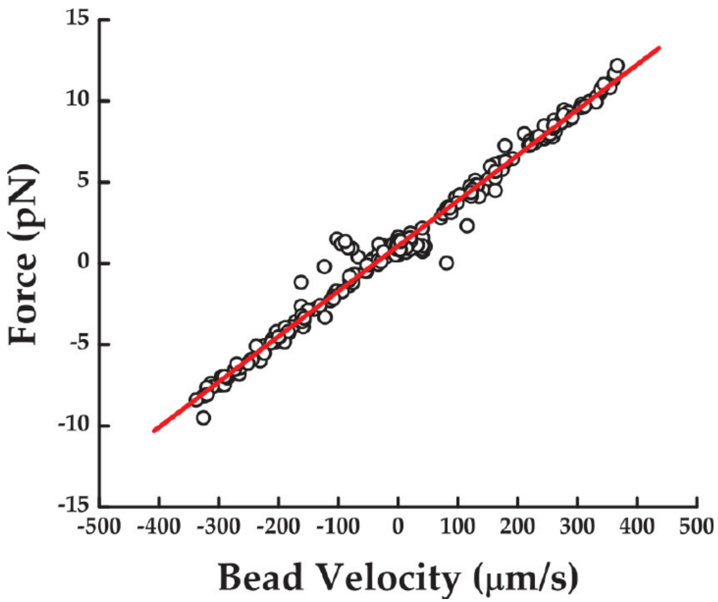
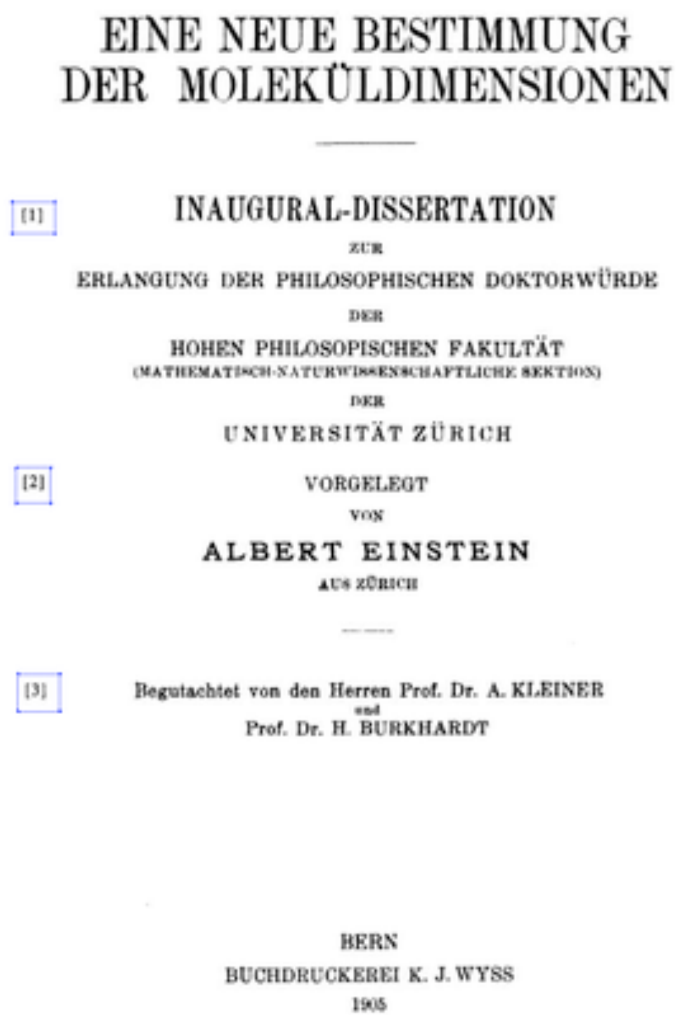


Neuman, K. C., & Nagy, A. (2008). Single-molecule force spectroscopy: optical tweezers, magnetic tweezers and atomic force microscopy. *Nature methods*, *5*(6), 491-505.



EINE NEUE BESTIMMUNG
DER MOLEKÜLDIMENSIONEN

[1] INAUGURAL-DISSERTATION

ZUR

ERLANGUNG DER PHILOSOPHISCHEN DOKTORWÜRDE

DER

HOHEN PHILOSOPHISCHEN FAKULTÄT
(MATHEMATISCH-NATURWISSENSCHAFTLICHE SEKTION)

DER

UNIVERSITÄT ZÜRICH

[2] VORGELEGT

VON

ALBERT EINSTEIN

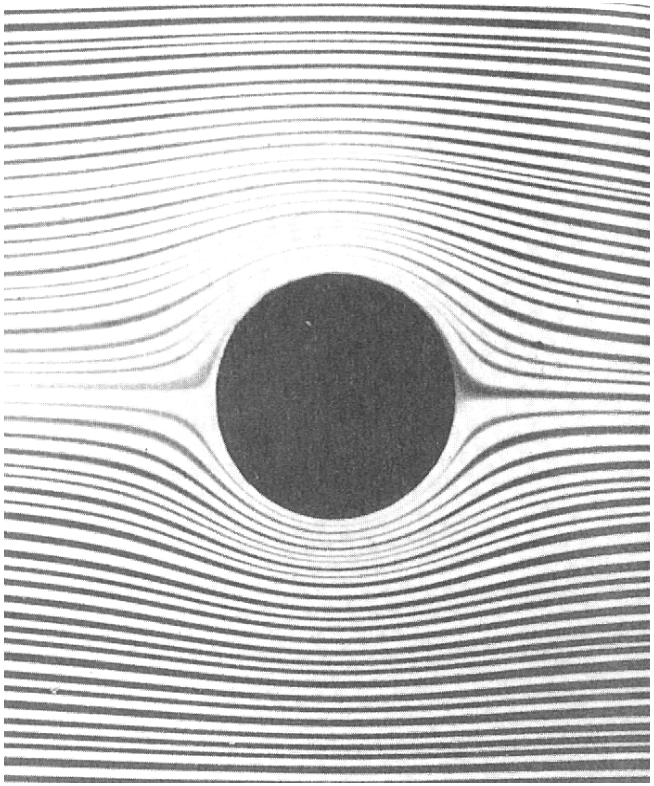
AUS ZÜRICH

[3] Begutachtet von den Herren Prof. Dr. A. KLEINER
und
Prof. Dr. H. BURKHARDT

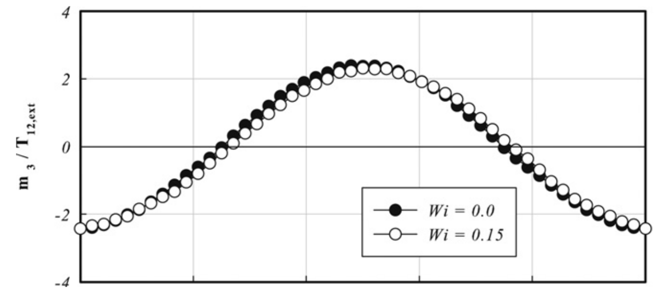
BERN
BUCHDRUCKEREI K. J. WYSS
1905

Disturbance velocity profile

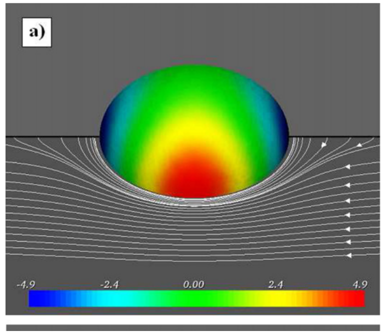
$$\mathcal{F}_s = \int_{A_s} \boldsymbol{n} \cdot \boldsymbol{\pi} dA$$

