Magnetic Field Simulation and Fitting of Superconducting Multipole Magnets at FRIB

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Introduction and Background:

Essential to the FRIB beam delivery system is the precise calibration of steering and focusing magnets along the beam line. To properly calibrate the superconducting quadrupole magnets (SQMs) used in the beam delivery system, the field throughout the bore of the magnet must be known. Specifically, when modeling or measuring SQMs with both large apertures and short lengths the fringing field at the edges of the field distribution must be accounted for. Moreover, the magnets in the beam line are not perfect quadrupoles and thus exhibit behavior associated with dipole, sextuple, and other harmonics. This further complicates field analysis. Here we present an analysis of the mathematical methods used to simulate and study the magnetic field within an SQM. The goal of the project was to minimize the error between simulated or experimental data and the final fitted data.

Creating a Model:

- Field data inside an SQM were simulated and measured on a cylindrical grid at a fixed radius
- From the grid, the field at any point within the radius can be found with a Fourier analysis of the grid
- Reduce field map to a simple on-axis gradient function, and extract polar magnetic field components with derivatives. The gradient is represented as a product of 5th order Enge functions, as in eq. (1)

$$b_{n,0}(z) = \frac{B_0}{\left(1+e^{\sum_k^5 c_{i,k} z^k}\right)\left(1+e^{\sum_k^5 c_{e,k}(-z)^k}\right)}$$

- "n" is the multipole harmonic of the magnet (n = 1, dipole; n = 2, quadrupole; …)
- Each coefficient in eq. (1), $c_{i,k}, c_{e,k}$, and B_0 behave approximately as 2nd order polynomials in current through the SQM coils, as in eq. (2)

$$B_0, c_{i,k}, c_{e,k} = p_{k,0} + p_{k,1} I + p_{k,2} I^2$$

- Field data is fitted to either parameter set: *local* -> $\{c_{i,k}, c_{e,k}, B_0\}$ or *global* -> $\{p_{k,l}\}$

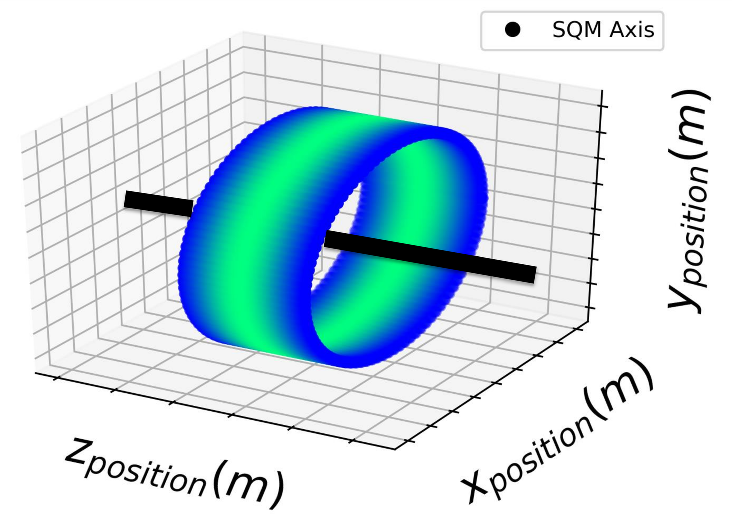


Figure (1): cylindrical grid on which data was simulated and measured.

