Ant Colony Optimization

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CSC 499 Independent Research
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Topics

- Inspiration
- How to apply ACO to solve TSP
- Parameters

Inspiration

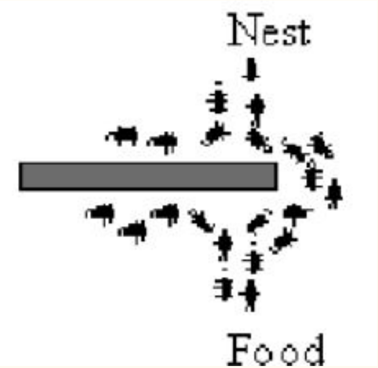
Real ants:

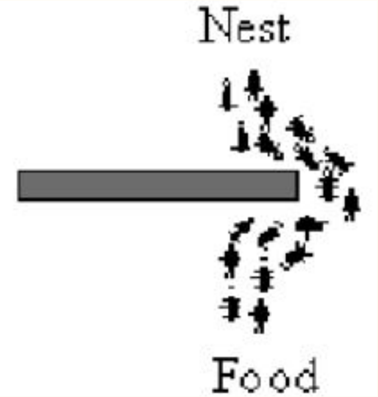
- start from nest to food and go back

Inspiration

Real ants:

- chose randomly
- communicate through pheromone
- are more likely to take the path with more pheromone(NOT guaranteed)





Inspiration

Artificial ants:

- start from a point and finally return to the point
- <u>memory</u>
- chose randomly
- communicate through pheromone
- <u>distance is given</u>
- are more likely to take the path with more pheromone/shorter distance(NOT guaranteed)

How to apply ACO to solve TSP?

- Edge Selection
- Pheromone Update
- Termination Criteria

Pseudo-code

```
procedure ACO_MetaHeuristic
    while(not_termination)
        edgeSelection()
        daemonActions()
        pheromoneUpdate()
    end while
  end procedure
```

Detailed Pseudo-Code

Edge Selection

Pheromone Update

Algorithm 8.1: Ant System for the traveling salesman problem

Input:

a complete non-directed graph $G = (V, E)$

a distance function $d : E \to \mathbb{R}$

a set of numerical parameters $\{\alpha, \beta, \rho, \tau_0, Q, nbAnts\}$

Postrelation: returns a Hamiltonian cycle of G

begin

  for *each edge* $(i, j) \in E$ **do** $\tau_{ij} \leftarrow \tau_0$

  while *stopping criteria not reached* **do**

    for *each ant* $k \in \{1, \ldots, nbAnts\}$ **do**

      put ant k on a randomly chosen vertex of V

      while *ant* k *has not visited all vertices of* V **do**

        let i be the vertex on which ant k is currently located

        let *Cand* be the set of unvisited vertices

        randomly choose $j \in Cand$ with respect to probability

$$p_{ij} = \frac{[\tau_{ij}]^{\alpha} \cdot [1/d_{ij}]^{\beta}}{\sum_{l \in Cand} [\tau_{il}]^{\alpha} \cdot [1/d_{il}]^{\beta}}$$

        move ant k to vertex j

    for *each edge* $(i, j) \in E$ **do** $\tau_{ij} \leftarrow \tau_{ij} \cdot (1 - \rho)$

    for *each ant* $k \in \{1, \ldots, nbAnts\}$ **do**

      let l_k be the length of the cycle built by ant k

      for *each edge* (i, j) *of the cycle built by ant* k **do**

        $\tau_{ij} \leftarrow \tau_{ij} + Q/l_k$

  return *the best Hamiltonian cycle built during the search process*

end

Edge Selection

$$p_{xy}^{k} = \frac{(\tau_{xy}^{\alpha})(\eta_{xy}^{\beta})}{\sum_{z \in \mathrm{allowed}_x} (\tau_{xz}^{\alpha})(\eta_{xz}^{\beta})}$$

Pheromone Update

$$\tau_{xy} \leftarrow (1-\rho)\tau_{xy} + \sum_k \Delta\tau_{xy}^k$$

$$\Delta\tau_{xy}^k = \begin{cases} Q/L_k & \text{if ant } k \text{ uses curve } xy \text{ in its tour} \\ 0 & \text{otherwise} \end{cases}$$

Termination Criteria

Improvement

Algorithm 8.1: Ant System for the traveling salesman problem

Input:
- a complete non-directed graph $G = (V, E)$
- a distance function $d : E \rightarrow \mathbb{R}$
- a set of numerical parameters $\{\alpha, \beta, \rho, \tau_0, Q, nbAnts\}$

Postrelation: returns a Hamiltonian cycle of G

begin
- **for** *each edge* $(i, j) \in E$ **do** $\tau_{ij} \leftarrow \tau_0$
- **while** *stopping criteria not reached* **do**
  - **for** *each ant* $k \in \{1, \ldots, nbAnts\}$ **do**
    - put ant k on a randomly chosen vertex of V
    - **while** *ant k has not visited all vertices of V* **do**
      - let i be the vertex on which ant k is currently located
      - let $Cand$ be the set of unvisited vertices
      - randomly choose $j \in Cand$ with respect to probability

$$p_{ij} = \frac{[\tau_{ij}]^{\alpha} \cdot [1/d_{ij}]^{\beta}}{\sum_{l \in Cand} [\tau_{il}]^{\alpha} \cdot [1/d_{il}]^{\beta}}$$

