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Problem Chosen

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2017

MCM/ICM

Summary Sheet

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Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Abstract

We have developed a model to anticipate the optimization of existing toll plaza. Our goal is to ensure the safety of the plaza as much as possible without congestion, while maximizing throughput and reducing the cost. The model mainly takes the following factors into consideration: design for physical shape of lanes, merging pattern of vehicles, the shape and area of toll plaza. According to the functional classification of different parts of the toll plaza, we divide the design into three models.

The first one is toll channel allocation model which is used to achieve the best allocation of toll station location speed up the vehicle deceleration process. Based on a large number of data provided by New Jersey Turnpike Authority official website, we calculate the proportion of vehicle type, and use the inverse method to find the best toll lane distribution. We first simulate the toll lane distribution effect, and then work out the most excellent lane allocation according to this effect. According to mathematical statistics theory, the results satisfy chi-square test. At the same time, we have added an angle of inclination to the buffer to speed up the deceleration and save construction costs.

The second one is the design model of the front square of the toll plaza, which is mainly to maximize the traffic flow with the lane design. We use the theory of high-speed public transit to study the relationship between three parameters of traffic flow: speed, density, flow, and analyze the random distribution function of vehicle arrival and the finally choose Poisson distribution. Based on related data, we can obtain the number of vehicles usually stranded in the plaza, and then according to the number of square to find the best area.

Inspired by the divide and conquer algorithm, the third model is intelligent merging model which uses a batch import, divide and rule method to deal with merged vehicles. First of all, according to the proportion of different vehicles, we divided different number of dedicated channels to improve the merging efficiency of classified vehicles. Second, the merged location and time can be staggered to minimize congestion at the junction. In addition, we also take into account the impact of future technology factors, when all vehicles share the speed, geographical location and other information between each other. The intelligent program can choose the best merging lane, accelerating passage speed and guaranteeing safety. At the same time, similar to the previous design, we set a certain angle in the acceleration section to reduce the vehicle in the toll plaza retention time.

Considering that the proportion of electronic payments will increase in the future, we design to open more EZPass channel. Our model can flexibly adapt to a variety of situations by adjusting the number of lanes to respond.

Content

| | |
|--|----|
| 1. Introduction..... | 2 |
| 1.1 Restatement of the Problem | 2 |
| 1.2 Our Approach..... | 2 |
| 2. Assumptions..... | 3 |
| 3. Notations..... | 3 |
| 4. The Model..... | 4 |
| 4.1 Layout Model of Lane..... | 4 |
| 4.1.1 The relational parameter..... | 4 |
| 4.1.2 Presupposition..... | 5 |
| 4.1.3 Model | 5 |
| 4.1.4 Limiting condition..... | 11 |
| 4.2 The area model of the toll plaza..... | 11 |
| 4.2.1 Related parameters | 11 |
| 4.2.2 Presupposition..... | 12 |
| 4.2.3 The model..... | 12 |
| 4.3 Shape Design..... | 14 |
| 4.3.1 Thought of model----- Divide and conquer, partial import | 14 |
| 4.3.2 Intelligent processing | 17 |
| 4.3.3 Further optimization..... | 18 |
| 5 Sensitivity Analysis..... | 19 |
| 6. Conclusion | 19 |
| Strengths: | 19 |
| Weaknesses: | 20 |
| 7. References..... | 22 |

1. Introduction

1.1 Restatement of the Problem

Recently, problems about toll plazas on freeways have attracted considerable public concerns. The government builds those tollbooths in two major purposes: to regular the traffic and to collect tolls. As we all know, vehicles have to slow down on entering the tollbooths and must “fan in” from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes when exiting. Hence it is obvious that the design of highways can significantly affect its throughput and cost. If unreasonably designed, toll booth will cause serious traffic congestion, bring out safety problems and lead to waste of resources in light traffic. The model aims to illustrate the shape and size of toll plazas, as well as how to merge after toll.

We need consider the following:

- 1 Considerations about important factors and accident prevention
- 2 Explanation of throughput and cost
- 3 To analyze performance of toll plaza and find a better solution (including shape, size and charge mode)
4. Performance of the toll station should consider light and heavy traffic situation
5. Corresponding changes when more private cars enter
6. Changes related to proportion of human-staffed tollbooths, exact-change tollbooths and electronic toll collection booths

1.2 Our Approach

In order to solve the problems about the cost and safety of the toll plaza and the tollbooths, and some other issues, we have designed the area and the shape of the toll plaza; the proportions and the locations of different tollbooths; the traffic lane where vehicles travel. On the basis of our design of each part, we have established three models. Respectively, they are lane and toll plaza shape design model, the toll plaza area model and the function import model. The three models are based on lots of documents, theorems and formulas, and there also exists many assumptions and constraints.

1. Through the design of the shape of the toll plaza, the distances between the driveways and the tollbooths for each car will be the same, thereby improving the efficiency of vehicles passing through the toll plaza.

2. Through the design of the proportion of the toll roads in different fees and charges in the charge buffer, we'll ensure that the max time interval of the different types of vehicles passing through the toll station can be the shortest, which is another

way to improve the efficiency.

3. Through the design of the area of the toll plaza, we are likely to ensure that the plaza is as small as possible. Besides, by the means of adding certain angle to the charge buffer, the speed of the automobiles can be reduced to specified speed. This approach can also reduce the first cost.

4. According to the average proportion of the compacts and full-size vehicles, we arrange different lanes for different types of vehicles. To guarantee the safety of the vehicles, proper traffic flow import mode is accordingly created. The use of the Internet technology also tunes up in the detection of the number and the type of vehicles through the tollbooths, with the help of which, changing the type of tollbooths flexibly according to the variation of the number and the proportion of cars and large vehicles is likely to become reality.