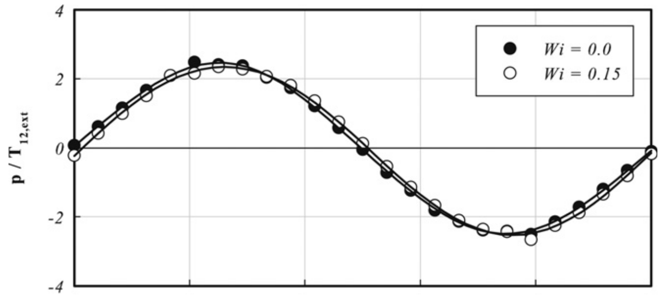


TANEDA S, 1956

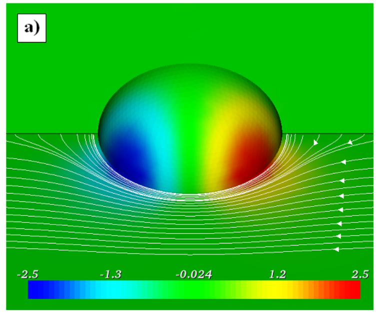


θ/π





θ/π



- What will you solve for?
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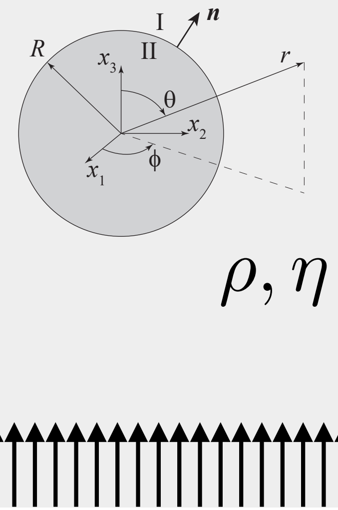
D'Avino, G., et al. J. Rheol. 52, 1331–1346 2008

Solid sphere in an incompressible viscous liquid

$$\boldsymbol{V} = V\boldsymbol{\delta}_3$$



$$\rho, \eta$$

$$r, \theta, v_r, v_\theta, \mathcal{P}, \eta, \rho, R, V$$

$$\nabla \cdot \boldsymbol{v} = 0$$

`Incompressibillity, constraint on the velocity`

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = \eta\nabla^2\boldsymbol{v} - \nabla\mathcal{P}$$

Navier Stokes Equations

$$\mathcal{P} = p^{\mathrm{L}} + \rho\phi$$

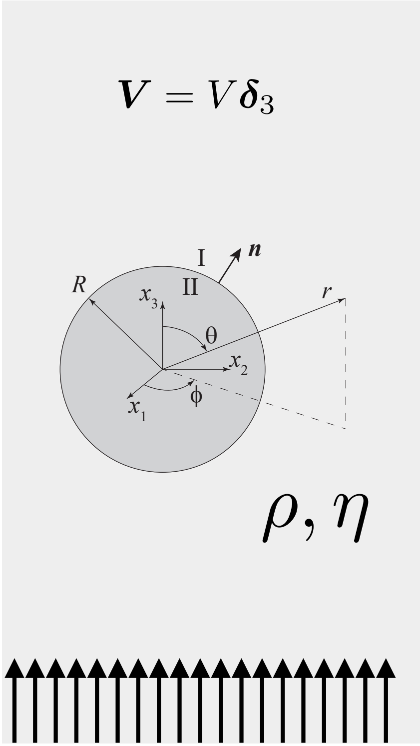
$$\nabla p^{\mathrm{L}} = \eta\nabla^2\boldsymbol{v}$$

Stokes equation

$$N_{\mathrm{Re}} \ll 1$$

$$N_{\mathrm{Re}} = \frac{\rho V R}{\eta}$$

$$v_r = v_r(r,\theta), \;\; v_\theta = v_\theta(r,\theta), \;\; v_\phi = 0$$



Incompressibillity, constraint on the velocity

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$$

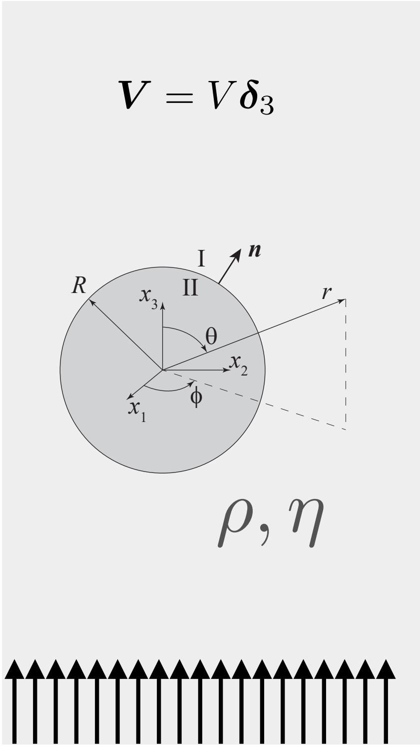
$$\boldsymbol{\nabla} p^{\mathrm{L}} = \eta \nabla^2 \boldsymbol{v}$$

$$\boldsymbol{\nabla}\boldsymbol{\nabla} p^{\mathrm{L}} = \boldsymbol{\nabla}(\eta \nabla^2 \boldsymbol{v})$$

$$\boldsymbol{\nabla}\boldsymbol{\nabla} p^{\mathrm{L}} = (\eta \nabla^2 \boldsymbol{\nabla} \cdot \boldsymbol{v})$$

$$\nabla^2 p^{\mathrm{L}} = 0$$

Pressure is a solution to a Laplace equation (same holds true for vorticity, can be shown by taking the curl)



$$\nabla^2 p^{\mathrm{L}} = 0$$

$$\boldsymbol{\nabla} p^{\mathrm{L}} = \eta \nabla^2 \boldsymbol{v}$$

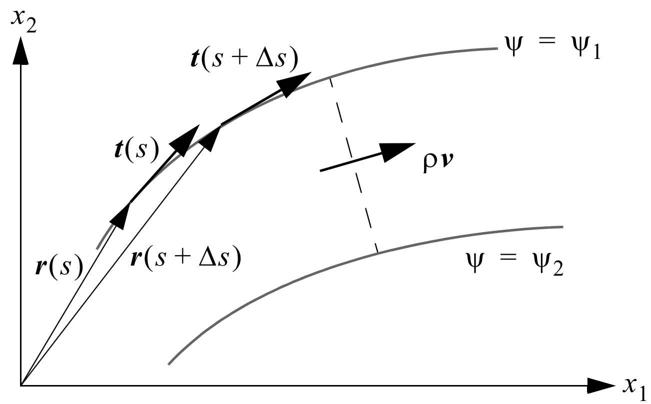
$$\nabla^4 \boldsymbol{v} = 0$$

$$\boldsymbol{v} = \boldsymbol{V} + \nabla\Phi$$
$$\boldsymbol{v} = \boldsymbol{V} + \boldsymbol{v}_i$$

$$\nabla^4 \Phi = 0$$

Pressure is a solution to a Laplace equation (same holds true for vorticity, can be shown by taking the curl)
the components of v are solutions to the BIHARMONIC equation

Stream function



time-independent, or constant density

$$\cancelto{0}{\frac{\partial \rho}{\partial t}} = -\nabla \cdot (\mathbf{v}\rho)$$

$$\frac{\partial}{\partial x_1}(\rho v_1) + \frac{\partial}{\partial x_2}(\rho v_2) = 0$$

Time independent,
2D or Axysymmetric flows

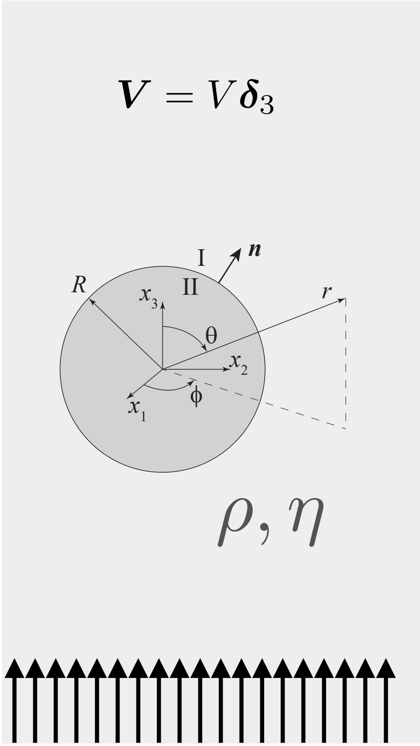
$$\frac{\partial}{\partial x_1}\left(\frac{\partial \psi}{\partial x_2}\right) - \frac{\partial}{\partial x_2}\left(\frac{\partial \psi}{\partial x_1}\right) = 0$$

