CIS 581

HW: Cameras and Convolution

Due: Sept. 13th, 2018 @ 3:00 pm

1.2 $I \odot f =$

Image I Kernel f output g

$$\begin{bmatrix} 0.5 & 2.0 & 1.5 \\ 0.5 & 1.0 & 0.0 \\ 2.0 & 0.5 & 1.0 \end{bmatrix} \quad \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix} \quad \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}$$

$g_{11} = (1)(0.5) + (0.5)(2) + (0)(0.5) + (0.5)(1) = \boxed{2}$

$g_{12} = (0)(0.5) + (1)(2.0) + (0.5)(1.5) + (0.5)(0.5) + (0)(1.0)$
$+ (0.5)(0) = \boxed{3}$

$g_{13} = (0)(2.0) + (1)(1.5) + (0.5)(1) + (0)(0) = \boxed{2}$

$g_{21} = (1)(0.5) + (0)(2.0) + (1)(0.5) + (0.5)(1) + (0)(2.0) + (0.5)(0.5)$
$= \boxed{1.75}$

$g_{22} = (0.5)(0.5) + (1)(2.0) + (0)(1.5) + (0)(0.5) + (1)(1) + (0.5)(0.0)$
$+ (0.5)(2.0) + (0)(0.5) + (0.5)(1.0) = \boxed{4.75}$

$g_{23} = (0.5)(2.0) + (1)(1.5) + (0)(1.0) + (1)(0.0) + (0.5)(0.5)$
$+ (0)(1.0) = \boxed{2.75}$

$$g_{31} = (1.0)(0.5) + (0.0)(1.0) + (1.0)(2.0) + (0.5)(0.5)$$
$$= \boxed{2.75}$$

$$g_{32} = (0.5)(0.5) + (1.0)(1.0) + (0.0)(0.0) + (0.0)(0.5)$$
$$+ (1.0)(1.0) + (0.5)(0.0) = \boxed{2.25}$$

$$g_{33} = (0.5)(1.0) + (1.0)(0.0) + (0.0)(0.5) + (1.0)(1.0)$$
$$= \boxed{1.5}$$

So $$I \odot f = \begin{bmatrix} 2 & 3 & 2 \\ 1.75 & 4.75 & 2.75 \\ 2.75 & 2.25 & 1.5 \end{bmatrix}$$

$$f \odot I =$$

f

$$\begin{bmatrix} 0.5 & 1.0 & 0.0 \\ 0.0 & 1.0 & 0.5 \\ 0.5 & 0.0 & 0.5 \end{bmatrix}$$

I

$$\begin{bmatrix} 0.5 & 2 & 1.5 \\ 0.5 & 1 & 0 \\ 2 & 0.5 & 1 \end{bmatrix}$$

output g

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{23} & g_{33} \end{bmatrix}$$

Assuming zero padding along the boundary, and 'same' output size,

$$g_{11} = (1.0)(0.5) + (0.0)(1.0) + (0.5)(0.0) + (1.0)(1.0) = \boxed{1.5}$$

$$g_{12} = (0.5)(0.5) + (1.0)(1.0) + (0.0)(0.5) + (2.0)(0.0) + (0.5)(1.0) + (1.0)(0.5) = \boxed{2.25}$$

$$g_{13} = (0.5)(1.0) + (1.0)(0.0) + (2.0)(1.0) + (0.5)(0.5) = \boxed{2.75}$$

$$g_{21} = (2.0)(0.5) + (1.5)(1.0) + (1.0)(0.0) + (0.0)(1.0) + (0.5)(0.5) + (1.0)(0.0) = \boxed{2.75}$$

$$g_{22} = (0.5)(0.5) + (2)(1) + (1.5)(0) + (0.5)(0) + (1)(1) + (0)(0.5) + (2)(0.5) + (0.5)(0) + (1)(0.5) = \boxed{4.75}$$

$$g_{23} = (0.5)(1) + (2.0)(0) + (0.5)(1) + (1)(0.5) + (2)(0) + (0.5)(0.5) = \boxed{1.75}$$

$$g_{31} = (2.0)(0.0) + (1.5)(1) + (1)(0.5) + (0)(0) = \boxed{2}$$

$$g_{32} = (0.5)(0) + (2)(1) + (1.5)(0.5) + (0.5)(0.5) + (1.0)(0) + (0)(0.5) = \boxed{3}$$

$$g_{33} = (0.5)(1.0) + (2)(0.5) + (0.5)(0) + (1)(0.5) = \boxed{2}$$

So $f \odot I = \begin{bmatrix} 1.5 & 2.25 & 2.75 \\ 2.75 & 4.75 & 1.75 \\ 2 & 3 & 2 \end{bmatrix}$

Therefore $I \odot f \equiv f \odot I$ doesn't hold.

