Scalar Sequences $\{x_k\}$, $x_k \in \mathbb{R}$.

(below) (increasing)

- If $\{x_k\}$ is bounded above and non-decreasing it converges

(smallest) (inf)

- The largest limit point of $\{x_k\}$ is $\limsup_{k\to\infty} x_k$

- $\{x_k\}$ converges $\iff$ $-\infty < \liminf_{k\to\infty} x_k = \limsup_{k\to\infty} x_k < \infty$

Definition (continuity) A real-valued function f is continuous at x if for every $\{x_k\}$ converging to x

$$\lim_{k\to\infty} f(x_k) = f(x)$$

Equivalently, given $\varepsilon > 0$, $\exists\, \delta > 0$ s.t.

$$|f(x) - f(y)| < \varepsilon \quad \forall \quad \|y - x\| < \delta$$

f is continuous if it is continuous at all points x

Definition (coercive) A real-valued function $f : \mathcal{S} \to \mathbb{R}$ is coercive if for every $\{x_k\} \subset \mathcal{S}$ s.t. $\|x_k\| \to \infty$, $f(x_k) \to \infty$

Examples

1) $x \in \mathbb{R}^2$, $f(x) = x_1^2 + x_2^2$ — coercive

2) $x \in \mathbb{R}$, $f(x) = 1 - e^{-|x|}$ — not coercive

3) $x \in \mathbb{R}^2$, $f(x) = x_1^2 + x_2^2 - 2x_1x_2$

$$= (x_1 - x_2)^2 \text{ — not coercive}$$