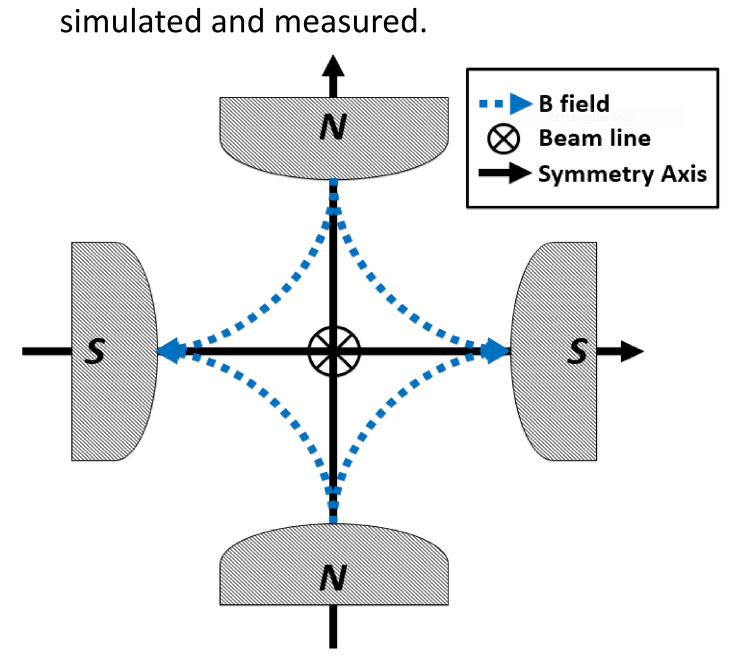


Figure (2): Magnetic field for a SQM

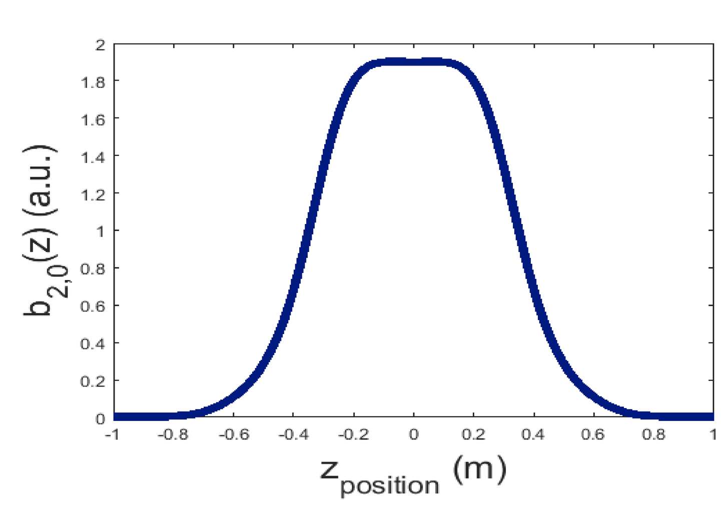


Figure (3): On-axis gradient function for the quadrupole component of an SQM.

