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Temporal product bundling with myopic and strategic consumers: Manifestations and relative effectiveness

Ashutosh Prasad[1] · **R. Venkatesh**[2] · **Vijay Mahajan**[3]

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Abstract Bundling in this era of eCommerce and high technology is a potent and widespread selling tool. The literature has focused on three *static* bundling strategies under which the products are sold separately (pure components or PC) or only in a bundled form (pure bundling or PB) or both (mixed bundling or MB). In a generalization, and motivated by real world examples, this paper examines the relative effectiveness of temporal bundling. We consider a firm that sells to a market of myopic and strategic consumers, and a selling season consisting of two stages. We compare four strategies – PC-PC (i.e., pure components in each of two stages), PB-PB, PB-PC and PC-PB – relative to MB. Our results show that PB-PB maximizes profits under low marginal costs; PC-PC prevails under high marginal costs given a large proportion of myopic consumers; and PB-PC is profit maximizing under moderate marginal costs when most consumers are strategic. These temporal strategies dominate MB except when the market is comprised entirely of strategic consumers. Finally, while temporal mixed bundling – MB-MB – is weakly superior to other temporal strategies, the latter are much easier to implement, as shown by real-world uses, and suffice to capture most of the profits. Related interesting pricing implications are discussed. Three extensions to the main model are also proposed.

Keywords Pricing · Bundling · Inter-temporal pricing · Analytical modeling

✉ R. Venkatesh
rvenkat@katz.pitt.edu

Ashutosh Prasad
aprasad@utdallas.edu

Vijay Mahajan
vijay.mahajan@mccombs.utexas.edu

[1] Jindal School of Management, The University of Texas at Dallas, Richardson, TX 75080, USA

[2] Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260, USA

[3] McCombs School of Business, The University of Texas at Austin, Austin, TX 78712, USA

1 Introduction

Bundling is receiving renewed attention as a selling strategy on Internet-based platforms and for related digital and hi-tech products and services (Crawford 2008; Goh and Bockstedt 2013; Hitt and Chen 2005; Pang and Etzion 2012). Three *static* forms of bundling are pervasive: pure components (PC), under which only the individual products are offered; pure bundling (PB) under which only the bundle in offered; and mixed bundling (MB) under which both the bundle and the individual products are on sale. Bundling research has demonstrated the power of MB in achieving second-degree price discrimination (e.g., Bhargava 2013; Ghosh and Balachander 2007; Schmalensee 1984). In contrast to academic research, the marketplace is witnessing a rise in hybrid strategies under which bundling is implemented temporally. Our research objective is to delineate the relative effectiveness of temporal bundling and examine their normative pricing implications.

We are drawn to two *temporal* forms of MB in this study:

- PB-PC, as when the bundle alone is offered in the first stage and the individual products alone are offered in the second stage. As an example, at a neighborhood Wal-Mart, "Toy Story 1–3", previously available as a boxed DVD set, is available only as three individually priced and packaged DVDs.
- PC-PB, as when the individual products alone are offered in the first stage and the bundle alone is offered in the second stage. At the above noted Wal-Mart, "The Lord of the Rings" movie trilogy, which was sold as individual DVDs in the past, is presently available only as a bundle.

Further, given that PC and PB are part of the static bundling triad, we examine the potency of their temporal variants, namely, PC-PC and PB-PB.[1]

We analyze the above noted strategies by developing a stylized model rooted in the literature. Specifically, we consider a monopolist who offers two products. The products can have negligible or sizable marginal costs relative to valuations. Consumers are heterogeneous in their preferences for the products. As with research on temporal pricing (e.g., Elmaghraby and Keskinocak 2003, Prasad, Venkatesh, and Mahajan 2015, Shen and Su 2007), we assume a population of consumers who are either myopic or strategic in nature. "Myopic" consumers in our model are unaware of or unwilling to wait for temporal price discounts (e.g., Varian 1980) while strategic consumers have the awareness of and the willingness to wait for lower prices, which reduces the seller's ability to price discriminate (e.g., Coase 1972).

Analogous to Schmalensee's (1984) approach with traditional bundling in which he compares PC and PB first before introducing MB, we begin by comparing PC-PC and PB-PB. PC-PB and PB-PC are introduced next. Among these strategies we find that PB-PB and PC-PC are profit maximizing as long as the proportion of myopic consumers is under about 75%. Between PC-PC and PB-PB, the latter is more profitable

[1] Amazon.com follows PC-PC in its offering of Blu-ray versions of "Argo" and "Life of Pi." Both were released at a price of $30 and later discounted by over 33%. By contrast, certain music bands such as AC/DC have adopted PB-PB. A price tracker of their album "Black Ice" reveals a launch price of about $11 and a subsequent drop to half that price in six months.

only when marginal costs are low. When a large majority of consumers are strategic, PB-PC is the most profitable, whereas interestingly enough PC-PB is never the best.

Significant pricing implications emerge. For example, compared to static PC or PB, the presence of myopic consumers requires that the first stage premium under PC-PC and PB-PB be significantly higher in magnitude than the second stage discount. Further, contrary to conventional bundling wisdom, the optimal bundle price under PB-PC can actually be higher than the sum of the individual product prices.

We compare the appeal of the above four strategies relative to (a) traditional (static) mixed bundling (MB) and (b) generalized temporal mixed bundling (MB-MB). We find that MB is dominated by the temporal bundling strategies except when the market is comprised entirely of strategic consumers. MB-MB nests the other strategies we examine and is the first-best, but we find that the simpler variants of dynamic bundling preferred by retailers recover on average 98% of the profits that accrue from the cumbersome MB-MB strategy.

In the next section 2 we briefly review the bundling and inter-temporal pricing literature streams and position our work. In section 3 we present our model and state our propositions comparing PC-PC, PB-PB, PB-PC and PC-PB. Later in this section we compare the strategies against the two benchmarks. We consider three extensions to the mail model in section 4. We conclude in section 5 with a discussion of the study's implications, limitations and future directions.

2 Literature overview and positioning

Bundling and inter-temporal pricing are the twin pillars of our study. The bundling literature is rooted in the seminal studies of Adams and Yellen 1976, McAfee et al. 1989, and Schmalensee 1984. As PC and PB are nested within MB, it is generally the case that MB is the most profitable (McAfee et al. 1989). MB is thus seen as the be-all-and-end-all even though the simpler pure strategies are often seen in practice. PB is more (respectively, less) profitable than PC when the products have low (respectively, high) marginal costs in relation to the market's willingness to pay (Schmalensee 1984; Venkatesh and Kamakura 2003). There are several excellent surveys of the bundling literature that clarify the strategy's rationales and benefits (e.g., Guiltinan 1987; Rao et al. 2017; Stremersch and Tellis 2002; Venkatesh and Mahajan 2009).