In our model, we also discuss the situations where the traffic is heavy or the opposite. Afterwards, we design the proportion of different types of the lanes. Solutions on the situation where there is increase in the number of the compacts are especially put forward.

2. Assumptions

1. Drivers have a certain quality and are able to make clear judgment. .
2. Vans and buses are classified as large vehicles, autonomous (self-driving) vehicles and cars are classified as cars. We only study the large vehicles and cars.
3. Large vehicles can use the same toll station to pay through human-staffed and exact-change.
4. Vans and cars can use the same EZ Pass toll station.
5. We take no account for other types of vehicles apart from cars, bus and vans.
6. In this model, the area of the tollbooths and constructions which have nothing to do with the toll plaza will not be calculated or considered.
7. Providing that traffic jams occur, the distance between the vehicles is 2m, which is a safe distance.
8. The vehicles are distributed evenly even when traffic congestion occurs.

3. Notations

All the variables and constants used in this paper are listed in Table1.

| Symbol | Definition | Units |
|--------|-----------------------------------|-------|
| L_E | Number of lanes for EZPass | S |
| L_T | Number of lanes for large vehicle | S |
| L_C | Number of lanes for cars | S |
| Q | Average flow rate | S/h |
| V | Average speed | Km/h |

| | | |
|-------|----------------------------------|------|
| D | Average density | S/Km |
| L_1 | Original lane number | P |
| L_2 | Number of lanes in toll plaza | P |
| M | Total number of vehicles | S |
| B | Number of toll booth | S |
| a | Fee stations of each toll booth | S |
| L_e | Distance between each toll booth | m |
| W | Length of lane | m |
| T | Payment time | min |
| V_c | Entering speed | Km/h |
| T_r | Number of large vehicles | S |
| C | Number of cars | S |
| D | Diameter of circle plaza | m |
| Z_c | Car length | M |
| E_2 | Number of vehicles using EZPass | S |

Table3.1

4. The Model

4.1 Layout Model of Lane

4.1.1 The relational parameter

The parameters used in this model are listed in table 2 during the process of establishing the model.

| Symbol | Significance | Unit |
|----------|---|------|
| $Q(t)$ | Function of arriving traffic | — |
| K_{ij} | K_{ij} ——Different way to charge of distinct vehicle types I ——Vehicle types .When ‘i’ equals 1,2, it represents large vehicles and cars respectively. J ——The way to charge. When ‘j’ equals 1、2、3,it represents artificial charge channel, charge in no change and EZPass charge channel singly. | Set |
| M | The shortest serving time in toll booth | |
| E_i | E_i ——Disposing ability of toll booths through different charge channels. When ‘i’ equals 1、2、3、4,it represents the disposing ability of cars human-staffed tollbooths, cars exact-change tollbooths, electronic toll collection booths, as well as human-staffed and exact-change tollbooths of large-vehicle | P/h |

| | | |
|--|---------------|--|
| | respectively. | |
|--|---------------|--|

Table 4.1

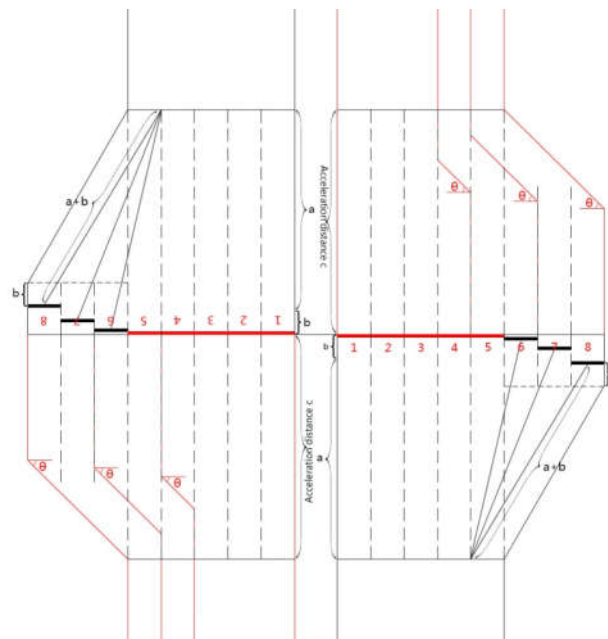
4.1.2 Presupposition

The establishment of model is under the corresponding presupposition with qualification. Here are the several situations limited.

1. Drivers have a certain quality and are able to make clear judgment.。
2. Vans and buses are classified as large vehicles, autonomous (self-driving) vehicles and cars are classified as cars. We only study the large vehicles and cars.
3. Large vehicles can use the same toll station to pay through human-staffed and exact-change.
4. Vans and cars can use the same EZ Pass toll station.
5. Considering the distance of getting into tollbooth for each car is same, we have designed the shape that corresponds to toll plaza and we will further give elaboration in 4.1.3.
6. The entering pattern of vehicles is obeying Poisson distribution, we will further give elaboration in 4.1.3.
7. We take no account for other types of vehicles apart from cars, bus and vans.

4.1.3 Model

1. Design for toll plaza (The concrete model will be displayed in the next model design)



2. Model overview

- (1) As depicted in figure 1, we can see that the shape of toll plaza is similar to two semicircles, which ensures that no matter from which direction the distance

between highway and the toll station is equal. Hence we can make every lane layout correction coefficient is roughly equal.

| Lane | Lane opposite the main road | First lane from inside to outside | Second lane from inside to outside | Third lane from inside to outside | Fourth lane from inside to outside | Fifth lane from inside to outside |
|--|-----------------------------|-----------------------------------|------------------------------------|-----------------------------------|------------------------------------|-----------------------------------|
| Common reduction factor | 1.0 | 0.85 | 0.79 | 0.61 | 0.60 | 0.60 |
| Conversion coefficients after the model adjustment | 1.0 | 0.97 | 0.96 | 0.93 | 0.92 | 0.91 |

Table 4.2 Lane number correction coefficient

(2) Meanwhile, toll station electronic signs will display toll free position on time, which means that vehicle can evenly enter into each toll booth.。

(3) In addition, the distance between the entrance and the toll booth should meet the request for vehicles to slow down. The deceleration area is the gradual slope with a certain angle. The height of the slope is equal to that of toll island, which is aimed to speed up the process of deceleration. Hence we can reduce the deceleration time and distance. Similarly, when vehicles drive out of the toll plaza, the slope is used to accelerate the departure process.

