

# Decomposição Espectral, SVD e PCA

Davi Wentrick Feijó - 200016806

2023-05-23

## Decomposição Espectral

A decomposição espectral consiste em decompor uma matriz A no produto de 3 matrizes da seguinte forma:

$$A_{n \times n} = P_{n \times n} D_{n \times n} P_{n \times n}^T$$

Onde P é uma matriz onde as cada coluna é composta por um autovetor de A e D é uma matriz diagonal com os autovalores correspondentes a cada autovetor de A.

Podemos realizar isso dentro do R por meio da função `eigen()`. Vamos utilizar a seguinte matriz e usaremos a matriz de covariância dela (com isso garantimos que ela é positiva definida)

99.00000	61.33333	20.50000	-29.66667
61.33333	40.33333	20.16667	-12.00000
20.50000	20.16667	30.25000	24.16667
-29.66667	-12.00000	24.16667	72.33333

Aqui vamos fazer nossa decomposicao, o resultado saira como um objeto onde `$vectors` é nosso P e `$values` é a matriz D (temos que usar a função `diag()` para que tenhamos a matriz diagonal pois o resultado sai em um vetor)

```
spectral_decomposition = eigen(A)
```

```
P = spectral_decomposition$vectors
```

-0.7943006	0.1158358	0.4105382	0.4325818
-0.4936169	0.1906772	-0.1244255	-0.8393467
-0.1506798	0.5193732	-0.7771652	0.3218096
0.3205066	0.8249096	0.4604248	-0.0693452

```
D = diag(spectral_decomposition$values)
```

152.9751	0.00000	0.000000	0
0.0000	80.60931	0.000000	0
0.0000	0.00000	8.332241	0
0.0000	0.00000	0.000000	0

Podemos confirmar a decomposição tentando obter a matriz A novamente

```
A_recuperado = P %*% D %*% solve(P)
```

99.00000	61.33333	20.50000	-29.66667
61.33333	40.33333	20.16667	-12.00000
20.50000	20.16667	30.25000	24.16667
-29.66667	-12.00000	24.16667	72.33333

Vale notar que quando A é simétrica  $P^{-1} = P^T$

```
P_transposto = t(P)
```

-0.7943006	-0.4936169	-0.1506798	0.3205066
0.1158358	0.1906772	0.5193732	0.8249096
0.4105382	-0.1244255	-0.7771652	0.4604248
0.4325818	-0.8393467	0.3218096	-0.0693452

```
P_inversa = solve(P)
```

-0.7943006	-0.4936169	-0.1506798	0.3205066
0.1158358	0.1906772	0.5193732	0.8249096
0.4105382	-0.1244255	-0.7771652	0.4604248
0.4325818	-0.8393467	0.3218096	-0.0693452

## Single Value Decomposition (SVD)

O SVD assim como a decomposição espectral busca representar uma matriz A pelo produto de 3 matrizes da seguinte forma:

$$A_{m \times n} = U_{m \times n} \Sigma_{n \times n} V_{n \times n}^T$$

Onde U e V são matrizes ortogonais e  $\Sigma$  (ou D ou S) é uma matriz diagonal. O cálculo da SVD consiste em encontrar os autovalores e autovetores de  $AA^T$  e  $A^T A$ . Os autovetores de  $A^T A$  compõem as colunas de V, os autovetores de  $AA^T$  compõem as colunas de U. Além disso, os valores singulares em S (Matriz de valores singulares) são raízes quadradas dos autovalores de  $AA^T$  ou  $A^T A$ . Os valores singulares são as entradas diagonais da matriz S e estão dispostos em ordem decrescente. Os valores singulares são sempre números reais. Se a matriz A for uma matriz real, então U e V também são reais. Resumindo:

$$U = AA^T$$

$$V = A^T A$$

S = Raiz quadrada dos autovalores de U ou V

Vale notar que ao aplicar essas transformações nós estamos rotacionando os dados (ortonormalidade) com U, escalando os vetores com S e rotacionando novamente (voltando ao estado inicial) com  $V^T$ .

```
sv_decomp = svd(A)
```

```
## $d
## [1] 1.529751e+02 8.060931e+01 8.332241e+00 1.091777e-14
##
## $u
##      [,1]      [,2]      [,3]      [,4]
## [1,] -0.7943006 -0.1158358  0.4105382  0.43258175
## [2,] -0.4936169 -0.1906772 -0.1244255 -0.83934669
## [3,] -0.1506798 -0.5193732 -0.7771652  0.32180960
## [4,]  0.3205066 -0.8249096  0.4604248 -0.06934517
##
## $v
##      [,1]      [,2]      [,3]      [,4]
## [1,] -0.7943006 -0.1158358  0.4105382 -0.43258175
## [2,] -0.4936169 -0.1906772 -0.1244255  0.83934669
## [3,] -0.1506798 -0.5193732 -0.7771652 -0.32180960
## [4,]  0.3205066 -0.8249096  0.4604248  0.06934517
```

Podemos verificar o SVD tentando recuperar a matriz A

```
sv_decomp$u %*% diag(sv_decomp$d) %*% t(sv_decomp$v)
```

```
##      [,1]      [,2]      [,3]      [,4]
## [1,] 99.00000 61.33333 20.50000 -29.66667
## [2,] 61.33333 40.33333 20.16667 -12.00000
## [3,] 20.50000 20.16667 30.25000 24.16667
## [4,] -29.66667 -12.00000 24.16667 72.33333
```

*#verificar*

```
sv_decomp$u %o% diag(sv_decomp$d) %o% t(sv_decomp$v)
```

```
## , , 1, 1, 1, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 96.51406 14.07501 -49.88377 -52.562242
## [2,] 59.97852 23.16884 15.11872 101.987529
## [3,] 18.30883 63.10811 94.43196 -39.102515
## [4,] -38.94419 100.23330 -55.94540 8.426008
##
## , , 2, 1, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 3, 1, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 4, 1, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 1, 2, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 2, 2, 1, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 50.857495 7.416739 -26.285951 -27.697353
## [2,] 31.605314 12.208681 7.966716 53.741707
## [3,] 9.647728 33.254435 49.760347 -20.604832
## [4,] -20.521404 52.817327 -29.480086 4.440034
##
## , , 3, 2, 1, 1
##
```

```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 1, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  5.2569230 0.7666368 -2.7170670 -2.8629576
## [2,]  3.2669070 1.2619595  0.8234855  5.5550517
## [3,]  0.9972446 3.4373696  5.1435156 -2.1298339
## [4,] -2.1212103 5.4595025 -3.0472311  0.4589474
##
## , , 4, 3, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##

```

```

## , , 2, 4, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 1, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 1, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 6.888167e-15 1.004527e-15 -3.560183e-15 -3.751344e-15
## [2,] 4.280641e-15 1.653550e-15 1.079016e-15 7.278806e-15
## [3,] 1.306693e-15 4.503999e-15 6.739568e-15 -2.790730e-15
## [4,] -2.779430e-15 7.153607e-15 -3.992799e-15 6.013606e-16
##
## , , 1, 1, 2, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 14.075007 2.052611 -7.274738 -7.665349
## [2,] 8.746892 3.378799 2.204819 14.873223
## [3,] 2.670046 9.203293 13.771367 -5.702466
## [4,] -5.679377 14.617398 -8.158727 1.228796
##
## , , 2, 1, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0

```

```

## [4,]    0    0    0    0
##
## , , 1, 2, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 2, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  7.416739 1.081611 -3.833379 -4.0392087
## [2,]  4.609121 1.780438  1.161816  7.8373544
## [3,]  1.406964 4.849619  7.256738 -3.0048798
## [4,] -2.992713 7.702549 -4.299191  0.6475067
##
## , , 3, 2, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 2, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  0.7666368 0.1118015 -0.3962401 -0.41751584