STREAM function

$$\rho v_1 = \frac{\partial \psi}{\partial x_2}, \qquad \rho v_2 = -\frac{\partial \psi}{\partial x_1}$$

spherical coordinates

$$v_r = \frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}, \qquad v_\theta = -\frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}.$$



stream function as a solution method

$$\nabla \cdot \boldsymbol{v} = 0 \qquad \left[\frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]^2\psi = E^4\psi = 0$$

Problem transformed from a system of a first-order and two second-order differential equations to a the single fourth-order differential equation

Boundary conditions (4 needed)

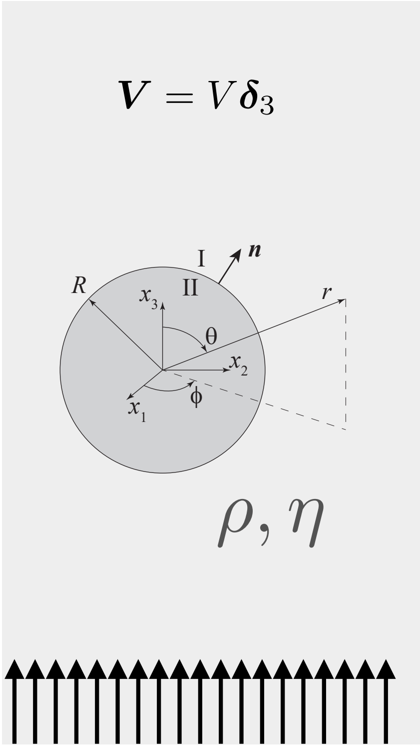
Far field conditions

$$v_r(\infty,\theta) \to V\cos\theta, \qquad \frac{\partial\psi}{\partial\theta}(\infty,\theta) \to Vr^2\sin\theta\cos\theta$$

$$v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}$$

$$v_\theta(\infty,\theta) \to -V\sin\theta, \qquad \frac{\partial\psi}{\partial r}(\infty,\theta) \to Vr\sin^2\theta$$

$$v_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}$$



Boundary conditions (4 needed)

- solid boundary condition + absence of mass transfer

$$\boldsymbol{v}^{\mathrm{s}} = \boldsymbol{0}$$
$$\boldsymbol{n} = \boldsymbol{\delta}_r$$
$$\longrightarrow \quad v_r(R,\theta) = 0, \qquad \frac{\partial \psi}{\partial \theta}(R,\theta) = 0$$

- jump balance for momentum (in absence mass transfer)

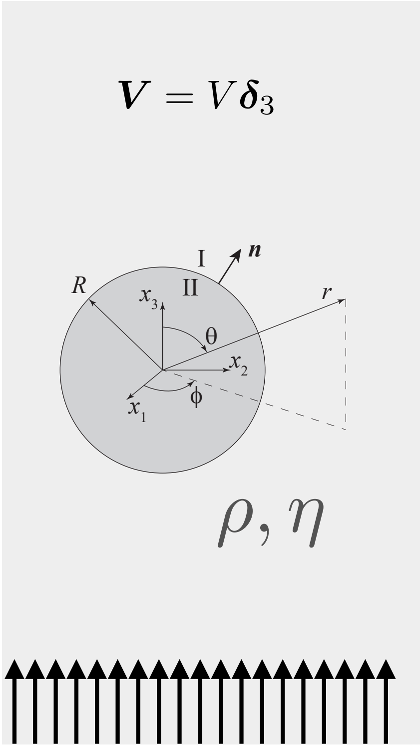
$$\boldsymbol{n} \cdot \boldsymbol{\pi}^{\mathrm{I}} = \boldsymbol{n} \cdot \boldsymbol{\pi}^{\mathrm{II}}$$

Newtonian

$$\boldsymbol{\xi}_{\|} = \xi_{\mathrm{slip}} \boldsymbol{\delta}_{\|}$$

$$v_\theta(R,\theta) = \xi_{\mathrm{slip}} \eta \left[r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]_{r=R}$$

$$\frac{\partial \psi}{\partial r}(R,\theta) = \xi_{\mathrm{slip}} \eta \left[r^2 \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial \psi}{\partial r}\right)\right]_{r=R}$$



stream function as a solution method

$$E^4\psi = 0 \qquad \psi(r,\theta)$$

separation of variables

Far field conditions

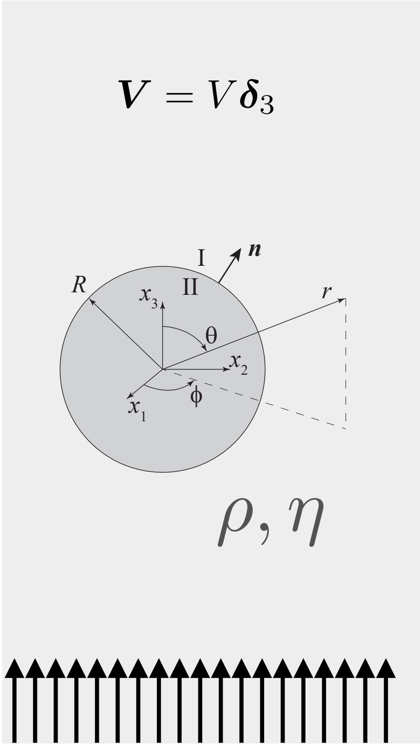
$$\frac{\partial\psi}{\partial r}(\infty,\theta) \to V r \sin^2\theta$$

$$\frac{\partial\psi}{\partial\theta}(\infty,\theta) \to V r^2 \sin\theta\cos\theta$$

$$\psi(r,\theta) = f(r)\sin^2\theta$$

$$[E^2]^2\psi = E^4(f\sin^2\theta) = \sin^2\theta\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right)^2 f$$

$$\boxed{f(r) = C_1 r^{-1} + C_2 r + C_3 r^2 + C_4 r^4}$$



stream function as a solution method

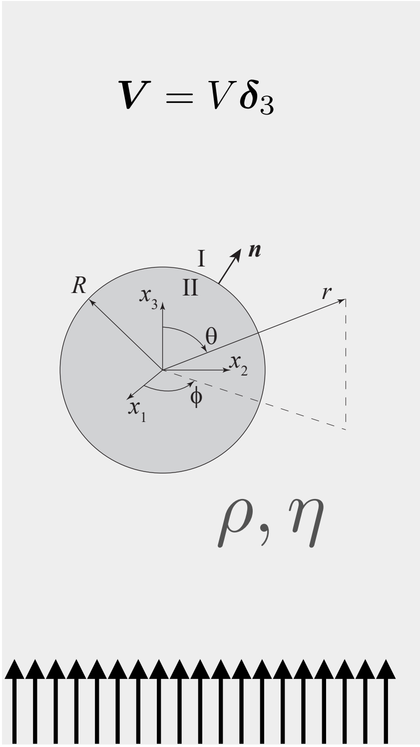
$$f(r) = C_1 r^{-1} + C_2 r + C_3 r^2 + C_4 r^4$$

4 B.C.

$$\psi(r,\theta) = V\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{3}{4}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{r}{R}\right) + \frac{1}{4}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)\right] R^2 \sin^2\theta$$

