Hw 2

1. (Modified from #1, p.62, [Alpaydin]) Assume the COVID-19 disease is seen in one person out of every ten thousand, (i.e., $P(d=1)=10^{-4}$). If the virus test has 96% chance to be positive (i.e., $P(t=1|d=1)=0.96$) on an infected person and has 2% chance to be positive on a healthy person, (i.e., $P(t=1|d=0)=0.02$). Assume that a new patient arrives.
   (a) If the test result of the new patient is positive, what is the probability that the patient has been infected?
   (b) If the test result of the new patient is negative, what is the probability that the patient has been infected?

2. (#6, #7, p.91-92, [Alpaydin]) In a two-class one-dimensional problem, given $P(C_1)$, $P(C_2)$, $p(x|C_1) \sim N(\mu_1, \sigma_1^2)$, and $p(x|C_2) \sim N(\mu_2, \sigma_2^2)$,
   (c) What is the likelihood ratio $\frac{p(x|C_1)}{p(x|C_2)}$?
   (d) What is the Bayes' discriminant function $g(x)=g_1(x)-g_2(x)$ and the decision boundary points?
   (e) What will be the decision boundary if we have $\sigma_1^2=\sigma_2^2=\sigma^2$.

3. (#9, p. 92, [Alpaydin]) If we have a two-class one-dimensional problem where both classes are normally distributed with the same mean $(\mu_1=\mu_2=\mu)$ but different variances $(\sigma_1^2>\sigma_2^2)$. What will be the discriminant function look like in this case?

4. (Ch2, [Theodoridis]) Given the data $X=\{x_1,\ldots,x_N\}$ sampled from a lognormal distribution $p(x|\theta)=\frac{1}{\sigma x\sqrt{2\pi}}\exp\left(-\frac{(\ln x-\theta)^2}{2\sigma^2}\right), x>0$. Show that the ML estimate is given by $\hat{\theta}_{ML}=\frac{1}{N}\sum_{i=1}^{N}\ln x_i$.