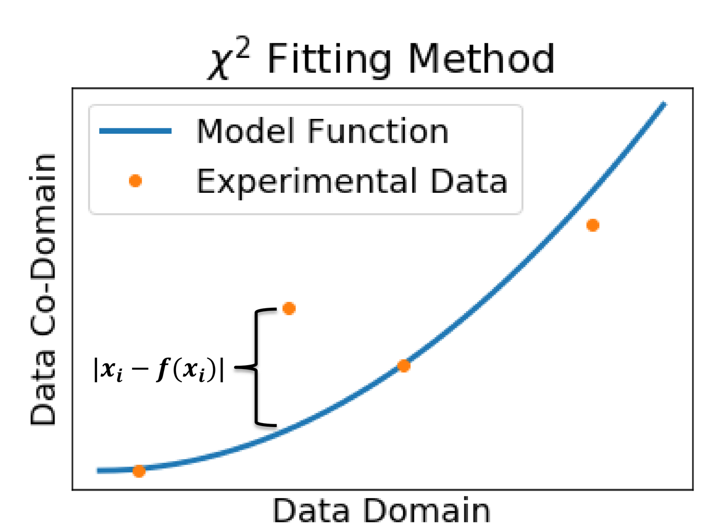


Figure (4): LSQs minimization technique

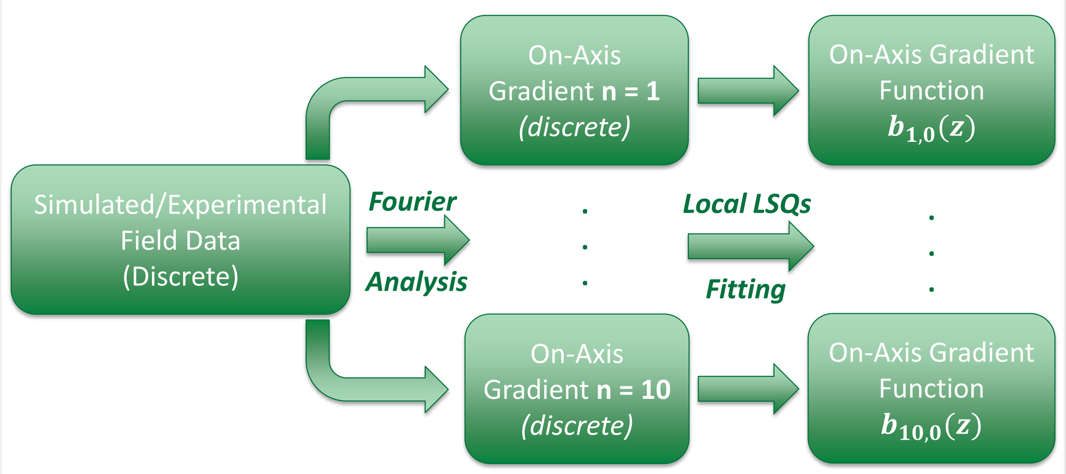
Fitting Method:

- Fitting was performed by the MINUIT software in ROOT, written in C++.
- MINUIT utilizes a least-squares (LSQs) technique, minimizing eq. (3).

$$\chi^2 = \sum_i (x_i - f(x_i))^2$$

SQM Fits for Multiple Harmonics:

- To perform fits at multiple poles, we consider only a single current
- The model consists of eq. (1)
- Use Fourier analysis to reduce field data to a discrete on-axis gradient for all harmonics $n \in \{1, \ldots, 10\}$ and determine the field contribution for each
- Perform a LSQs fit for harmonic data sets to obtain the parameters $\{c_{i,0}, \ldots, c_{i,5}, c_{e,0}, \ldots, c_{e,5}, B_0\}$