3. Purpose of the model

(1) In order to maximize throughput capacity.

(2) In order to make vehicles evenly enter each station and to improve the utilization rate.

(3) To allow vehicles to accept service at roughly the same time point.

(4) The slope can help vehicles decelerate, which reduces the distance of the slowdown area.

(5) To reduce the time spent on slowdown area, for according to the original model vehicles arriving at the lateral toll station accept longer service than the inside ones.

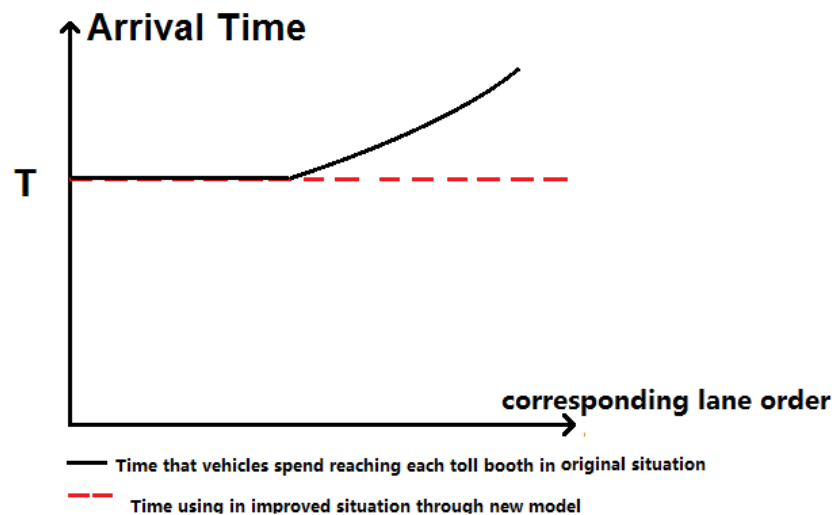


Table 4.3

4. Design toll lane allocation model and the distribution of each type of toll lane number to make the processing capacity of toll station strongest.

(1) Thought of model

As we all know, each toll plaza has the shortest serving time M under ideal situation. We hope to create a model to make service time as close to M as possible. According to the traffic situation on New Jersey Turnpike, we can simulate different situations with different time intervals and different traffic flows and calculate the toll processing time to find out a suitable scheme to minimize the service time.

(2) Related data

| Charge Type | Capacity of each lane (puc/h) |
|---------------|-------------------------------|
| Human-staffed | 350 |
| Exact-change | 650 |
| EZ Pass | 1800 |

Table 4.4 US road traffic capacity of different charging system

| Charge Type | Human-staffed | Exact-change | EZPass |
|--------------------|---------------|--------------|--------|
| Usage Rate of 2016 | 5.43% | 11.97% | 82.6% |
| Usage Rate of 2015 | 5.92% | 12.58% | 81.5% |
| Usage Rate of 2014 | 6.298% | 12.502% | 81.2% |

Table 4.5 Proportion of each charge type in New Jersey Turnpike

| Type of vehicle | Car | Large vehicles |
|--------------------|-------|----------------|
| Proportion of 2015 | 87.3% | 12.7% |
| Proportion of 2014 | 87.1% | 12.9% |
| Proportion of 2013 | 87.0% | 13.0% |
| Proportion of 2012 | 87.2% | 12.8% |
| Proportion of 2009 | 87.7% | 11.3% |

Table 4.6 Proportion of different types of vehicles in New Jersey Turnpike

(3) Design goal

To be clear, every toll booth has its own fixed year. According to US administration of traffic rules, we expected the toll station to be used for 20 years, which is to say our toll plaza should adjust to the future situation in 20 years. According to data provided above, we can roughly work out the traffic situation of New Jersey Turnpike in 20 years.

According to data, we can use linear fitting to calculate:

The characteristic function of human-staffed usage rate:

$$Y = -0.4350x + 6.3183 \quad x \text{ is the result of current year minus 2014}$$

The characteristic function of exact-change usage rate:

$$Y = -0.2660x + 12.6167 \quad x \text{ is the result of current year minus 2014}$$

The characteristic function of EZPass usage rate:

$$Y = 0.7000x + 81.0667 \quad x \text{ is the result of current year minus 2014}$$

Chi-square test:

| | | | | Usage Rate | | | | |
|--------------------------|--------------------------|----------|----------------------|--------------------------|----------|--------------------------|--------------------------|----------|
| | Human-staffed | | | Exact-change | | | EZPass | |
| Chi-square test | Mathematical expectation | Variance | Chi-square test | Mathematical expectation | Variance | Chi-square test | Mathematical expectation | Variance |
| P=0.9999 q=1.7936e-04 | 5.8827 | 0.0011 | p=0.9984 q=0.0032 | 12.3507 | 0.0394 | P=0.9997 q=6.5281e-04 | 81.7667 | 0.0533 |

Table 4.7 The prediction of charge type usage rate

We can draw to the conclusion that it is reasonable to use linear distribution to measure charge type and can pass chi-square test at 0.05 significant level.

