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A Bertrand model of pricing of complementary goods under information asymmetry

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Abstract

Parties in a supply chain, being independent firms, have private information about various aspects of the business not normally available to other parties. We consider a market where customers need to buy two complementary goods as mixed bundle, offered by two separate firms. The demand for each firm is dependent on the pricing strategy of both firms, which, in turn, depends on the quantities offered as per their own forecasts. We present a profit maximization model to obtain optimal strategies for a firm making decisions under information asymmetry. The model follows a simultaneously played Bertrand type game. We contrast and compare three scenarios: (1) when forecast information is asymmetric between the firms; (2) when forecast information is shared between the firms; and (3) when the firms form a strategic alliance.

Keywords: Asymmetric information; Information sharing; Game theory

1. Introduction

In recent times there is a growing interest in the mechanism of information sharing between the members of a supply chain. All parties in a supply chain seek information about the market and about each other's decisions (Li, 2002). In many cases, part of the information is private to a party and other parties make decisions with limited information. The effect of this information asymmetry on optimum decisions is studied in many recent articles. Raju and Roy (2000), Cachon and Marshall (2000), and Lee et al. (2000) deal with different aspects of asymmetric information in a supply chain. Value of timely demand information to reduce inventory level is shown by Bourland et al. (1996). Sharing of forecast information between a manufacturer and a supplier is studied by Cachon and Lariviere (2001). Li (2002) studies a supplier–retailer supply chain. Retailers have private information about their forecast and costs. He examines the incentives for sharing information and its effect on optimal decisions. When two firms are marketing their products (either substitute or complementary) to the same market, each needs information about the other firm's decision. More often than not, this information is private. This is the scenario of our paper.

The case of marketing substitutable products in a competitive setting has been extensively studied in the literature. This is the traditional competition model where customers choose between the competing products depending on their preferences and the marketing strategy of the firms. The case of complementary products arises when customers have to buy more than one product at the same time to get the full utility of the goods. This case has recently gained interest. For example, Cisco and HP decided to deliver co-branded support services (HP, 2004). HP and Canon engage in jointly marketing their products (Lewis, 1999). Other examples are IBM and Microsoft (Sengupta and Bucklin, 1995), washer and dryer etc. The marketing paradigm of complementary goods is different from that of substitutable goods in that the goods benefit from each other's sales rather than losing sales to the other firm. The goods can be thought of as

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a bundle. Bundling is a common pricing strategy especially in information industry (Parker and Van Alstyne, 2000).

We illustrate the scenario, with the example of HP and Canon. HP leads the world in laser printers. Canon, on the other hand, supplies components to the same market (Lewis, 1999). Pricing of the individual products plays an important role in the customers' buying decision and thus on the respective demand. The two firms can follow a number of possible cooperation strategies. One is complete non-cooperation, the other is cooperative decision. Lewis (1999) mentions that for the HP and Canon cooperative case "the two companies share resources and engage in joint activities and achieve more than they can on their own." For the market segment who wants both products as a bundle, the demand for Canon products, say, will depend on the prices of both products. The decision that Canon would make to fix the price of its own product, therefore, would depend on the HP's pricing decision. The power of bundling as a price discrimination device is studied by Venkatesh and Kamakura (2003). HP would take this decision considering its own demand forecast, which is private. Canon, therefore, will need to make its pricing decision given this information asymmetry. In this paper, we consider this scenario and develop a model to yield optimal decisions.

2. The model scenario

We consider two suppliers selling products, P1 and P2, that are complementary to each other. The target market is divided into three segments: customers that are interested in P1 only (Segment 1,) customers who need both P1 and P2 (Segment 2) and customers that want P2 only (Segment 3.) This kind of segmentation necessitates a mixed bundling strategy. An example of this scenario can be found in the marketing of Web Services which is currently being hailed as a revolutionary technology. Web services normally conform to universally agreed specifications and can be integrated dynamically without "hardwiring" the code. This has become possible because of the recent development of technology like XML (extensible markup language) etc. One characteristic of these services is that the integration needed by the user is no longer prohibitively expensive or time-consuming. Users are no longer locked into products from one firm and can opt for the "best of breed" strategy. For the firms selling these products, most selling is done by making these products available via Internet with little marginal cost. The Segment 2 customers can buy P1 and P2 separately from two different firms and would integrate the two products easily to form their own bundle. If a customer deems the price of one product higher than what he is willing to pay, he is free to buy only the other one and can make alternative arrangement for the other product which may include buying from another vendor or developing the product in-house.

The decision variable for each firm is the price of the product. While segments 1 and 3 are independent, the fact that segment 2 customers would consider both prices, would make the decisions dependent on the other firm's decision. This is the setting of a game theoretic approach to the problem. Sharing of information or lack thereof is also an important factor in this setting. We assume a Bertrand game model where the two firms make decisions simultaneously, rather than sequentially, to maximize their own profit. Simultaneous games can be modelled either as a Bertrand game or as a Cournot game. Cournot equilibrium is the Nash equilibrium in quantities while Bertrand equilibrium is one in prices. In a pioneering work, Singh and Vives (1984) show that under duopoly Bertrand price is lower than Cournot price and the Bertrand equilibrium is more efficient in the sense that both consumer surplus and total surplus are greater under price competition regardless of whether the goods are substitutes or complements. Also, a Cournot model realistically represents firm behaviour in a market where competing firms' marginal costs are relatively 'steep' and capacity constraints exist whereas the Bertrand model may be used for representing firm behaviour in a market where competing firms' marginal costs are relatively 'flat' and excess capacity exists. Bertrand, therefore, is an appropriate model when analyzing the competition in Information Technology industry. Usually the information product or system, specifically software product, has negligible marginal cost and no capacity limitation. So information product vendors are more concerned about pricing decision, rather than quantity decision, of their product. Vives (1984) studies a duopoly model where firms have private information about an uncertain linear demand, and uses both Bertrand and Cournot models. The results show that if the products are substitutes then sharing information is a dominant strategy for each firm in Bertrand competition. The result is reversed for complementary products.

Gal-Or (1985) and Balboa et al. (2004) focused on firms' preferences over sequential versus simultaneous play in markets and study the case of strategic substitutes when firms choose quantity strategies. Raju and Roy (2000) study two firms offering substitute products under Stackelberg and Bertrand modes. These papers do not address the issue of information asymmetry and sharing of information. Our paper considers simultaneous moves by two firms where there is information asymmetry. We get Bayesian Bertrand equilibrium following "an expectation approach to simultaneous-choice models" proposed by Cyert et al. (1978). They say that the equilibrium can be expressed as an infinite regress which characterizes a simultaneous-decision problem, referring to "I think that he thinks that I think...". A game with simultaneous moves involves a logical circle. So far, Nash's notion of equilibrium remains an incomplete solution to

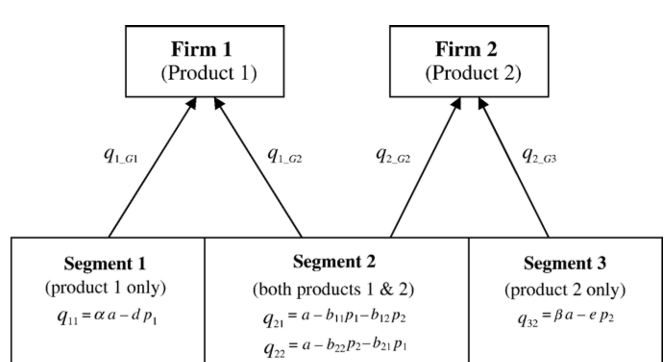

Fig. 1. Demand functions for two products from three market segments.

the problem of circular reasoning in simultaneous move games. However, some special cases of the problem can be developed in which this chain of circle can be broken and an equilibrium solution obtained.

