Bargaining

Econ 220B, Spring 2024

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Why bargaining?

Economics has many models of

- Competitive markets – No agents have bargaining power.
- Mechanism design – One agent has all the bargaining power.

"Bargaining" covers the case where multiple agents have some bargaining power.

Why bargaining?

example: monopoly pricing

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- with perfectly personalized pricing there would be no monopoly distortion.
- In vertical relationships, double marginalization disappears with two-part tariffs.

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What else might be going on?

- Asymmetric information about consumers values.
- Bargaining power for consumers (especially B-to-B).
- ...?

Why bargaining?

countless applications

Oligopoly models

- Both Cournot and Bertrand assume one side of the market sets the price. Why?

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Platforms and intermediaries: facilitate interaction between two (or more) sides of a market.

- Pharmacy benefits managers (PBMs).
- Health insurers.
- Amazon.
- ...But who sets prices? Even better, how is the market structure decided upon?

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We will use bargaining to refer to "a process to determine the terms of trade that is not adequately captured by off-the-shelf oligopoly models." (Loertscher and Marx, 2021).

Without modeling bargaining, can't think about shifts bargaining power, buyer power, or countervailing power

Strategic, or non-cooperative, models explicitly specify the game played by the bargaining parties, and analyze equilibria.

- Strengths: conclusions drawn from *clearly stated* assumptions about behavior, information, actions, etc.
- Weaknesses: conclusions drawn from *one set of* assumptions about behavior, information, actions, etc. Also, need to solve the game.

Axiomatic of cooperative models abstract from the details of the bargaining process. Instead, specify a set of “reasonable” properties, and consider only outcomes or agreements that satisfy these.

- Strengths: conclusions do not depend on *specific* assumptions about behavior, information, actions, etc. Generally easy to compute. By construction, outcomes should be “reasonable”.
- Weaknesses: what, if any, assumptions about behavior, information, actions, etc are the conclusions consistent with?

Robust predictions are all those that are consistent with a clearly specified **set** of games, i.e. a set of assumptions on behaviour, information, actions, etc.

- Strengths: combines the benefits of both cooperative and non-cooperative models.
- Weaknesses: generally won't give a unique prediction. Also, may be difficult to compute (although can in fact be easier that characterizing equilibria of a single game).

We'll see a bit of this approach later, but in general it is very much a work in progress.

Structure for today

1. Axiomatic complete information bargaining models.
    - Bilateral negotiations: Nash bargaining solution.
    - Multiple simultaneous negotiations: Nash-in-Nash.
    - Microfoundations.
2. Incomplete information bargaining
    - Impossibility results: Myerson and Satterthwaite (1983) and extensions.
    - Recent developments from mechanism design: Loertscher and Marx (2022).
3. Incomplete information, dynamics, and transparency (time permitting)
    - Valenzuela-Stookey (2023): a simple model showing that funny things can happen.

Axiomatic complete information: Nash-on-Nash-on-Nash

Proposed by Nash in his 1950 paper, where he states "One states as axioms several properties that would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely."

The first question to answer is: What are some reasonable axioms?

Let X to denote the set of possible agreements.

Let D be the disagreement outcome.

- This means that either party has the option to unilaterally enforce D.

Each player i has preferences represented by a utility function u_i over $X \cup \{D\}$. We denote the set of possible payoffs by set U defined by

$$U := \{(u_1(x), u_2(x)) : x \in X\},$$

and $d := (u_1(D), u_2(D))$.

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We can simply define a bargaining problem to be a pair (U, d) where $U \subseteq \mathbb{R}^2$ and $d \in U$.

Maintain the assumptions that

- U is a convex and compact set.
- There exists some $v \in U$ such that $v > d$ (i.e., $v_i > d_i$ for all i).

Denote the set of all such bargaining problems by B.

A bargaining solution is a function f on B where $f(U, d) \in U$.

Pareto Efficiency. A bargaining solution $f(U, d)$ is Pareto efficient if there does not exist a $(v_1, v_2) \in U$ such that $v \geq f(U, d)$ and $v_i > f_i(U, d)$ for some i.

Idea: an inefficient outcome is unlikely, since it leaves space for renegotiation.

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Symmetry. Let (U, d) be such that $d_1 = d_2$ and $(v_1, v_2) \in U$ if and only if $(v_2, v_1) \in U$. Then $f_1(U, d) = f_2(U, d)$.

Idea: if the players are indistinguishable, the agreement should not discriminate between them.

- Obviously ignores bargaining power. Can be easily relaxed.