Similarly, we use the linear fitting to work out:

The characteristic function of car proportion

$$Y = -0.0900x + 87.4400 \quad x \text{ is the result of current year minus 2009}$$

The characteristic function of large vehicles proportion

$$Y = 0.2900x + 11.9600 \quad x \text{ is the result of current year minus 2009}$$

| | | Proportion | | | |
|--------------------------|--------------------------|------------|-----------------------|--------------------------|----------|
| | Cars | | | Large vehicles | |
| Chi-square test | Mathematical expectation | Variance | Chi-square test | Mathematical expectation | Variance |
| P=0.9825 q=4.8028e-04 | 87.4500 | 0.0840 | P= 0.8612 q=0.0306 | 12.0500 | 0.7321 |

Table 4.8 Proportion of vehicles

We can draw to the conclusion that the proportion of vehicles roughly Obey the linear distribution and can pass chi-square test at 0.05 significant level.

(3) Prediction

We can predict the traffic situation of New Jersey Turnpike in 20 years through fitting the function.

| | |
|------------------------------|---------|
| Proportion of cars | 84.92% |
| Proportion of large vehicles | 20.08% |
| Usage rate of human-staffed | -3.678% |
| Usage rate of exact-change | 6.4987% |
| Usage rate of EZPass | 97.1667 |

Table 4.9 Traffic situation of New Jersey Turnpike in 20 years

Analysis of predicted data:

After 20 years, human-staffed charge has already been replaced. Also, there are only a few vehicles remain to choose exact-change. Of course, we can't deny that fitting has error which means that this may not match the reality.

(4) Design of toll lane

① Characteristics of the vehicle reaching the toll plaza

According to the related studies, when traffic density is not large and there is no other interference, the mutual influence between vehicles is small, hence traffic flow is random:

Basic formula

$$P_k = \frac{(\lambda t)^k}{K!} e^{-\lambda t}$$

In this formula: P_k : The probability of arriving k vehicles within t time

λ : The average arrival rate of vehicles

t : The duration of the each counting interval

e : Napierian base

In this formula, $m = \lambda t$ is the parameter of Poisson distribution, which represents the average number of vehicles arriving within time t .

The recursive formula

$$P_0 = e^{-m} \quad P_{k+1} = \frac{m}{k+1} P_k$$

The mean value M and variance D of the distribution are equal to t .

② According to the characteristics of vehicles, we can assign different numbers of lanes to different types of vehicles, and then simulate the capacity of each type of toll station to decide the exact allocated number.

According to the New Jersey Turnpike traffic data, we can simulate the number of vehicles reaching toll plaza in several arrival time.

Time spent to deal with the throughput of these vehicles is:

$$T = \text{Max} \left\{ \lambda t * K_{21} / (E_1 * L_1), \lambda t * K_{22} / (E_2 * L_2), \lambda t * K_{i3} / (E_3 * L_3), \lambda t * (K_{11} + K_{12}) / (E_4 * L_4) \right\}$$

In the expression:

$\lambda t * K_{21} / (E_1 * L_1)$ represents the time that cars go through the tollbooths in

human-staffed way.(h)

$\lambda t * K_{22} / (E_2 * L_2)$ represents the time that cars go through the tollbooths in exact-change way.(h)

$\lambda t * K_{13} / (E_3 * L_3)$ represents the time that cars and large-vehicles go through the electronic tollbooths.(h)

$\lambda t * (K_{11} + K_{12}) / (E_4 * L_4)$ represents the time that large-vehicles go through the tollbooths in human-staffed and exact-change way.(h)

We hope to work out a smallest T,

So set $\lambda t * K_{21} / E_1 = a$

$\lambda t * K_{22} / E_2 = b$

$\lambda t * K_{13} / E_3 = c$

$\lambda t * (K_{11} + K_{12}) / E_4 = d$

$a/L_1 = x \quad b/L_2 = y \quad c/L_3 = z \quad d/L_4 = w \quad M = \max\{x, y, z, w\}$

So $a/x + b/y + c/z + d/w = L_1 + L_2 + L_3 + L_4 = L$

$a/x \geq a/M \quad b/y \geq b/M \quad c/z \geq c/M \quad d/w \geq d/M$

So $L \geq (a+b+c+d)/M$

So $M \geq (a+b+c+d)/L$

According to the above, the smallest M is: $(a+b+c+d)/L$

Due to the data about traffic flow provided by New Jersey Turnpike Authority, we can input several t and λ in different allocation ways. Observe the processing time and then choose the optimal solution which has the nearest result to $(a+b+c+d)/L$ in reality.

Here is the data and result.

| Year | 2016 | 2015 | 2014 | 2015 |
|--------------|---------|---------|---------|---------|
| Traffic flow | 12899/h | 12818/h | 13465/h | 13962/h |

Table 4.10 Average traffic flow on New Jersey Turnpike

We can use an average value as per hour average flow value and change the time period t:

| Time period | 1min | 5min | 10min | 30min | 60min |
|-------------|----------|----------|----------|----------|-----------|
| (3,1,2,2) | 2.83min | 7.94min | 27.59min | 73.57min | 134.78min |
| (2,2,2,2) | 1.38min | 7.33min | 15.74min | 50.23min | 100.52min |
| (4,2,1,1) | 3.72min | 13.76min | 30.98min | 80.75min | 157.98min |
| (2,3,1,2) | 1.26min | 6.49min | 14.57min | 50.43min | 98.64min |
| (1,3,2,2) | 1.369min | 7.27min | 18.68min | 58.67min | 126.53min |
| (1,4,2,2) | 2.03min | 7.88min | 20.07min | 85.74min | 153.88min |

| | | | | | |
|---------|--------|-------|----------|-------|-----------|
| 1) | n | n | | min | |
| (2,1,3, | 2.53mi | 10.57 | 24.58min | 79.56 | 117.85min |
| 2) | n | min | | min | |

Table 4.11

We can draw to the conclusion that the best allocation of toll booth channels on New Jersey Turnpike is (2,3,1,2).

