Math Background

Linear Algebra

Vector $x \in \mathbb{R}^n$ – n-dim Euclidean Space

$$x = (x_1, \dots, x_n) \equiv [x_1\ x_2\ \dots\ x_n]^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Norm of x $\quad \|x\|$ satisfies properties:

(a) $\|x\| \geq 0$

(b) $\|x\| = 0 \iff x = 0$

(c) $\|cx\| = |c|\,\|x\|$, for $c \in \mathbb{R}$

(d) $\|x+y\| \leq \|x\| + \|y\|$ ← Triangle Ineq.

Inner Product $\quad x^T y = \sum_{i=1}^{n} x_i y_i$

Euclidean Norm $\quad \|x\| = \sqrt{x^T x} = \sqrt{\sum_{i=1}^{n} x_i^2}$

Two important results for Euclidean norm:

1) Pythagorean Theorem: If $x^T y = 0$,

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2$$

2) Cauchy-Schwarz Inequality:

$$|x^T y| \leq \|x\|\,\|y\|$$

"=" iff $x = \alpha y$ for some $\alpha \in \mathbb{R}$