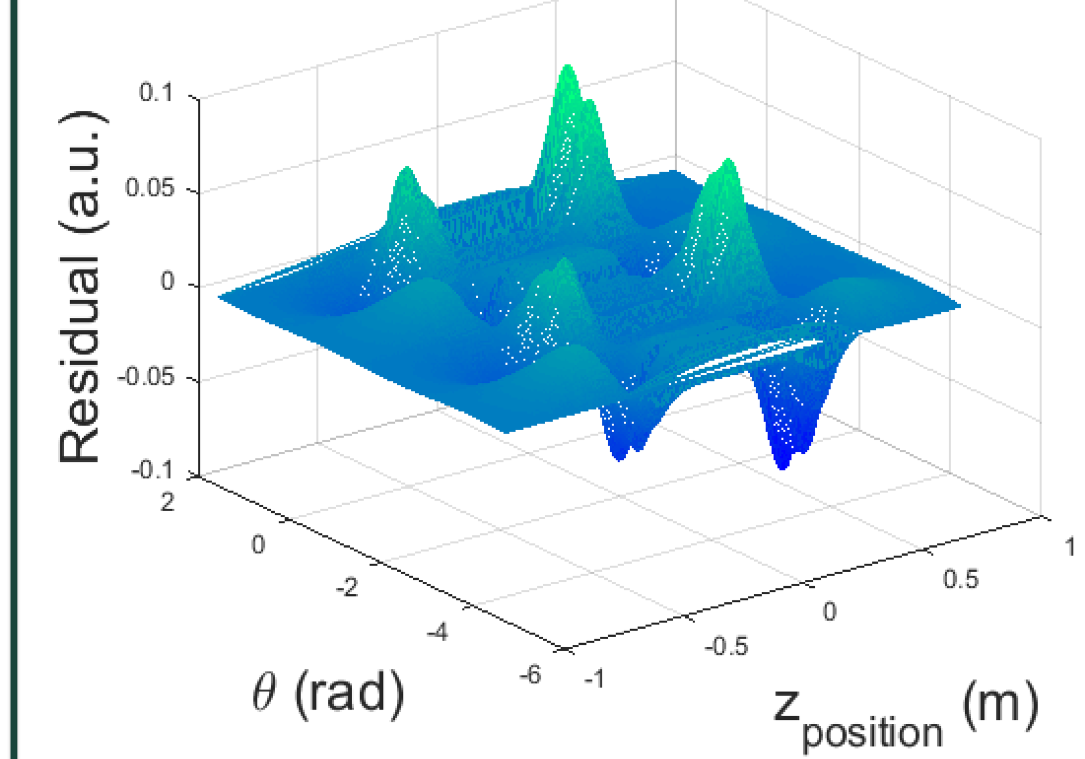


Figure (5): Total field residual for an SQM over the polar domain $z \in [-1,1]$, $\theta \in [0, 2\pi]$, $r = 0.2$. Residuals are given as $Exp(r, \theta, z)$ - $Fit(r, \theta, z)$. The non-zero values