Lista Entrega 5

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Questao 1 (9.1)

A questao nos dá a matriz de covariancia ρ e a matriz de erros Ψ

A matriz ρ

1.00	0.63	0.45
0.63	1.00	0.35
0.45	0.35	1.00

A matriz Ψ

0.19	0.00	0.00
0.00	0.51	0.00
0.00	0.00	0.75

Sabemos que na analise fatorial temos a seguinte relacao:

$$\Sigma = LL^T + \Psi$$
$$LL^T = \Sigma - \Psi$$

Calculando LL^T

0.81	0.63	0.45
0.63	0.49	0.35
0.45	0.35	0.25

Podemos encontrar a comunalidade na diagonal da matriz LL^T já que subtraimos o Ψ

x
0.81
0.49
0.25

Com essas informacoes podemos escrever nossa matriz Σ como:

$$\Sigma = LL^T + \Psi$$

```r
p_construido = LLT + psi
```

1.00	0.63	0.45
0.63	1.00	0.35
0.45	0.35	1.00

Questao 2 (9.2)

A) As comunalidades sao:

```r
comu
```

```
## [1] 0.81 0.49 0.25
```

Podemos perceber que F1 detem a maior comunalidade logo é o fator que mais explica a variancia dos dados

B) Sabemos que:

$$Cor(X,Y) = \frac{Cov(X,Y)}{S_x S_y}$$

$$Cov(X,F) = L$$

Logo

$$Cor(X_i,F_i) = \frac{Cov(X_i,F_i)}{S_i S_f} = \frac{L_i}{S_x S_f}$$

```r
cor_xf = Lestimado[1]/(1*comu[1])
```

```
## [1] -1.141896
```

Questao 3 (9.3)

A) Para realizar por meio de componentes principais primeiro precisamos encontrar os autovalores e autovetores da matriz de correlacao aplicando a decompisicao espectral em ρ dada na questao 9.1

$$\rho = CDC^T$$

```r
eigen_p = eigen(p)

autoval <- eigen_p$values

autovet <- eigen_p$vectors

D <- matrix(0, nrow = 3, ncol = 3)
diag(D) <- sqrt(autoval)
```

```r
autoval
```

```
## [1] 1.9632830 0.6794930 0.3572239
```

```r
autovet
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.6250027  0.2186276  0.7493822
## [2,] -0.5931510  0.4910833 -0.6379726
## [3,] -0.5074875 -0.8432314 -0.1772492
```

Em seguida podemos encontrar nossa matriz L

$$L = CD^{1/2}$$

Aqui temos nossa matriz dos loadings

```r
Lestimado
```

```
## [1] -0.8757363 -0.8311066 -0.7110772
```

Para calcular a matriz Ψ temos que seguir a equacao:

$$\Psi = \Sigma - LL^T$$

Na diagonal obteremos nosso Ψ

```r
psiestimado <- diag(p-LLT)
```

```r
psiestimado
```

```
## [1] 0.2330860 0.3092618 0.4943692
```

Para comparar com os resultados anteriores podemos aproximar a matrix Σ de correlacoes por meio da formula:

$$\Sigma = LL^T + \Psi$$

```
##           [,1]      [,2]      [,3]
## [1,] 0.9569140 0.7278302 0.6227161
## [2,] 0.7278302 1.2007382 0.5909810
## [3,] 0.6227161 0.5909810 1.2556308
```

B) A variancia explicada é:

```
## [1] 0.6544277 0.2264977 0.1190746
```

Podemos notar que a primeira componente exxplica 65% da variancia dos dados

Questao 4 (9.19)

	x1	x2	x3	x4	x5	x6	x7
x1	1.0000000	0.9260758	0.8840023	0.5720363	0.7080738	0.6744073	0.9273116
x2	0.9260758	1.0000000	0.8425232	0.5415080	0.7459097	0.4653880	0.9442960
x3	0.8840023	0.8425232	1.0000000	0.7003630	0.6374712	0.6410886	0.8525682
x4	0.5720363	0.5415080	0.7003630	1.0000000	0.5907360	0.1469074	0.4126395
x5	0.7080738	0.7459097	0.6374712	0.5907360	1.0000000	0.3859502	0.5745533
x6	0.6744073	0.4653880	0.6410886	0.1469074	0.3859502	1.0000000	0.5663721
x7	0.9273116	0.9442960	0.8525682	0.4126395	0.5745533	0.5663721	1.0000000

```r
AF2 <- principal(cor_data, nfactors = 2, rotate = 'none',
                   covar = F,n.obs = 50)
AF3 <- principal(cor_data, nfactors = 3, rotate = 'none',
                 covar = F,n.obs = 50)
```

```r
AF2
```