2.1. The model

Fig. 1 shows our model scenario. The three market segments are shown as Segments 1, 2 and 3. The demand for P1 by Segment 1 is denoted by q_{11}, and that for P2 by Segment 3 by q_{32}. Demands of Segment 2 for both products are given by q_{21} and q_{22}, respectively.

Decision variables for the two firms are the product prices p_1 and p_2 respectively, regardless of which segments are buying. The demand of product 1 from segment 1 is given by:

$$q_{11} = \alpha\, a - d\, p_1 \tag{1}$$

Similarly, demand for product 2 from segment 3 is:

$$q_{32} = \beta\, a - e\, p_2 \tag{2}$$

Segment 2 will demand both products. The respective demands for products 1 and 2 are:

$$\begin{aligned} q_{21} &= a - b_{11}p_1 - b_{12}p_2 \\ q_{22} &= a - b_{22}p_2 - b_{21}p_1 \end{aligned} \tag{3}$$

We assume that the industry primary demand is a for segment 2, αa for the segment 1 and βa for the segment 3, where α (β) is the proportion of segment 1(3)'s primary demand as a percentage of the segment 2's primary demand. To keep the analytical model tractable, we assume linear model as is usually used in literature. b_{11}, b_{22}, e and d are price sensitivities of a product's demand to its own price; and b_{12} and b_{21} are cross-price sensitivities. The demand for a product decreases from a base level by an amount proportional to its price. For a complementary product, in a similar fashion, segment 2's demand for product 1 decreases with its own price. Note that segment 2 wants to buy both the products. When product 2's price is very high, some customers will be unable to afford to buy product 2 (say, when p_2 is higher than their reservation price). Similar explanation can be given for segment 2's demand for product 2 decreasing with p_1. Martin (1999) uses inverse demand curves for complementary products which, when converted to demand functions, would show a structure of demand equations similar to ours as given above. He mentions earliest use of this type of model by Spence (1976) and Dixit (1979). We assume that b_{11}, b_{22}, e and d are larger than b_{12} and b_{21}, i.e., own price effects are greater than cross-price effects. Combining the demands of different segments for products P1 and P2 respectively, we have:

$$\begin{aligned} q_1 &= q_{11} + q_{21} = (\alpha a - dp_1) + (a - b_{11}p_1 - b_{12}p_2), \\ q_2 &= q_{22} + q_{32} = (\beta a - ep_2) + (a - b_{22}p_2 - b_{21}p_1). \end{aligned} \tag{4}$$

The market demand is uncertain and is represented by assuming the primary level demand a to be a random variable, given by,

$$a = \bar{a} + e \tag{5}$$

where e is assumed to be distributed normally with mean zero and variance V. To make production decisions given this uncertain demand, each firm obtains a forecast for a using the market information-gathering techniques at its disposal. Suppose the forecasts independently obtained by the two firms are f_i, $i=1, 2$ respectively. As is usual, the forecasts will have errors, characterized by,

$$f_i = a + \varepsilon_i, \quad i = 1, 2 \tag{6}$$

Where ε_i is normally distributed, independent of the primary demand a, with mean zero and variance s_i. A higher (lower) variance implies a less (more) precise forecast. It is possible that the forecast errors ε_1 and ε_2 can be correlated. The extent of correlation depends on the data and methodology used by the two firms in their forecasting processes. For example, similar data and methodology will result in higher correlation between forecasts. Therefore, we further assume that the forecast errors ε_1 and ε_2 follow a bivariate normal distribution. The covariance matrix of forecast errors is represented by:

$$\sum = \begin{pmatrix} s_1 & s_{12} \\ s_{12} & s_2 \end{pmatrix}$$

The covariance between the two forecasts is s_{12} and the coefficient of correlation is $\rho = \frac{s_{12}}{\sqrt{s_1}\sqrt{s_2}}$

As in existing literature (e.g., Vives, 1984; Raju and Roy, 2000; Li, 2002), we assume that all parameters of the model except the forecasts are common knowledge to both firms. The pricing decision would be conditional on the forecast (private information of the firm) and known demand function parameters. Therefore, the demands and in turn profits would also be conditional on the forecasts and other known parameters. These expressions for profits are shown below. π_{ij} defines the profit of firm j from Segment i, $i=1, 2, 3, j=1, 2$. π_1 and π_2 are the total profits of firm 1 and firm 2 respectively.

$$\begin{aligned} E(\pi_1|f_1) &= E[(\pi_{11} + \pi_{21})|f_1], \\ E(\pi_2|f_2) &= E[(\pi_{22} + \pi_{32})|f_2], \\ E(\pi_1|f_1,f_2) &= E[(\pi_{11} + \pi_{21})|f_1,f_2], \\ E(\pi_2|f_1,f_2) &= E[(\pi_{22} + \pi_{32})|f_1,f_2]. \end{aligned} \tag{7}$$

where

$$\pi_{11} = p_1(\alpha a - dp_1), \ \ \pi_{21} = p_1(a - b_{11}p_1 - b_{12}p_2),$$
$$\pi_{22} = p_2(a - b_{22}p_2 - b_{21}p_1), \ \ \pi_{32} = p_2(\beta a - ep_2).$$

After substitutions, we get the two profit functions as:

$$\begin{aligned} E_1(\pi_1|f_1) &= [E_1(\pi_{1\_G1} + \pi_{1\_G2})|f_1] \\ &= p_1[E_1(\alpha a - dp_1|f_1)] \\ &\quad + p_1[E_1(a - b_{11}p_1 - b_{12}p_2|f_1)] \end{aligned} \tag{8}$$

$$\begin{aligned} E_2(\pi_2|f_2) &= [E_2(\pi_{2\_G3} + \pi_{2\_G2})|f_2] \\ &= p_2[E_2(\beta a - e p_2|f_2)] \\ &\quad + p_2[E_2(a - b_{22}p_2 - b_{21}p_1|f_2)] \end{aligned} \tag{9}$$

We use results from Cyert et al. (1978), Vives (1984) and Winkler (1981), which show that the expected value of the primary demand, given forecast f_i (and f_j), is a convex combination of the average demand (the prior), $\bar{a}$, and the observed forecast f_i (and f_j). We, therefore, have

$$\begin{aligned} E(a|f_1) &= (1-t_1)\,\bar{a} + t_1 f_1 = A_1(\text{say}), \\ E(a|f_2) &= (1-t_2)\,\bar{a} + t_2 f_2 = A_2(\text{say}), \text{and} \\ E(a|f_1,f_2) &= (1-J-K)\,\bar{a} + Jf_1 + Kf_2 = A\ (\text{say}) \end{aligned} \tag{10}$$

where,

$$t_1 = \frac{V}{V+s_1}, t_2 = \frac{V}{V+s_2}, J = \frac{t_1(d_2-1)}{d_2d_1-1}, \text{and } K = \frac{t_2(d_1-1)}{d_2d_1-1}.$$

t_i is referred to as the precision parameter and it is inversely proportional to the error variance s_i Note that in Eq. (10), while A_1 or A_2 captures a consensus prediction of the primary demand using one firm' information and the prior information ($\bar{a}$), A captures a consensus prediction of the primary demand using two firms' information and the prior information ($\bar{a}$). Using the results in Vives (1984), the conditional expectation of one firm's forecast given the other firm's forecast can be expressed as:

$$\begin{aligned} E(f_2|f_1) &= (1-d_1)\,\bar{a} + d_1 f_1, \\ E(f_1|f_2) &= (1-d_2)\,\bar{a} + d_2 f_2, \end{aligned} \tag{11}$$

where,

$$d_1 = \frac{V+s_{12}}{V+s_1}, \quad \text{and} \quad d_2 = \frac{V+s_{12}}{V+s_2}$$

We assume that both firms operate by choosing prices to maximize their respective profits. Each recognizes that the other also does the same. The optimal price depends not only on known demand function parameters (e.g., price sensitivity, cross-sensitivity, etc.), but also on the forecasts.

