Multi-Stage Choice Model (with Decision Theory Foundation)

Wenxiao Yang

University of Illinois at Urbana-Champaign
Department of Mathematics

2022-05-04

Rational Inattention Foundation for Multinomial Logit Model

- Matějka, F., & McKay, A. (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. American Economic Review, 105(1), 272-98. (Too complicated, the proof can be simplified to the format below. However, the proof cost function over purchase probability is omitted in this paper and I can't reproduce it.)
- Ben Hebert Stanford Graduate School of Business: Rational Inattention Theory and Evidence (presentation at Israel Institute for Advanced Studies $https://www.youtube.com/watch?v=l4gx_3KxHzo$).

Rational Inattention Foundation for Multinomial Logit Model

- Gabaix, X. (2019). Behavioral inattention. In Handbook of Behavioral Economics: Applications and Foundations 1 (Vol. 2, pp. 261-343). North-Holland.
  (Don't have full proof, change the cost function in the first paper to Kullback–Leibler distance (which is also deduced from information theory, I will use this setting))

I also adjust some settings to make it close to assortment optimization settings (and can be easily understood).

Model

- A consumer makes an action on action set $A = \{1, ..., N\}$ (Buy which product)
- A consumer makes an action on action set $A = \{1, ..., N\}$ (Buy which product)
- **If consumer knows v**, this problem is simply:

$$i \in_i v_i$$

However, we need to discuss the situation that the consumer doesn't know **v**. She has to infer the **v** from some information.
- **When consumer doesn't know v**, she has belief $b \in \Delta(V)$. ($\Delta(V)$ is the set of all possibility distributions on V) Then,

$$i(b) \in_i \mathbb{E}_b[v_i]$$

(Choose the product with the highest expected value under belief b) Set the map from beliefs to utility (not actual utility, the utility under belief) is

$$\pi(b) = \max_{i} \mathbb{E}_b[v_i]$$

- The related boundary constraints for the mufflers are specified.

Derivation of Four Pole Matrices and an expression for STL

- For the ease of theoretical derivation on muffler, two kinds of muffler elements, are identified,
- On the basis of plane wave theorem, a transfer matrix between inlet and outlet can then be deduced in each muffler element

Figure 1: Four poles matrix between point 1 and point 2 with mean flow

Four poles matrix between point 2 and point 3 with mean flow is:

Figure 2: Four poles matrix between point 2 and point 3 with mean flow

Figure 3: Space constraints for two-segments muffler

Derivation of Expression for STL....3

After multiplying all the above matrices, we will obtain the final transfer matrix

$$\begin{bmatrix} p_1 \\ \rho_0 c_0 u_1 \end{bmatrix} = \begin{bmatrix} T_{11}^* & T_{12}^* \\ T_{21}^* & T_{22}^* \end{bmatrix} \times \begin{bmatrix} p_4 \\ \rho_0 c_0 u_4 \end{bmatrix}$$

The sound transmission loss (STL) of muffler is defined as

Figure 4: Final expression for STL

Genetic Algorithm

- Search algorithms based on the mechanics of natural selection and natural genetics
- Based on “survival of fittest” concept
- Simulates the process of evolution
- KEY IDEA: “Evolution is an optimizing process”

Figure 5: The Evolution cycle

Genetic Algorithm : Initialization

- Population, whose individuals represent solution to problems
- (d_1,d_2)=(5.4064,3.8005) is a member in our population!
- A member/Design vector (d_1,d_2)=(5.4064,3.8005) may be represented using binary numbers like this

Figure 6: Design vector coded to string structure

Genetic Algorithm : Ranking the Genomes

- Each individual/ String is evaluated to find the fitness value
- Roulette Wheel Selection is implemented

Figure 7: A roulette wheel marked for five individuals according to their fitness [Figure Courtesy: Optimization for Engineering Design: Algorithms and Examples, Kalyanmoy Deb]

Genetic Algorithm : Reproduction Operators

Single Point Crossover

- Each chromosome of parent is divided into two parts and then joined stochastically

Figure 8: Single point Crossover

Mutation

- To make sure that sufficient variety of strings are there to assure that GA will go through the entire problem space
- Prevents premature convergence

Genetic Algorithm : Reproduction Operators...2

Elitism

- The elitism scheme to keep best gene in the parent generations
- To prevent the best gene from the disappearing and improve the accuracy of optimization during reproduction

A numerical case of noise elimination

- With the spectrum analysis in sound, it is found that the sound energy at 500 Hz is highly remarkable.
- The minimal diameters at each segment are specified to be no less than 0.0762 m
- The design volume flow rate is confined to 0.8 CMS.
- For optimization of a Two segments muffler, 3 parameters were selected
  - Diameter, D_1
  - Diameter, D_2
  - Length, L_1

- The maximal value of STL is 38.5 dB

Figure 9: Tabulation of finally obtained results

Figure 10: Optimal shape in a two segment muffler

Results and Discussion...2

- The performance curves for different GA control parameter are plotted.

Figure 11: STL of two-segments muffler at four sets of GA parameters.

Conclusion

- Because of no first derivative and starting design data of objective function as required in traditional gradient method, GA becomes easier.
- The case study reveals that by increasing the segments in muffler, the performance in STL can be improved efficiently.
- Results are sensitive to the GA control parameters like, probability of crossover p_c and probability of mutation p_m

Thanks