indicate error in the fitting analysis corresponding to approximately 5% of the maximum field value. Much of the residual lies outside the SQM, $z \in [-0.5, 0.5]$.

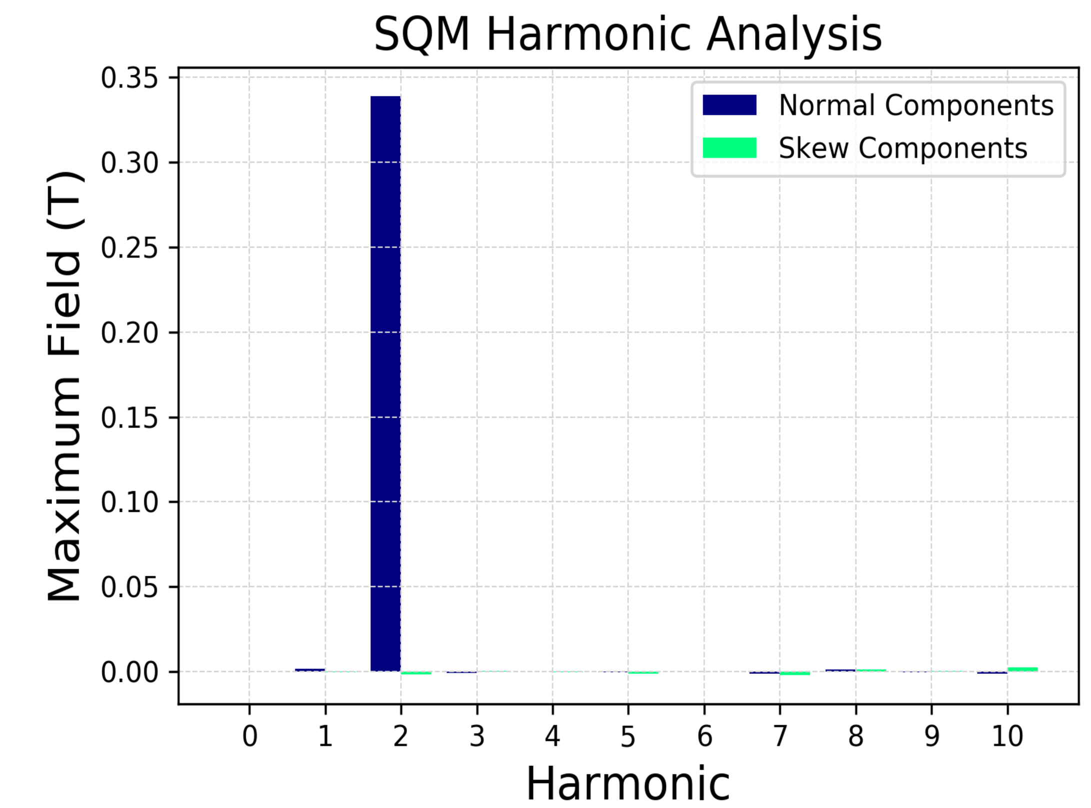
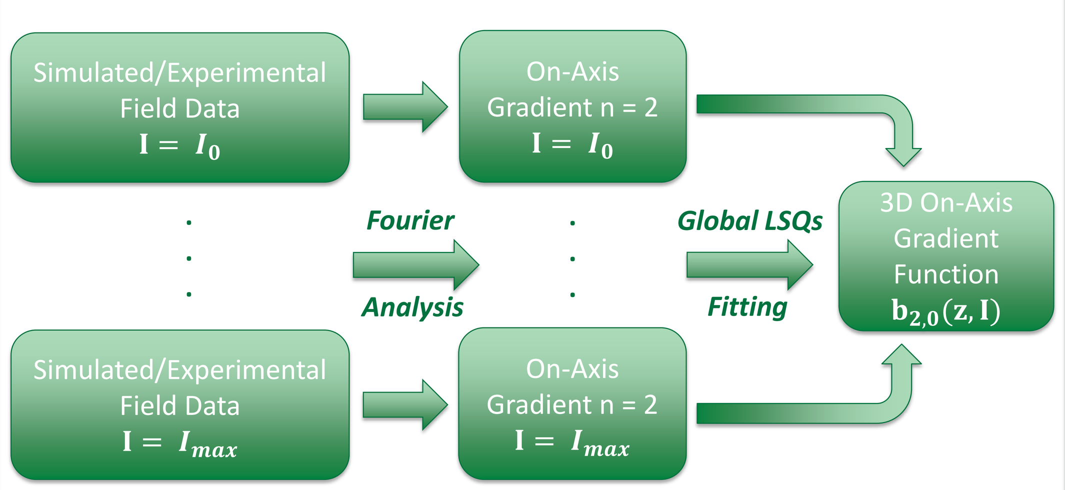