Invariance to Equivalent Payoff Representations. Given a bargaining problem (U, d), consider a different bargaining problem (U', d') where for some $\alpha > 0$ and some β:

$$U' = \{(\alpha_1 v_1 + \beta_1, \alpha_2 v_2 + \beta_2) : (v_1, v_2) \in U\}$$

$$d' = (\alpha_1 d_1 + \beta_1, \alpha_2 d_2 + \beta_2)$$

Then, $f_i(U', d') = \alpha_i f_i(U, d) + \beta_i$.

Idea: utility functions are just one possible cardinal representation of ordinal preferences. They have no cardinal meaning. Therefore monotone (in particular linear) transformations should not affect the outcome.

Independence of Irrelevant Alternatives. Let (U, d) and (U', d') be two bargaining problems such that $U' \subseteq U$. If $f(U, d) \in U'$, then $f(U', d) = f(U, d)$.

Idea: taking away options that we didn't choose anyway shouldn't change what we do

- Arguably more about behavior than the rest of Nash's axioms, and therefore most in need of a micro-foundation.

Definition
We say that a pair of payoffs (v_1^*, v_2^*) is a Nash bargaining solution if it solves the following optimization problem:

$$\max_{v_1, v_2} (v_1 - d_1)(v_2 - d_2) \quad \text{subject to} \quad (v_1, v_2) \in U,\ (v_1, v_2) \geq (d_1, d_2)$$

Denote the Nash bargaining solution by $f_N(U, d)$.

Remarks

- A solution exists: the set U is compact and the objective function of the problem is continuous.
- ... and is unique: the objective function of the problem is strictly quasi-concave.

Nash bargaining
solution

Theorem (Nash, 1950)
$f_N(U, d)$ *is the unique bargaining solution that satisfies Nash's 4 axioms.*

Nash bargaining

remarks

- Can easily generalize to allow for unequal bargaining weights

$$f_\beta(U, d) = \operatorname{argmax}\{(v_1 - d_1)^\beta (v_2 - d_2)^{1-\beta} : (v_1, v_2) \in U, (v_1, v_2) \geq (d_1, d_2)\}$$

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  - This is a property of Nash bargaining which often plays a large role in empirical work.

Primitives

- Suppose that instead of just two agents, we have a finite set $\{1, \ldots, N\}$.
- Let $\mathcal{G}$ be the set of pairs, ij, that can feasibly form agreements to work together.
- Let p_{ij} be the transfer from j to i if they reach agreement.
- The value to agent i if a set $A \subset \mathcal{G}$ of agreements is reached is $\pi_i(A)$. The net payoff to i is $\pi_i(A) + p_i$, where p_i is the total payment received by i.
    - Important: there can be externalities from agreements, but not from payments.
- For $B \subset A \subset \mathcal{G}$, define $\Delta\pi_i(A, B) = \pi_i(A) - \pi_i(A \setminus B)$.
    - The marginal value of adding the set B of agreements, given that $A \setminus B$ is already realized.

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Assumption (Gains from trade)
For all $ij \in \mathcal{G}$, $\Delta\pi_i(\mathcal{G}, ij) + \Delta\pi_j(\mathcal{G}, ij) > 0$

Definition
In the Nash-in-Nash solution, under the gains-from-trade assumption, all agreements are reached. The transfer from j to i is

$$p_{ij}^N = \frac{b_i \Delta\pi_j(\mathcal{G}, ij) - b_j \Delta\pi_i(\mathcal{G}, ij)}{b_i + b_j}$$

which is the solution to $\max_p \left(\Delta\pi_i(\mathcal{G}, ij) + p\right)^{b_i} \left(\Delta\pi_j(\mathcal{G}, ij) - p\right)^{b_j}$.

In other words, the outcome between each pair is the bilateral Nash bargaining solution, given the "equilibrium" conjecture that all other agreements are reached.

nash equlibrium in nash bargaining

Nash-in-Nash

remarks

- Inherits the limitations of the Nash solution.
- What if agreements are not binary?
    - To some extent, the same logic of Nash bargaining over transfers with "equilibrium" conjectures about agreements can be extended. Not clear what the microfoundation is though.
- Externalities only over agreements, not lump sum payments.
- "Passive beliefs": if the negotiation between i and j breaks down, i negotiates with k as if the agreement between i and j was in place.
    - Usually justified by imagining that each bilateral negotiation is delegated to a separate manager, and managers cannot communicate.
    - Other micro-foundations available (Collard-Wexler et al., 2019). Still require a form of passive beliefs.