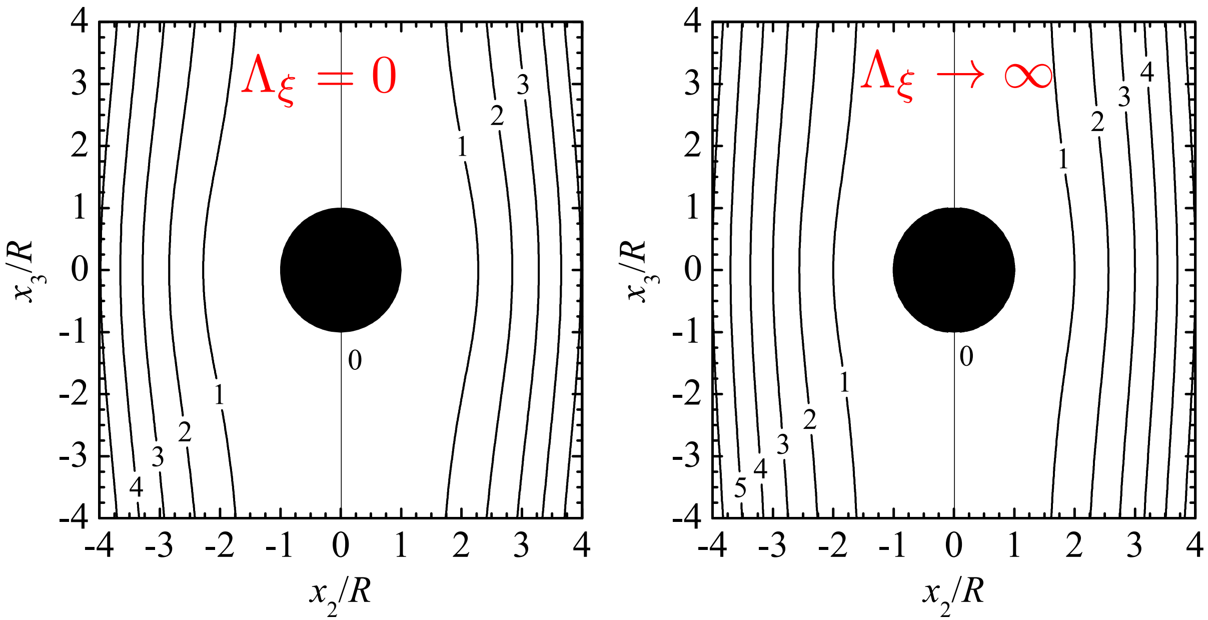
$$\Lambda_\xi = \frac{\xi_{\mathrm{slip}}\eta}{R}$$ dimensionless slip coefficient

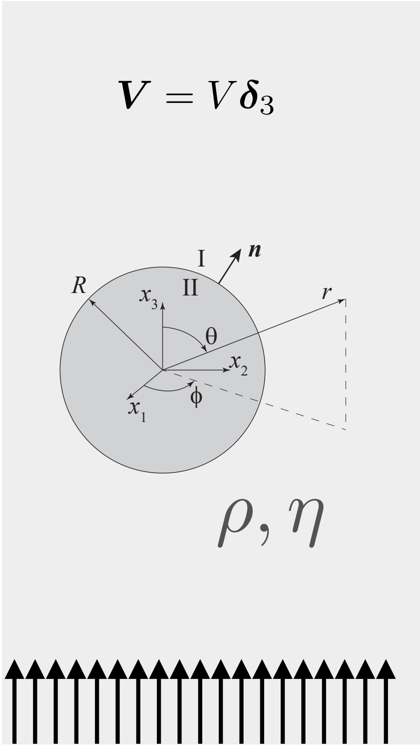
$$\Lambda_\xi = 0$$ Stokes' solution (1851)



$$\psi(r,\theta) = V\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{3}{4}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{r}{R}\right) + \frac{1}{4}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)\right]R^2\sin^2\theta$$

$$\psi/(VR^2)$$



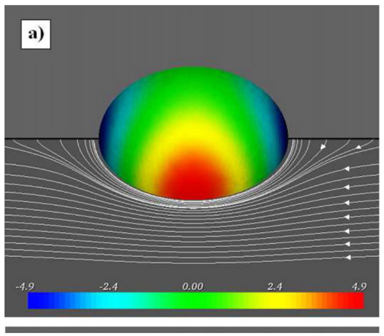


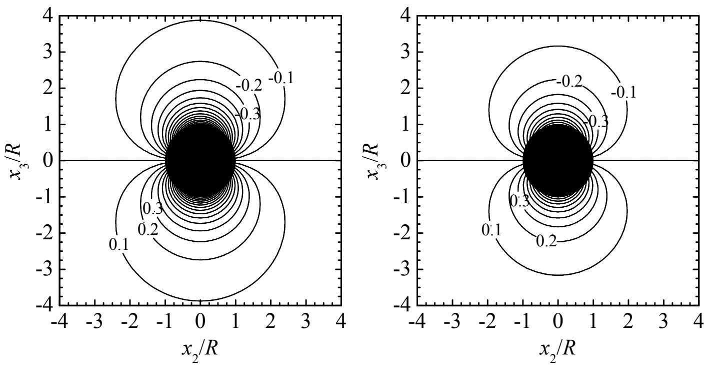
$$\psi(r,\theta) = V\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{3}{4}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{r}{R}\right) + \frac{1}{4}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)\right]R^2\sin^2\theta$$

$$\frac{v_r}{V} = \left[1 - \frac{3}{2}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{R}{r}\right) + \frac{1}{2}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)^3\right]\cos\theta$$

$$\frac{v_\theta}{V} = -\left[1 - \frac{3}{4}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{R}{r}\right) - \frac{1}{4}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)^3\right]\sin\theta$$

$$\frac{p^{\mathrm{L}}}{\eta V/R} = -\frac{3}{2}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)^2\cos\theta \qquad p^{\mathrm{L}}(\infty,\theta)\to 0.$$





$\Lambda_\xi = 0$ $\qquad$ $\Lambda_\xi \to \infty$

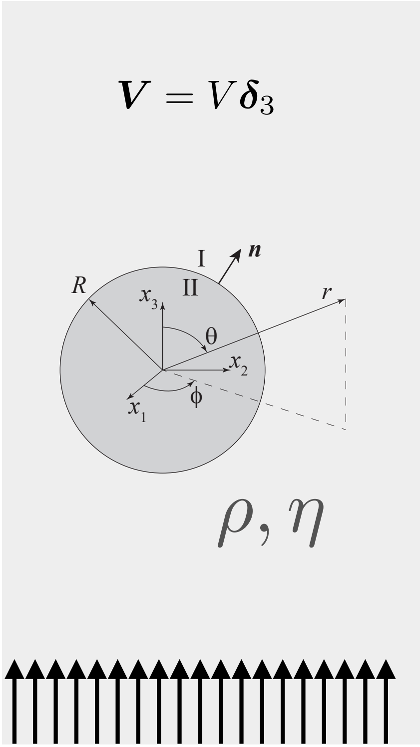
Assignment 3 :

Plot velocity and pressure profiles and then calculate absolute values of the pressures which occur.

What is the meaning of the two terms in the flow field (Stokeslet and stresslet)

Visser, A. W. (2001). Hydromechanical signals in the plankton. Marine Ecology Progress Series, 222, 1-24.