Figure (6): Harmonic analysis of an SQM. Skew field components arise from misalignment of detectors with respect to the symmetry axis and should be zero. Since the n = 2 component constitutes the majority of field magnitude, we can conclude that the magnet is a quadrupole. Past n = 10 the harmonics become negligible.

SQM Fits Over Multiple Currents:

- In performing fits over multiple currents, we consider only the $n = 2$ harmonic
- The model employs eqs. (1,2) with each c_{i,e_k} depending on I through $p_{k,0}, p_{k,1}$, and $p_{k,2}$
- Reduce field data for currents $I \in \{I_0, \ldots, I_{max}\}$ into discrete on-axis gradient data using Fourier Analysis
- Perform a global LSQs fit for combined on-axis gradient data sets to obtain the parameters $\{p_0, \ldots, p_{38}\}$



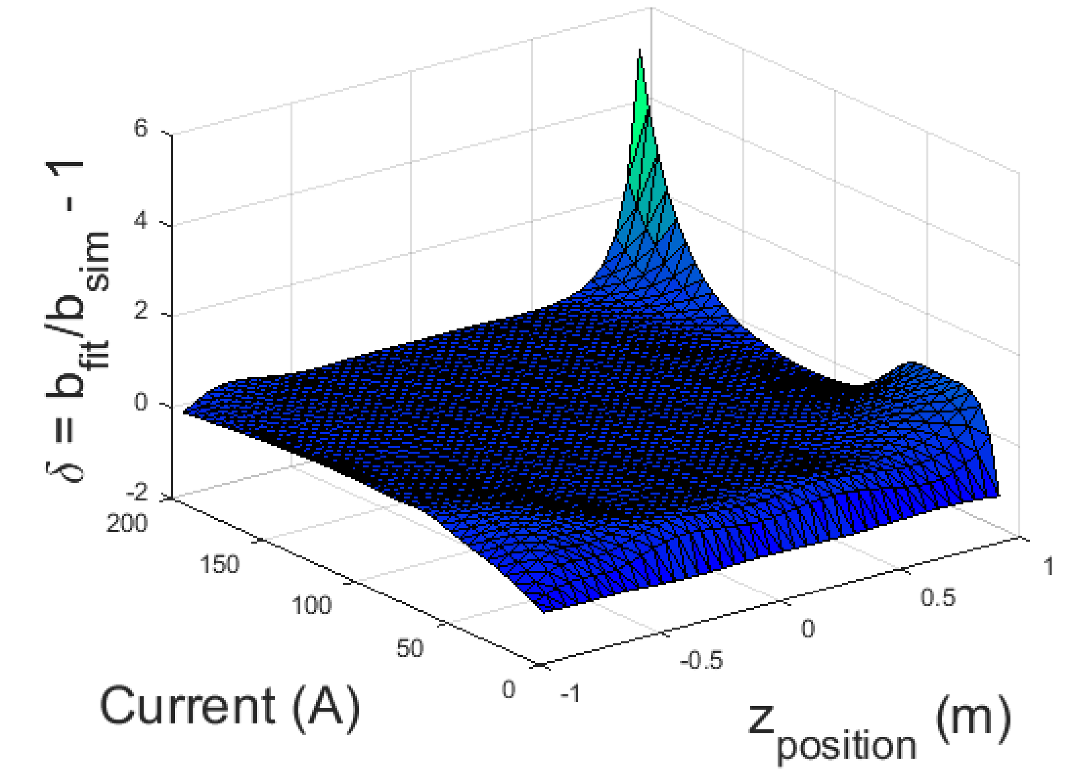


Figure (7): Quotient of a simulated 2D on-axis gradient function [eqs. (1,2)] and a globally fitted model function over the domain $z \in [-1,1]$ and $I \in [0, 200]$. It yields the expected $\delta = 0$ over the center of the domain. The majority of the residual comes from edges of the domain. The global fitting algorithm is inaccurate in the domain of zero and maximum currents.