The demand side explanation for the usefulness of PB is that the bundle results in a flattening of the demand curve due to reduced heterogeneity in willingness to pay for the bundle. This allows a single price to extract a greater fraction of the consumer surplus. With MB, second-degree price discrimination occurs such that consumers who prefer one product predominantly purchase the individual product and those who have intermediate or high valuations for both products buy the discounted bundle (Schmalensee 1984). This discriminatory power of MB dwindles when marginal costs are higher. Higher costs cause the feasible price of the bundle to be high, diminishing its role in the product line (Venkatesh and Kamakura 2003).

To our knowledge, the lone exception to the static analysis of bundling is the empirical study of DeGraba and Mohammed (1999) who consider sales of tickets for sports events (or concerts) that have limited seating over the playing season. They allow ticket sales in two stages: In particular, they find a benefit from having PB in the first stage, which appeals to customers with high valuation for both products, and PC in the second stage, for the remaining unsold seats. Interestingly, in their context, the bundle price in the first stage is more than the sum of the later component prices. However, the key driver of their results is the capacity constraint in event seating. This is not a factor in the present study. Further, we consider a broader strategy space.

Inter-temporal price discrimination occurs with the use of regular and markdown prices (e.g., Lazear 1986; Pashigian 1988). Pashigian (1988) argues that for clothing with seasonal demand, the market of potential customers would deplete and inventory would be left at the end of the selling season leading to the offer of markdown prices. In support of this conjecture, there is a correlation between the historical increase in percentage of fashion clothing and the occurrence of markdown pricing. Dynamic bundling has not been deeply examined, in contrast to dynamic pricing under inventory constraints, including yield management (see review by Elmaghraby and Keskinocak 2003).

Prasad et al. (2015, PVM) examine a temporal pricing tactic called "reserved product pricing" (RPP). The main idea of RPP is that the seller, who has information in the second period on who purchased its product in the first period, offers a promotion to those consumers only. In the present paper no such identification is made and prices are available to all consumers. In contrast to ours, temporal bundling strategies are not examined in PVM. (RPP is dominated by the temporal bundling strategies we propose).

The standard argument for increase in profitability resulting from charging two prices in stage 1 and stage 2 (instead of a single price in both stages) is easily shown under linear demand. Here, for demand $D(p) = 1 - p$, the optimal single price is at the value of $p^* = \frac{1+c}{2}$ (where c is the marginal cost) and it extracts 50% of the potential surplus. However, two prices at the optimal values of $p_1^* = \frac{1+c}{2} + \frac{1-c}{6}$ in stage 1 and $p_2^* = \frac{1+c}{2} - \frac{1-c}{6}$ in stage 2 extract 67% of the potential surplus. Thus, price skimming over two stages allows for a 33% increase in profits. This price discrimination method becomes difficult to apply if the market has a large number of strategic consumers who wait for the lower price (Coase 1972).

These studies and findings provide the backdrop to our study.

3 Model and analysis

We rely on a set of commonly invoked assumptions in bundling studies to characterize the product-market structure. As in Schmalensee (1984) and Venkatesh and Kamakura (2003), among others, the seller is a profit maximizing monopolist who sells two products X and Y. From a practical perspective, bundle pricing is complex enough that there are a number of examples of two-product bundles (e.g., on Amazon.com) even when larger bundles can be created.

The market is normalized to a unit square with uniform density which means that consumers' reservation prices R_X and R_Y for the two products X and Y are uniform and independent in the interval $[0, 1]$. This assumption is the de facto standard in the bundling literature (e.g., Carbajo, de Meza, and Seidmann 1990; Matutes and Regibeau 1992; Nalebuff 2004; Prasad et al. 2010; Seidmann 1991; Venkatesh and Kamakura 2003). The assumption not only makes the model analytically more tractable, but also helps position our findings relative to the literature. Consumers maximize their surplus by deciding which product to purchase, or to buy neither. Each consumer buys at most one unit and may wait until the second stage. The product is a durable, requiring no repurchase, and is consumed in the purchase period (e.g., books, movies, events etc. but not cars or tools).[2] The products have constant marginal costs $c \in [0, 1)$ where the half-open interval means that each product can always generate positive profit with appropriate pricing. For the bundle, we use the subscript B, and the marginal cost of the bundle is the sum of the component costs.

Following the conceptualization in Prasad et al. (2015) and the finding of Li et al. (2014), we assume that a proportion α of consumers are myopic and the remaining $1-\alpha$ are strategic.[3] While awareness of later stage discounts is one measure of consumer myopia or foresight, myopic consumers can also include those who are impatient and unwilling to wait for future discounts. Myopic consumers maximize their surplus at each stage independently, while strategic consumers decide based on their awareness of prices in both stages. The firm decides the pricing strategy at the outset and only strategic consumers have an awareness of the price path.

3.1 Analysis of temporal strategies

3.1.1 Analysis of PC-PC

Recall that reservation prices are independently and uniformly distributed. At price P in the first stage, the demand for each product from myopic consumers is $(1-P)\alpha$. Let δ represent the second stage discount so that the second stage price is $P-\delta$. Demand from myopic consumers in the second stage is $\alpha\delta$. From strategic consumers the demand is $(1-\alpha)(1-P+\delta)$ since they pay $P-\delta$ in the second stage. Thus, the total first stage profit to the seller is $2(P-c)(1-P)\alpha$ and second stage profit is $2(P-\delta-c)[\alpha\delta+(1-\alpha)(1-P+\delta)]$. Maximizing the profits with respect to the price and discount gives the following result.

Proposition 1 With PC-PC, i.e., pure components selling in each stage, and given a proportion α of consumers are myopic and the remaining are strategic, optimal prices are $P_1 = \frac{1+c}{2} + \frac{(2-\alpha)(1-c)}{2(4-\alpha)}$ in the first stage and $P_2 = \frac{1+c}{2} - \frac{\alpha(1-c)}{2(4-\alpha)}$ in the second stage. Total profit is $\frac{2(1-c)^2}{4-\alpha}$.

The pricing results in the Proposition are illustrated graphically in Fig. 1 as a function of marginal cost c and three levels of the mix of consumers, corresponding to all myopic ($\alpha = 1$), half strategic ($\alpha = 0.5$) or all strategic ($\alpha = 0$) consumers. Figure 1(a) contains the plot of the optimal prices P_1 and P_2 in each stage. The ratio of these prices is graphed in Fig. 1(b).