Could interfacial slip be measured in this way?



$$\nabla^2 p^{\mathrm{L}} = 0$$

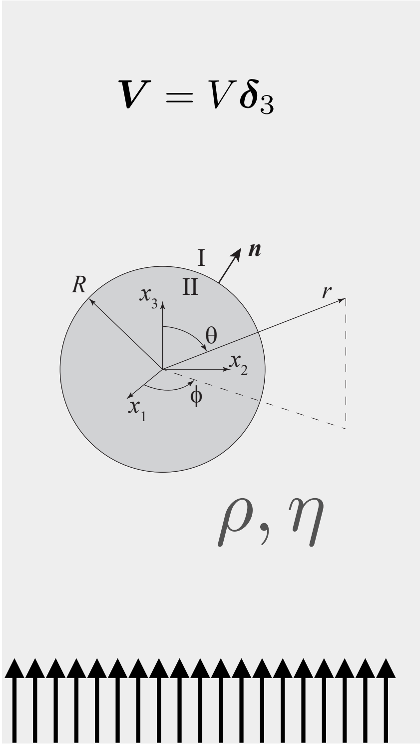
$$\boldsymbol{\nabla} p^{\mathrm{L}} = \eta \nabla^2 \boldsymbol{v}$$

$$\nabla^4 \boldsymbol{v} = 0$$

$$\boldsymbol{v} = \boldsymbol{V} + \nabla \Phi$$
$$\boldsymbol{v} = \boldsymbol{V} + \boldsymbol{v}_i$$

$$\nabla^4 \Phi = 0$$

Pressure is a solution to a Laplace equation (same holds true for vorticity, can be shown by taking the curl)
the components of v are solutions to the BIHARMONIC equation



The fundamental solutions to Laplace's equation are *harmonic* functions:

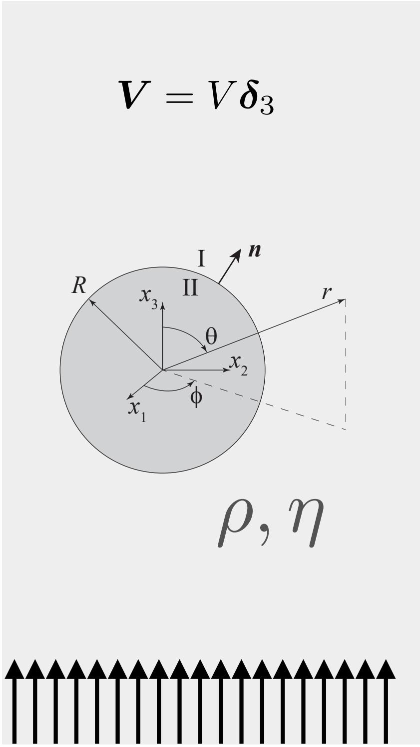
$$g(r) = \frac{1}{r} = \frac{1}{\sqrt{x_i x_i}}$$

$$\nabla^2 g(r) = \frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}\frac{1}{\sqrt{x_i x_i}} = \frac{\partial}{\partial x_j}\left(-\frac{1}{2} * 2\frac{x_i}{|\mathbf{x}|^{\frac{3}{2}}}\delta_{ij}\right) = -\frac{\partial}{\partial x_j}\frac{x_j}{|\mathbf{x}|^{\frac{3}{2}}}$$

$$= -\frac{3}{|\mathbf{x}|^{\frac{3}{2}}} + \frac{3}{2} * 2x_j\frac{x_j}{|\mathbf{x}|^{\frac{5}{2}}} = 0$$

but, $\nabla\frac{1}{r} = \frac{x_i}{r^3}$ is also a solution.

and so is $\nabla\frac{x_i}{r^3} = \frac{x_i x_j}{r^5} - \frac{\delta_{ij}}{3r^3}$ and so on.



- Decaying harmonics:

$$\phi_{-(n+1)} = \frac{(-1)^n}{1\cdot 3\cdot 5\cdots(2n-1)} \frac{\partial^{n-1}(1/r)}{\partial x_i \partial x_j \partial x_k \cdots}, \quad n = 0,1,2, \ldots$$

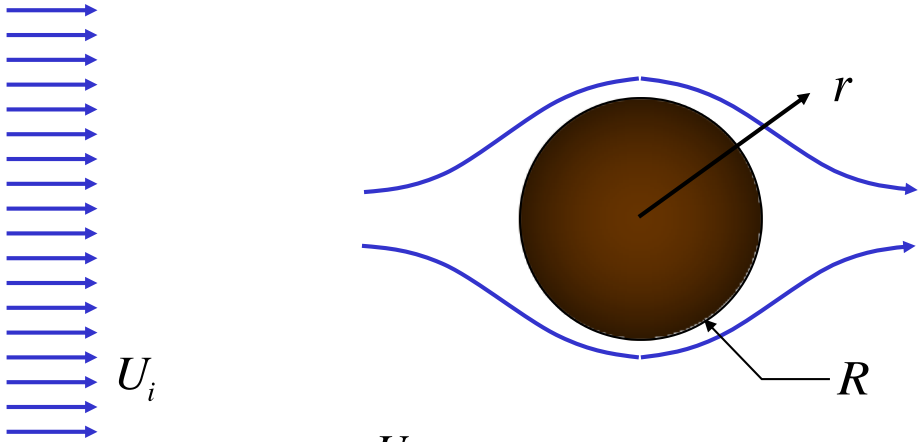
$$\phi_{-(n+1)} = \frac{1}{r}, \frac{x_i}{r^3}, \frac{x_i x_j}{r^5} - \frac{\delta_{ij}}{3r^3}, \cdots$$

- Growing harmonics:

$$r^{2n+1}\phi_{-(n+1)} = 1,\ x_i, \left(x_i x_j - \frac{r^2}{3}\delta_{ij}\right), \cdots$$

$$\boldsymbol{v}_i = \frac{\boldsymbol{x}_i}{2\eta} p^L + \boldsymbol{v}_i^{(h)} \qquad \nabla^2 \boldsymbol{v}_i^{(h)} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v}_i = 0$$



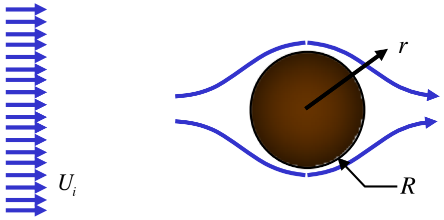
$$p = \mu A \frac{U_i x_i}{r^3}; \quad v_i = U_i + \frac{x_i p}{2\mu} + v_i^{(H)}$$

added the far field velocity to the disturbance flow

$$v_i = U_i + \frac{x_i}{2} A \frac{U_k x_k}{r^3} + v_i^{(H)}$$

$$v_i^{(H)} = B' U_k \frac{\delta_{ik}}{r} + C' U_k \left(\frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} \right)$$

$$v_i = U_i + \frac{U_k}{2} \left[A \frac{x_k x_i}{r^3} + B \frac{\delta_{ik}}{r} + C \left(\frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} \right) \right]$$



$$p = \mu A \frac{U_i x_i}{r^3}$$

$$v_i = U_i + \frac{U_k}{2}\left[A\frac{x_k x_i}{r^3} + B\frac{\delta_{ik}}{r} + C\left(\frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} \right) \right]$$

$$\text{continuity equation}: \quad \frac{\partial v_i}{\partial x_i} = 0$$

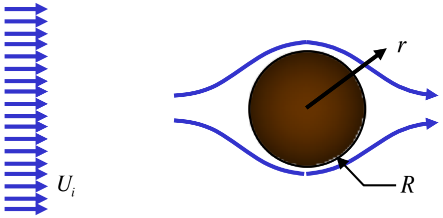
$$\frac{\partial v_i}{\partial x_i} = \frac{U_k}{2}\left[A\frac{\partial}{\partial x_i}\left(\frac{x_k x_i}{r^3} \right) + B\frac{\partial}{\partial x_i}\frac{\delta_{ik}}{r} + C\frac{\partial}{\partial x_i}\left(\frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} \right) \right]$$

$$\text{but,} \quad \frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} = \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_k}\frac{1}{r}, \quad \text{so} \quad \frac{\partial}{\partial x_i}\left(\frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} \right) = \frac{\partial}{\partial x_k}\nabla^2 \frac{1}{r} = 0$$

$$\frac{\partial v_i}{\partial x_i} = \frac{U_k}{2}\left[A\frac{\partial}{\partial x_i}\left(\frac{x_k x_i}{r^3} \right) + B\frac{\partial}{\partial x_i}\frac{\delta_{ik}}{r} \right] = \frac{U_k}{2}\left[3A\frac{x_k}{r^3} + A\frac{\delta_{ik} x_i}{r^3} - 3A x_k x_i \frac{x_i}{r^5} - B\frac{\delta_{ik} x_i}{r^3} \right]$$