In this section, we developed the profit expressions of both firms for a mixed bundle of products targeted to three segments of the market, given that the demands are uncertain and given the firms' forecasts of the demand. In the next section, we model a Bertrand type game played by the two firms where they make the decisions simultaneously, and find optimal solutions to the model.

3. The Bertrand game

The firms make decisions simultaneously, without knowing each other's decision. For this kind of game, we develop optimal policies under three scenarios: a first best case where the firms cooperate to act jointly as if they are in a strategic alliance (Case A), asymmetric information when the firms do not know the other firm's forecast information (Case BN) and full information for both firms (Case BI). As discussed in Section 2, Bertrand game makes sense in our scenario because more often the firms do not want to be the follower or the second mover. Our model can be used for any complementary products with mixed bundling. Next, we present our first scenario.

3.1. Strategy alliance-cooperative game (A)

In this subsection, we analyze the strategy alliance – the first best case– as a benchmark, wherein two firms maximize their joint profits using both firms' private information collectively. This first best case provides upper bound on firms' expected profits. Our strategy alliance is relatively easy to implement as both firms' profits have been explicitly determined based on their respective first best prices. This case is of particular interest currently because we see more and more instances where two firms are entering into a strategy alliance for the purpose of benefiting from each other's core competency and unique advantages. Examples include SAP going into strategic alliance with Catalyst Inc. for their warehouse management software. HP and Apple "announced a strategic alliance to deliver an HP-branded digital music player based on Apple's iPod" (Apple, 2004). SBC Communications and HP announced strategic alliance for integrated IT and Telecom managed services (SBC, 2005).

In recent literature in the area of asymmetric information, it has become a norm to include the case of strategic alliance (SA) for two important reasons. One is that it is used as a base case to show the quantum of profit reduction when the supply chain is not integrated. The second is that, in real life, firms are treating supply chain integration and strategic alliance as one of their major thrusts in setting their operations strategy. Many global strategy researchers are advocating firms to explore strategic alliance with other firms and in some cases, even with their competitors.

In this cooperative form of game, there is no information asymmetry. The two firms know both the forecasts and simultaneously set the prices together. Also, because of the alliance, the firms effectively act like a single firm. They, therefore, maximize a single objective function which is the sum of the two profit functions, as follows:

$$\begin{aligned} \pi^A &= E(\pi_1|f_1,f_2) + E(\pi_2|f_1,f_2) \\ &= E[(\pi_{1\_G1} + \pi_{1\_G2} + \pi_{2\_G2} + \pi_{2\_G3})|f_1,f_2] \end{aligned} \tag{12}$$

The equilibrium prices and the corresponding profits are derived in the next proposition. Proof of this proposition follows the same line as Proposition 2 given in the Appendix.

Proposition 1. *The equilibrium prices for the strategy alliance case are*:

$$\begin{aligned} p_1^A &= \frac{2(1+\alpha)(b_{22}+e)-(1+\beta)(b_{12}+b_{21})}{4(d+b_{11})(e+b_{22})-(b_{12}+b_{21})}A, \text{and} \\ p_2^A &= \frac{2(1+\beta)(b_{11}+d)-(1+\alpha)(b_{12}+b_{21})}{4(d+b_{11})(e+b_{22})-(b_{12}+b_{21})^2}A \end{aligned}$$

The corresponding profits for each firm are:

$$\begin{aligned} \pi_1^A &= \frac{[2(1+\alpha)(b_{22}+e)-(1+\beta)b_{12}][(1+\beta)(b_{21}-b_{12})+2(1+\alpha)(e+b_{22})]}{[4(d+b_{11})(e+b_{22})-(b_{12}+b_{21})^2]^2}A^2, \\ \pi_2^A &= \frac{[2(1+\beta)(b_{11}+d)-(1+\alpha)b_{21}][(1+\alpha)(b_{12}-b_{21})+2(1+\beta)(d+b_{11})]}{[4(d+b_{11})(e+b_{22})-(b_{12}+b_{21})^2]^2}A^2. \end{aligned}$$

The strategy alliance's equilibrium prices and corresponding profits in Proposition 1 represent the best price settings and highest profits (derived from the most effective coordination of prices to manage consumer demand). We observe that the accuracy of the forecast plays a vital role in the profitability of the alliance. As proved in the next Proposition, profitability increases as any of the forecast becomes more accurate. We found this to be true also for the BN and BI cases.

Proposition 1.1. The expected equilibrium profit of Firm i ($i=1$, 2) for the strategy alliance case

(a) increases with the accuracy of its own forecast.
(b) increases with the accuracy of the other firm's forecast.

Proposition 1.1 indicates that better forecast of any firm would lead to profitability for each firm, thus there is an incentive for firms to improve their forecast accuracies.

3.2. Non-cooperation-Bayesian Bertrand game (BN)

In this mode of the Bertrand game, information about the firms' forecasts is not shared. This is where the firms are truly separate and act on their own interest. In this case, there is no known co-operation agreement between these two firms, and they experience information asymmetry. The firms announce their prices simultaneously by maximizing their respective profit functions as given in Eq. (7). In the next proposition, we derive both the optimal prices and profit of each firm. Proofs are shown in the Appendix.

Proposition 2. *The Bayesian Bertrand equilibrium prices in a non-cooperation setting are given by*:

$$p_1^{\mathrm{BN}} = U\,\overline{a} + U'\,t_1(f_1 - \overline{a}) \text{ and}$$
$$p_2^{\mathrm{BN}} = V\,\overline{a} + V'\,t_2(f_2 - \overline{a})$$

where

$$U = \frac{2(b_{22}+e)(1+\alpha) - b_{12}(1+\beta)}{4(b_{11}+d)(b_{22}+e) - b_{12}b_{21}},$$
$$U' = \frac{2(b_{22}+e)(1+\alpha) - b_{12}(1+\beta)d_2,}{4(b_{11}+d)(b_{22}+e) - b_{12}b_{21}d_1d_2}$$
$$V = \frac{2(b_{11}+d)(1+\beta) - b_{21}(1+\alpha)}{4(b_{11}+d)(b_{22}+e) - b_{12}b_{21}},$$
$$V' = \frac{2(b_{11}+d)(1+\beta) - b_{12}(1+\alpha)d_1}{4(b_{11}+d)(b_{22}+e) - b_{12}b_{21}d_1d_2},$$