[2] Our conceptualization echoes with Bulow (1982, p.316) who notes that durable goods can be broadly defined and writes, "A ticket to a first-run movie has the durable quality that once someone has seen the film that person is unlikely to buy a second ticket. … Similarly, the expectation that a viewing will be available at a cheaper price on a second-run affects demand for the first-run film. (The market for hardcover and paperbacks works this way also.)"

[3] The empirical analysis by Li et al. (2014) suggests that less than 20% of consumers are strategic in the air-travel industry.

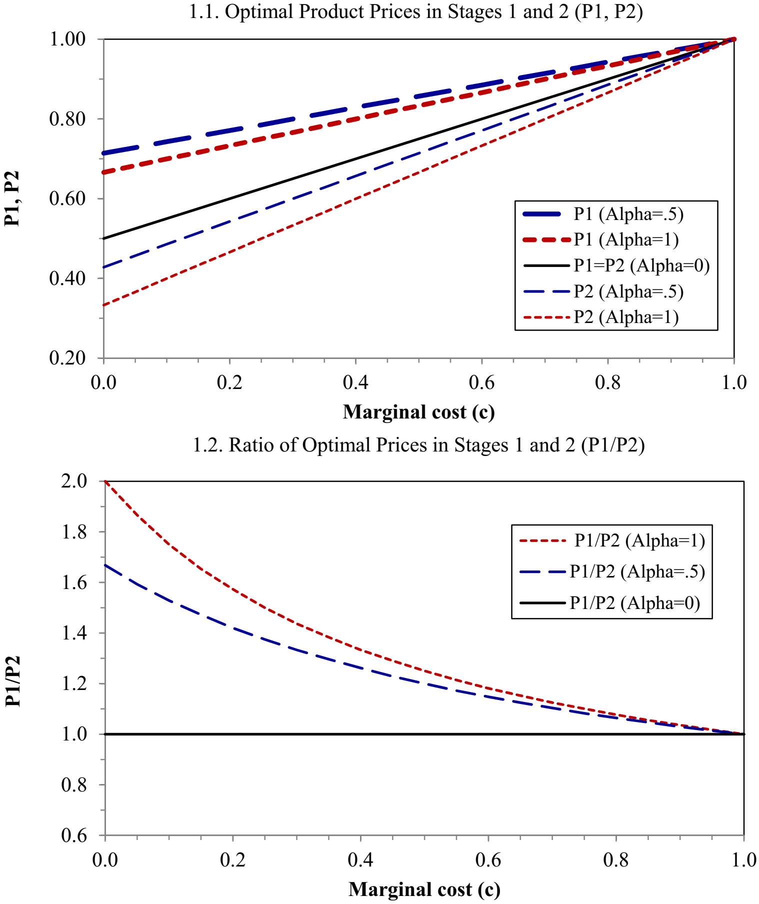


Fig. 1 Optimal prices under PC – PC. (Alpha (α) represents the proportion of myopic consumers in the market)

The intuition behind the proposition in three snapshots:

- Case 1 – Strategic consumers only (α=0): Here the utility of temporal price discrimination is lost. The customers will patiently wait for the lower price in the second period. As such, the seller can do no better than charge the static PC price for each component.
- Case 2 – Myopic consumers only (α=1): Conditions are ripe for temporal price discrimination. Accordingly, the first stage prices are significantly higher and the second stage prices are lower that the static PC price. Doing so balances the ability to skim the high valued customers in the first stage while ensuring adequate clearing of the untapped market in the second stage.
- Case 3 – Equal proportions of strategic and myopic consumers (α=0.5): The constraint here is on the second period prices. The seller has to balance the need not to leave a large surplus for the strategic consumers who have delayed their purchase, while keeping the price low enough to tap a sizable pool of untapped myopic consumers. These countervailing factors cause the second stage price for

α=0.5 to be higher than that for α=1 (preceding bullet). This higher second stage price relative to Case 2 pushes up the optimal first stage price to achieve optimal temporal price discrimination among high and low valued myopic consumers. This yields the plots in Fig. 1.

3.1.2 Analysis of PB-PB

PB-PB implements pure bundling in each stage. The products are not sold individually. The first and second stage bundle prices are denoted P_{B1} and P_{B2}, respectively, and $P_{B1} > P_{B2}$ due to price skimming. Three cases are possible, with distinct demand expressions (see Fig. 2). The cases are, Case (a): $P_{B1} > P_{B2} \geq 1$, Case (b): $P_{B1} \geq 1 \geq P_{B2}$, and Case (c): $1 \geq P_{B1} > P_{B2}$. The derivations of the demand and profit expressions in each case are lengthy and hence relegated to Appendix 1.

We find that Case (a) applies for moderate to high marginal costs, and Case (b) for low marginal costs. Case (c) is never part of the optimal solution. The method of proof follows Prasad et al. (2010) and is to examine the intermediate Case (b) first and derive phase boundaries between this and the other two cases. The boundary between Cases (b) and (c) exists at out-of-range marginal costs, indicating that Case (c) is not optimal. The boundary between Cases (a) and (b) is given by $c = 0.25 + \alpha/9$.

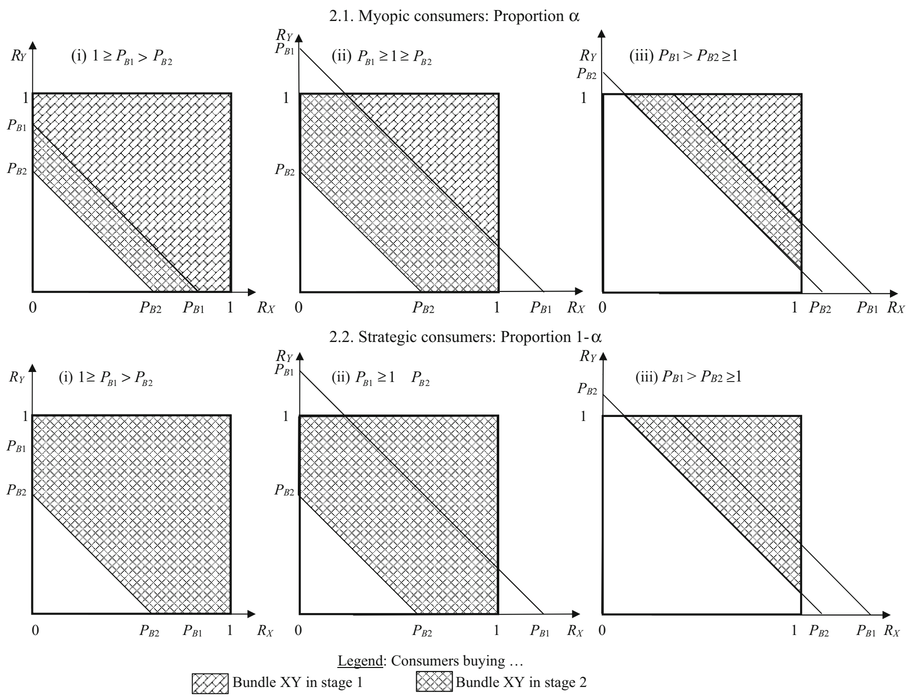


Fig. 2 Schematic representation of prices and penetration under PB – PB. (Pure bundling is implemented temporally over two stages)

