Causal Inference

Author: Wenxiao Yang

Institute: Haas School of Business, University of California Berkeley

Date: 2023

All models are wrong, but some are useful.

Chapter 1 Causal Inference

The fundamental problem of causal inference:

(a). Never see the same person treated and untreated

(b). Missing data problem

(c). "Solve" by finding a comparison group

Definition 1.1 (Notations and Estimands)

- Treatment: $T \in \{0, 1\}$
- Potential Outcome with treatment $Y(1), Y(0)$
- Other Variable X
- Individual Treatment Effect (ITE) $= Y_i(1) - Y_i(0)$
- Conditional Average Treatment Effect (CATE) $= \mathbb{E}[Y(1) - Y(0)|X = x] := \tau(x)$
- Average Treatment Effect (ATE) $= \mathbb{E}[Y(1) - Y(0)] := \tau$
- Average Treatment Effects on Treated (ATT) $= \mathbb{E}[Y(1) - Y(0) \mid T = 1]$

♣

Difference in Means

$$\hat{\tau} = \bar{Y}_1 - \bar{Y}_0 = \frac{1}{n_1}\sum_{i=1}^{n} Y_i T_i - \frac{1}{n_0}\sum_{i=1}^{n} Y_i(1 - T_i)$$

By the Law of Large Numbers,

$$\begin{aligned}
\lim_{n\to\infty} \frac{1}{n_1}\sum_{i=1}^{n} Y_i T_i &= \lim_{n\to\infty} \frac{n}{n_i}\frac{1}{n}\sum_{i=1}^{n} Y_i T_i \\
&= (P[T = 1])^{-1}\,\mathbb{E}[YT] \\
&= (P[T = 1])^{-1}\,\mathbb{E}[YT \mid T = 1]P[T = 1] \\
&= \mathbb{E}[YT \mid T = 1] \\
\bar{Y}_1 &\xrightarrow{P} \mathbb{E}[YT \mid T = 1]
\end{aligned}$$

Causal Effect

Assumption

(1). SUTVA: Only your treatment matters;

(2). Consistency: Observed outcome matches treatment "assignment": $Y = TY(1) + (1 - T)Y(0)$.

Only yields $\hat{\tau} = \bar{Y}_1 - \bar{Y}_0 \xrightarrow{P} \mathbb{E}[Y(1) \mid T=1] - \mathbb{E}[Y(0) \mid T=0]$

$$\begin{aligned}
&\mathbb{E}[Y(1) \mid T=1] - \mathbb{E}[Y(0) \mid T=0] \\
=&\underbrace{\mathbb{E}[Y(1) \mid T=1] - \mathbb{E}[Y(0) \mid T=1]}_{\text{ATT}} + \underbrace{\mathbb{E}[Y(0) \mid T=1] - \mathbb{E}[Y(0) \mid T=0]}_{\text{selection bias}}
\end{aligned}$$

To get the ATT (eliminate the selection bias), we need exclusion/independence: Randomization.

Assume $Y(t) = \mu(t) + \epsilon_t$ (SUTVA). Consider the consistency assumption:

$$\begin{aligned}
Y &= TY(1) + (1-T)Y(0) \\
&= Y(0) + T(Y(1) - Y(0)) \\
&= \underbrace{\mu_0}_{\alpha} + T\underbrace{(\mu_1 - \mu_0)}_{\beta T} + \underbrace{\epsilon_0 + T(\epsilon_1 - \epsilon_0)}_{\epsilon}
\end{aligned}$$

Consider the covariate between T and X. Why important?

$$\begin{aligned}
&\mathbb{E}[Y|X=1,T=1] - \mathbb{E}[Y|X=1,T=0] \\
=&\underbrace{\mathbb{E}[Y(1)|X(1)=1] - \mathbb{E}[Y(0)|X(1)=1]}_{\text{ATE}|X(1)=1} + \underbrace{\mathbb{E}[Y(0)|X(1)=1] - \mathbb{E}[Y(0)|X(0)=1]}_{\text{selection bias}}
\end{aligned}$$

$Y(t) = \mu(t, X) + \epsilon_t$. Then,

$$\begin{aligned}
Y &= Y(1)T + Y(0)(1-T) \\
&= \underbrace{\mu(0, X)}_{\alpha(X)} + T\underbrace{(\mu_1(X) - \mu_0(X))}_{\beta(X)} + \epsilon
\end{aligned}$$