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Major issue: **complete information!**

Incomplete information

Recall our three approaches to bargaining:

1. Axiomatic/cooperative
2. Specify a game form, analyze equilibria.
3. Specify the set of equilibrium outcomes within a class of games.

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We used 1 for complete information, but difficult with incomplete info.

- Much harder to motivate axioms without thinking about how information is revealed in the game.

What about 2 and 3?

Simplest case: bilateral trade

what can happen?

In a simple setting, let’s try to map out all possible outcomes for a reasonable set of games.

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Bilateral trade

- A buyer, B, has value $v \in [\underline{v}, \bar{v}]$ for a good.
- A seller, S, owns a good for which they have value $c \in [\underline{c}, \bar{c}]$ (equivalently, c is production cost).
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Equivalently, by the revelation principal we can consider all incentive compatible direct mechanisms. A direct mechanism consists of

- Allocation rule $q : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}] \to [0, 1]$ determining the probability of trade.
- Payment rule $p_j[\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}] \to \mathbb{R}$ determining the payment made by j, for $j \in \{A, B\}$

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BIC, Interim IR, and no-deficit seem reasonable conditions which should be satisfied, at a minimum.

Efficiency can't happen in bilateral trade

Myerson and Satterthwaite (1983)

Theorem (Myerson and Satterthwaite (1983))
In the bilateral trade setting, if $\underline{v} < \bar{c}$ then there is no efficient, BIC, and interim IR mechanism that does not run a deficit in expectation.

In other words, however agents interact, if they have the option to walk away and they play a Bayes-Nash equilibrium, trade cannot be efficient.

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Proof.
Recall the Revenue Equivalence theorem, which said that we can write the payments made by agent i as a function of the allocation rule, plus the surplus that the mechanism leaves to i when they have the least-favorable type ($\underline{v}$ for buyer, $\bar{c}$ for the seller).

Consider the following mechanism which implements the efficient allocation rule

- If $v \geq c$ there is trade. The buyer pays $p_B = \max\{c, \underline{v}\}$, and the seller receives $-p_S = \min\{v, \bar{c}\}$. Otherwise, no payments are made and there is no trade.

This mechanism is DSIC, and so a fortiori BIC. Moreover, the payoff of a type $\underline{v}$ buyer and type $\bar{c}$ seller are both zero. Moreover, $p_B(v, c) + p_S(v, c) \leq 0$ for all c, strictly so for almost all types.

By revenue equivalence, any other efficient, IR, and BIC mechanism has weakly lower revenue (strictly so if either agents IR constraint doesn't bid). □

Okay, so what can happen? Let's get some traction by specifying a simple game.

k-double auction, proposed in Chatterjee and Samuelson (1983) "Bargaining Under Incomplete Information" as a non-cooperative foundation of bargaining in bilateral trade.

- buyer submits a sealed bid b.
- seller submits a sealed offer s.
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Assume that agents play a Bayes-Nash equilibrium, and can walk away before the game begins.

- These are exactly the BIC and Interim-IR constraints for the direct mechanism.
- Note that the outcome of the k-double auction is budget balanced by construction.

What can happen?

k-double auction

Here's one thing that can happen.

Fix $p \in (\underline{c}, \bar{v})$, and consider the following strategies

- The buyer bids p whenever $v \geq p$, and $\underline{c}$ otherwise
- The seller proposes p whenever $c \leq p$, and $\bar{v}$ otherwise.

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Fixing the strategy of agent i, the specified strategy is optimal for j ex-post (even if j knew i's type).

- Thus we can implement this as a DSIC direct mechanism.

Any $p \in (\underline{c}, \bar{v})$ will do.

[picture on the board]

What is everything that can happen?

DSIC version

With dominant strategy implementation the k-double auction gives us trade at a posted price. What else could we get?

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With dominant strategy implementation the k-double auction gives us trade at a posted price. What else could we get?

Nothing, exept for some Pareto-dominated things

Theorem
If $\underline{c} \leq \overline{v}$, the only budget-balanced, Ex-post-IR, DSIC, and Pareto undominated mechanisms are randomizations over posted prices.

Proof.
[picture on the board] $\square$

What is everything that can happen?

BIC version

Can we get more if we relax to BIC and Interim-IR?

Can we get more if we relax to BIC and Interim-IR? No!

Theorem

If $\underline{c} \leq \overline{v}$, then any budget-balanced, Interim-IR, BIC, and Pareto-undominated mechanism has the same interim allocation and interim payments as some randomization over posted prices.