```

```

## [2,] 0.4764253 0.1840363 0.1200920 0.81011400
## [3,] 0.1454319 0.5012845 0.7500981 -0.31060165
## [4,] -0.3093440 0.7961797 -0.4443891 0.06693002
##
## , , 4, 3, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 2, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 2, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 1.004527e-15 1.464940e-16 -5.191949e-16 -5.470726e-16
## [2,] 6.242619e-16 2.411434e-16 1.573570e-16 1.061496e-15
## [3,] 1.905600e-16 6.568350e-16 9.828565e-16 -4.069825e-16
## [4,] -4.053347e-16 1.043237e-15 -5.822848e-16 8.769866e-17
##
## , , 1, 1, 3, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -49.883774 -7.274738 25.782678 27.167058
## [2,] -31.000197 -11.974933 -7.814185 -52.712765
## [3,] -9.463012 -32.617744 -48.807633 20.210330
## [4,] 20.128500 -51.806083 28.915658 -4.355025
##
## , , 2, 1, 3, 1
##

```



```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 3, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -26.285951 -3.833379 13.586025 14.315515
## [2,] -16.335365 -6.310118 -4.117637 -27.776670
## [3,] -4.986476 -17.187721 -25.718884 10.649710
## [4,] 10.606590 -27.298899 15.236930 -2.294854
##
## , , 3, 2, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##

```

```

## , , 1, 3, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 3, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -2.7170670 -0.3962401  1.4043296  1.4797340
## [2,] -1.6885172 -0.6522501 -0.4256226 -2.8711563
## [3,] -0.5154309 -1.7766217 -2.6584517  1.1008153
## [4,]  1.0963582 -2.8217712  1.5749766 -0.2372093
##
## , , 4, 3, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 3, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0

```

```

## [4,]    0    0    0    0
##
## , , 4, 4, 3, 1
##
##          [,1]          [,2]          [,3]          [,4]
## [1,] -3.560183e-15 -5.191949e-16  1.840099e-15  1.938901e-15
## [2,] -2.212471e-15 -8.546458e-16 -5.576950e-16 -3.762087e-15
## [3,] -6.753711e-16 -2.327914e-15 -3.483380e-15  1.442403e-15
## [4,]  1.436562e-15 -3.697378e-15  2.063698e-15 -3.108162e-16
##
## , , 1, 1, 4, 1
##
##          [,1]          [,2]          [,3]          [,4]
## [1,]  52.56224  7.665349 -27.167058 -28.625771
## [2,]  32.66473 12.617917  8.233761  55.543133
## [3,]   9.97112 34.369127  51.428318 -21.295507
## [4,] -21.20928 54.587768 -30.468261  4.588864
##
## , , 2, 1, 4, 1
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 4, 1
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 4, 1
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 4, 1
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 4, 1
##
##          [,1]          [,2]          [,3]          [,4]
## [1,]  27.697353  4.039209 -14.31552 -15.084175

```

```

## [2,] 17.212479 6.648935 4.33873 29.268115
## [3,] 5.254221 18.110602 27.09984 -11.221538
## [4,] -11.176103 28.764691 -16.05506 2.418074
##
## , , 3, 2, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 4, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 2.8629576 0.4175158 -1.4797340 -1.559187
## [2,] 1.7791807 0.6872721 0.4484761 3.025321
## [3,] 0.5431065 1.8720159 2.8011951 -1.159923
## [4,] -1.1552262 2.9732838 -1.6595437 0.249946
##
## , , 4, 3, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 4, 1
##

```

```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 4, 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 4, 1
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 3.751344e-15 5.470726e-16 -1.938901e-15 -2.043009e-15
## [2,] 2.331267e-15 9.005353e-16 5.876399e-16 3.964089e-15
## [3,] 7.116345e-16 2.452910e-15 3.670417e-15 -1.519851e-15
## [4,] -1.513697e-15 3.895905e-15 -2.174507e-15 3.275052e-16
##
## , , 1, 1, 1, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 59.97852 8.746892 -31.000197 -32.66473
## [2,] 37.27356 14.398244 9.395504 63.38000
## [3,] 11.37800 39.218443 58.684603 -24.30020
## [4,] -24.20181 62.289835 -34.767183 5.23633
##
## , , 2, 1, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
##

```

```

## , , 4, 1, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 1, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 31.605314  4.609121 -16.335365 -17.212479
## [2,] 19.641075  7.587067  4.950904  33.397703
## [3,]  5.995566 20.665919 30.923494 -12.804842
## [4,] -12.752996 32.823249 -18.320355  2.759252
##
## , , 3, 2, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 1, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0