2.1 $g_1 = I \otimes f_x \otimes f_y$

$$I \otimes f_x = \begin{bmatrix} 0.0 & 1.0 & -1.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 3.0 & -1.0 \end{bmatrix} \otimes \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ -1.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0 & 1.0 & -1.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 3.0 & -1.0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times (-1) \quad +$$

$$\begin{bmatrix} 0.0 & 1.0 & -1.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 3.0 & -1.0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ -3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

$$I \otimes f_x \otimes f_y = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ -3 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

$$f_{xy} = f_x \otimes f_y$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$g_2 = I \otimes f_{xy}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \\ 0 & 3 & -1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= I \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times (-1) + I \otimes \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times (-1) +$$

$$I \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times (-1) + I \otimes \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$I \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + I \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ -3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ -3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

2.2 addition and multiplication shown in 2.1

one $\otimes$ is (9) (9 m (multiplication) and 8 a (addition))

g_1 $(9)(9m+8a)(5) + 9m + 6a$

$= 405m + 360a + 9m + 6a = $ $\boxed{\begin{array}{l}\text{414 multiplication} \\ \text{366 addition}\end{array}}$

g_2 $(9)(9m+8a)(9) + (3)(9m) + 14a$

$= $ $\boxed{\begin{array}{l}\text{756 multiplication} \\ \text{662 addition}\end{array}}$

2.3 Edge detection

3.

$$I = \begin{bmatrix} 1 & 5 & 2 \\ 7 & 8 & 6 \\ 3 & 9 & 4 \end{bmatrix} \qquad g = \begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix}$$

possible filters

$$\begin{array}{ccccccccc} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 \\ \begin{bmatrix} 1&0&0\\0&0&0\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&1&0\\0&0&0\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&0&1\\0&0&0\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\1&0&0\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\0&1&0\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\0&0&1\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\0&0&0\\1&0&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\0&0&0\\0&1&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\0&0&0\\0&0&1 \end{bmatrix} \end{array}$$

resulting output

$$\begin{array}{ccccccccc} g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 & g_9 \\ \begin{bmatrix} 8&6&0\\9&4&0\\0&0&0 \end{bmatrix} & \begin{bmatrix} 7&8&6\\3&9&4\\0&0&0 \end{bmatrix} & \begin{bmatrix} 0&7&8\\0&3&9\\0&0&0 \end{bmatrix} & \begin{bmatrix} 5&2&0\\8&6&0\\9&4&0 \end{bmatrix} & \begin{bmatrix} 1&5&2\\7&8&6\\3&9&4 \end{bmatrix} & \begin{bmatrix} 0&1&5\\0&7&8\\0&3&9 \end{bmatrix} & \begin{bmatrix} 0&0&0\\5&2&0\\8&6&0 \end{bmatrix} & \begin{bmatrix} 0&0&0\\1&5&2\\7&8&6 \end{bmatrix} & \begin{bmatrix} 0&0&0\\0&1&5\\0&3&9 \end{bmatrix} \\ a & b & c & d & e & f & g & h & i \end{array}$$

$$\begin{cases} 4e + 9f + 6h + 9i = 20 \\ 8a + 7b + 5d + e = 29 \\ 6a + 8b + 7c + 2d + 5e + f = 43 \\ 6b + 8c + 2e + 5f = 10 \\ 9a + 3b + 8d + 7e + 5g + h = 62 \\ 4a + 9b + 3c + 6d + 8e + 7f + 2g + 5h + i = 52 \\ 4b + 9c + 6e + 8f + 2h + 5i = 30 \\ 9d + 3e + 8g + 7h = 15 \end{cases}$$

$$4d + 9e + 3f + 6g + 8h + 3i = 45$$

Solving:

$a = 3$

$b = 0$

$c = 0$

$d = 0$

$e = 5$

$f = 0$

$g = 0$

$h = 0$

$y = 0$

$$f = 3 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 5 \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4.

4.1

$$I_Q = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

since $I_Q \otimes f_1 = I_M$

assume flipped f_1 is $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$\begin{cases} e+f+h+i = 0.25 \\ d+e+f+g+h+i = 0.5 \\ d+e+g+h = 0.5 \\ d+g = 0.25 \\ b+e+h+c+f+i = 0.5 \\ a+b+c+d+e+f+g+h+i = 1 \\ a+b+d+e+g+h = 1 \\ a+d+g = 0.5 \\ b+c+e+f = 0.5 \\ a+b+c+d+e+f = 1 \\ a+b+d+e = 1 \end{cases}$$

$b+e = 0.5$

$b+c = 0.25$

$a+b+c = 0.5$

$a+b = 0.5$

$a = 0.25$

$$\left.\begin{aligned}a&=0.25\\ b&=0.25\\ c&=0\\ d&=0.25\\ e&=0.25\\ f&=0\\ g&=0\\ h&=0\end{aligned}\right\}$$

$$f_1 \text{ flipped} = \begin{bmatrix} 0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.25 & 0.25 \\ 0 & 0.25 & 0.25 \end{bmatrix}}$$