In other words, all budget-balanced, interim-IR, BIC, and Parato-undominated mechanism is "interim equivalent" to a randomization over posted prices.

- We started by specifying a specific (indirect) game form, the k-double auction.
- We saw that for any k, the set of equilibria of this game was exactly the set of all “reasonable” outcomes (BIC, IIR, BB, Pareto-undominated).
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Which posted prices should we expect to see? Depends on bargaining power!

Remark on bargaining power

Suppose we took the k-double auction as our model of incomplete information bargaining power.

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Need to take a step back and ask ourselves: What is bargaining power?

- To me, bargaining power is whatever determines which from among the set of reasonable outcomes is selected.
- So k is just not the right notion of bargaining power in the k-double auction.
- What is?

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- What is? p!
- Alternatively, we could say that the mechanism maximizes weighted sum of buyer and seller surplus, where the weights will tell us which p to choose
- This is really the same as choosing p, except it gives us a way to tie together predictions for across different bargaining scenarios.

More generally, what can happen?

Loertscher and Marx (2022)

We saw that for the bilateral trade problem assuming only that

i. agents play a Bayes-Nash equilibrium of some game, and
ii. no money gets magically lost or created,

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Loertscher and Marx (2022) do the same thing in a more general setting.

- Market with n^S suppliers and n^B buyers.
- Sets of suppliers and buyers denoted by $\mathcal{N}^S \equiv \{1, \dots, n^S\}$ and $\mathcal{N}^B \equiv \{1, \dots, n^B\}$, respectively.
- Each supplier j can produce up to k_j^S units, with constant marginal cost, and each buyer i has a constant marginal value for up to k_i^B units of the good, where k_j^S and k_i^B are positive integers.
- Total demand is $K^B \equiv \sum_{i \in \mathcal{N}^B} k_i^B$, total supply is $K^S \equiv \sum_{j \in \mathcal{N}^S} k_j^S$, and define $K \equiv \min\{K^B, K^S\}$.
- Supplier j draws constant marginal cost c_j independently from distribution G_j with support $[\underline{c}, \overline{c}]$ and density g_j positive on the interior of the support.
- Buyer i draws constant marginal value v_i independently from distribution F_i with support $[\underline{v}, \overline{v}]$ and density f_i positive on the interior of the support.
- Assume $\overline{v} \geq \underline{c}$, otherwise problem is trivial.
- Primitives $G_1, \dots, G_{n^S}, F_1, \dots, F_{n^B}, k_1, \dots, k_{n^S}, k_1, \dots, k_{n^B}$ are common knowledge.
- Realized costs and values are private information.

Quasilinear preferences for suppliers and buyers.

- Payoff of supplier j with type c_j when producing $q \in \{0, \ldots, k_j^S\}$ units and receiving payment m is $m - c_j q$.
- Payoff of buyer i with type v_i when receiving $q \in \{0, \ldots, k_i^B\}$ units and making the payment m is $v_i q - m$.
- Normalize the value of the outside option of not trading to 0 for every agent.

- Model bargaining a direct mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$ "operated by the market".
  - Allocation rule $\mathbf{Q} = (\mathbf{Q}^S, \mathbf{Q}^B)$ with $\mathbf{Q}_j^S : [\underline{v}, \bar{v}]^{n^B} \times [\underline{c}, \bar{c}]^{n^S} \to \{0, \ldots, k_j^S\}$ and $\mathbf{Q}_i^B : [\underline{v}, \bar{v}]^{n^B} \times [\underline{c}, \bar{c}]^{n^S} \to \{0, \ldots, k_i^B\}$. (quantities provided by suppliers and received by buyers)
  - Payment rule $M = (\mathbf{M}^S, \mathbf{M}^B)$ with $\mathbf{M}^S : [\underline{v}, \bar{v}]^{n^B} \times [\underline{c}, \bar{c}]^{n^S} \to \mathbb{R}^{n^S}$ and $\mathbf{M}^B : [\underline{v}, \bar{v}]^{n^B} \times [\underline{c}, \bar{c}]^{n^S} \to \mathbb{R}^{n^B}$. (payments from buyers and to sellers)
- Feasibility requires $\forall$ type realizations, $\sum_{j \in n^S} \mathbf{Q}_j^S(v, c) \geq \sum_{i \in n^B} \mathbf{Q}_i^B(v, c)$.
- Conditions imposed on the mechanism
  - Interim IC (BIC).
  - Interim IR.
  - No deficit in expectation. Note: not imposing budget balance.

  Let $\mathcal{M}$ be the set of such mechanisms.