```

```

## [4,]    0    0    0    0
##
## , , 3, 3, 1, 2
##
##          [,1]      [,2]      [,3]      [,4]
## [1,]  3.2669070  0.4764253 -1.6885172 -1.7791807
## [2,]  2.0302145  0.7842428  0.5117539  3.4521786
## [3,]  0.6197362  2.1361482  3.1964301 -1.3235821
## [4,] -1.3182230  3.3927997 -1.8936972  0.2852122
##
## , , 4, 3, 1, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 1, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 1, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 1, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 1, 2
##
##          [,1]      [,2]      [,3]      [,4]
## [1,]  4.280641e-15  6.242619e-16 -2.212471e-15 -2.331267e-15
## [2,]  2.660198e-15  1.027596e-15  6.705531e-16  4.523403e-15
## [3,]  8.120427e-16  2.799003e-15  4.188295e-15 -1.734295e-15
## [4,] -1.727273e-15  4.445599e-15 -2.481319e-15  3.737146e-16
##
## , , 1, 1, 2, 2
##
##          [,1]      [,2]      [,3]      [,4]
## [1,] 23.168845   3.378799 -11.974933 -12.61792

```

```

## [2,] 14.398244  5.561836   3.629349  24.48279
## [3,]  4.395158 15.149524  22.669024  -9.38682
## [4,] -9.348813 24.061673 -13.430066   2.02272
##
## , , 2, 1, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 2, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 12.208681  1.780438 -6.310118 -6.648935
## [2,]  7.587067  2.930776  1.912463 12.901056
## [3,]  2.316001  7.982949 11.945304 -4.946327
## [4,] -4.926300 12.679152 -7.076891  1.065860
##
## , , 3, 2, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 2, 2
##

```



```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 2, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  1.2619595 0.1840363 -0.6522501 -0.6872721
## [2,]  0.7842428 0.3029418  0.1976832  1.3335273
## [3,]  0.2393952 0.8251635  1.2347353 -0.5112808
## [4,] -0.5092107 1.3105900 -0.7315082  0.1101734
##
## , , 4, 3, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
##

```

```

## , , 3, 4, 2, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 2, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 1.653550e-15 2.411434e-16 -8.546458e-16 -9.005353e-16
## [2,] 1.027596e-15 3.969458e-16 2.590251e-16 1.747326e-15
## [3,] 3.136805e-16 1.081215e-15 1.617878e-15 -6.699333e-16
## [4,] -6.672208e-16 1.717271e-15 -9.584980e-16 1.443606e-16
##
## , , 1, 1, 3, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 15.118717 2.204819 -7.814185 -8.233761
## [2,] 9.395504 3.629349 2.368314 15.976124
## [3,] 2.868039 9.885748 14.792561 -6.125324
## [4,] -6.100523 15.701328 -8.763725 1.319916
##
## , , 2, 1, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0

```

```

## [4,]    0    0    0    0
##
## , , 2, 2, 3, 2
##
##          [,1]      [,2]      [,3]      [,4]
## [1,]  7.966716  1.161816 -4.117637 -4.3387300
## [2,]  4.950904  1.912463  1.247969  8.4185212
## [3,]  1.511296  5.209235  7.794850 -3.2277020
## [4,] -3.214633  8.273719 -4.617991  0.6955216
##
## , , 3, 2, 3, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 3, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 3, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 3, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 3, 2
##
##          [,1]      [,2]      [,3]      [,4]
## [1,]  0.8234855  0.1200920 -0.4256226 -0.44847608
## [2,]  0.5117539  0.1976832  0.1289972  0.87018675
## [3,]  0.1562162  0.5384565  0.8057205 -0.33363383
## [4,] -0.3322830  0.8552192 -0.4773421  0.07189311
##
## , , 4, 3, 3, 2
##
##          [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0

```

```

## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 3, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 3, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 1.079016e-15 1.573570e-16 -5.576950e-16 -5.876399e-16
## [2,] 6.705531e-16 2.590251e-16 1.690256e-16 1.140209e-15
## [3,] 2.046907e-16 7.055416e-16 1.055739e-15 -4.371617e-16
## [4,] -4.353917e-16 1.120597e-15 -6.254632e-16 9.420182e-17
##
## , , 1, 1, 4, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -101.98753 -14.87322 52.71276 55.543133
## [2,] -63.38000 -24.48279 -15.97612 -107.771409
## [3,] -19.34716 -66.68708 -99.78735 41.320082
## [4,] 41.15278 -105.91770 59.11815 -8.903861
##
## , , 2, 1, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 4, 2
##