$i=0$

$$I_T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

since $I_T \otimes f_2 = I_M$

assume f_2 flipped $= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$\left\{\begin{aligned}i&=0.25\\ h+i&=0.5\\ g+h+i&=0.5\\ g+h&=0.25\\ f+i&=0.5\\ e+f+h+i&=1\\ d+e+f+g+h+i&=1\\ d+e+g+h&=0.5\end{aligned}\right.$$

$$\begin{aligned}c+f+i&=0.5\\ b+c+e+f+h+i&=1\\ a+b+c+d+e+f+g+h+i&=1\\ a+b+d+e+g+h&=0.5\\ c+f&=0.25\\ b+c+e+f&=0.5\\ a+b+c+d+e+f&=0.5\\ a+b+d+e&=0.25\end{aligned}$$

$$\begin{cases} a = 0 \\ b = 0 \\ c = 0 \\ d = 0 \\ e = 0.25 \\ f = 0.25 \\ g = 0 \\ h = 0.25 \\ i = 0.25 \end{cases}$$

$$\underset{\text{flipped}}{f_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.25 & 0.25 \\ 0 & 0.25 & 0.25 \end{bmatrix}$$

so $\boxed{f_2 = \begin{bmatrix} 0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$

4.2

$$f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.25 & 0.25 \\ 0 & 0.25 & 0.25 \end{bmatrix} \quad f_2 = \begin{bmatrix} 0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f = \frac{f_1 + f_2}{2} = \begin{bmatrix} 0.125 & 0.125 & 0 \\ 0.125 & 0.25 & 0.125 \\ 0 & 0.125 & 0.125 \end{bmatrix}$$

$$I_Q' = I_Q \otimes f$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0.125 & 0.125 & 0 \\ 0.125 & 0.25 & 0.125 \\ 0 & 0.125 & 0.125 \end{bmatrix}$$

since f flipped $= f$

$$= \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

$a = 1/4 + (1/8)(3) = .625$

$b = 1/4 + (1/8)(4) = .75$

$c = 1/4 + (1/8)(2) = .5$

$d = 1/8 = .125$

$e = 1/4 + (1/8)(4) = .75$

$f = 1/4 + (1/8)(6) = .1$

$g = 1/4 + (1/8)(4) = .75$

$h = (1/8)(2) = .25$

$i = 1/4 + (1/8)(2) = .5$

$j = 1/4 + (1/8)(4) = .75$

$k = 1/4 + (1/8)(3) = .625$

$l = (1/8)(2) = .25$

$m = 1/8 = .125$

$n = (1/8)(2) = .25$

$o = (1/8)(2) = .25$

$p = (1/8) = .125$

$$\boxed{I'_Q = I_Q \otimes f = \begin{bmatrix} 0.625 & 0.75 & 0.5 & 0.125 \\ 0.75 & 1 & 0.75 & 0.25 \\ 0.5 & 0.75 & 0.625 & 0.25 \\ 0.125 & 0.25 & 0.25 & 0.125 \end{bmatrix}}$$

$$I'_T = I_T \otimes f$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0.125 & 0.125 & 0 \\ 0.125 & 0.25 & 0.125 \\ 0 & 0.125 & 0.125 \end{bmatrix}$$

since I_T is I_Q flipped

I'_T is going to be I'_Q flipped

$$\boxed{I'_T = \begin{bmatrix} 0.125 & 0.25 & 0.25 & 0.125 \\ 0.25 & 0.625 & 0.75 & 0.5 \\ 0.25 & 0.75 & 1 & 0.75 \\ 0.125 & 0.5 & 0.75 & 0.625 \end{bmatrix}}$$

$$D(I'_Q, I'_T) = (.625-.125)^2 +$$
$$(.75-.25)^2 +$$
$$(.5-.25)^2 +$$
$$(.125-.125)^2 +$$
$$(.75-.25)^2 +$$
$$(1-.625)^2 +$$
$$(.75-.75)^2 +$$
$$(.25-.5)^2 +$$
$$(.5-.25)^2 +$$
$$(.75-.75)^2 +$$
$$(.625-1)^2 +$$
$$(.25-.75)^2 +$$
$$(.125-.125)^2 +$$
$$(.25-.5)^2 +$$
$$(.25-.75)^2 +$$
$$(.125-.625)^2 = \boxed{2.03125}$$

2.03125 is much smaller than $D(I'_Q, I'_T)$ which is 10. filter f is a form of blurring in both f_1 and f_2 directions. When you blur I_Q and I_T with f, it will make them appear more similiar, therefore smaller D. The smaller the D the easier for it to recognize.