Proposition 2 Define $c^* \equiv 0.25 + \alpha/9$. For PB-PB, i.e., pure bundling in each stage:

- If $c \geq c^*$, optimal prices are $P_{B1} = \frac{2(1+P_{B2})}{3}$ and $P_{B2} = \frac{18(1+2c)-8\alpha}{27-4\alpha}$. The solution falls under Case (a): $P_{B1} > P_{B2} \geq 1$, with the demand and profit as derived for that case.
- If $c \leq c^*$, optimal prices are $P_{B1} = \frac{2(1+P_{B2})}{3}$ and $P_{B2} = \frac{18c+8\alpha+\sqrt{(18c+8\alpha)^2+(27+4\alpha)(18-16\alpha)}}{27+4\alpha}$. The solution falls under Case (b): $P_{B1} \geq 1 \geq P_{B2}$, with the demand and profit as derived for that case.

Figure 3 contains the plots of optimal prices as a function of marginal cost c and three levels of α, namely 0, 0.5 and 1. Figure 3(a) contains the plot of the optimal prices P_{B1} and P_{B2} in each stage. The ratio of these prices is in Fig. 3(b).

Note that when all consumers are strategic, only the discounted second stage price P_{B2} is relevant, and the PB-PB case reduces to traditional (static) PB, which is also verifiable by comparing the second stage price to the optimal PB price and noting that they are identical. The introduction of some myopic consumers introduces a benefit from price skimming and hence, in the same manner that PC-PC prices in the first and second stage lie above and below the optimal PC price, the PB-PB prices in the first and second stages also sandwich the optimal PB price.

Pricing distinctions between PC-PC and PB-PB arise because bundling reduces the heterogeneity in reservation prices and there is greater density of consumers around the mean bundle reservation price. Consequently, the ratio of the first stage to second stage price is weakly lower under PB-PB relative to PC-PC.[4]

3.1.3 Analysis of PC-PB

PC in stage 1 followed by PB in stage 2 represents a type of (temporal) mixed bundling, because the products and bundle are both available for sale, albeit in different stages. Prices and demand are shown schematically in Fig. 4.

The analysis of the strategic consumer segment (Fig. 4(b)) when the product prices are lower than the bundle price is the same as for MB and if the product price is higher than the bundle price, the derivation is the same as for PB. For the segment of myopic consumers, when $P_B \geq P$, the product and bundle demands are shown in Fig. 4(a) (panel ii or iii). However, when $P_B \leq P$ the derivations do not parallel the static analysis. Here myopic consumers with higher willingness to pay than P purchase the components in the first stage, but the bundle is offered at a significantly low price in the second stage. Some customers who purchased a single product in a first stage may actually find it advantageous to purchase the bundle so as to obtain the other product that was let go in the first stage. While this will cause a subset of consumers to obtain two units of one product, we assume free disposal without arbitrage.

[4] Both the first and second stage bundle prices show a non-linear increase in marginal costs with a kink at $c = c^* = 0.25 + \alpha/9$. The kink per se is not a major result and is consistent with extant findings under static pure bundling PB (e.g., Venkatesh and Kamakura 2003, p. 222). Further, there is no discontinuity in the trajectory of optimal prices and in the profit function. The non-linear rise in optimal bundle prices as a function of marginal cost is attributable to the unimodal distribution of the reservation prices for the bundle (a point that resonates with Schmalensee's 1984 work with the normal distribution).

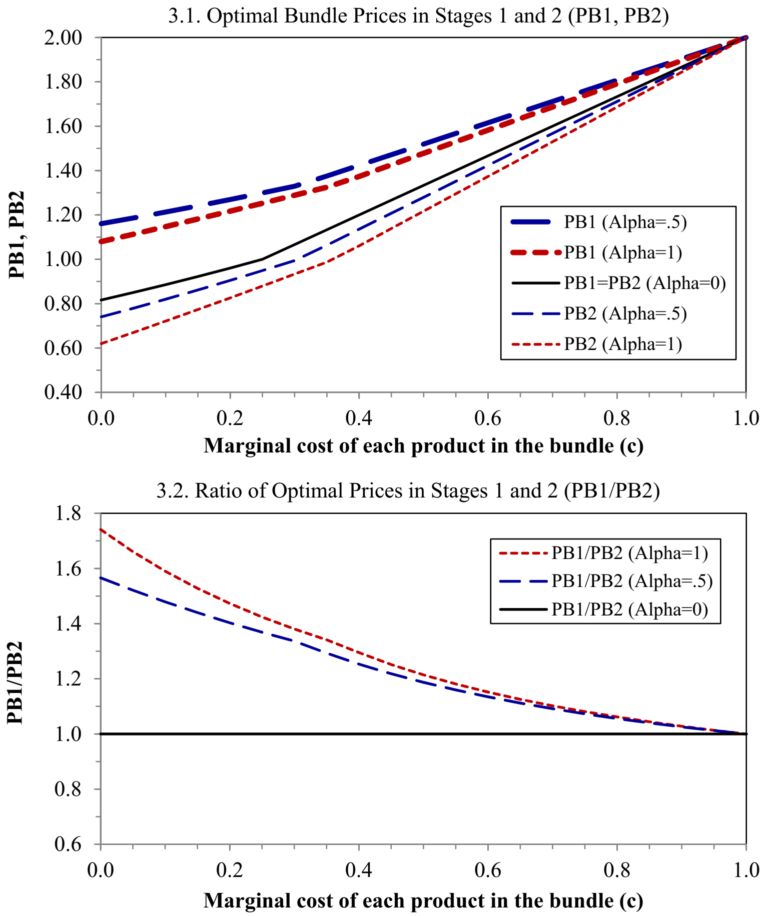


Fig. 3 Optimal prices under PB–PB. (Alpha (α) represents the proportion of myopic consumers in the market)

An explicit expression for the optimal prices cannot be obtained because the problem is akin to traditional MB which also requires a numerical solution. Although approximations may be useful for implementing these solutions, they are less useful when comparing different strategies because the order of profits might be determined by the approximation. Thus, we rely on high precision numerical calculations. Notably, however, the boundary between the cases $P_B \geq P$ and $P_B \leq P$ can still be expressed analytically. Doing so yields the next proposition which refers to two profit expressions (1) and (2) given in Appendix 1.