```

```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 4, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -53.74171 -7.837354 27.776670 29.268115
## [2,] -33.39770 -12.901056 -8.418521 -56.789486
## [3,] -10.19487 -35.140349 -52.582338 21.773365
## [4,] 21.68521 -55.812683 31.151949 -4.691835
##
## , , 3, 2, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
##

```

```

## , , 2, 3, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 4, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -5.555052 -0.810114  2.8711563  3.0253206
## [2,] -3.452179 -1.333527 -0.8701867 -5.8700878
## [3,] -1.053800 -3.632308 -5.4352127  2.2506202
## [4,]  2.241508 -5.769120  3.2200446 -0.4849751
##
## , , 4, 3, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 4, 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 4, 2
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -7.278806e-15 -1.061496e-15  3.762087e-15  3.964089e-15
## [2,] -4.523403e-15 -1.747326e-15 -1.140209e-15 -7.691599e-15
## [3,] -1.380798e-15 -4.759428e-15 -7.121780e-15  2.948996e-15

```

```

## [4,] 2.937056e-15 -7.559300e-15 4.219237e-15 -6.354647e-16
##
## , , 1, 1, 1, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 18.308833 2.670046 -9.463012 -9.971120
## [2,] 11.377997 4.395158 2.868039 19.347157
## [3,] 3.473208 11.971685 17.913857 -7.417794
## [4,] -7.387760 19.014377 -10.612909 1.598424
##
## , , 2, 1, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 1, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 9.647728 1.406964 -4.986476 -5.2542211
## [2,] 5.995566 2.316001 1.511296 10.1948662
## [3,] 1.830186 6.308406 9.439598 -3.9087613
## [4,] -3.892935 10.019510 -5.592408 0.8422797
##
## , , 3, 2, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0

```

```

## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 1, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  0.9972446 0.1454319 -0.5154309 -0.54310650
## [2,]  0.6197362 0.2393952  0.1562162  1.05379998
## [3,]  0.1891785 0.6520731  0.9757311 -0.40403204
## [4,] -0.4023961 1.0356742 -0.5780634  0.08706287
##
## , , 4, 3, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 1, 3
##

```



```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 1, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 1, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 1.306693e-15 1.905600e-16 -6.753711e-16 -7.116345e-16
## [2,] 8.120427e-16 3.136805e-16 2.046907e-16 1.380798e-15
## [3,] 2.478813e-16 8.544140e-16 1.278504e-15 -5.294047e-16
## [4,] -5.272612e-16 1.357048e-15 -7.574387e-16 1.140788e-16
##
## , , 1, 1, 2, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 63.10811 9.203293 -32.617744 -34.369127
## [2,] 39.21844 15.149524 9.885748 66.687077
## [3,] 11.97168 41.264806 61.746683 -25.568150
## [4,] -25.46463 65.540032 -36.581286 5.509555
##
## , , 2, 1, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
##

```

```

## , , 1, 2, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 2, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 33.254435  4.849619 -17.187721 -18.110602
## [2,] 20.665919  7.982949  5.209235  35.140349
## [3,]  6.308406 21.744238 32.537038 -13.472981
## [4,] -13.418430 34.535920 -19.276286  2.903226
##
## , , 3, 2, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 2, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 3.4373696 0.5012845 -1.7766217 -1.8720159
## [2,] 2.1361482 0.8251635  0.5384565  3.6323084
## [3,] 0.6520731 2.2476094  3.3632153 -1.3926447