$$= \frac{U_k}{2}\left[A\frac{x_k}{r^3} - B\frac{x_k}{r^3} \right] = 0$$

$$\boxed{A = B}$$



$$p = \mu A \frac{U_i x_i}{r^3}$$

$$v_i = U_i + \frac{U_k}{2}\left[A\left(\frac{x_k x_i}{r^3} + \frac{\delta_{ik}}{r} \right) + C\left(\frac{x_i x_k}{r^5} - \frac{\delta_{ik}}{3r^3} \right) \right]$$

$$\text{boundary condition: } v_i = 0 \text{ on } r = R$$

$$0 = U_i + \frac{U_k}{2}\left[A\left(\frac{x_k x_i}{R^3} + \frac{\delta_{ik}}{R} \right) + C\left(\frac{x_i x_k}{R^5} - \frac{\delta_{ik}}{3R^3} \right) \right]_{r=R}$$

$$0 = U_i + \frac{U_k}{2}\left[\left(A + \frac{C}{R^2} \right)\left(\frac{x_k x_i}{R^3} \right) + \left(A - \frac{C}{3R^2} \right) \frac{\delta_{ik}}{R} \right]_{r=R}$$

$$0 = U_i\left[1 + \frac{1}{2}\left(A - \frac{C}{3R^2} \right) \right] + \frac{U_k}{2}\left[\left(A + \frac{C}{R^2} \right)\left(\frac{x_k x_i}{R^3} \right) \right]$$

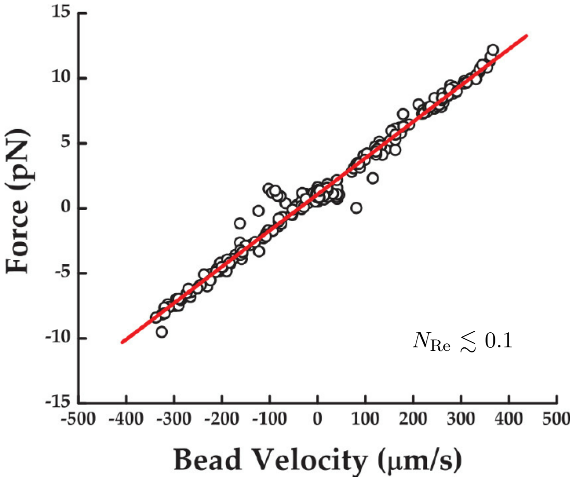
$$1 + \frac{1}{2R}\left(A - \frac{C}{3R^2} \right) = 0, \quad A + \frac{C}{R^2} = 0 \qquad A = -\frac{3}{2}R; \quad C = \frac{3}{2}R^3$$

vector form of the velocity and pressure field

$$\boldsymbol{v} = \left[\left(1 - \frac{3}{4}\frac{R}{r} - \frac{1}{4}\frac{R^3}{r^3} \right) \boldsymbol{\delta} - \frac{3}{4}\left(\frac{R}{r^3} - \frac{R^3}{r^5} \right) \boldsymbol{r}\boldsymbol{r} \right] \cdot \boldsymbol{V},$$

$$p^{\mathrm{L}} = -\frac{3}{2}\,\eta R\,\frac{\boldsymbol{r}\cdot\boldsymbol{V}}{r^3}.$$

optical tweezer



$$\boldsymbol{\mathcal{F}}_{\mathrm{s}} = \int_{A_{\mathrm{s}}} \boldsymbol{n} \cdot \boldsymbol{\pi} dA$$

$$\boldsymbol{\pi} = p^{\mathrm{L}}\boldsymbol{\delta} - \eta\left[\boldsymbol{\nabla v} + (\boldsymbol{\nabla v})^T\right]$$

$$\boldsymbol{n} = \boldsymbol{\delta}_r$$

$$\begin{aligned}\boldsymbol{\mathcal{F}}_{\mathrm{s}} &= -\int_0^{2\pi}\int_0^{\pi}\left([p^{\mathrm{L}}(R) + \tau_{rr}(R)]\boldsymbol{\delta}_r + \tau_{r\theta}(R)\boldsymbol{\delta}_\theta + \tau_{r\phi}(R)\boldsymbol{\delta}_\phi\right)R^2 \sin\theta d\theta d\phi \\ &= -2\pi R^2\int_0^{\pi}\left([p^{\mathrm{L}}(R) + \tau_{rr}(R)]\cos\theta + \tau_{r\theta}(R)\sin\theta\right)\sin\theta d\theta\ \boldsymbol{\delta}_3,\end{aligned}$$

$$\boldsymbol{\tau} = -\eta\left[\boldsymbol{\nabla v} + (\boldsymbol{\nabla v})^T\right]$$

$$\frac{v_r}{V} = \left[1 - \frac{3}{2}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{R}{r}\right) + \frac{1}{2}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)^3\right]\cos\theta$$

$$\tau_{rr}(R) = -6\frac{\eta V}{R}\frac{\Lambda_\xi}{1+3\Lambda_\xi}\cos\theta,$$

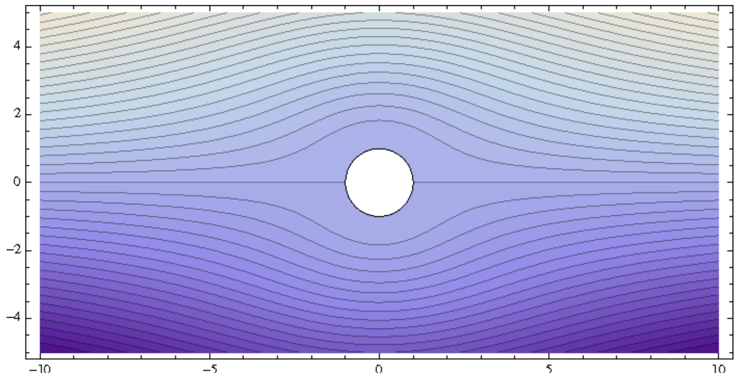
$$\frac{v_\theta}{V} = -\left[1 - \frac{3}{4}\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\left(\frac{R}{r}\right) - \frac{1}{4}\frac{1}{1+3\Lambda_\xi}\left(\frac{R}{r}\right)^3\right]\sin\theta$$

$$\tau_{r\theta}(R) = \frac{3}{2}\frac{\eta V}{R}\frac{1}{1+3\Lambda_\xi}\sin\theta,$$

$$\begin{aligned}\boldsymbol{\mathcal{F}}_{\mathrm{s}} &= \frac{3\pi R\eta V}{1+3\Lambda_\xi}\int_0^{\pi}\left([1+6\Lambda_\xi]\cos^2\theta + \sin^2\theta\right)\sin\theta d\theta\ \boldsymbol{\delta}_3 \\ &= 6\pi R\eta V\frac{1+2\Lambda_\xi}{1+3\Lambda_\xi}\boldsymbol{\delta}_3,\end{aligned}$$

Assignment 4 :

What if we have a cylinder? explain the Stokes paradox.



$$\boldsymbol{\nabla} p^{\mathrm{L}} = \eta \nabla^2 \boldsymbol{v}$$

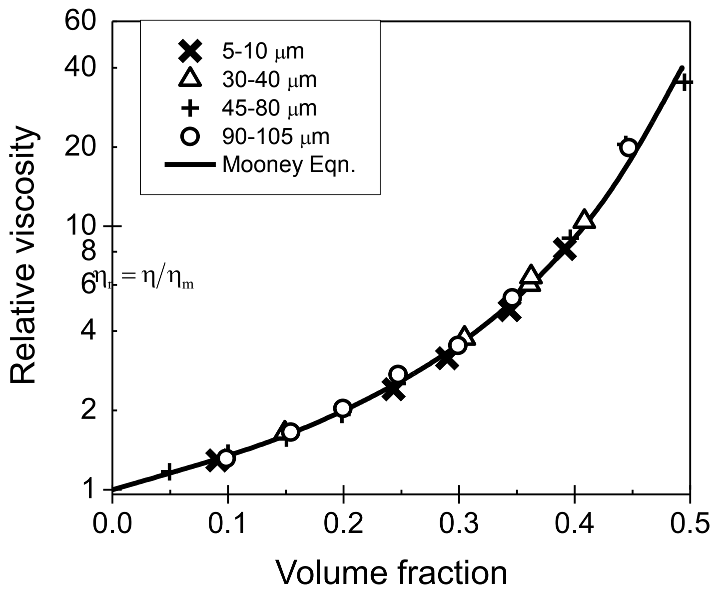
Flow around a cylinder - no zero Re solution?