The corresponding profits at equilibrium are given as:

$$\pi_1^{\mathrm{BN}} = [U\,\overline{a} + U'(A_1 - \overline{a})]\{A + \alpha A - (b_{11}+d)\,[U\,\overline{a} + U'(A_1 - \overline{a})] - b_{12}[V\,\overline{a} + V'(A_2 - \overline{a})]\} \text{ and}$$
$$\pi_2^{\mathrm{BN}} = [V\,\overline{a} + V'(A_2 - \overline{a})]\{A + \beta A - (b_{22\_}e)\,[V\,\overline{a} + V'(A_2 - \overline{a})] - b_{21}[U\,\overline{a} + U'(A_1 - \overline{a})]\}\cdot$$

In these expressions, we derive a firm's price directly from the market parameters and the firm's own forecast. Let us analyze the price function obtained in Proposition 2. Note that given the market parameters, U and U' are constants. t_1 is a measure of Firm 1's forecast accuracy, with higher accuracy meaning higher t_1. Optimal price as given in the Proposition uses a combination of $\bar{a}$ and f_1. If the forecast is completely non-informative, ($t_1=0$), then $p_1=U\bar{a}$, i.e., the firm would not use forecast at all. On the other hand, if the forecast is completely accurate ($t_1=1$, $U=U'$), then $p_1=Uf_1$, i.e., the firm use the forecast only foregoing the prior information. For any other case in between these two extreme cases, p_1 will be set using both the prior and the forecast. Similar explanation holds for Firm 2.

In the next proposition, we derive an insight how the expected profit of a firm is affected by the accuracy of the firms' forecast. We are able to prove the proposition for a special case where

(i) all the price sensitivities and cross-sensitivity parameters are symmetric (i.e., $b_{11}=b_{22}=d=e=b$, and $b_{12}=b_{21}=c$),
(ii) the forecast errors of the two firms are uncorrelated ($s_{12}=0$), and
(iii) primary demands for the singular markets namely segments 1 and 2 are equal (i.e., $\alpha=\beta=r$(say)).

Proposition 2.1. The expected equilibrium profit of a firm in a noncooperative setting

(a) increases with the accuracy of its own forecast.
(b) decreases with the accuracy of the other firm's forecast.

Proposition 2.1 (a) indicates the expected result that the more accurate the forecast, the better performance of the firm. However, Proposition 2.1 (b) shows that one firm's inaccurate forecast would contribute to the other firm's profitability. This is because one firm (Firm 2, say)'s ability to manage and control the demand declines with its forecast accuracy. Consequently, the other firm (Firm 1, say) is able to exploit demand more to its own benefit.

3.3. Information sharing-Bertrand game (BI)

We now take up the third mode of the game where the two firms share information about each other's forecast. As Lewis (1999) says, "By pairing up with others, companies can achieve more than they can on their own, as the lucrative partnership between Hewlett-Packard and Canon demonstrates." Presumably and intuitively, this will increase the quality of the decisions made by the firms because now they know how the other firm is going to act. The expected values for firms, in this case, are taken given not just their own forecasts, as in Proposition 2, but given both f_1 and f_2. The results are given in Proposition 3. The proof comes from the first order condition and algebraic manipulation and therefore is omitted.

Proposition 3. *The Bayesian Bertrand equilibrium prices in a cooperative setting are given by*:

$$p_1^{\mathrm{BI}} = UA \text{ and } p_2^{\mathrm{BI}} = VA$$

where U and V are as defined in Proposition 2.

The corresponding profits at equilibrium are given as:

$$\pi_1^{\mathrm{BI}} = UA[A - \alpha A - (b_{11}+d)UA - b_{12}VA] \text{ and}$$
$$\pi_2^{\mathrm{BI}} = VA[A - \beta A - (b_{22}+e)VA - b_{21}UA]$$

As seen in Proposition 3, both the firm's price is directly proportion to A (defined in Eq. (10), which is a function of the prior $\bar{a}$ and the forecasts of both the firms f_1 and f_2. These are weighted by $(1-J-K)$, J and K respectively. These weights, in turn, depend on the respective forecast accuracy. For example, if he forecast accuracy for Firm 1 is low, a higher variance will give a lower weight J resulting in a lower weight given to f_1 in calculating p_1. In contract, we saw that the equilibrium price of Firm 1 in the non-cooperative case depends also on Firm 1's own forecast.

Proposition 3.1. The expected equilibrium profit of Firm i ($i=1, 2$) in a cooperative setting (information sharing)

(a) increases with the accuracy of its own forecast.
(b) increases with the accuracy of the other firm's forecast.

Proof of Proposition 3.1 is omitted as it is similar to that of Proposition 1.1.

As in the strategic alliance case, Proposition 3.1 indicates that, in a cooperative setting too, the improvements in forecast precision of any firm would lead to increased profits of both firms. This is because as each firm makes its pricing decision based on both parties' forecasts, more accurate forecast of any firm would lead to higher profitability of each firm.

In BN and BI cases, the firms take decisions to maximize their own profits given the competition in the market manifested by the cross-price sensitivities of the customers in the common group, the Segment 2. Information sharing brings both advantage and disadvantage to each firm. The advantage is that sharing other firm's information would contribute to boosting the forecast accuracy, and thus the firm would make better decision for its own good. However, at the same time, information sharing would provide the opportunity for the other firm to better exploit the information at the expense of the firm, i.e., the other firm may increase its price for its own good, thus reducing the demand and the profit for the firm. It will be interesting to find out the conditions under which information sharing would be beneficial for firms.

3.4. Analysis of the three cases

In this subsection, we will derive the differences between the optimum strategies in the three cases and develop some managerial guidelines. To focus on analyzing how the forecast information sharing impact the firms offering complementary goods and maintain analytical tractability, we simplify the model here by assuming that all the price sensitivities and cross-sensitivity parameters are symmetric (i.e., $b_{11}=b_{22}=d=e=b$, and $b_{12}=b_{21}=c$). Also, we assume that $\alpha=\beta=r$ (say), i.e., primary demands for the singular markets namely segments 1 and 2 are equal and $\rho=0$. The above assumptions are without loss of generality as the same approach can be applied to the case in which these parameters are different. Table 1 shows the expression for prices and profits for the three cases under these assumptions.

Next, we will introduce two propositions in which we will derive some important results regarding the profits and optimum pricing strategies of Firms 1 and 2 under the three different cases considered above. First, Proposition 4 shows how the two Firm's profits are affected by the information asymmetry.

Proposition 4. *Profit and price comparison*

When two firms form a strategy alliance, their individual expected profits and their combined expected profit would be higher than when they do not cooperate to share information or even when they share information, i.e., $E(\pi_i^A)>E(\pi_i^{\mathrm{BI}})$, $E(\pi_i^A)>E(\pi_i^{\mathrm{BN}})$ ($i=1,2$) and, $E(\pi^A)>E(\pi^{\mathrm{BI}})$, $E(\pi^A)>E(\pi^{\mathrm{BN}})$.

Also, a strategy alliance would be able to lower its price compared to the individual prices in a cooperative case, i.e., $p_1^A<p_1^{\mathrm{BI}}$ and $p_2^A<p_2^{\mathrm{BI}}$.

Proposition 4 indicates that strategy alliance gives a higher profit to both firms compared to the case of information sharing and of non-cooperation. This is expected as strategy alliance is the first best solution. Also, under strategy alliance, both firms would price their products lower than the optimal prices under information sharing. This means that prices will decrease from the scenario of non-coordination to that of coordination. This result makes sense as coordination contributes to alleviating the conflict and the price competition between the two firms, and they both achieve higher profits. The lower price would increase the demand. This increased revenue due to demand increase will more than offset the decrease in revenue due to lower price. Lower price will have quite a salutary effect on the market performance of both the products. This also gives powerful incentives to the firms to form strategic alliance, as in the examples mentioned earlier.