```

```

## [4,] -1.3870060 3.5698312 -1.9925077 0.3000942
##
## , , 4, 3, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 2, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 2, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 4.503999e-15 6.568350e-16 -2.327914e-15 -2.452910e-15
## [2,] 2.799003e-15 1.081215e-15 7.055416e-16 4.759428e-15
## [3,] 8.544140e-16 2.945051e-15 4.406834e-15 -1.824788e-15
## [4,] -1.817399e-15 4.677564e-15 -2.610791e-15 3.932145e-16
##
## , , 1, 1, 3, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 94.43196 13.77137 -48.80763 -51.428318
## [2,] 58.68460 22.66902 14.79256 99.787355
## [3,] 17.91386 61.74668 92.39479 -38.258957
## [4,] -38.10405 98.07097 -54.73849 8.244234
##
## , , 2, 1, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0

```

```

## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 3, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 49.760347  7.256738 -25.71888 -27.099839
## [2,] 30.923494 11.945304  7.79485  52.582338
## [3,]  9.439598 32.537038 48.68687 -20.160324
## [4,] -20.078697 51.677900 -28.84411  4.344249
##
## , , 3, 2, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 3, 3
##

```

```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 3, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  5.1435156 0.7500981 -2.6584517 -2.8011951
## [2,]  3.1964301 1.2347353  0.8057205  5.4352127
## [3,]  0.9757311 3.3632153  5.0325546 -2.0838870
## [4,] -2.0754495 5.3417248 -2.9814933  0.4490465
##
## , , 4, 3, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 3, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
##

```

```

## , , 4, 4, 3, 3
##
##          [,1]          [,2]          [,3]          [,4]
## [1,]  6.739568e-15  9.828565e-16 -3.483380e-15 -3.670417e-15
## [2,]  4.188295e-15  1.617878e-15  1.055739e-15  7.121780e-15
## [3,]  1.278504e-15  4.406834e-15  6.594176e-15 -2.730525e-15
## [4,] -2.719470e-15  6.999283e-15 -3.906662e-15  5.883874e-16
##
## , , 1, 1, 4, 3
##
##          [,1]          [,2]          [,3]          [,4]
## [1,]  39.102515  5.702466 -20.210330 -21.295507
## [2,]  24.300199  9.386820  6.125324  41.320082
## [3,]   7.417794 25.568150 38.258957 -15.842321
## [4,] -15.778176 40.609360 -22.666187  3.413784
##
## , , 2, 1, 4, 3
##
##          [,1] [,2] [,3] [,4]
## [1,]      0    0    0    0
## [2,]      0    0    0    0
## [3,]      0    0    0    0
## [4,]      0    0    0    0
##
## , , 3, 1, 4, 3
##
##          [,1] [,2] [,3] [,4]
## [1,]      0    0    0    0
## [2,]      0    0    0    0
## [3,]      0    0    0    0
## [4,]      0    0    0    0
##
## , , 4, 1, 4, 3
##
##          [,1] [,2] [,3] [,4]
## [1,]      0    0    0    0
## [2,]      0    0    0    0
## [3,]      0    0    0    0
## [4,]      0    0    0    0
##
## , , 1, 2, 4, 3
##
##          [,1] [,2] [,3] [,4]
## [1,]      0    0    0    0
## [2,]      0    0    0    0
## [3,]      0    0    0    0
## [4,]      0    0    0    0
##
## , , 2, 2, 4, 3
##
##          [,1]          [,2]          [,3]          [,4]
## [1,] 20.604832  3.004880 -10.649710 -11.221538
## [2,] 12.804842  4.946327  3.227702 21.773365
## [3,]  3.908761 13.472981 20.160324 -8.348014

```

```

## [4,] -8.314214 21.398854 -11.943809 1.798872
##
## , , 3, 2, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 4, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  2.1298339 0.3106016 -1.1008153 -1.1599226
## [2,]  1.3235821 0.5112808  0.3336338  2.2506202
## [3,]  0.4040320 1.3926447  2.0838870 -0.8628987
## [4,] -0.8594049 2.2119086 -1.2345808  0.1859418
##
## , , 4, 3, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0

```

```

## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 2, 4, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 3, 4, 4, 3
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 4, 4, 4, 3
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 2.790730e-15 4.069825e-16 -1.442403e-15 -1.519851e-15
## [2,] 1.734295e-15 6.699333e-16 4.371617e-16 2.948996e-15
## [3,] 5.294047e-16 1.824788e-15 2.730525e-15 -1.130660e-15
## [4,] -1.126082e-15 2.898272e-15 -1.617676e-15 2.436403e-16
##
## , , 1, 1, 1, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -38.94419 -5.679377 20.128500 21.209284
## [2,] -24.20181 -9.348813 -6.100523 -41.152781
## [3,] -7.38776 -25.464626 -38.104050 15.778176
## [4,] 15.71429 -40.444936 22.574413 -3.399961
##
## , , 2, 1, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 3, 1, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 4, 1, 1, 4
##