Proposition 3 With PC-PB, i.e., pure components in the first stage and pure bundling in the second stage, the optimal solution has the following properties:

- If the marginal cost is low[5] then the solution has $P_B \leq P$ and profit is given by (2) in Appendix 1.

[5] Specifically, either $c \leq 0.108$ or [$c \in (0.108, 0.261)$ and $\alpha \geq \frac{7c^2-10c+1}{14c^2-8c}$].

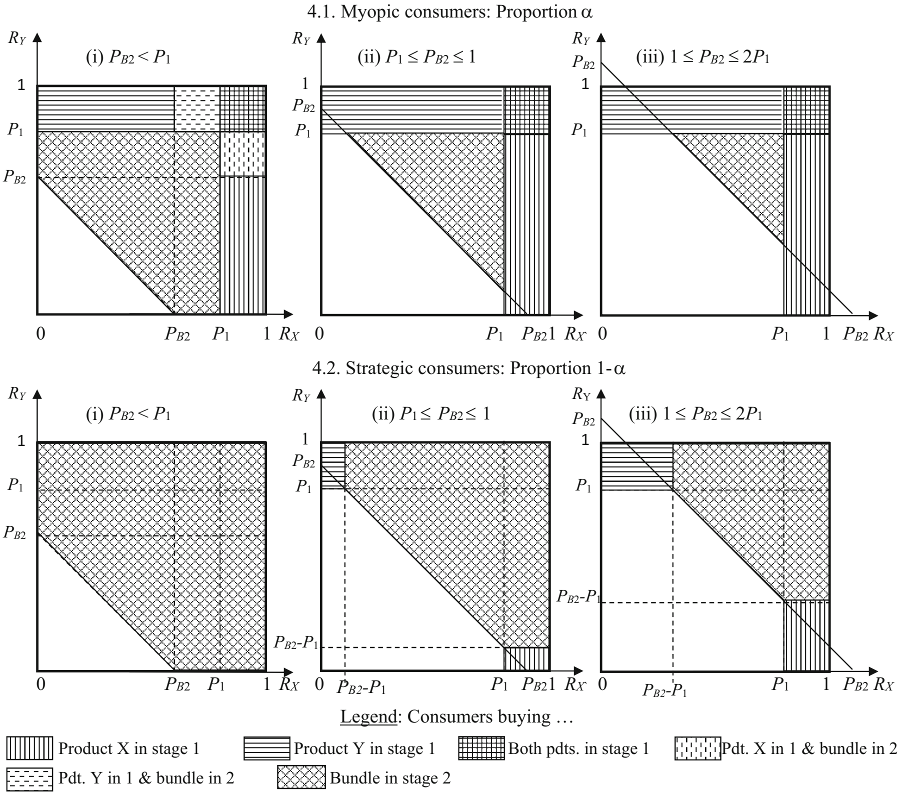


Fig. 4 Schematic representation of prices and demand under PC–PB

- Else, the solution has $P_B \geq P$ and profit is given by (1) in Appendix 1.

Excluding a small range of costs, the relationship between the second-stage bundle price and first-stage component price is independent of the consumer mix, because for $c < 0.108$ it is always optimal that $P_B < P$ and for $c > 0.261$ it is always optimal that $P_B > P$. Between these values, we need to also consider the proportion of myopic and strategic consumers in determining which of these two relationships occurs.[6] Pricing implications are discussed with those of PB-PC.

3.1.4 Analysis of PB-PC

As with PC-PB, PB-PC is a type of (temporal) mixed bundling because the products and bundle are available, albeit in different stages. Thus, both these strategies allow for some amount of second degree price discrimination along two dimensions of product preferences and customer type. Figure 5 represents this case.

[6] The function $(7c^2-10c + 1)/(14c^2-8c)$ in the interval of c = 0.108 to 0.261, is a slightly curved line from 0 to 1. Thus at some point between c = 0.108 to 0.261 it must intersect with the value of α (the proportion of myopic consumers) and the transition between cases occurs.

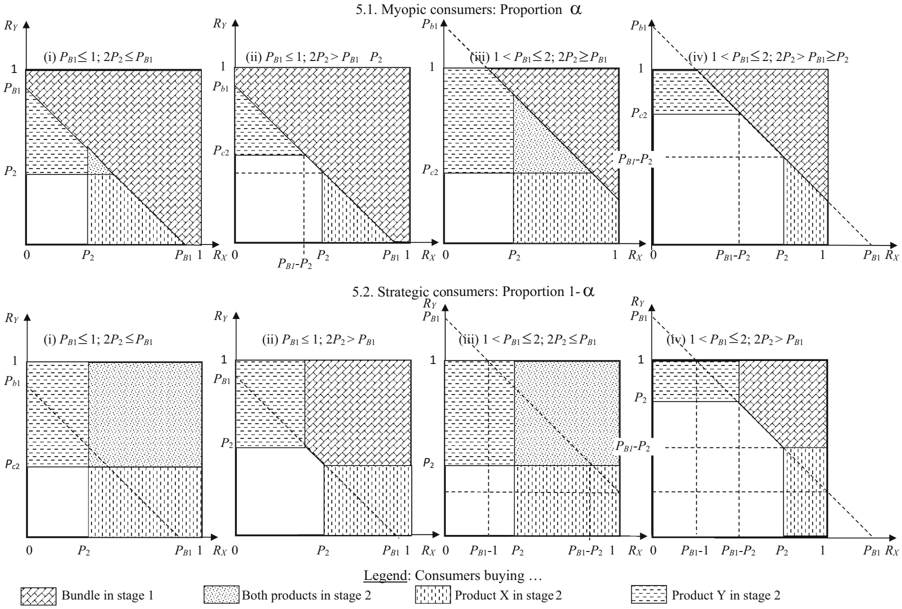


Fig. 5 Schematic representation of prices and demand under PB–PC

From strategic customers the same demand expressions apply as from PC-PB or MB as long as the same set of prices for the components and the bundle emerge in the profit maximizing solution. A new set of demand expressions arise from myopic consumers. Four scenarios arise for each of myopic and strategic segments (Fig. 5(a) and (b)).[7]

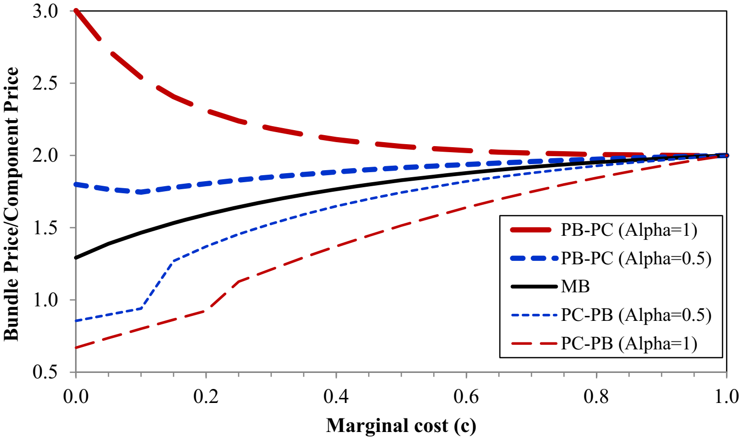
Overall, the expressions for demand and profit, like those from traditional MB, are higher order polynomials and, consequently, the case of PB-PC is also analytically intractable. Insights are based on numerical solution search of the necessary conditions for optimal prices once the demand and profit expressions are derived.

