Examples 1) $A = [1,2]$, $\inf A = \min A = 1$
$\sup A = \max A = 2$

2) $A = (1,2]$, $\inf A = 1$, not achieved
$\sup A = \max A = 2$

3) $A = (1, \infty)$, $\sup A = \infty$, no maximum

Extrema of Functions Let $S \subseteq \mathbb{R}^n$, $f: S \to \mathbb{R}$

$$\inf_{x \in S} f(x) = \inf \{ f(x) : x \in S \}.$$

If $\exists\, x^* \in S$ s.t. $\inf\limits_{x \in S} f(x) = f(x^*)$ then

f achieves (attains) its minimum and

$$f(x^*) = \min_{x \in S} f(x)$$

x^* is called a minimizer of f, written as

$$x^* \in \arg\min_{x \in S} f(x)$$

If x^* is unique we write $x^* = \arg\min\limits_{x \in S} f(x)$.

Similarly, supremum and maximum of f.

Example 1 $f(x) = x$, $x \in (-1, 2)$

$\sup f(x) = 2$
$\inf f(x) = -1$
neither achieved

Example 2 $f(x) = x^2$, $x \in \mathbb{R}$

$\inf f(x) = \min f(x) = 0$, $x^* = 0$
$\sup f(x) = \infty$, max does not exist.