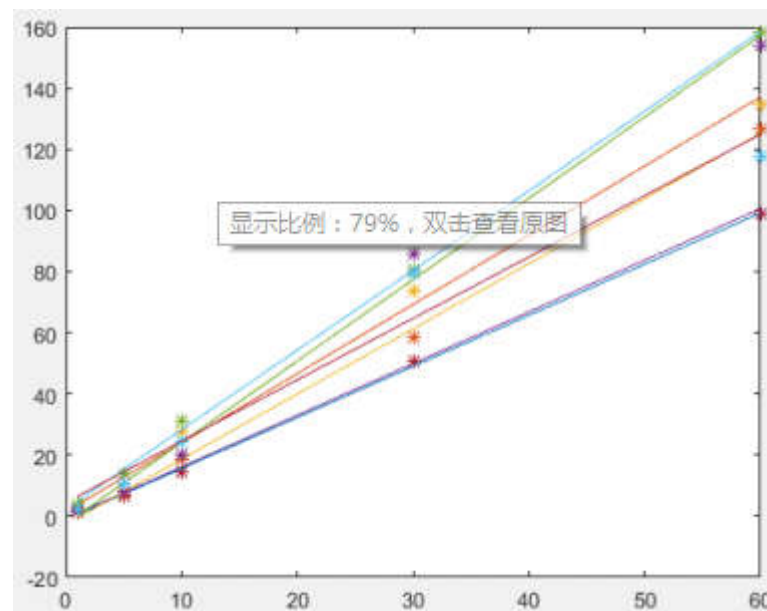


Figure 1

4.1.4 Limiting condition

According to the statement of problem B, the total of lanes through various channels in toll plaza is equal to L . That is:

$$L_1 + L_2 + L_3 + L_4 = L$$

4.2 The area model of the toll plaza

4.2.1 Related parameters

The parameters of Table 3 are used in the process of establishing the model.

| | |
|-------|------------------------------|
| H_c | proportion of compacts |
| H_t | proportion of large vehicles |
| L_s | safe distance |
| D_j | blocking flow density |
| V_f | free stream velocity |
| V | vehicle speed |

| | |
|-------|--|
| D | traffic density |
| C | vehicle flow |
| V_l | limit speed of the vehicle entering the toll plaza |
| R | vehicle staged at toll plaza |

Table 4.12

4.2.2 Presupposition

1. In this model, the area of the tollbooths and constructions which have nothing to do with the toll plaza will not be calculated or considered.
2. Providing that traffic jams occur, the distance between the vehicles is 2m, which is a safe distance.
3. The vehicles are distributed evenly even when traffic congestion occurs.

4.2.3 The model

1. The design thought of the model.

In our real life, when the toll stations fail to make it possible for the smooth passage of vehicles, the toll plaza congestion occurs. We are considering that how to design the area of the toll plaza to avoid the congestion, in other words, how to increase the vehicle flow rate to satisfy the need of toll station. And the best situation we can expect is to make the vehicle flow of the toll plaza reach its maximum. What's more, with the increase in the proportion of electronic payment, the toll station will be reconstructed and the throughput will accordingly increase, in which case the action to add the vehicle flow of the toll plaza seems more necessary.

2. The design of the model

2.1 The state of the cars entering the toll plaza

Within a certain interval of time, the number of the cars arriving at the toll plaza is random. By the means of discrete distribution, such as Poisson distribution, binomial distribution and negative two distribution, we'll be able to describe the statistical law of this kind of random variable. Besides, in the actual situation, the vehicles always pass through the tollbooths without causing traffic jams. Therefore, in the real life, the traffic density is not large, the interferences between vehicles are slight, the external interference factors don't exist and the number of vehicles is random, in which case passion distribution provides a standard way of representing the amounts of the vehicles reaching the tollbooths.

The basic formula:

$$P_k = \frac{(\lambda t)^k}{K!} e^{-\lambda t}$$

2.2 The volume of the vehicles

Considering that different types of the cars cannot have the same volume, we need a unified standard. Consequently, the vehicle reduction factor is introduced.

| Type of vehicles | Car | Medium vehicles | Large vehicles |
|------------------------------------|-----|-----------------|----------------|
| MTC vehicle conversion coefficient | 1.0 | 1.7 | 2.0 |
| ETC vehicle conversion coefficient | 1.0 | 1.3 | 1.7 |
| Integrated conversion coefficient | 1.0 | 1.4 | 1.8 |

Table 4.13 The vehicle reduction factor of different types of vehicles

The average number of the cars passing through the tollbooths in the time T is λ , so (λT) is the total of the cars through the tollbooths within T . And $(\lambda T * H_c * Z_c + \lambda T * H_t * Z_t)$ equals the number of the vehicles entering the toll plaza.

Meanwhile, the formula $\int_0^T Q_c(t) dt$ can be used to calculate the throughput of the toll plaza, in other words, the number of the vehicles pulling out of the plaza. Therefore, the number of the cars stagnating at the plaza is $M = (\lambda T * H_c * Z_c + \lambda T * H_t * Z_t) - \int_0^T Q_c(t) dt$. S_1 is the area of the toll plaza. Accordingly, in the time T , the number of the vehicle at the toll plaza is $D = \frac{M}{S_1}$.