3.1.5 Pricing implications: PC-PB vs. PB-PC

To clarify the pricing inferences under PC-PB and PB-PC we have included Fig. 6 graphing the ratios of the bundle price to the component product price for same three levels of α as before. Recall that $\alpha = 0$ where all consumers are strategic has the same prices as under traditional MB, our baseline.

Under traditional MB the individual products and the bundle are offered at a premium relative to PC and PB respectively. Thus, while we associate the bundle in MB as the discounted offering, the discount is only relative to the sum of the prices of

[7] There are four scenarios because in Fig. 5(b), panels (i) and (iii) are analytically equivalent, as are (ii) and (iv). To illustrate, we expand one case: From Fig. 5(a) (i) where $2P_2 \le P_{B1} \le 1$, a total of $\alpha(1 - P_{B1})^2/2$ myopic consumers buy the first stage bundle, providing a profit of $\alpha(P_{B1} - 2c)(1 - P_{B1})^2/2$. Then, in the second stage, $\alpha(P_{B1} - P_2)^2$ myopic consumers buy components, yielding a profit of $\alpha(P_2 - c)(P_{B1} - P_2)^2$. For Fig. 5(b) (i) for strategic consumers, the demand occurs only in the second stage and is $2(1 - \alpha)(1 - P_2)$ and their profit contribution is $2(1 - \alpha)(1 - P_2)(P_2 - c)$. Thus the total profit for the case $2P_2 \le P_{B1} \le 1$ is given by $\alpha(P_{B1} - 2c)(1 - P_{B1})^2/2 + 2(1 - \alpha)(1 - P_2)(P_2 - c)$. Similar expressions for the remaining three cases can be derived.



Legend:

- PB-PC (Alpha = 1) and PB-PC (Alpha = 0.5) represents the cases when the bundle alone is offered in the first stage and the individual products alone in the second stage.
- PC-PB (Alpha = 1) and PC-PB (Alpha = 0.5) represents the cases when the individual products alone are offered in the first stage and the bundle alone in the second stage.
- MB represents traditional mixed bundling. The results are equivalent to the case when Alpha = 0 (i.e., all consumers are strategic.)

Fig. 6 Ratio of bundle price to component price under PB-PC, PC-PB and MB

the individual products under MB and not relative to the bundle price under PB. As seen in Fig. 6, the ratio for MB is increasing in cost, which merely underscores the burden on the bundle from the cost side.

With both PC-PB and PB-PC, the ability to skim and the desirability to do so make the first stage prices of the individual products (under PC-PB) or the bundle (under PB-PC) higher than those under MB. This explains why the price ratio graphs of PC-PB (or PB-PC) are above (or below) that of MB.

It is an interesting facet of PB-PC that whereas the normal price skimming argument (for example, as seen under PC-PC) is that price should decline over time, we find that under PB-PC the price of buying both products (in the first stage as a bundle or in the second stage as components) can actually increase over time. This occurs when there are a significant proportion of strategic consumers. For example, it may be observed in Fig. 6 for the case when half of consumers are strategic (PB-PC, alpha =0.5; ratio of bundle to component price is below 2) but not for the case where there are only myopic consumers (PB-PC, alpha = 1; ratio of bundle to component price is above 2). And the reason is that it helps manage the expectations that otherwise might drive the bundling price lower in the first period if customers anticipate they can form their own bundle at a lower price next period.

3.2 Comparison of strategies

We address here a principal research question of the study: Which of the four temporal bundling strategies – PC-PC, PB-PB, PC-PB, or PB-PC – is the most profitable strategy?

The related phase diagram is in Fig. 7. Figure 7 is obtained by comparing the profits across strategies at each of 441 distinct nodes, corresponding to 21 equally spaced levels of c and α in the interval [0, 1]. We summarize the observations.

***Result* 1**: *Among PC-PC, PB-PB, PB-PC and PC-PB:*

(i) *PB-PB is profit maximizing when marginal costs are low and the proportion of myopic consumers is moderate to high ($c < 0.15$, $\alpha > 0.15$).*
(ii) *PC-PC is profit maximizing when the marginal costs and the proportion of myopic consumers are moderate to high ($c > 0.15$, $\alpha > 0.3$).*
(iii) *PB-PC is profit maximizing when marginal costs are low to moderate and the proportion of strategic consumers is high ($0.1 < c < 0.7$, $0.7 < 1\text{-}\alpha < 1$).*
(iv) *PC-PB is weakly dominated.*

Between PC-PC and PB-PB, the relative attractiveness interestingly resembles that between PC and PB in the traditional bundling landscape. Specifically, the bundle is relatively unburdened on the cost side when marginal costs are lower. The power of the bundle to force consumers to buy both products and the ability to skim under PB-PB make this strategy prevail over PC-PC. In the second stage, the low levels of marginal cost ensure that even the discounted bundle is able to yield a positive margin. Conversely, at higher levels of marginal cost, the floor price of the second stage bundle is pushed high enough that its ability to clear a sufficiently sizable segment of the market is hampered. PC-PC reigns over a large domain. (Note that the thresholds for marginal cost and proportion of myopic consumers stated in the result are approximate. See Fig. 7 for the boundaries demarcating profit maximizing strategies.)

