

Other norms: l_1 -norm: $\|x\|_1 = \sum_{i=1}^n |x_i|$

sup-norm or l_∞ -norm: $\|x\|_\infty = \max_i |x_i|$

Sequences $\{x_k\}_{k=1, \dots}$ or $\{x_k\}$, $x_k \in \mathbb{R}^n$

Definition (convergence) $x_k \rightarrow x$, $\lim_{k \rightarrow \infty} x_k = x$

Given $\varepsilon > 0$, $\exists N_\varepsilon$ s.t. $\|x_k - x\| < \varepsilon \quad \forall k \geq N_\varepsilon$

Definition (Cauchy sequence) $\{x_k\}$ is Cauchy if
given $\varepsilon > 0$, $\exists N_\varepsilon$ s.t. $\|x_k - x_m\| < \varepsilon \quad \forall k, m \geq N_\varepsilon$.

$\{x_k\}$ converges $\iff \{x_k\}$ is Cauchy

Definition (subsequence) infinite subset of $\{x_k\}$.

$\{x_k: k \in K\}$ or $\{x_k\}_K$, K : infinite subset of \mathbb{Z}^+

Definition (Limit point) x is a limit point of $\{x_k\}$
if \exists a subsequence of $\{x_k\}$ that converges to x .

Definition (bounded sequence) $\|x_k\| \leq b, \forall k$

Results about Bounded sequences

1. Every bounded has at least one limit point
2. A bounded sequence converges iff it has a unique limit point