Explain and rationalize the stokes paradox.

Excercis 17.9

1. Temperature fields around a single sphere
2. Effective medium theory
3. Stokes flow around a sphere
4. Einstein's law for the viscosity of suspensions

$$\eta = \eta_m(1 + 2.5\phi)$$



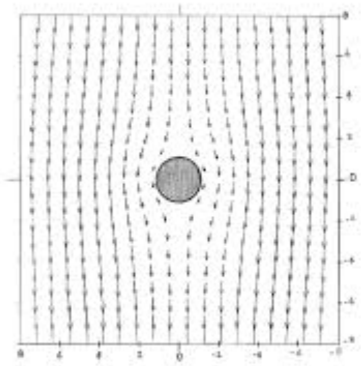
(after Lewis and Nielsen, 1968)

Effective medium theory : volume average of a quantity

$$\langle(\ldots)\rangle = \frac{1}{V_{\text{eff}}}\int_{V_{\text{eff}}}(\ldots)dV$$

rate of work done by the fluid

$$\dot{W} = -\int_{A_{\text{eff}}} v_i \pi_{ij} n_j dA$$



mechanical energy balance

Consider flow within a volume V that is bound by surface A with outward unit normal vector $\boldsymbol{n}$. The surface $A = A_1 + A_2 + A_s$ has an entrance with area A_1, an exit with area A_2, and an impermeable, non-entrance-exit surface with area A_s.

$$\frac{\partial}{\partial t}\Big(\frac{1}{2}\rho v^2 + \rho\phi\Big) = -\boldsymbol{\nabla}\cdot\Big(\boldsymbol{v}\frac{1}{2}\rho v^2 + \boldsymbol{v}\rho\phi + \boldsymbol{\pi}\cdot\boldsymbol{v}\Big) + \boldsymbol{\pi}:\boldsymbol{\nabla}\boldsymbol{v}$$

$$\int_V \frac{\partial}{\partial t}\Big(\frac{1}{2}\rho v^2+\boldsymbol{v}\rho\phi\Big)dV = -\int_V \boldsymbol{\nabla}\cdot\Big(\boldsymbol{v}\frac{1}{2}\rho v^2+\boldsymbol{v}\rho\phi+\boldsymbol{\pi}\cdot\boldsymbol{v}\Big)\Big]dV+\int_V \boldsymbol{\nabla}\cdot(\boldsymbol{\pi}\cdot\boldsymbol{v})dV.$$

rearrange and use DT

$$K_{\text{tot}} = \int_V \tfrac{1}{2}\rho v^2 dV$$

$$\Phi_{\text{tot}} = \int_V \rho\phi dV$$

$$\frac{d}{dt}(K_{\text{tot}} + \Phi_{\text{tot}}) = -\int_A \Big[(\boldsymbol{v}-\boldsymbol{v}_A)\frac{1}{2}\rho v^2 + (\boldsymbol{v}-\boldsymbol{v}_A)\rho\phi + p\boldsymbol{v} + \boldsymbol{\tau}\cdot\boldsymbol{v}\Big]\cdot\boldsymbol{n}dV + \int_V p\boldsymbol{\nabla}\cdot\boldsymbol{v}dV + \int_V \boldsymbol{\tau}:\boldsymbol{\nabla}\boldsymbol{v}dV$$

input/output and impermeable surface

$$\frac{d}{dt}(K_{\text{tot}} + \Phi_{\text{tot}}) = -\int_{A_1+A_2}\Big(\frac{1}{2}\rho v^2 + \rho\phi + p\Big)\boldsymbol{v}\cdot\boldsymbol{n}dV - \int_{A_{\text{sm}}}(\boldsymbol{\pi}\cdot\boldsymbol{v})\cdot\boldsymbol{n}dV + \int_V p\boldsymbol{\nabla}\cdot\boldsymbol{v}dV + \int_V \boldsymbol{\tau}:\boldsymbol{\nabla}\boldsymbol{v}dV$$

rate of work done by fluid

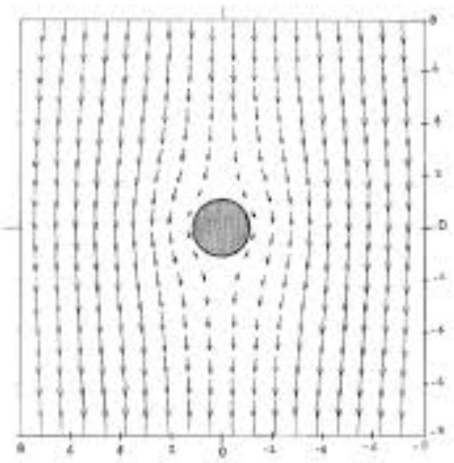
ρ and ϕ are uniform over A_1 and A_2,

$$\frac{d}{dt}(K_{\text{tot}}+\Phi_{\text{tot}}) = -\Delta\Big[\rho\langle v\rangle\Big(\frac{1}{2}\frac{\langle v^3\rangle}{\langle v\rangle}+\phi+\frac{p}{\rho}\Big)A\Big]+\int_V p\boldsymbol{\nabla}\cdot\boldsymbol{v}dV+\int_V \boldsymbol{\tau}:\boldsymbol{\nabla}\boldsymbol{v}dV+\dot{W}, \tag{8.46}$$

$$\dot{W} = -\int_{A_{\text{sm}}}(\boldsymbol{\pi}\cdot\boldsymbol{v})\cdot\boldsymbol{n}dA$$

rate of work done by the fluid

$$\dot{W} = -\int_{A_{\mathrm{eff}}} v_i \pi_{ij} n_j dA$$



$$\boldsymbol{v} = \boldsymbol{v}^{(0)} + \boldsymbol{v}^{(1)}$$

$$v_i^{(0)} = \kappa_{ij}^{(0)} x_j$$

unperturbed flow

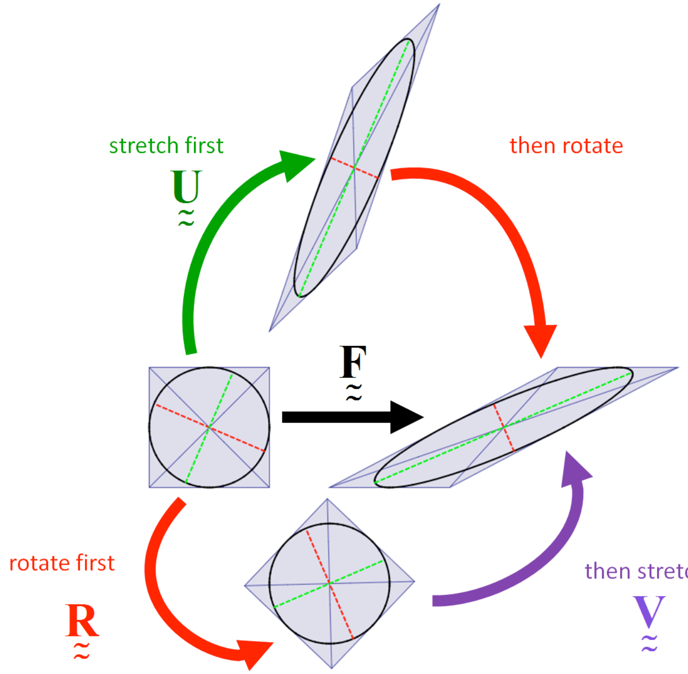
$$\boldsymbol{\kappa} = (\boldsymbol{\nabla} \boldsymbol{v})^T$$