The advantages of PB-PB and PC-PC are diminished when the proportion of strategic consumers is large. Here PB-PC, which is akin to mixed bundling implemented in two stages, emerges as the profit maximizing strategy, while its mirror strategy PC-PB is dominated. Under PC-PB, myopic consumers have the pick of their most

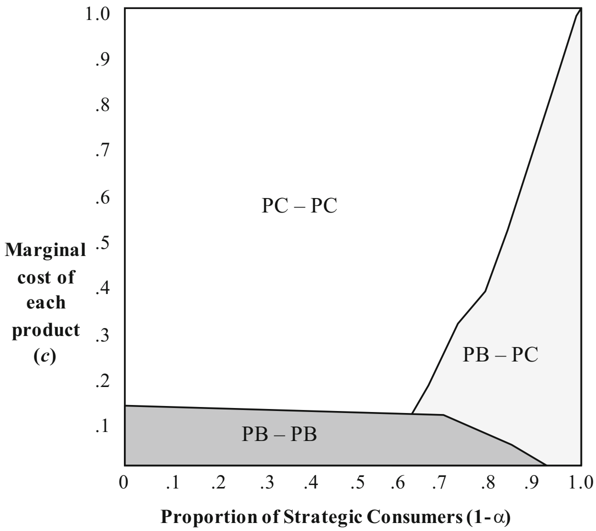


Fig. 7 Profit maximizing strategies among PC-PC, PB-PB, PB-PC, PC-PB

preferred product in the first stage. Consequently, the upper limit of the reservation price distribution of the bundle is brought down. This, in effect, reduces the market's valuation in relation to the marginal cost of the bundle. The ineffectiveness of the bundle under high marginal cost is established in the literature (see Schmalensee 1984, for example). By contrast, PB-PC prevails by reversing the above noted problem of PC-PB. Further, while the extant literature emphasizes the discounted bundle and premium-priced components, PB-PC succeeds from offering a premium-priced bundle aimed at the high-valued myopic consumers in the first stage and the discounted individual products targeted at the untapped market in the second stage. Where PB-PC is the most profitable for the seller, the bundle price in the initial stage typically *exceeds* the sum total of the component prices in the second stage. Of course, this would work only when the proportion of myopic consumers is non-trivial. In the limit, when all consumers are strategic, the outcome under PB-PC will converge to that under a familiar strategy. This is discussed next.

3.3 Benchmarking against MB and MB-MB

Our two benchmarks are MB (traditional mixed bundling) and MB-MB (mixed bundling offered at each of two stages). MB is a relevant benchmark as the vast literature on static bundling has underscored that it is the dominant form of traditional bundling. MB-MB is relevant as it is the most general albeit a cumbersome form of temporal bundling.

3.3.1 Comparison with MB

The domains over which PC-PC, PB-PB and PB-PC are profit maximizing as noted in Result 1 and Fig. 7 are are unaffected by the inclusion of MB, with the exception that MB is not dominated in the corner case where the market is comprised entirely of strategic consumers.

Our main proposal to add a temporal dimension to traditional bundling leads to a significant increase in seller's profits. Across the scenarios we have examined (defined by various levels of c and α), the profit maximizing form of temporal bundling – *excluding MB-MB* – enhances seller's profits by 11.9% (increases ranging 0% to 32.6%). As seen in Fig. 8, the bigger gains are when a larger proportion of consumers are myopic.

The weakness of MB relative to temporal bundling strategies at higher marginal cost levels is because the appeal of the bundle diminishes and the individual products have to deliver much of the sales and profits. Relative to this, when a sizable fraction of consumers is not strategic, PC-PC can do better by its ability to skim the myopic segment of the market. At lower levels of marginal cost, PB-PB is more effective because (a) the bundle in the first stage skims the sub-segment of myopic consumers who value both products and (b) in the second stage, faced with the low marginal cost advantage, the discounted bundle can not only clear consumers who primarily value one of the two products, but also force the sale of the other product. These advantages of PC-PC and PB-PB relative to MB are greater when the proportion of myopic consumers α is larger.

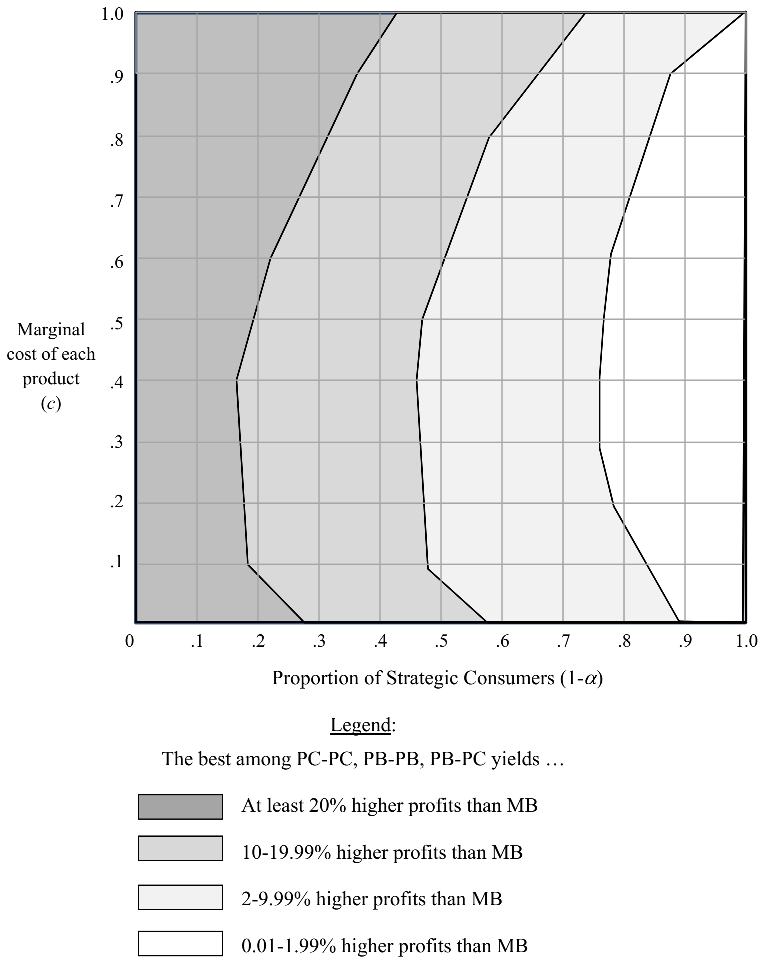


Fig. 8 Profits from temporal bundling vs. traditional mixed bundling. (The best among PC-PC, PB-PB and PB-PC vs. MB)

On the flip side, when the proportion of strategic consumers increases but is below 1, PB-PC combines the advantages of PC-PC and PB-PB, and yields higher profits than MB. Only in a market of strategic consumers is MB able to retain its hold as the profit maximizing strategy. Profits from PB-PC converge to those under MB when all consumers are strategic (α = 0).

3.3.2 Comparison with MB-MB

On offering MB at each stage as MB-MB, some intuitions can be confirmed: (i) We find profits from MB-MB are weakly greater than those from other strategies we have examined. (ii) While MB is analytically intractable without restrictive assumptions, MB-MB is even more so. (Derivations for MB-MB are given in the Appendix.)