In the next proposition, we study the sensitivity of a firm's profit to changes in the market parameters.

Proposition 4.1.

(1) Higher primary demands in Segment 1 and 3 (r) would result in higher profits for both firms, in cases of cooperation and strategy alliance.
(2) Profit under all three scenarios would increase for both firms if market becomes less price sensitive (decreasing b or c).

Proof: The results are derived by having derivatives of expected profits with respect to r, b and c.

Proposition 4.1(2) leads to an important managerial guideline. As lower sensitivity is preferable from the profitability point of view, the manager should use any operational means to prevent the market from being very sensitive to price. This can be achieved by offering non-price differentiating values to the customers, for example, enhancing the quality of product, offering more functionality, increasing customer support, etc.

3.4.1. Comparison between information sharing and non-information sharing

Proposition 4 established the profitability difference of the strategy alliance over the other two cases. Now we assess the impact of information sharing on each firm's performance over

Table 1
The equilibrium price and profit for three modes

Bayesian Bertrand no information sharing (BN)	Bertrand info sharing (BI)	Strategy alliance (*A*)
$p_1^{\mathrm{BN}}=W\bar{a}+W_1(A_1-\bar{a})$	$p_1^{\mathrm{BI}}=\dfrac{(1+r)}{(4b+c)}A$	$p_1^{A}=\dfrac{(1+r)}{2(2b+c)}A$
$p_2^{\mathrm{BN}}=W\bar{a}+W_2(A_2-\bar{a})$	$p_2^{\mathrm{BI}}=\dfrac{(1+r)}{(4b+c)}A$	$p_2^{A}=\dfrac{(1+r)}{2(2b+c)}A$
$\pi_1^{\mathrm{BN}}=[W\bar{a}+W_1(A_1-\bar{a})][(1+r)A-2bW\bar{a}-2bW_1(A_1-\bar{a})-cW\bar{a}-cW_2(A_2-\bar{a})]$	$\pi_1^{\mathrm{BI}}=\dfrac{2(1+r)^2b}{(4b+c)^2}A^2$	$\pi_1^{A}=\dfrac{(1+r)^2A^2}{4(2b+c)}$
$\pi_2^{\mathrm{BN}}=[W\bar{a}+W_2(A_2-\bar{a})][(1+r)A-2bW\bar{a}-2bW_2(A_2-\bar{a})-cW\bar{a}-cW_1(A_1-\bar{a})]$	$\pi_2^{\mathrm{BI}}=\dfrac{2(1+r)^2b}{(4b+c)^2}A^2$	$\pi_2^{A}=\dfrac{(1+r)^2A^2}{4(2b+c)}$
$\pi^{\mathrm{BN}}=[W\bar{a}+W_1(A_1-\bar{a})][(1+r)A-2bW\bar{a}-2bW_1(A_1-\bar{a})\text{-}cW\bar{a}-cW_2(A_2-\bar{a})]+$ $[W\bar{a}+W_2(A_2-\bar{a})][(1+r)A-2bW\bar{a}\text{-}2bW_2(A_2-\bar{a})-cW\bar{a}\text{-}cW_1(A_1-\mathrm{v}\bar{a})]$	$\pi^{\mathrm{BI}}=\dfrac{4(1+r)^2b}{(4b+c)^2}A^2$	$\pi^{A}=\dfrac{(1+r)^2A^2}{2(2b+c)}$

Where, $W=\dfrac{(1+r)(4b-c)}{16b^2-c^2}=\dfrac{(1+r)}{(4b+c)}$, and $W_1=\dfrac{(1+r)(4b-cd_2)}{16b^2-c^2d_1d_2}$,

that when information is not shared. Eqs. (13) and (14) below give the expression for the individual expected profit difference between the BI and BN cases, for Firm 1 and Firm 2 respectively. Eq. (15), on the other hand gives the increase in the total profit when information is shared.

$$E(\pi_1^{\mathrm{BI}})-E(\pi_1^{\mathrm{BN}})=2b(1+r)^2\Bigg[-\frac{V^2(s_1+V)(cV-4b(s_2+V))^2}{(c^2V^2-16b^2(s_1+V)(s_2+V))^2}+\frac{(s_1+s_2)V^2}{(s_2V+s_1(s_2+V))(4b+c)^2}\Bigg] \tag{13}$$

$$E(\pi_2^{\mathrm{BI}})-E(\pi_2^{\mathrm{BN}})=2b(1+r)^2\Bigg[-\frac{V^2(s_2+V)(cV-4b(s_1+V))^2}{(c^2V^2-16b^2(s_1+V)(s_2+V))^2}+\frac{(s_1+s_2)V^2}{(s_2V+s_1(s_2+V))(4b+c)^2}\Bigg] \tag{14}$$

$$E(\pi_1^{\mathrm{BI}})+E(\pi_2^{\mathrm{BI}})-E(\pi_1^{\mathrm{BN}})-E(\pi_2^{\mathrm{BN}})=2b(1+r)^2V^2\Bigg[-\frac{(s_1+V)(cV-4b(s_2+V))^2+(s_2+V)(cV-4b(s_1+V))^2}{(c^2V^2-16b^2(s_1+V)(s_2+V))^2}+\frac{2(s_1+s_2)}{(s_2V+s_1(s_2+V))(4b+c)^2}\Bigg] \tag{15}$$

Condition 1. Eq. (13)>0 and Eq. (14)>0.

Condition 2. Eq. (15)>0.

Given these two conditions, we can come up with a number of insights. For example, if condition (1) is satisfied, then the Bertrand game with information sharing equilibrium, as established in Proposition 3, will be reached.

To have both firms benefit from the information sharing (i.e., Eqs. (13) and (14) both >0), it can be intuitively inferred that the forecast precision of both firm are relatively "equal" in magnitude. Otherwise, if Firm 1 has better information, say, then Firm 2 will benefit asymmetrically much more if information is shared. Note that if Condition (1) is satisfied, Condition (2) is automatically satisfied, and total profit is improved. In case, Eqs. (13) and (14) both <0, then the BN equilibrium as characterized in Proposition 2 will be reached. There is one other possibility where either Eq. (13) or Eq. (14), but not both, is less than zero (however, they are not less than zero at the same time) and Condition (2) is satisfied. In that situation, one of the firm will be benefited and one not, but total profit increases. This situation gives rise to a unique guideline. The firm who gets the benefit can offer a part of its own benefit to the other firm to compensate for the loss of benefit so that information sharing is induced. This is a contract-signing game guaranteeing the participation in the information sharing game, to move from the BN to the BI equilibrium.

Thus we can formulate the following proposition.

Proposition 5. The information sharing equilibrium (i.e., information sharing-Bertrand game) can be reached if and only if one of the following conditions are satisfied

(i) Eq. (13) ≥ 0 and Eq. (14) ≥ 0 or
(ii) Eq. (15) ≥ 0 and a contract exists for splitting of the benefit.

Otherwise, the no information sharing equilibrium (BN) (i.e., non-cooperation-Bayesian Bertrand game) is reached.

4. Computational analysis

The purpose of our computational study is to explore the magnitude of profits of Firm 1 and Firm 2 under the three strategy alternatives, and to illustrate the value of information sharing, which can be thought of as the difference in profits between non-cooperation and information sharing cases. We also study the impact of model parameters on expected profits. This computational study will complement our analytical results and give us more managerial insights.