2.3 The exact calculations

According to Greenshields' research about the linear relationship between speed and density

$$V = V_f * \left(1 - \frac{D}{D_j}\right)$$

In the formula: V ——Speed、The length of car

D ——Density

V_f ——Free stream velocity

D_j ——Blocking density

$$D_j = \frac{1000}{L_1 + L_2}$$

L_1 ——The length of car

L_2 ——The safety distance of the automobile braking

According to the formula above, we can know that:

$$C = V_f * \left(1 - \frac{D}{D_j}\right) * D$$

$$C = D_j * \left(1 - \frac{V}{V_f}\right) * V$$

Make $\frac{dC}{dD} = 0$, we can get the maximum capacity under the condition of the vehicle

critical density: $D_L = \frac{D_j}{2}$

Make $\frac{dC}{dV} = 0$, we can get the maximum capacity under the condition of the vehicle

critical density: $V_L = \frac{V_f}{2}$

So the theory of maximum traffic capacity is:

$$C_{\max} = 0.25 * V_f * D_f$$

Consider that New Jersey Turnpike traffic safety bureau made regulations that the speed of the vehicles in front of the tollbooths cannot exceed $V_f = 25$ mile per hour.

Consequently, $V_f = V_1 * 80\% = 20$ mile per hour

$$V_f = 32.8688$$

Therefore, we can obtain the optimum front square area of toll plaza is 5144.88.

4.3 Shape Design

S_i ——The corresponding toll booth of each lane $i=1,2,3,\dots,8$.

Z_i ——Convert coefficient, $i=1,2$ represent cars and large vehicles respectively.

C ——Distance between toll booth and entrance of highway

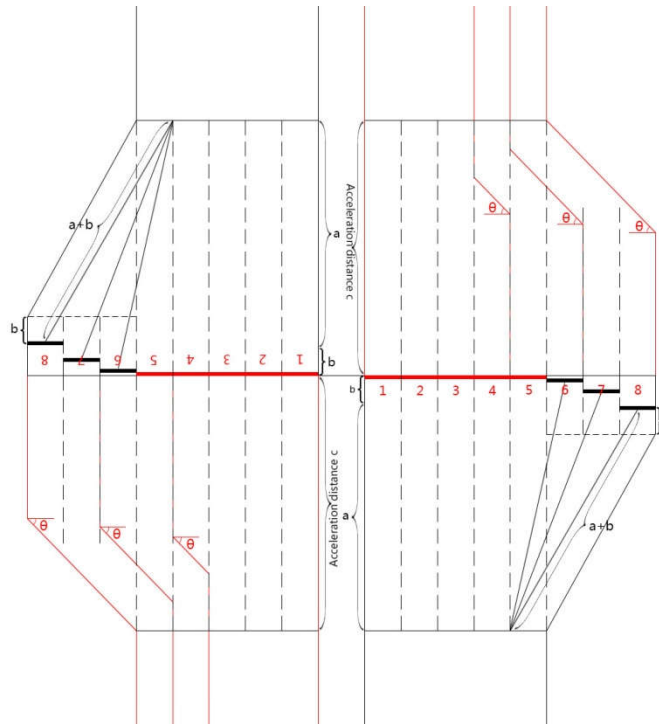
a ——Angle of the lane when merging

X_i ——Distance between each toll passageway and highway

Assume that the width of toll road is 3.2m.

4.3.1 Thought of model----- Divide and conquer, partial import

What is a good merging mode that vehicles can adapt to converge to the highway in attempt to avoid traffic congestion? Obviously, we should prevent too large density, which is related to the number of other merging vehicles from other lanes. Hence we think about a new merging mode to let vehicles merge at different locations at different time. Then what merging speed and density can satisfy the condition? An ideal situation is that we can maximize the traffic flow on the highway after merging. Thus we can promise that more vehicles can pass the intersection as soon as possible to avoid congestion. What's more, we also know every type of the vehicle driving out of each toll booth, according to which we can reduce the probability of congestion when merging. While different charge way determines the service time, vehicles also have different starting time. When using EZ-pass, drivers don't have to stop their cars. However, if using human-staffed or exact-change to pay, drivers have to stop their cars, which is to say the initial velocity of their cars is 0 when driving out of toll booth. Due to different charge way, the speed and density of vehicles on different lanes are not the same. To convergence different vehicles on different lanes, for example, we make vehicles on 5 and 6 lane merge into 4 lane, make vehicles on 1,2,3,4 lane merge into 1,2,3 lanes, make vehicles on 7 and 8 lane merge into 5 lane, thus 1,2,3,4,5 lanes will have roughly equal traffic density and speed after merging.



1. Match for merging lanes

When congestion occurs on the front part of toll plaza, toll station is on full load operation, so the number of vehicles driving out of toll booth is equal to toll station largest throughput capacity. In a period of time T , the number of vehicles driving out of one toll booth is:

$$M = E_i * T$$

So the density of vehicles driving out of this toll booth in this period of time T is:

$$D_i = \frac{M}{S_i}$$

The accelerating model when driving out of toll booth is:

$$V = G(t)$$

So the traffic flow on each road is:

$$C_i = D_i * G(t)$$

Among these lanes, the two lanes on the most right sides are for large vehicles only. So the traffic density on 7 and 8 lanes should be multiplied by a conversion coefficient:

$$C_7 = D_7 * G(t) * Z_2$$

$$C_8 = D_8 * G(t) * Z_2$$

| Length of time | Traffic flow in 1 min | Traffic flow in 5 min | Traffic flow in 10 min | Traffic flow in 30 min | Traffic flow in 60 min | Traffic flow in 120 min |
|----------------|-----------------------|-----------------------|------------------------|------------------------|------------------------|-------------------------|
| Lane 1 | 30pcu | 150pcu | 300pcu | 900pcu | 1800pcu | 3600pcu |
| Lane 2 | 30pcu | 150pcu | 300pcu | 900pcu | 1800pcu | 3600pcu |
| Lane 3 | 11pcu | 54pcu | 108pcu | 325pcu | 650pcu | 1300pcu |
| Lane 4 | 11pcu | 54pcu | 108pcu | 325pcu | 650pcu | 1300pcu |
| Lane 5 | 11pcu | 54pcu | 108pcu | 325pcu | 650pcu | 1300pcu |

| | | | | | | |
|--------|-------|-------|--------|--------|---------|---------|
| Lane 6 | 6pcu | 29pcu | 58pcu | 175pcu | 350pcu | 700pcu |
| Lane 7 | 16pcu | 82pcu | 166pcu | 500pcu | 1000pcu | 2000pcu |
| Lane 8 | 10pcu | 50pcu | 100pcu | 300pcu | 600pcu | 1200pcu |

