Computer experiment 4 (Ch14, [Theodoridis 2009])

(EM algorithm for Gaussian mixtures)

1. Consider three Gaussian pdfs $N(\mu_i, \Sigma_i), (i = 1,2,3)$ with mean vectors $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}5\\5\end{bmatrix}, \begin{bmatrix}9\\1\end{bmatrix}$ and covariance matrices $\begin{bmatrix}1 & 0.4\\0.4 & 1\end{bmatrix}, \begin{bmatrix}1 & -0.6\\-0.6 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, respectively. Generate 500 samples according to the following rule. The first two samples are generated from the 2nd Gaussian, the 3rd sample from the 1st one, and the 4th sample from the last Gaussian. This rule repeats until all 500 samples are generated. The pdf underling the random samples is modeled as a mixture $\sum_{i=1}^{3} N(\mu_i, \Sigma_i)P_i$.
   (a) Use EM algorithms and the generated samples to estimate the unknown parameters $\boldsymbol{\mu}_i, \Sigma_i, P_i$ $(i = 1,2,3)$. Please specify your experimental settings (e.g., initialization, stopping criterion) in the report.
   (b) Repeat the mixture density estimation by EM when the mean vectors are $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}3.5\\3.5\end{bmatrix}, \begin{bmatrix}6\\1\end{bmatrix}$.
   (c) Repeat the mixture density estimation by EM when the mean vectors are $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}, \begin{bmatrix}3\\1\end{bmatrix}$.
   (d) Compare the results (in terms of confusion matrices or 2-D visualization) and draw your conclusion.

(K-means algorithm)

2. Use k-means algorithm on the data set of the previous experiment, for $k = 2,3,4$. Compare the results and draw your conclusion.

(Fuzzy k-means algorithm)

3. Use fuzzy k-means algorithm on the data set of the previous experiment, for $k = 3$ and the fuzzifier $q = 2, 3$. Compare the results and draw your conclusion.