4.1. Design of the simulation analysis

The values we use for the various parameters are shown in Table 2. As mentioned in the table, we vary some of the parameters to find their effect on the optimum policies. We simulate 100 * 100 batches of 100 trials, for a total of 1 million trials each simulation. This was done to bring the standard deviation of our profit estimate to within 1% of the mean. The simulation program was coded in Fortran and run on a PC. For each run, we simulate three pricing strategies alternatives: Non-cooperation, Information Sharing and Strategy Alliance. In the non-cooperation mode, we generate primary demand a and two correlated forecasts f_1 and f_2. We simulate the cases of no information sharing, information sharing and strategy alliance. The values of the parameters used in the simulation study are given in Table 2.

Effect of varying forecasting errors.

Fig. 2A and B illustrate the impact of varying $\sqrt{s_1}$ (the standard deviation of Firm 1's forecast error) on the firms' profits. Note that the value of information sharing is reflected in the gap between the cases of information sharing (BI) and no information sharing (BN). In Fig. 2A, Firm 1's profits π_1^{SN}, π_1^{SI} and π_1^{A} are all decreasing as error increases. This result is intuitive. But at some reasonably low value of the error, non-cooperation strategy (BN) actually dominates information sharing (BI) for Firm 1. This is expected as Firm 1's high precision makes Firm 2 to exploit the information opportunistically (as explained in Section 3). But, as the error increases, Firm 1's profit drastically decreases (because the forecast becomes increasingly less accurate) and BI catches up with BN at some certain level. This is because sharing Firm 2's forecast information becomes increasingly more valuable for Firm 1, which makes the advantage of BI overcome its disadvantage side. The

Table 2
Base values and range of values used in the simulation study

Parameters	Base value and (range of values)
A	100
$\sqrt{V}$	20 (15–25)
$\sqrt{s_1}, \sqrt{s_2}$	10 (1–16)
ρ	0
b_{11}, b_{22}, d, e	1 (1–2)
b_{12}, b_{21}, c	0.5
α, β, r	0.5 (0–1)

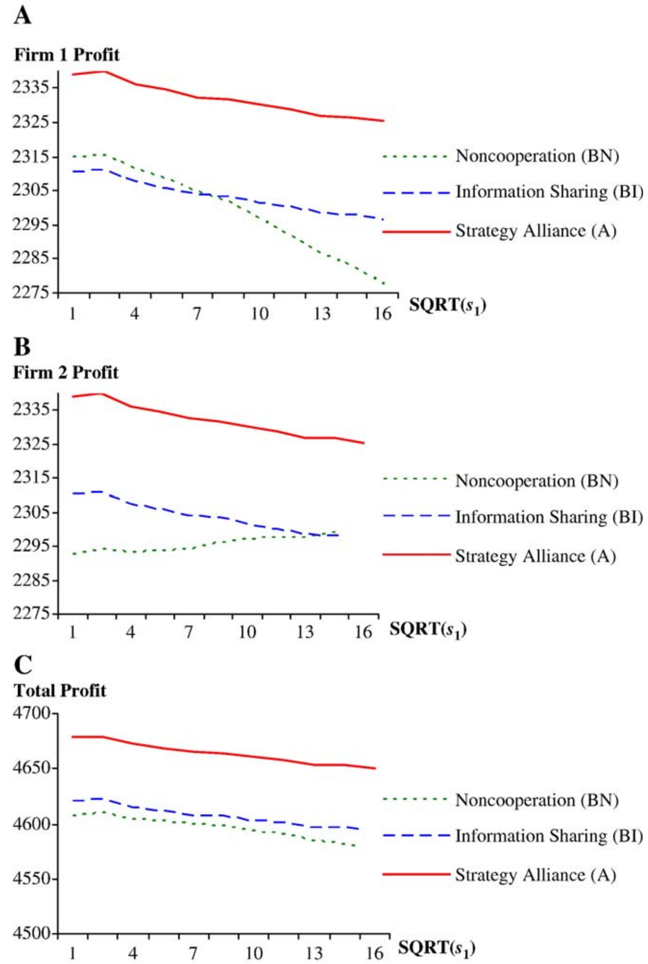

Fig. 2. (A) $\sqrt{s_1}$ versus Firm 1 profit. (B) $\sqrt{s_1}$ versus Firm 2 profit. (C) $\sqrt{s_1}$ versus total profit from Firm 1 and Firm 2.

same reasoning applies to the analysis of Firm 2 performance. In Fig. 2B, we can observe that value of information sharing decreases for firm 2 when firm 1's forecast accuracy decreases. This is expected because, (when $\sqrt{s_1}$ increases), firm 1's forecast information becomes increasingly less valuable for firm 2. Also, we can observe that given $\sqrt{s_2} = 10$ (in this computational study), when $\sqrt{s_1}$<7, Firm 1 (2) is worse off (better off) from information sharing; when 7<$\sqrt{s_1}$<13, both Firm 1 and Firm 2 are better off from information sharing (obviously, 7<$\sqrt{s_1}$<13 satisfying Condition (1)); when $\sqrt{s_1}$>13, Firm 1 (2) is better off (worse off) from information sharing. Therefore, when 7<$\sqrt{s_1}$<13, both firms have incentive to share forecast with each other so that the information sharing equilibrium (i.e., information sharing-Bertrand game) can be reached. When $\sqrt{s_1}$>13 *or* $\sqrt{s_1}$<7, the no information sharing equilibrium (BN) (i.e., non-cooperation-Bayesian Bertrand game) is reached. We also notice that a firm benefits more from information sharing when its forecast accuracy is low or the other firm's forecast accuracy is high (this makes sense as the lower forecast accuracy or the higher the other firm's forecast accuracy, the more benefit the firm can obtain).

Thus, we can conclude that, if both firms' forecast variances are not very far away from each other, forecast information

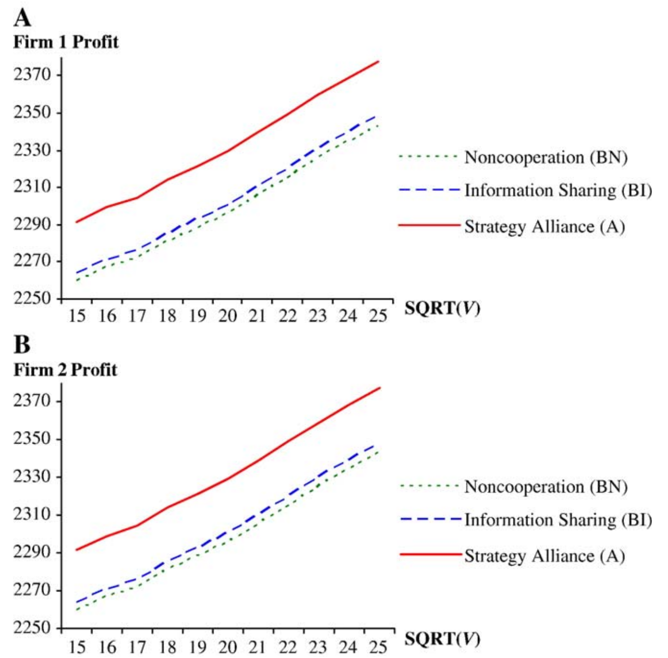

Fig. 3. (A) $\sqrt{V}$ versus Firm 1 profit. (B) $\sqrt{V}$ versus Firm 2 profit.

can be shared between the two firms. Otherwise, information sharing cannot be achieved.

