Industrial Organization Papers

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All models are wrong, but some are useful.

Contents

Chapter 1 Berry, S. T. (1994). Estimating discrete-choice models of product differentiation.

Based on

- Berry, S. T. (1994). Estimating discrete-choice models of product differentiation. *The RAND Journal of Economics*, 242-262.

1.1 Problem

The problem of estimating supply-and-demand models in markets with product differentiation.

1.2 What's New?

1. Discrete-choice model.
2. Unobserved demand factors are considered.
3. Estimation by "inverting" the market-share equation to find the implied mean levels of utility for each good.

Note ***The problem of previous empirical model:*** *a system of N goods gives N^2 elasticizes to estimate.*

This paper put some structure based by making assumptions on consumer utility. It uses the aggregation of consumers' individual choice to estimate demand.

1.3 Model

Data: The econometrician is assumed to observe the market outcomes of price and quantities sold by each firm.

1.3.1 Settings

1. There are R independent markets.
2. There are N_r firms in market r, with each firm producing one product. (For simplicity, we omit r in following notations).
3. For product j, the observed characteristics are denoted by $z_j = (x_j, w_j) \in \mathbb{R}^k$.

   Specifically, z_j includes two parts:

   characteristics that affect demand x_j;

characteristics that affect marginal cost w_j.

4. For product j, the unobserved characteristics are denoted by $(\xi_j, \omega_j) \in \mathbb{R}^k$.

   Specifically, (ξ_j, ω_j) includes two parts:

   unobserved characteristics that affect demand ξ_j;

   unobserved characteristics that affect marginal cost ω_j.

   Unobserved characteristics are mean independent of observed characteristics and independent across markets.

5. Price of product j is denoted by p_j.

6. **Discrete Choice Model:** the utility of consumer i for product j:

$$u_{ij} = U(x_j, \xi_j, p_j, v_i, \theta_d)$$

   v_i captures the consumer i's consumer-specific terms that are not observed by the econometrician;

   θ_d are demand parameters.

   Consider a simple random coefficients specification for utility,

$$\begin{aligned} u_{ij} &= U(x_j, \xi_j, p_j, v_i = \tilde{\beta}_i, \theta_d = \alpha) \\ &= x_j\tilde{\beta}_i - \alpha p_j + \xi_j + \epsilon_{ij} \end{aligned} \tag{(2)}$$

   where ϵ_{ij} represents the distribution of consumer preferences about this mean, and α is invariant across consumers (although not necessary).

   Random coefficients: (to avoid unreasonable substitution effects) Decompose consumer i's taste parameter for characteristic k as

$$\tilde{\beta}_{ik} = \beta_k + \sigma_k \xi_{ik} \tag{(3)}$$

   where β_k is the mean level of taste for characteristic k and ξ_{ik} has mean zero.

   Combing (2) and (3), we can write

$$\begin{aligned} u_{ij} &= \sum_k x_{jk}\tilde{\beta}_{ik} - \alpha p_j + \xi_j + \epsilon_{ij} \\ &= \sum_k x_{jk}\left(\beta_k + \sigma_k \xi_{ik}\right) - \alpha p_j + \xi_j + \epsilon_{ij} \\ &= x_j\beta - \alpha p_j + \xi_j + \sum_k x_{jk}\sigma_k\xi_{ik} + \epsilon_{ij} \\ &= x_j\beta - \alpha p_j + \xi_j + v_{ij} \end{aligned}$$

   with $v_{ij} = \sum_k x_{jk}\sigma_k\xi_{ik} + \epsilon_{ij}$, which has mean zero.

   The mean utility level of product j is

$$\delta_j \equiv x_j\beta - \alpha p_j + \xi_j \tag{(5)}$$

Then, the utility of consumer i for product j can be written as

$$u_{ij} = \delta_j + v_{ij}$$

7. **Discrete-choice Market Share Function:**

   Define the set of consumer unobservables that lead to the consumption of good j as

$$A_j(\boldsymbol{\delta}) \triangleq \{\boldsymbol{v}_i : \delta_j + v_{ij} > \delta_k + v_{ik}, \forall k \neq j\}$$

Then, the market share of j is the probability that $\boldsymbol{v}_i$ falls into the region $A_j(\boldsymbol{\delta})$, $P(\boldsymbol{v}_i \in A_j(\boldsymbol{\delta}))$.

Given $\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\theta}$, and the distribution of $\boldsymbol{v}$ follows $F(\cdot, \boldsymbol{x}, \sigma)$ (p.d.f f), the market share is

$$\natural_j(\boldsymbol{\delta}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\xi}), \boldsymbol{x}, \boldsymbol{\theta}) = \int_{A_j(\boldsymbol{\delta})} f(\boldsymbol{v}, \boldsymbol{x}, \sigma_v) d\boldsymbol{v}$$

The measure of consumers in a market is denoted by M (which is assumed to be observed). The observed output quantity of the firm is

$$q_j = M \natural_j(\boldsymbol{x}, \boldsymbol{\xi}, \boldsymbol{p}, \theta_d)$$

In addition to competing products $j = 1, ..., N$, there is also an outside good $j = 0$.

8. **The Supply Side:**

   The **total costs for firm** j are given by the function $C_j(q_j, w_j, \omega_j, \boldsymbol{\gamma})$;

   The **marginal costs** are given by $c_j(q_j, w_j, \omega_j, \boldsymbol{\gamma})$, where $\boldsymbol{\gamma}$ is a vector of unknown parameters.

   Profits for firm j are

$$\begin{aligned} \pi_j(\boldsymbol{p}, \boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{w}), \boldsymbol{\xi}, \omega_j, \boldsymbol{\theta} = (\theta_d, \boldsymbol{\gamma})) &= p_j q_j - C_j(q_j, w_j, \omega_j, \boldsymbol{\gamma}) \\ &= p_j M \natural_j(\boldsymbol{x}, \boldsymbol{\xi}, \boldsymbol{p}, \theta_d) - C_j(q_j, w_j, \omega_j, \boldsymbol{\gamma}) \end{aligned}$$

By first-order conditions:

$$q_j + \frac{\partial q_j}{\partial p_j} p_j - \frac{\partial q_j}{\partial p_j} \frac{\partial C_j}{\partial q_j} = 0$$

we have

$$p_j = c_j - \frac{\natural_j}{\frac{\partial \natural_j}{\partial p_j}} = c_j + \frac{\natural_j}{|\frac{\partial \natural_j}{\partial p_j}|}$$