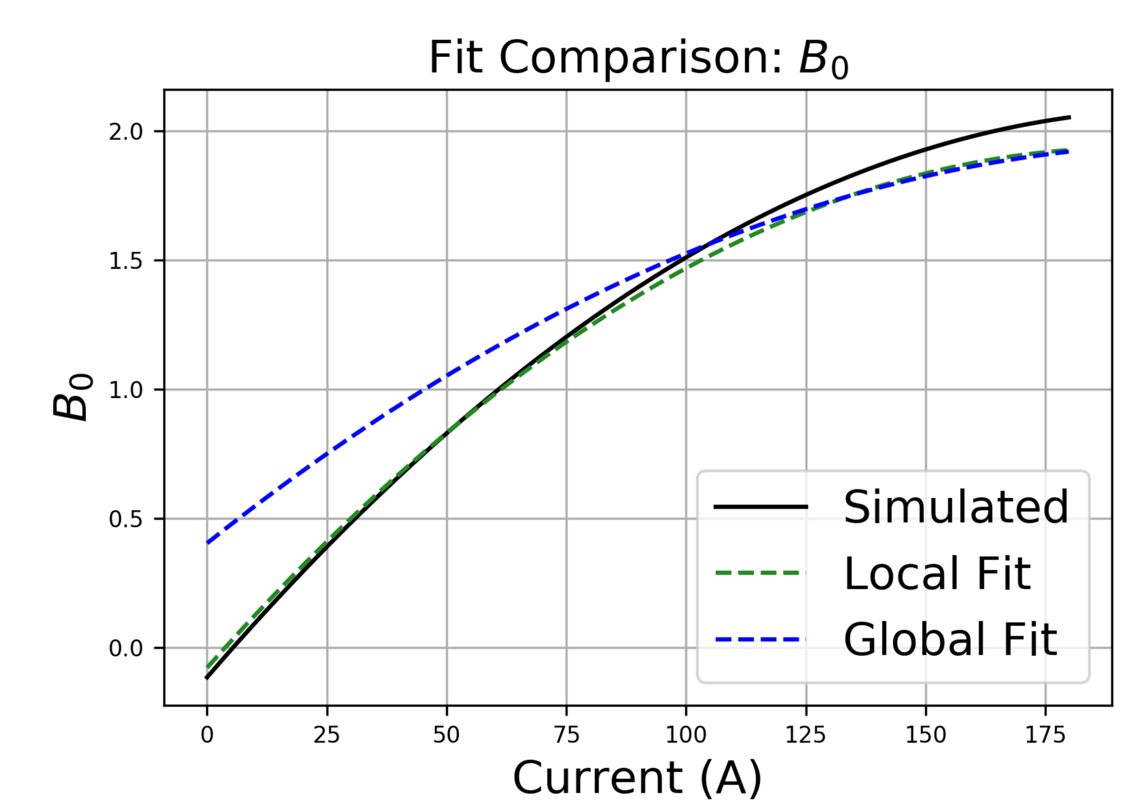


Figure (8): Quadratic behavior of B_0 as found by local and global fits. The local method fit B_0 to currents in the range $I \in [0, 200]$ and then fit a polynomial to the resultant data. The global method fit directly to the parameters $\{p_0, \ldots, p_{38}\}$. The local fit yields a better fit to the simulated polynomial, but the residual of the gradient was smaller for the global fit.

Application of Iterative Fits:

Residual Reduction			
Fitting Algorithm	χ^2 $i = 1$ (x 10^{-2})	χ^2 $i = 30$ (x 10^{-2})	%Δ
Global Fit $\{p_0, \ldots, p_{38}\}$	5.97	5.11	14.4

Table (1): Results of an iterative algorithm on a global field fit. Re-initializing fit parameters decreases error over iterations.

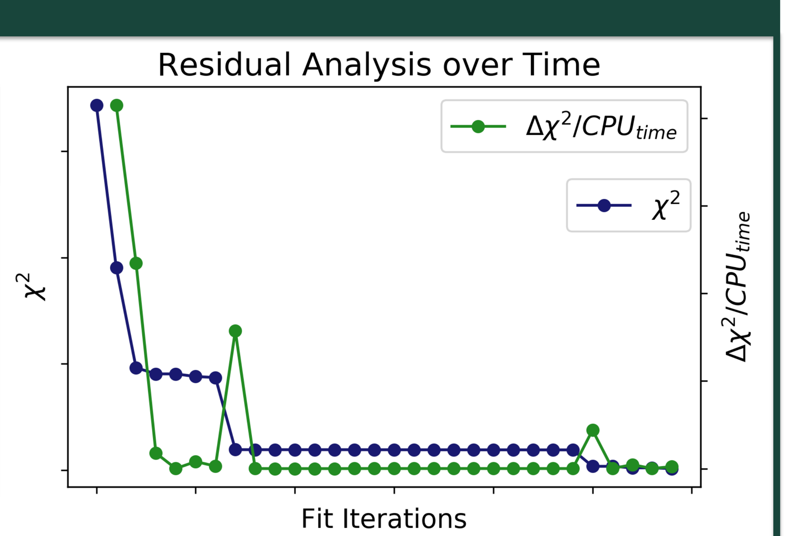


Figure (9): Plot of χ^2 and the ratio of $\Delta\chi^2$ and computation time versus iterations. There are decreasing returns with iteration number.

Conclusions and Future Goals:

Completed Analyses:

- Developed analysis methods for magnets of arbitrary multipole accounting for fringe fields
- Applied fitting algorithms for harmonic and global analyses to simulated and experimental data

Moving forward:

- Test whether a reduction in field measurement step size decreases error significantly
- Apply the fitting algorithms to data taken from multipole magnets on the FRIB beam line
- Use the fitted parameters to accurately install FRIB multipole magnets

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References:

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