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ON THE THEORY OF TWO COUPLED CAVITIES

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This work presents research results on a novel analytical model of electromagnetic systems coupling through small size holes. The key problem regarding interactions of two cavities through an aperture in separating screen of finite thickness without making assumption on smallness of any parameters is considered. We are the first to calculate on the base of rigorous electromagnetic approach the coupling coefficients of the cylindrical cavities within the limit of small aperture and infinitely thin separating screen. The numeric results of electromagnetic characteristic dependencies that have been impossible to perform on the base of previous models are given.

INTRODUCTION

The problem of electromagnetic systems coupling has been in the focus of scientific attention for over 40 years. The approach of tackling this problem with the use of the concepts of equivalent electric and magnetic dipole moments, suggested in [1,2], proved to be fruitful. On its base various electromagnetic characteristics of interacting objects have been studied (see [3-10] and literature cited therein). The key element this approach is employment of the "static" analysis used for determination the fields in the immediate vicinity of the hole. Clearly, this procedure is valid only if the hole dimensions are small compared to the wave length. Besides, the apertures have to be placed at a remote distance from the borders of the electromagnetic systems being considered. This notwithstanding, the developed methods allowed not only to calculate the number of important characteristics, but formulate (or lay the basis) for entirely new approaches for consideration of different RF-devices. This approach exerted considerable influence on the theory of slow-wave structures based on utilization of resonant properties of electromagnetic systems (disk-loaded waveguides, coupled-cavity chains, etc.)

However, even to this day, there have not been developed general methods of calculations of small aperture coupling coefficients from which the "static" results could be obtained by means of the limit transition $\omega a / c \to 0$. Development of such methods would permit not only to assess the region of applicability of "static" results, but also to expand the frontiers of problems regarding RF-interactions that can be rigorously solve (correct evaluation of the separating screen thickness, the vicinity of walls, etc.) It must be noted that several efforts were made to push forward the frontier of applicability of the "static" approach [4,10]. However, the accurateness of the proposed techniques cannot be proven within the framework of the models considered.

Development of novel analytical method for investigation of electromagnetic systems coupling through small-size apertures is also important considering the fact that there are difficulties of utilization the widely developed electromagnetic simulations techniques in this particular area. These difficulties are associated with the requirements of very high precision mathematical models to be used for small coupling holes, since the relative correctness of a model has to be smaller than the coupling coefficients.

BASIC EQUATIONS AND RESULTS

Let us consider two ideal conducting co-axial cylindrical cavities coupled through a cylindrical aperture of the radius a in the separating planar screen of the thickness t. The radii and lengths of the first and second cavities will be designated b_1, d_1 and b_2, d_2, respectively. To construct a mathematical model of the electromagnetic system under consideration, we will use a relatively novel method of partially crossed regions (see, for instance [11,12]. As the first and second regions, we will take the cylindrical cavity volumes; for the third, a cylinder that is co-axial with the coupling hole , its radius being equal $b_3 = a$. This cylinder projects into the area of the first cavity for the length d_{1*} and into the second one for the length d_{2*}, the cylinder length being $\ell_* = d_{1*} + d_{2*} + t$.

In each region, we expand the electromagnetic fields in terms of the orthonormal complete set of field functions without the hole. We can get a set of equations for field amplitudes only in the 1-st and 2-nd regions:

$$a^{(i)}_{k,l} = \sum_{n',s'} (a^{(1)}_{n',s'} V^{(i,1)}_{n',s',k,l} + a^{(2)}_{n',s'} V^{(i,2)}_{n',s',k,l}), \quad i = 1,2 \quad (1)$$

This uniform set of equations describes the interaction of two infinite sets of oscillators, which are eigenmodes of closed cavities (without the coupling hole in the separating screen), being, in principle, fit to be used for calculations of necessary electromagnetic characteristics of coupled cavities. However, this set of equations has three drawbacks that make it difficult to carry out both analytical investigations and numerical calculations. Firstly, the structure of this set of equations does not yield a possibility to obtain analytical results, in particular, in the well studded limit t=0 and $a \to 0$. Secondly, this set is two-dimensional, and it is necessary to have great calculative resources to solve it. Thirdly, owing to the presence of field peculiarities at acute angles of the hole in the screen the coefficients $V^{(i,j)}_{n',s',k,\ell}$ decrease slowly with increasing indices. Our studies show that this set of equations can be reduced to such a form that has no first or second drawbacks:

$$(\omega^{(1)2}_{0,1} - \omega^2) a^{(1)}_{0,1} = -\omega^{(1)2}_{0,1} \frac{2a^3}{3\pi b_1^2 d_1 J_1^2(\lambda_1)} \times \left[\Lambda_{1,1} a^{(1)}_{0,1} - \frac{b_1^2 \sqrt{d_1}}{b_2^2 \sqrt{d_2}} \Lambda_{1,2} a^{(2)}_{0,1} \right], \quad (2.1)$$