Fixing a mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$

- Supplier j's ex post surplus is $U^S_{j;\mathbf{Q},\mathbf{M}}(v,c) \equiv M^S_j(v,c) - cQ^S_i(v,c)$.
- Buyer i's ex post surplus is $U^B_{i;\mathbf{Q},\mathbf{M}};(v,c) \equiv vQ^B_i(v,c) - M^B_i(v,c)$.
- Budget surplus generated by the mechanism is $R_{\mathbf{M}}(v,c) \equiv \sum_{i \in \mathcal{N}^B} M^B_i(v,c) - \sum_{j \in \mathcal{N}^S} M^S_j(v,c)$.
- Welfare or social surplus generated by the mechanism: $W_{\mathbf{Q}}(v,c) \equiv \sum_{i \in \mathcal{N}^B} v_i Q^B_i(v,c) - \sum_{j \in \mathcal{N}^S} c_j Q^S_j(v,c)$.

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To capture bargaining power, endow the agents with bargaining weights $\mathbf{w} = (\mathbf{w}^S, \mathbf{w}^B)$, where $w^S_j \in [0, 1]$ is supplier j's bargaining weight and $w^S_j \in [0, 1]$ is buyer i's bargaining

- Define weighted welfare with weights $\mathbf{w}$ as

$$W^w_{\mathbf{Q},\mathbf{M}}(v, c) \equiv \sum_{i \in \mathcal{N}^B} w^B_i U^B_{i;\mathbf{Q},\mathbf{M}}(v, c) + \sum_{i \in \mathcal{N}^S} w^S_i U^S_{i;\mathbf{Q},\mathbf{M}}(v, c)$$

(so $W_{\mathbf{Q}}(v, c) = W^1_{\mathbf{Q},\mathbf{M}}(v, c) + R_{\mathbf{M}}(v, c)$).

The prediction is that the market maximizes

$$\mathbb{E}\left[W^{w}_{\mathbf{Q},\mathbf{M}}(v,c)\right] \quad s.t. \quad \mathbb{E}\left[R_{\mathbf{M}}(v,c)\right] \geq 0$$

The bulk of the paper is devoted to characterizing the solution.

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An alternative way to think about this

- By varying the weights $\mathbf{w}$, we pick up all points on the Pareto-undominated frontier in $\mathcal{M}$.
- The bargaining weights just pick one of these points.

- In this setting, the set of BIC and Interim-IR mechanisms is interim-equivalent to the set of DSIC and Ex-post IR mechanisms, so could switch these conditions.
- It's possible that the predicted mechanism generates a strict surplus in expectation.
  - In this case, specify a sharing rule for the surplus, to balance the budget.
  - Since budget balance only needs to hold ex-ante this doesn't mess with incentives.
  - But this could be an issue if we wanted budget balance ex-post.

  The results do *not* carry over if we require no deficit ex-post.
- This is an independent private values setting: no externalities.
  - They use the model to talk about horizontal and vertical integration, but without externalities it's hard to take this seriously as a model of competing firms.

Transparency

The paper is called “Pay Transparency and Discrimination”, but you can view it as a model of bargaining between many sellers and one buyer.

It can serve as a parable for why passive beliefs, as in Nash-in-Nash, might be a bad assumption, especially with incomplete information, and how it could hide interesting phenomena.

Here's the simplest version of the model

Primitives.

- N sellers. Each offers a single item at zero marginal cost.
- One buyer. Buyer's demand is additive (can buy from many sellers).
- Buyer's value for goods is either high, h, or low, l. Initially unknown to sellers.

Timing. Two periods.

- Period 1.
  1. Sellers simultaneously make take-it-or-leave-it price offers to the buyer.
  2. Buyer observes all offers and chooses which to accept.
- Information revelation stage (transparency). Some buyers (perhaps randomly) observe the the outcomes of others.
- Period 2. Sellers who observed that the seller bought from someone at a price above l in period 1 infer that the buyer is type h, and demand a price of h. All other buyers demand l.

Valenzuela-Stookey (2023)

what happens

The interesting thing is that the buyer has incentives to maintain a reputation for being a low type.

This means that the buyer's actions in the first period are not independent across workers.

In particular, for a fixed set of period-1 offers, the buyer is more willing to accept a price above l from seller i if they are already doing so with seller k.

In equilibrium sellers anticipate this. It turns out that sellers price in period 1 knowing that if the buyer rejects their offer, it will also reject all the other offers from sellers who priced above l.

- Beliefs are very much not passive!

This has interesting implications for prices and the role of transparency.

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