A)

```
## Principal Components Analysis
## Call: principal(r = cor_data, nfactors = 2, rotate = "none", n.obs = 50,
##     covar = F)
## Standardized loadings (pattern matrix) based upon correlation matrix
##     PC1   PC2   h2    u2 com
## x1 0.97 -0.11 0.96 0.041 1.0
## x2 0.94  0.03 0.89 0.110 1.0
## x3 0.94  0.01 0.89 0.107 1.0
## x4 0.66  0.65 0.85 0.147 2.0
## x5 0.78  0.28 0.69 0.305 1.3
## x6 0.65 -0.62 0.81 0.194 2.0
## x7 0.91 -0.19 0.87 0.127 1.1
##
##                        PC1  PC2
## SS loadings           5.03 0.93
## Proportion Var        0.72 0.13
## Cumulative Var        0.72 0.85
## Proportion Explained  0.84 0.16
## Cumulative Proportion 0.84 1.00
##
## Mean item complexity =  1.3
## Test of the hypothesis that 2 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.08
##  with the empirical chi square  11.93  with prob <  0.15
##
## Fit based upon off diagonal values = 0.99
```

```r
AF3
```

```
## Principal Components Analysis
## Call: principal(r = cor_data, nfactors = 3, rotate = "none", n.obs = 50,
##     covar = F)
## Standardized loadings (pattern matrix) based upon correlation matrix
##     PC1   PC2   PC3   h2    u2 com
## x1 0.97 -0.11 -0.05 0.96 0.039 1.0
## x2 0.94  0.03 -0.31 0.99 0.013 1.2
## x3 0.94  0.01  0.14 0.91 0.087 1.0
## x4 0.66  0.65  0.32 0.95 0.045 2.4
## x5 0.78  0.28  0.00 0.69 0.305 1.3
## x6 0.65 -0.62  0.43 0.99 0.012 2.7
## x7 0.91 -0.19 -0.31 0.97 0.033 1.3
##
##                        PC1  PC2  PC3
## SS loadings           5.03 0.93 0.50
## Proportion Var        0.72 0.13 0.07
## Cumulative Var        0.72 0.85 0.92
## Proportion Explained  0.78 0.14 0.08
## Cumulative Proportion 0.78 0.92 1.00
##
## Mean item complexity =  1.6
## Test of the hypothesis that 3 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.04
##  with the empirical chi square  3.95  with prob <  0.27
##
## Fit based upon off diagonal values = 1
```

```r
#b)
AF2_rotated <- principal(cor_data, nfactors = 2, rotate = 'varimax',
                 covar = F,n.obs = 50)
AF3_rotated <- principal(cor_data, nfactors = 3, rotate = 'varimax',
                 covar = F,n.obs = 50)
```

B)

```r
#c)

AF2_comu = AF2$loadings^2
AF2_psi = diag(AF2$uniquenesses)
AF2_LLT = AF2$loadings %*% t(AF2$loadings)
resAF2 <- round(cor(data) - AF2_LLT - AF2_psi,3)


AF3_comu = AF3$loadings^3
```

```r
AF3_psi = diag(AF3$uniquenesses)
AF3_LLT = AF3$loadings %*% t(AF3$loadings)
resAF3 <- round(cor(data) - AF3_LLT - AF3_psi,3)
```

C)

```r
#d)

#m=2
dim(AF2$loadings)
```

D)

```
## [1] 7 2
```

```r
n = dim(data)[1]
p = dim(AF2$loadings)[1]
m = dim(AF2$loadings)[2]

AF2_teste_stat = AF2$chi
AF2_pvalue = AF2$PVAL

#m=3
dim(AF3$loadings)
```

```
## [1] 7 3
```

```r
n = dim(data)[1]
p = dim(AF3$loadings)[1]
m = dim(AF3$loadings)[2]

AF2_teste_stat = AF3$chi
AF2_pvalue = AF3$PVAL
```