Strategy alliance profit is larger than the other two for both firms. This is one of the reasons why supply chain integration is becoming an important operations strategy. Note that individually all the profits decline as error increases except for the non-cooperation case for Firm 2 which benefits from the errors suffered by Firm 1.

Fig. 2C shows that the numerical sum of the profits of the two firms for the BN and BI cases and the common objective function value for the strategy alliance case. We find that $E(\pi^{A})>E(\pi^{\mathrm{BI}})>E(\pi^{\mathrm{BN}})$ from the total profit point of view, i.e., Alliance (A) is the best strategy, Information Sharing (BI) strategy is the second and Non-cooperation (BN) is the worst. $E(\pi^{\mathrm{BI}})>E(\pi^{\mathrm{BN}})$ indicates that, given the set of parameters, if both firms agree to sign the contract, information sharing equilibrium (i.e., information sharing-Bertrand game) can be reached.

We skip the analysis of impact of varying $\sqrt{s_1}$ on the firms' profits, because it follows the exactly same reasoning line.

Effect of varying demand variance

In this set of experiments, we study the impact of varying $\sqrt{V}$ on the firms' profits. Note that $\sqrt{V}$ represents the inherent volatility of the demand in the market. Fig. 3A and B illustrate the impact of varying $\sqrt{V}$ on the firms' profits. As the market is more volatile, both the firms would enjoy increasing profit in all the three cases. This is counter-intuitive in the sense that it would be expected that more uncertain demand would be detrimental to the firms' profitability. The results we get can be explained this way. When $\sqrt{V}$ increases, the probability of a very high as well as very low primary demand increases (for the same mean.) However, the larger profits resulting from larger primary demand dominate the small profits resulting from a small primary demand. We also see the familiar pattern where, for any value of the market variance, profits for strategy alliance dominate the other two cases for both the firms. For the other two strategies, the information sharing case does show better profits compared to non-cooperation case, but the profit differences, i.e., the value of information is not substantial.

5. Conclusion

In this paper, we consider a scenario very often encountered in information technology industry. Customers need to buy more than one product, often from different vendors, to get the full benefit of the products, giving rise to the notion of product bundling. The firms do not know the forecast information of the other firm. The firms need to make marketing decisions with market uncertainty and incomplete information.

We first derived the equilibrium prices and profits under three cases (non-cooperation, cooperation, and strategy alliance) and studied how the forecast precision would impact the firms' expected profit. We then investigated the impact of demand forecast sharing on firms' performances. We found that information sharing is not always good for each firm all the time, and further identified conditions under which information sharing can be achieved. We also showed that if the firm get into strategy alliance and behave like a single firm optimizing a single objective function, the profits would be higher than any other scenarios. This strategy alliance is relatively easy to implement as both firms' profits have been split based on their individual prices, comparing to the usual integration case in which firms have to bargain on how to divide the total integration profit. Our computational study illustrated and verified our analytical findings and provided more managerial interpretations and insights.

A further research path could be to study the same scenario but with a sequential game rather than the simultaneous game considered in this paper. In a sequential game, one firm will act as a leader, setting and declaring the pricing decision. Then the other firm, who now can make a guess about the private information of the leader firm would follow. We can show the differences between the two modes of game and get managerial insights into under what conditions, a firm would opt for sequential game or otherwise.

Appendix A

Proof of Proposition 1.1. We know that $a=\bar{a}+e$ and $f_i=a+\varepsilon_i$; $(i=1,2)$, so $f_i^2=a^2+2a\varepsilon_i+\varepsilon_i^2$,

$$\begin{aligned} E[f_i] &= \bar{a} \\ E[f_i^2] &= E[a^2]+E[\varepsilon_i^2] = \bar{a}^2+V+s_i \\ &\quad (\text{Note}: \ E[a^2]=(E[a])^2+\mathrm{Var}[a]=\bar{a}^2+V) \\ E[f_1 f_2] &= E[(a+\varepsilon_1)((a+\varepsilon_2)] = E[a^2+a\varepsilon_1+a\varepsilon_2+\varepsilon_1\varepsilon_2] \\ &= \bar{a}^2+V+s_{12} \end{aligned}$$

Substituting $E[f_i]$, $E[f_i^2]$and $E[f_i f_2]$ into expressions of, $E(\pi_1^{\mathrm{A}})$, then

$$\frac{dE(\pi_1^A)}{dt_1} = \frac{2(-1+d_2)[-2(b_{22}+e)(1+\alpha)+b_{12}(1+\beta)][-2(b_{22}+e)(1+\alpha)+(b_{12}-b_{21})(1+\beta)]}{(-1+d_1d_2)^2[(b_{12}+b_{21})^2-4(b_{11}+d)(b_{22}+e)]^2} \times((-1+d_1)s_1t_1+(-1+d_2)t_1V+(-1+d_1)t_2(V+\sqrt{s_1s_2\rho}))\geq 0$$

(we assume that α is not very far away from β)

$$\frac{dE(\pi_1^A)}{dt_2} = \frac{2(-1+d_2)[-2(b_{22}+e)(1+\alpha)+b_{12}(1+\beta)][-2(b_{22}+e)(1+\alpha)+(b_{12}-b_{21})(1+\beta)]}{(-1+d_1d_2)^2[(b_{12}+b_{21})^2-4(b_{11}+d)(b_{22}+e)]^2} \times\left((-1+d_1)t_2(s_2+V)+(-1+d_2)t_1(V+\sqrt{s_1s_2\rho}\right)\geq 0$$

In the same way, we can prove $\frac{dE(\pi_2^A)}{dt_2}\geq 0, \frac{dE(\pi_2^A)}{dt_1}\geq 0$

Proof of Proposition 2. Firm 1 and Firm 2 would set a price to optimize its own profit (8) and (9), respectively. We can get best response function p_1 and p_2 by taking first order condition (FOC) of profit function with respect to price and setting it equals zero, respectively.

From (8),

$$\frac{\partial E_1(\pi_1|f_1)}{\partial p_1} = 0 \Rightarrow \quad p_1 = \frac{(1+\alpha)E_1(a|f_1)-b_{12}E_1(p_2|f_1)}{2(b_{11}+d)}$$

From (9),

$$\frac{\partial E_2(\pi_2|f_2)}{\partial p_2} = 0 \Rightarrow \quad p_2 = \frac{(1+\beta)E_2(a|f_2)-b_{21}E_2(p_1|f_2)}{2(b_{22}+e)}$$

For ease of exposition, we use the abbreviated forms, $E_1(a)=E_1(a|f_1)$, $E_2(a)=E_2(a|f_2)$, $E_1(p_2)=E_1(p_2|f_1)$ and $E_2(p_1)=E_2(p_1|f_2)$.