We can see that only adjacent lanes can be merged. According to the table above, 1,2,3,4 lanes can be coalesced into 1,2,3 lanes on the highway, 5 and 6 lanes can be coalesced into 4 lane, 7 and 8 lanes can be coalesced into 5 lane.

1. Design for the length of merging lane

We can know that:

the distance between 4 toll booth channel and highway lane by geometric features:

$$X4 = C + 1.6 * (1 - \sin(a)) / \cos(a)$$

the distance between 5,6 toll booth channels and highway lane by geometric features: $X5 = X6 = C + 4.8 * (1 - \sin(a)) / \cos(a)$

the distance between 7,8 toll booth channel and highway lane by geometric features:

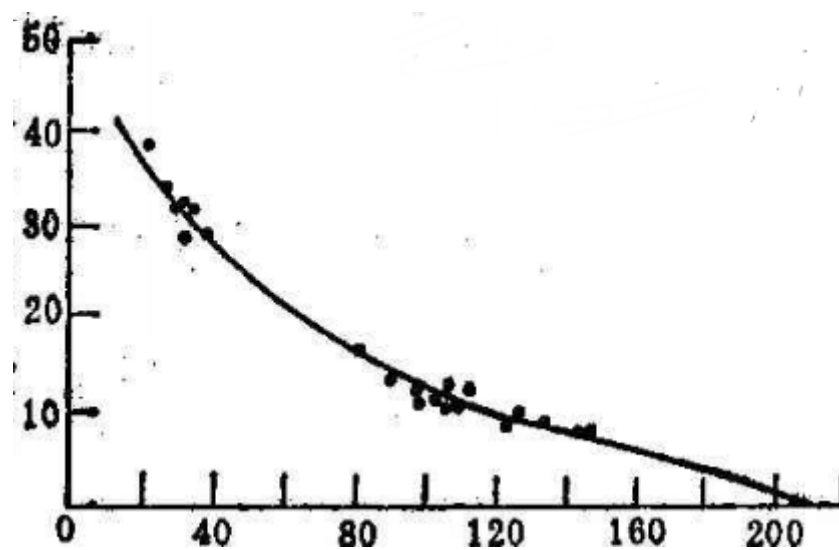
$$X7 = X8 = C + 8 * (1 - \sin(a)) / \cos(a)$$

1.1 The best speed and density when merging into highway:

Greenberg's Model: Logarithmic model

$$v_s = v_m \ln \left(\frac{K_j}{K} \right)$$

(speed, miles/hour)



(density, set/mile)

According to the density of vehicles when merging into highway :

$$D = \left(\sum_{i=1}^8 D_i \right) * \frac{1}{5}$$

So the corresponding optimal speed under the function above is:

VL

According to accelerating model, the corresponding distance of acceleration from different toll booth channels is:

$$S1 = 75.36m$$

$$S2 = 78.75m$$

$$S3 = 81.63m$$

$$S4=82.93\text{m}$$

$$S5=84.52\text{m}$$

$$S6=96.53\text{m}$$

$$S7=101.49\text{m}$$

$$S8=103.00\text{m}$$

So the design of the channel should be the average value :

$$C=87.77625\text{m}$$

4.3.2 Intelligent processing

In 2014, the U.S. department of transportation and the intelligent transportation system (ITS) project office jointly put forward ITS strategic plan 2015-2019, making the direction of development in the field of intelligent transportation clear for the next five years. Intelligent and snatched cars have become the core of the strategic plan and the key technology to solve the problem of transport system in the United States. Strategy book is based on six major projects: workshop networking, automatic driving, new features, enterprise data, interoperability, and accelerating deployment. In fact, with the development of technology and science, we can predict that higher and higher proportion of scientific factors will be taken into consideration in the future. Hence we build the assumption that this application can be used to design the merging pattern after toll.

Considering the popularity of the future car networking, we might as well assume that all vehicles have been connected to the Internet, sharing information between each other, including speed, length, and the geographical position, which can be calculated by the computer to choose the best way for vehicles to enter the highway. When all the vehicles driving out of toll station, they will be calculated by the computer according to their own speed before reaching the intersection. The computer will work out the best driving route, which means the shortest time and the most secure way to quickly enter the highway.

Using car network system will have the following benefits:

1. Making the vehicles and lanes safer;
2. Strengthen maneuverability: improving traffic management, accident management and transportation management;
3. To reduce the environmental impact , to have better control of the traffic flow, vehicle speed and traffic congestion , with other advanced technology management;
4. Supporting transportation information sharing: to apply advanced wireless technology to make all vehicles, infrastructure, mobile devices connected to Internet realizing real-time transmission of information and applications

Although in short-term car networking construction needs a lot of financial support, but in the long run, the popularization of the new technology will greatly optimize the way people travel and ultimately achieve the goal of high security and fast circulation speed.

4.3.3 Further optimization

New Jersey is a state in the northeastern and mid-Atlantic regions of the United States. It is bordered on the north and east by New York, on the southeast and south by the Atlantic Ocean, on the west by Pennsylvania, and on the southwest by Delaware. New Jersey state is one of the major tourism states and has well-developed service industry and advanced manufacturing. According to these characteristics, we believe that the proportion of cars and large trucks (for transportation) are stable. We may take the future development of the state into consideration, and change the proportion of special toll appropriately according to the fluctuant ratio between cars and trucks.