      - move ant k to vertex j
  - **for** *each edge* $(i, j) \in E$ **do** $\tau_{ij} \leftarrow \tau_{ij} \cdot (1 - \rho)$
  - **for** *each ant* $k \in \{1, \ldots, nbAnts\}$ **do**
    - let l_k be the length of the cycle built by ant k
    - **for** *each edge* (i, j) *of the cycle built by ant k* **do**
      - $\tau_{ij} \leftarrow \tau_{ij} + Q/l_k$
- **return** *the best Hamiltonian cycle built during the search process*

end

Termination Criteria

Iteration Threshold

- Get the coordinates of the cities

Then for t=1 to interation_threshold

For k=1 to l ant

For move_count =1 to n

Let ant move based on P_{ij}^{k}

Loop

Calculate L_k

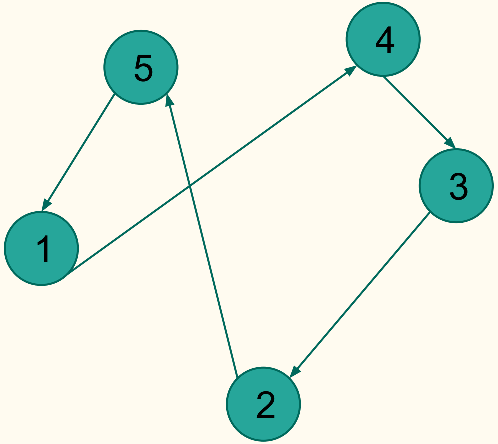
Loop

update pheromone by formula $\Delta\tau_{ij}$

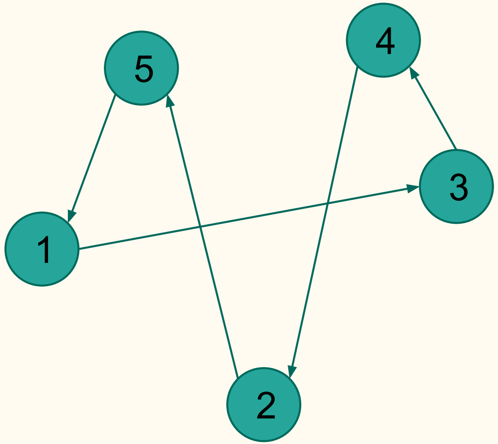
Loop

Visualization

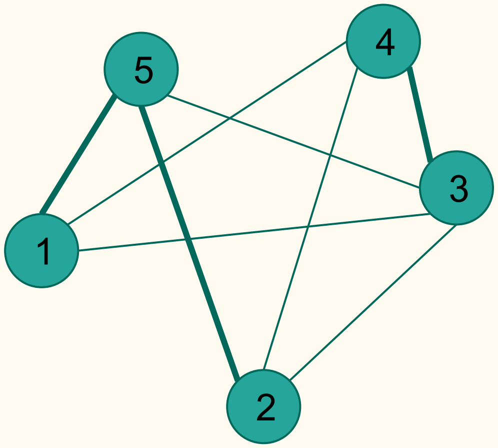
Iteration 1: 2 ants



Distance: 45



Distance: 55


5
4
3
1
2

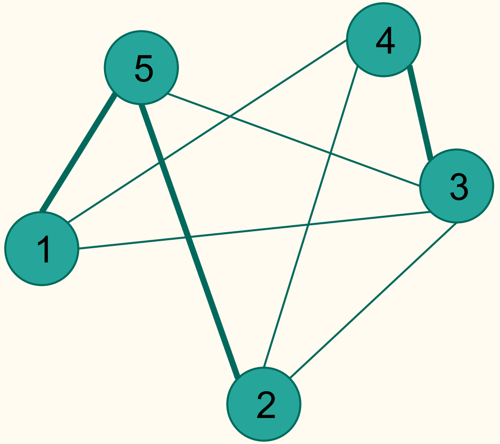
What will happen after the 3rd ant reached city 5?

He can choose 2 based on the pheromore

He can choose 3 based on the probability

He can choose 4 (shorter distance)

Better route found!



Parameters

Number of the ants

Pheromone evaporation speed

Impact of the pheromone

Impact of the distance

...

Reference

Solnon, C. (2013). *Ant Colony Optimization and Constraint Programming.* Somerset: Wiley.

Shekhawat, A., Poddar, P. & Boswal D.(2009). *Ant colony Optimization Algorithms: Introduction and Beyond*.

Malakar, G. (2017, October 15). Retrieved February 11, 2019, from https://www.youtube.com/watch?v=wfD5xlEcmuQ

ACO Implementation on Github.