The above equations become

$$p_1 = \frac{(1+\alpha)E_1(a)-b_{12}E_1(p_2)}{2(b_{11}+d)} \tag{A1}$$

$$p_2 = \frac{(1+\beta)E_2(a)-b_{21}E_2(p_1)}{2(b_{22}+e)} \tag{A2}$$

We substitute p_2 in Eq. (A2) into Eq. (A1)

$$p_1 = \frac{(1+\alpha)E_1(a)}{2(b_{11}+d)} - \frac{(1+\beta)b_{12}E_1E_2(a)}{4(b_{11}+d)(b_{22}+e)} + \frac{b_{12}b_{21}E_1E_2(p_1)}{4(b_{11}+d)(b_{22}+e)}, \tag{A3}$$

Next, we replace p_1 in the RHS of Eq. (A3) by the value given in Eq. (A2), we obtain

$$p_1 = \frac{(1+\alpha)E_1(a)}{2(b_{11}+d)} - \frac{(1+\beta)b_{12}E_1E_2(a)}{4(b_{11}+d)(b_{22}+e)} + \frac{(1+\alpha)b_{12}b_{21}E_1E_2E_1(a)}{8(b_{11}+d)^2(b_{22}+e)} - \frac{b_{12}^2b_{21}E_1E_2E_1(p_2)}{8(b_{11}+d)^2(b_{22}+e)}, \tag{A4}$$

We continue make substitutions in this fashion; we obtain the following infinite series:

$$p_1 = \frac{(1+\alpha)E_1(a)}{2(b_{11}+d)} - \frac{(1+\beta)b_{12}E_1E_2(a)}{4(b_{11}+d)(b_{22}+e)} + \frac{(1+\alpha)b_{12}b_{21}E_1E_2E_1(a)}{8(b_{11}+d)^2(b_{22}+e)} - \frac{b_{12}^2b_{21}E_1E_2E_1E_2(a)}{16(b_{11}+d)^2(b_{22}+e)^2} + \cdots \tag{A5}$$

Using Eqs. (1) and (2), we obtain

$$\begin{aligned}
E_1 &= \overline{a} + t_1(f_1-\overline{a}),\\
E_1E_2E_1 &= \overline{a} + t_1d_1d_2(f_1-\overline{a}),\\
E_1E_2E_1E_2E_1 &= \overline{a} + t_1d_1^2d_2^2(f_1-\overline{a}),\\
E_1E_2 &= \overline{a} + t_2(f_2-\overline{a}),\\
E_1E_2E_1E_2 &= \overline{a} + t_2d_2d_1(f_2-\overline{a}),\\
E_1E_2E_1E_2E_1E_2 &= \overline{a} + t_2d_2^2d_1^2(f_2-\overline{a}),
\end{aligned} \tag{A6}$$

Bringing (A6) back to (A5), we have an infinite series which is a combination of four infinite equal ratios with ratios less than 1. So we can obtain non-cooperation Bayesian Bertrand expected price equilibrium p_1^{BN} as follows.

$$\begin{aligned}
p_1 &= \frac{(1+\alpha)[\overline{a}+t_1(f_1-\overline{a})]}{2(b_{11}+d)} - \frac{(1+\beta)b_{12}[\overline{a}+t_2(f_1-\overline{a})]}{4(b_{11}+d)(b_{22}+e)} + \frac{(1+\alpha)b_{12}b_{21}[\overline{a}+t_1d_1d_2(f_1-\overline{a})]}{8(b_{11}+d)^2(b_{22}+e)} - \frac{b_{12}^2b_{21}[\overline{a}+t_2d_2d_1(f_1-\overline{a})]}{16(b_{11}+d)^2(b_{22}+e)^2}\\
&\quad + \cdots = \frac{(1+\alpha)\,\overline{a}}{2(b_{11}+d)} + \frac{(1+\alpha)t_1(f_1-\overline{a})}{2(b_{11}+d)} - \frac{(1+\beta)b_{12}\,\overline{a}}{4(b_{11}+d)(b_{22}+e)} - \frac{(1+\beta)b_{12}t_2(f_1-\overline{a})}{4(b_{11}+d)(b_{22}+e)}\\
&\quad + \frac{(1+\alpha)b_{12}b_{21}\,\overline{a}}{8(b_{11}+d)^2(b_{22}+e)} + \frac{(1+\alpha)b_{12}b_{21}t_1d_1d_2(f_1-\overline{a})}{8(b_{11}+d)^2(b_{22}+e)} - \frac{(1+\beta)b_{12}^2b_{21}\,\overline{a}}{16(b_{11}+d)^2(b_{22}+e)^2} - \frac{(1+\beta)b_{12}^2b_{21}t_2d_2d_1(f_1-\overline{a})}{16(b_{11}+d)^2(b_{22}+e)^2}\\
&\quad + \cdots
\end{aligned}$$

Which, after some algebraic manipulation, becomes:

$$p_1 = \frac{2(b_{22}+e)(1+\alpha)-b_{12}(1+\beta)}{4(b_{11}+d)(b_{22}+e)-b_{12}b_{21}}\,\overline{a} + \frac{2(b_{22}+e)(1+\alpha)-b_{12}(1+\beta)d_2}{4(b_{11}+d)(b_{22}+e)-b_{12}b_{21}d_1d_2}(f_1-\overline{a}).$$

The non-cooperation Bayesian Bertrand equilibrium price for Firm 2, p_2^{BN} is obtained in a symmetrical manner.

Proof of Proposition 2.1.

$$\frac{dE(\pi_1^{\mathrm{BN}})}{ds_1} = \frac{-(2bV^2(cV-4b(s_2+V))^2(c^2V^2+16b^2(s_1+V)(s_2+V)(1+r)^2)}{(-c^2V^2+16b^2(s_1+V)(s_2+V))^3} \le 0$$

$$\frac{dE(\pi_1^{\mathrm{BN}})}{ds_2} = \frac{16b^2cV^3(s_1+V)(-cV+4b(s_1+V))(-cV+4b(s_2+V))(1+r)^2}{(-c^2V^2+16b^2(s_1+V)(s_2+V))^3} \ge 0$$

Similarly, $\frac{dE(\pi_2^{\mathrm{BN}})}{ds_2} \le 0, \frac{dE(\pi_2^{\mathrm{BN}})}{ds_1} \ge 0$

Proof of Proposition 4.

$$E(\pi_1^{\mathrm{BN}}) = t_1W_1(J(1+r)(s_1+V)-2bt_1(s_1+V)W_1-ct_2VW_2 + K(1+r)V + W(1+r-(2b+c)W)\,\overline{a}^2$$

$$E(\pi_1^{\mathrm{BI}}) = \frac{2(1+r)^2(J^2(s_1+V)+K^2(s_2+V)+2JKV+\overline{a}^2}{(4b+c)^2}$$

$$E(\pi_1^{A}) = \frac{(1+r)^2(J^2(s_1+V)K^2(s_2+V)+2JKV+\overline{a}^2}{4(2b+c)}.$$

Thus, $E(\pi_1^A)-E(\pi_1^{\mathrm{BI}})>0$, and $E(\pi_1^A)-E(\pi_1^{\mathrm{BN}})>0$

Similarly, $E(\pi_2^A)-E(\pi_2^{\mathrm{BI}})>0$, $E(\pi_2^A)-E(\pi_2^{\mathrm{BN}})>0$, $E(\pi^A)-E(\pi^{\mathrm{BI}})>0$, $E(\pi^A)-E(\pi^{\mathrm{BN}})>0$

From Table 1,

$$p_1^{\mathrm{BI}} = \frac{(1+r)}{4b+c}\mathrm{A} \Rightarrow \quad p_1^{\mathrm{BI}} = \frac{1}{4b+c}(1+r)A$$

$$p_1^{A} = \frac{(1+r)}{2(2b+c)}\mathrm{A} \Rightarrow \quad p_1^{A} = \frac{1}{4b+2c}(1+r)A$$

So we can get $p_1^{\mathrm{BI}}>p_1^A$.

Using the same approach, we can get $p_2^{\mathrm{BI}}>p_2^A$.

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