```



```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 1, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -20.521404 -2.992713  10.606590  11.176103
## [2,] -12.752996 -4.926300  -3.214633 -21.685207
## [3,]  -3.892935 -13.418430 -20.078697   8.314214
## [4,]   8.280550 -21.312212  11.895449  -1.791589
##
## , , 3, 2, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
##

```

```

## , , 3, 3, 1, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -2.1212103 -0.3093440  1.096358  1.1552262
## [2,] -1.3182230 -0.5092107 -0.332283 -2.2415076
## [3,] -0.4023961 -1.3870060 -2.075450  0.8594049
## [4,]  0.8559253 -2.2029528  1.229582 -0.1851889
##
## , , 4, 3, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 1, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 1, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -2.779430e-15 -4.053347e-16  1.436562e-15  1.513697e-15
## [2,] -1.727273e-15 -6.672208e-16 -4.353917e-16 -2.937056e-15
## [3,] -5.272612e-16 -1.817399e-15 -2.719470e-15  1.126082e-15
## [4,]  1.121522e-15 -2.886538e-15  1.611126e-15 -2.426538e-16
##
## , , 1, 1, 2, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 100.23330  14.61740 -51.80608 -54.587768
## [2,]  62.28984  24.06167  15.70133 105.917695
## [3,]  19.01438  65.54003  98.07097 -40.609360

```

```

## [4,] -40.44494 104.09587 -58.10129 8.750711
##
## , , 2, 1, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 2, 2, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 52.81733  7.702549 -27.298899 -28.764691
## [2,] 32.82325 12.679152  8.273719  55.812683
## [3,] 10.01951 34.535920 51.677900 -21.398854
## [4,] -21.31221 54.852682 -30.616123  4.611134
##
## , , 3, 2, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0

```

```

## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 2, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,]  5.459503  0.7961797 -2.8217712 -2.9732838
## [2,]  3.392800  1.3105900  0.8552192  5.7691198
## [3,]  1.035674  3.5698312  5.3417248 -2.2119086
## [4,] -2.202953  5.6698885 -3.1646585  0.4766333
##
## , , 4, 3, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 2, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 2, 4
##

```

```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 2, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] 7.153607e-15 1.043237e-15 -3.697378e-15 -3.895905e-15
## [2,] 4.445599e-15 1.717271e-15 1.120597e-15 7.559300e-15
## [3,] 1.357048e-15 4.677564e-15 6.999283e-15 -2.898272e-15
## [4,] -2.886538e-15 7.429277e-15 -4.146664e-15 6.245344e-16
##
## , , 1, 1, 3, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -55.94540 -8.158727 28.915658 30.468261
## [2,] -34.76718 -13.430066 -8.763725 -59.118152
## [3,] -10.61291 -36.581286 -54.738488 22.666187
## [4,] 22.57441 -58.101295 32.429342 -4.884225
##
## , , 2, 1, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 1, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 2, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##

```

```

## , , 2, 2, 3, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -29.480086 -4.299191 15.236930 16.055064
## [2,] -18.320355 -7.076891 -4.617991 -31.151949
## [3,] -5.592408 -19.276286 -28.844112 11.943809
## [4,] 11.895449 -30.616123 17.088444 -2.573713
##
## , , 3, 2, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 2, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 3, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 3, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 3, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -3.0472311 -0.4443891 1.5749766 1.6595437
## [2,] -1.8936972 -0.7315082 -0.4773421 -3.2200446
## [3,] -0.5780634 -1.9925077 -2.9814933 1.2345808
## [4,] 1.2295820 -3.1646585 1.7663598 -0.2660337
##
## , , 4, 3, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0