1. EZPASS is a No-stop electronic toll collection system which uses radio frequency transmission and needs no parking. It has speed limit for 10 to 20 km/h through the toll booths. It can effectively improve the traffic capacity, cut down charging time, reduce the vehicles blocking phenomena, save manpower and reduce operating costs. According to New Jersey Turnpike Authority annual report, the proportion of using EZPass is increasing gradually year after year. So it is reasonable to increase the proportion of specialized electronic payment lanes. See the table below.

| | 2016 | 2015 | 2014 |
|-----------|-------|-------|-------|
| January | 83.2% | 82.3% | 82.3% |
| February | 83.2% | 82.4% | 82.2% |
| March | 82.8% | 82.3% | 81.7% |
| April | 82.6% | 81.8% | 81.4% |
| May | 82.4% | 81.0% | 81.0% |
| June | 82.2% | 81.3% | 80.4% |
| July | 80.9% | 80.4% | 80.0% |
| August | 81.9% | 80.5% | 79.6% |
| September | 82.7% | 81.9% | 81.7% |
| October | 83.1% | 82.5% | 82.0% |
| November | 83.3% | 82.3% | 81.8% |
| December | | 82.1% | 81.5% |
| Total | 82.6% | 81.7% | 81.2% |

Table 4.14 New Jersey Turnpike
Electronic Toll Collection Usage Rate

Percentage of All Toll Transactions Collected Electronically

2. With the popularization of car networking in the future, we can easily detect the type and the number of vehicles heading to toll plaza. We can set up the car and large vehicles ratio threshold ζ_0 . Within a reasonable period of time T_0 , when $\zeta \geq \zeta_0$ 时, we can change the proportion of specialized toll lanes to open more lanes for cars.; When $\zeta \leq \zeta_0$, similarly, we can change the original charge window specialized for cars to serve large vehicles.

5 Sensitivity Analysis

Model Performance Sensitivity Analysis:

As time goes by, the proportion of various charging methods will change, the type of vehicle also changes.

We can simulate their changes to test the model's sensitivity. But before we model is based on this trend of change designed, and through the chi-square test, so the sensitivity of this area is no problem.

We simulated the performance of the test plaza when different traffic flows into the toll plaza. The following table

| | | | | | | |
|----------------------|------------|------------|----------|----------|----------|----------|
| Import plaza traffic | 12000pcu/h | 24000pcu/h | 26000puc | 28000pcu | 30000pcu | 32000pcu |
| Square traffic | 8700pcu/h | 21000pcu | 22500pcu | 2400pcu | 25300pcu | 2600pcu |

Square car traffic can be imported from the toll station processing, get the number of vehicles divided by the square area to obtain a density, according to the speed corresponding to the density of the traffic flow rate, multiplied by the density of traffic flow available.

6. Conclusion

Strengths:

1. We apply the idea divide and conquer in the computer algorithm to our model. We sort different types of vehicles and use batch export mode in attempt to maximize the outflow as much as possible without congestion.

2. This model includes a large number of parameters. These parameters are related to two aspects: one includes parameters about traffic flow, the other includes parameters about disposing ability of tollbooths. When using these parameters, we input a large number of true and reliable data to work out a reasonable model.

3. The design of toll plaza is not only aimed to make the density of vehicles in an ideal range but also make the flow of vehicles driving into the square and out of the square largest. Our plaza design meets the above two requirements.

4. In terms of costs, the angle of the tilt design reduces the length of the buffer zone, as well as the amount of cement. Hence the reasonable planning of the square

improves the utilization of the site.

Weaknesses:

1. Sometimes the data processing is too idealistic, because a lot of things are rough calculated.
2. The model is simplified, and there are still some parameters that are not taken into account.
3. The model relies on the future development of the technological level, especially the intelligent processing of the vehicle network, and the specific algorithm of the corresponding intelligent software is not given in this model.

Letter to the New Jersey Toll Road Authority

New Jersey Turnpike Authority:

New Jersey, USA

Dear Sir and Madam,

With the economic development, more and more private cars have entered into people's daily lives. As one of the most important parts of transportation, how to design expressway to adapt to today's change becomes more and more important. The design of the expressway has seriously affected the traffic throughput, vehicle safety and the cost of the toll plaza. For this purpose, we have carried out a new model based on the analysis on the proportion of different types of lanes, the shape of the toll plaza, the area of the toll plaza and the merging pattern of vehicles order to optimize the existing toll plaza. Nowadays with the rapid development of Internet technology, we hope that the Internet technology can be applied to our design and thus can improve the efficiency of the toll plaza.

When the traffic volume is large, we can imagine that congestion of the toll station often occurs, let alone illegal phenomena that vehicles change lanes or seize lanes several times when entering toll plaza, which greatly improving vehicle insecurity and reducing the efficiency of the toll plaza. Accordingly, we work out a reasonable design for different types of toll stations. We also design the type and quantity ratio of corresponding lanes, as well as merging pattern of different types of vehicle. When the traffic flow is small, considering that the disposal capacity of toll booth is stable, the possibility of congestion is relatively small. Hence the traffic flow is steady and fast.

On the other hand, we have designed the shape and size of the toll plaza to ensure greater throughput while minimizing the toll plaza area to reduce car costs. In our design, we increase the angle of the way along toll plaza buffer to help vehicles decelerate and reduce the area space of the toll plaza.

Although there are many prerequisites in our model, we also take full account of the practicality of the model and economical efficiency of the toll station

We have submitted a detailed elaboration in our essay, hoping to help you.

Yours Sincerely

Team 56867

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