uniform, irrotational flow

$$\boldsymbol{\pi} = \boldsymbol{\pi}^{(0)} + \boldsymbol{\pi}^{(1)}$$

$$\pi_{ij}^{(0)} = -2\eta\kappa_{ij}^{(0)}$$

Polar decomposition

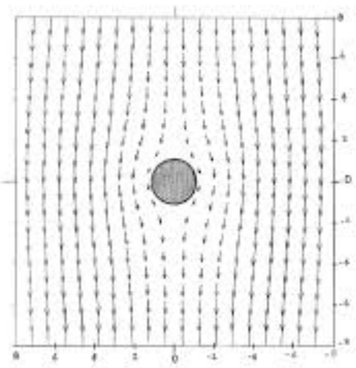


$$\underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{R}} \cdot \underset{\approx}{\mathbf{U}}$$

$$\underset{\approx}{\mathbf{F}} = \underset{\approx}{\mathbf{V}} \cdot \underset{\approx}{\mathbf{R}}$$

rate of work done by the fluid

$$\dot{W} = 2\eta\kappa_{ij}^{(0)}\kappa_{ij}^{(0)}V_{\mathrm{eff}} - \int_{A_{\mathrm{eff}}} v_i^{(0)}\pi_{ij}^{(1)}n_j\,dA - \int_{A_{\mathrm{eff}}} v_i^{(1)}\pi_{ij}^{(0)}n_j\,dA$$



disturbance velocity field in vectorial form

$$v_i^{(1)} = -\frac{5}{2}R^3\kappa_{jk}^{(0)}\frac{x_i x_j x_k}{r^5} - R^5\left(\kappa_{ij}^{(0)}\frac{x_j}{r^5} - \frac{5}{2}\kappa_{jk}^{(0)}\frac{x_i x_j x_k}{r^7}\right)$$

$$p^{\mathrm{L}(1)} = -5\eta R^3\kappa_{ij}^{(0)}\frac{x_i x_j}{r^5}$$

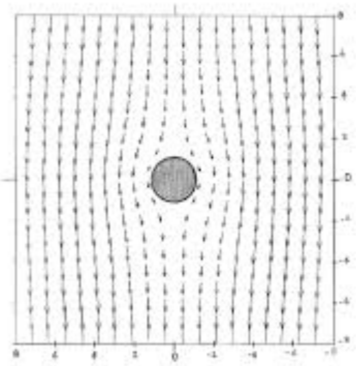
From disturbance velocity to stress acting a a surface (normal)

$$\tau_{il}^{(1)} = 5\eta\frac{R^3}{r^3}\left[n_i n_j \kappa_{jl}^{(0)} + \kappa_{ij}^{(0)}n_j n_l + (\delta_{il} - 5n_i n_l)n_j\kappa_{jk}^{(0)}n_k\right]$$

rate of work done by the fluid for a single large particle

$$\dot{W} = 2\eta\kappa_{ij}^{(0)}\kappa_{ij}^{(0)}V_{\text{eff}} - \int_{A_{\text{eff}}} v_i^{(0)}\pi_{ij}^{(1)}n_j\,dA - \int_{A_{\text{eff}}} v_i^{(1)}\pi_{ij}^{(0)}n_j\,dA$$

$$\tau_{il}^{(1)} = 5\eta\frac{R^3}{r^3}\left[n_i n_j \kappa_{jl}^{(0)} + \kappa_{ij}^{(0)}n_j n_l + (\delta_{il} - 5n_i n_l)n_j\kappa_{jk}^{(0)}n_k\right]$$



$$-\int_{A_{\text{eff}}} v_i^{(1)}\pi_{il}^{(0)}n_l\,dA = -5\eta\kappa_{il}^{(0)}\kappa_{jk}^{(0)}R^3\int_{A_{\text{eff}}} n_i n_j n_k n_l\,\frac{dA}{r^2}$$

$$-\int_{A_{\text{eff}}} v_i^{(0)}p^{\mathrm{L}(1)}n_i\,dA = 5\eta\kappa_{ij}^{(0)}\kappa_{kl}^{(0)}R^3\int_{A_{\text{eff}}} n_i n_j n_k n_l\,\frac{dA}{r^2}$$

$$-\int_{A_{\text{eff}}} v_i^{(0)}\tau_{il}^{(1)}n_l\,dA = 5\eta R^3\int_{A_{\text{eff}}}\left(3\kappa_{ij}^{(0)}\kappa_{kl}^{(0)}n_i n_j n_k n_l - \kappa_{ij}^{(0)}\kappa_{ik}^{(0)}n_j n_k\right)\frac{dA}{r^2}$$

$$\int_{A_{\text{eff}}} n_j n_k\,\frac{dA}{r^2} = 4\pi\,\frac{1}{3}\,\delta_{jk}$$

$$\int_{A_{\text{eff}}} n_i n_j n_k n_l\,\frac{dA}{r^2} = 4\pi\,\frac{1}{15}\,(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\dot{W} = 2\eta\kappa_{ij}^{(0)}\kappa_{ij}^{(0)}\,V_{\text{eff}} + \eta\kappa_{ij}^{(0)}\kappa_{ij}^{(0)}\,\frac{4}{3}\pi R^3$$

$$\dot{W} = 2\eta\kappa_{ij}^{(0)}\kappa_{ij}^{(0)}\left(V_{\text{eff}} + \frac{2}{3}\pi R^3\right) = 2\eta\kappa_{ij}^{(0)}\kappa_{ij}^{(0)}V_{\text{eff}}\left(1 + \frac{1}{2}\phi\right)$$

rate of work done by the fluid for any particles

$$r = |\boldsymbol{x}| \text{ by } |\boldsymbol{x} - \boldsymbol{x}'|$$

$$\frac{1}{V_{\text{eff}}} \int_{V_{\text{eff}}} \left(5 \frac{|\boldsymbol{x} - \boldsymbol{x}'|}{R} + \frac{R}{|\boldsymbol{x} - \boldsymbol{x}'|} \right) d^3 x' = \frac{4\pi R^3}{3V_{\text{eff}}} \left(\frac{3R_{\text{eff}}^2 - r^2}{2R^2} + \frac{15R_{\text{eff}}^4 + 10R_{\text{eff}}^2 r^2 - r^4}{4R^4} \right)$$

$$v_i^{(1)} = -\frac{4\pi R^3}{3V_{\text{eff}}} \kappa_{ij}^{(0)} x_j$$

$$v_i = \kappa_{ij}^{(0)} x_j (1 - \phi)$$

modified flow field

$$\dot{W} = 2\eta_{\text{eff}} \kappa_{ij}^{(0)} \kappa_{ij}^{(0)} V_{\text{eff}} (1 - 2\phi)$$

$$\eta = \eta_m (1 + \color{red}{2.5}\phi)$$

Assignment 5 :

Compare the Stresslet and the Stokeslet
with the solution for the disturbance velocity field in shear flow

and discuss the Stokes Einstein relation

Assignment 5 :

Mackay, M. E., Dao, T. T., Tuteja, A., Ho, D. L., Van Horn, B., Kim, H. C., & Hawker, C. J. (2003). Nanoscale effects leading to non-Einstein-like decrease in viscosity. *Nature materials*, *2*(11), 762-766.

- Systems containing spherical particles provide an important example of heterogeneous systems comprised of one material dispersed in a second material; detailed solutions of transport problems around spheres can be used to develop effective medium theories for heterogeneous systems.
- By solving the steady-state temperature equation around a single sphere with an imposed temperature gradient far from the sphere, we obtain an expression for the effective thermal conductivity of a heterogeneous material containing dispersed spheres at small concentration.
- By solving the steady-state Navier-Stokes equation without inertia for flow around a single sphere, we derive Stokes' law for the friction coefficient of a sphere in terms of the fluid viscosity and radius of the sphere.
- By evaluating the increase in the rate of energy dissipation resulting from flow around a sphere and neglecting interactions between multiple spheres, we obtain Einstein's expression for the effective viscosity of a suspension at small concentration.