```

```

## [4,]    0    0    0    0
##
## , , 1, 4, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 3, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 3, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -3.992799e-15 -5.822848e-16  2.063698e-15  2.174507e-15
## [2,] -2.481319e-15 -9.584980e-16 -6.254632e-16 -4.219237e-15
## [3,] -7.574387e-16 -2.610791e-15 -3.906662e-15  1.617676e-15
## [4,]  1.611126e-15 -4.146664e-15  2.314468e-15 -3.485850e-16
##
## , , 1, 1, 4, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -8.426008 -1.228796  4.355025  4.5888640
## [2,] -5.236330 -2.022720 -1.319916 -8.9038610
## [3,] -1.598424 -5.509555 -8.244234  3.4137836
## [4,]  3.399961 -8.750711  4.884225 -0.7356194
##
## , , 2, 1, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 1, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0

```

```

## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 4, 1, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 1, 2, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 2, 2, 4, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -4.4400337 -0.6475067 2.2948536 2.4180739
## [2,] -2.7592523 -1.0658597 -0.6955216 -4.6918353
## [3,] -0.8422797 -2.9032262 -4.3442489 1.7988725
## [4,] 1.7915890 -4.6111337 2.5737126 -0.3876302
##
## , , 3, 2, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 4, 2, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 1, 3, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,] 0 0 0 0
## [2,] 0 0 0 0
## [3,] 0 0 0 0
## [4,] 0 0 0 0
##
## , , 2, 3, 4, 4
##

```



```

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 3, 4, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -0.45894741 -0.06693002  0.23720926  0.24994602
## [2,] -0.28521218 -0.11017339 -0.07189311 -0.48497506
## [3,] -0.08706287 -0.30009415 -0.44904654  0.18594180
## [4,]  0.18518893 -0.47663329  0.26603373 -0.04006768
##
## , , 4, 3, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 1, 4, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 2, 4, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 3, 4, 4, 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
## [3,]    0    0    0    0
## [4,]    0    0    0    0
##
## , , 4, 4, 4, 4
##
##      [,1]      [,2]      [,3]      [,4]
## [1,] -6.013606e-16 -8.769866e-17  3.108162e-16  3.275052e-16
## [2,] -3.737146e-16 -1.443606e-16 -9.420182e-17 -6.354647e-16
## [3,] -1.140788e-16 -3.932145e-16 -5.883874e-16  2.436403e-16
## [4,]  2.426538e-16 -6.245344e-16  3.485850e-16 -5.250084e-17

```

## Diferenças entre a Decomposição espectral e o SVD

Considere a decomposição em autovalores  $A = PDP^{-1}$  e a SVD  $A = U\Sigma V^{-1}$ . Algumas diferenças-chave são as seguintes:

- Os vetores na matriz de decomposição em autovalores  $P$  não são necessariamente ortogonais, então a mudança de base não é uma simples rotação. Por outro lado, os vetores nas matrizes  $U$  e  $V$  na SVD são ortonormais, então eles representam rotações (e possivelmente reflexões).
- Na SVD, as matrizes não diagonais  $U$  e  $V$  não são necessariamente inversas uma da outra. Geralmente, elas não têm relação entre si. Na decomposição em autovalores, as matrizes não diagonais  $P$  e  $P^{-1}$  são inversas uma da outra.
- Na SVD, as entradas na matriz diagonal  $\Sigma$  são todos números reais e não negativos. Na decomposição em autovalores, as entradas de  $D$  podem ser qualquer número complexo - negativo, positivo, imaginário, qualquer coisa.
- A SVD sempre existe para qualquer tipo de matriz retangular ou quadrada, enquanto a decomposição em autovalores só existe para matrizes quadradas e, mesmo entre as matrizes quadradas, às vezes ela não existe.

## Análise de Componentes Principais (PCA)

Aplicar o SVD em uma matriz de Covariância ou Correlação podemos retirar informações importantes sobre um conjunto de dados.

Mas antes vamos entender o que ocorre com os dados ao obter essas matrizes.

- A matriz de covariância ela mantém a escala dos dados e não é centralizada.
- A matriz de correlação ela normaliza os dados (divide pela variância) ou seja ela controla a escala das variáveis deixando todas “iguais”
- A centralização dos dados pode ser feita, que nada mais é que subtrair a média de cada coluna, centralizando os dados em torno de 0.