$$(\omega^{(2)2}_{0,1} - \omega^2) a^{(2)}_{0,1} = -\omega^{(2)2}_{0,1} \frac{2a^3}{3\pi b_2^2 d_2 J_1^2(\lambda_1)} \times \left[\Lambda_{2,2} a^{(2)}_{0,1} - \frac{b_2^2 \sqrt{d_2}}{b_1^2 \sqrt{d_1}} \Lambda_{2,1} a^{(1)}_{0,1} \right]. \quad (2.2)$$

where the coefficients $\Lambda_{i,k}$, which determine the frequency shifts and cavities coupling, are defined by the expression

$$\Lambda_{i,k} = J_0^2(\lambda_1 \frac{a}{b_i}) \sum_{s=1}^{\infty} \sigma_{s,1,i} w^{(i,k)}_s \quad (3)$$

and $w^{(i,k)}_s$ are the solutions of some sets of linear algebraic equations.

First of all, let us become clear on the influence of electromagnetic field non-potentiality in the interaction region on $\Lambda_{i,k}$-values, since in all previous research studies [1-4,7] on coupling through small-size holes the assumption about field potentiality in the vicinity of the hole were made. In our model investigation of this problem is reduced to studying the dependence of the coefficients $\Lambda_{i,k}$ on the frequency f; the case f=0 corresponds to the assumption of field potentiality in the interaction region. Since frequency comes into the appropriate coefficients only in the form of expression $\Omega=\omega a/c$, then it follows that Λ-variation with increasing frequencies from 0 to f_{010} must be dependent on coupling aperture size - the smaller a the weaker dependence of Λ on frequency. This is confirmed by the calculations results (Tab.1.).

.Table 1. Dependence of $\Lambda_{i,k} = \Lambda$ coefficients and coupling coefficients $\tilde{\Lambda}$ on frequency f ($d_1 = d_2 = 3.5$ cm, $b_1 = b_2 = 4$ cm, t=0.0 cm, $f_{010} = 2.868563$ GHz)

	(a=1 cm)		(a=1.5 cm)	
f(GHz)	Λ	$\tilde{\Lambda}$	Λ	$\tilde{\Lambda}$
0	0.896590	0.012606	0.788984	0.037440
1	0.897783	0.012623	0.793784	0.037667
2	0.900862	0.012666	0.808207	0.038352
3	0.903614	0.012705	0.831250	0.039445

From Tab.1 it follows that an error in calculations of the coupling coefficients $\tilde{\Lambda}$ $\left(\tilde{\Lambda} = 2a^3 \Lambda / 3\pi b_1^2 d_1 J_1^2(\lambda_1)\right)$ at a=1 cm is on the order of 10^{-4} (the equivalent frequency shift being ≈300 kHz) and, consequently, all calculations can be made in static approximation. Yet, already for a=1.5 cm the error is of the order of 2×10^{-3} (the equivalent frequency shift being ≈ 6 MHz), which is inadmissible for precise calculations.

Of importance for applied use is the dependence of the coupled coefficients on the coupling aperture radius a. Analysis indicates that $\Lambda_{i,k}$ depends both on the above parameter $\Omega=\omega a/c$ and on relation of a to all cavity geometrical parameters and screen thickness (a/d_j, a/b_j, a/t, $j=1,2$). Results of the calculations of the relationship of interest on basic of our model in the static approach (f=0) are given in Fig.1. Fig.1 also shows the results of calculations for various values of the parameter a/t at $a/d_j \to 0$, $a/b_j \to 0$ [9].

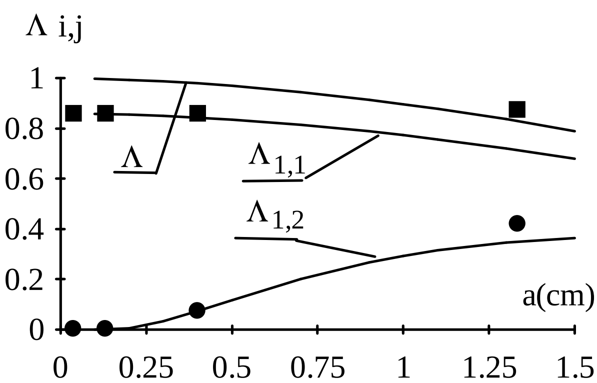

Fig.1. Dependence of coefficients $\Lambda_{i,k}$ on the coupling aperture radius a ($\Lambda_{i,k}=\Lambda$ - t=0, $\Lambda_{1,1}$, $\Lambda_{1,2}$ - t=0.4 cm, $d_1=d_2$ =3.5 cm, $b_1=b_2$ =4 cm, f=0, marks show the results of calculations from [9]).

It follows from Fig.1 that taking into account the finitness of parameters a/d and a/b lead not only to a drastic change of the numerical values of the coupling coefficients, but to change the functional dependence of $\Lambda_{i,k}$ on a. For instance, at a finite thickness of the screen the coefficient $\Lambda_{1,1}$ which determines the cavity eigenfrequency shift decreases with increasing a, contrary to what one can obtain from the results of the paper [9].

On this way, we put forward a novel analytical model for studding the coupling of two cavities through an aperture in separating screen of finite thickness without making assumption on smallness of any parameters.

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