While the best among PC-PC, PB-PB and PB-PC will yield a 11.9% increase in profits over MB, we find that MB-MB can raise the seller's profits by a further 1.8%, with a range of [0, 4.7%]. These findings show that the principal gains accrue from PC-PC, PB-PB or PB-PC. Figure 9 maps the regions where the inclusion of MB-MB most augments the seller's profits.

As seen in Fig. 9, the boost in profits from MB-MB is most pronounced when marginal costs are low, but not zero, and the market had a good mix of both myopic and strategic consumers. Key reasons: When marginal costs are zero, the power of pure bundling is high and MB-MB essentially converges to PB-PB. When consumers are entirely strategic, temporal bundling is not meaningful and MB-MB profits converge to those under MB. When consumers are mostly myopic, the targeting ability of MB-MB in which myopic (or strategic) consumers are the principal target in stage 1 (or 2) is rendered mostly redundant. That said, MB-MB still yields a modest advantage here by discriminating among myopic consumers with high vs. low reservation prices. When marginal costs get higher, the bundle in MB is less effective and the incremental profits from MB-MB fall in the 0–1.99% band. By contrast, in the darkly shaded "eye" of the figure, these forces that diminish the advantage of MB-MB are themselves weaker. The gain from MB-MB relative to the next best strategy approaches 5%.

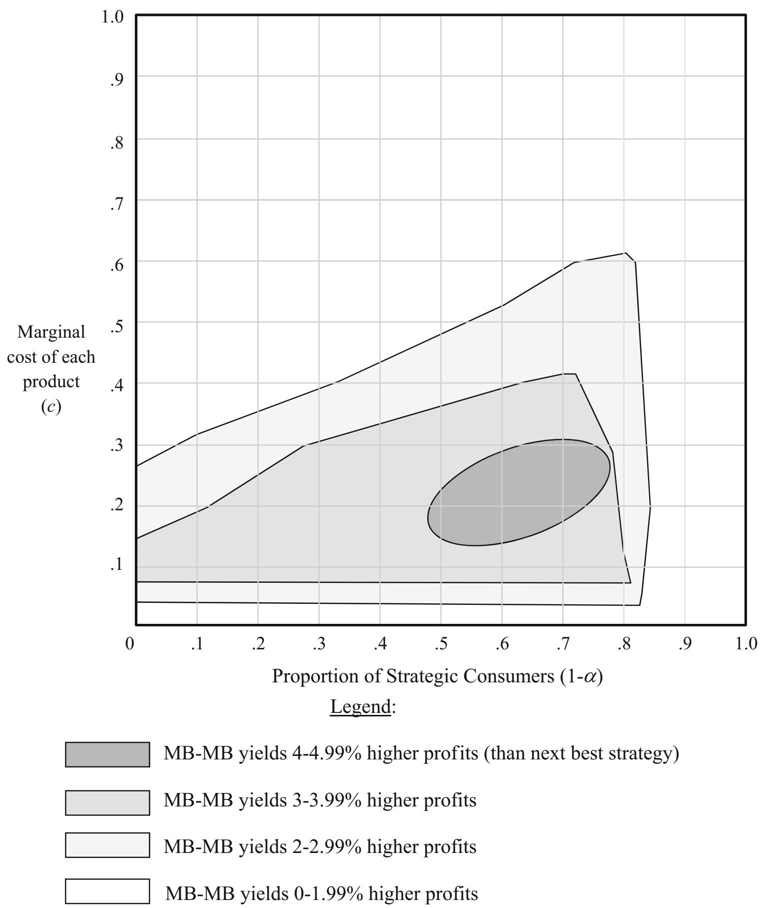


Fig. 9 Profits from MB-MB vs. next best strategy

Some observational evidence that the difference between MB-MB and PB-PC (as well as PB-PB and PC-PC) in terms of profitability may be relatively small is that complicated structures are not so widespread in practice. (Recall the motivating examples under "Introduction.") We see a reinforcing argument for this practice in Chu et al. (2011, p.263) who note "In reality firms almost never implement complex pricing structures. Indeed, the reverse seems more common: firms often employ remarkably few prices. [This is because] simple pricing strategies are often nearly optimal".[8] Similarly, Bakos and Brynjolfsson (1999) find that pure bundling approximates the optimal strategy as the number of products increases to infinity, assuming IID reservation prices and zero marginal costs. This point on the effectiveness of simpler strategies resonates with the findings of McAfee (2002) and Wilson (1993).[9] To conclude, while in theory we find that a PB-PC strategy can approach MB (see Fig. 8) and MB-MB in profitability, it would be worthwhile to explore the question empirically in future.

4 Extensions to the main model

The main model makes at least three salient assumptions: (i) Reservation prices for the two products are independent across consumers and uniformly distributed; (ii) The seller under inter-temporal pricing commits to second stage prices; (iii) The first and second stage purchases occur close enough that discounting is not significant. This section examines the implications of relaxing these assumptions.

4.1 Relaxing assumptions of uniformly and independently distributed reservation prices

As already noted, two widely invoked assumptions in analytical models of bundling are that consumers' reservation prices for two products are uniformly and independently distributed. While our paper follows these precedents, we *conjecture* how the results might change when the assumptions does not hold. A full-scale investigation is, unfortunately, not tractable.

Suppose reservation prices for the two products follow a bivariate normal distribution (as in Schmalensee 1984). Let the correlation coefficient ρ be 0 to begin with. All else being the same, given two supports equidistant from the mean, the reservation prices are less heterogeneous under the normal distribution relative to the uniform. This would lessen the power of inter-temporal pricing under the normal vis-à-vis the uniform. This is because skimming among myopic consumers yields a smaller incremental benefit when reservation prices are concentrated around a mean value than when they are more heterogeneously distributed. The domain of traditional MB in Fig.

[8] Chu et al. (2011) show via numerical and empirical work that the profitability of mixed bundle pricing for a large number of products can be approximated by pricing the bundles based solely on the number of items in the bundle."

[9] In the parallel universe of static bundling, there are contexts where MB is more common than the simpler PB or PC, such as sales of video games and video game consoles. In an empirical study, Derdenger and Kumar (2013) note that the profitability of MB could exceed that of PB by over 30%. However, the utility structure is different from the present study as video game buyers might want to buy new games but not necessarily new consoles on a purchase, and complementarity is a factor. For the music industry where complementarity is less of a factor, Elberse (2010) finds that MB reduces revenues and profits compared to PB.

7 is likely to be larger. The reduced heterogeneity further means that within MB the role of the bundle is likely to be more significant than that of the individual products because a larger proportion of consumers now value the products about equally.

Now consider a ρ value of +1 (i.e., perfect positive correlation in reservation prices). Here consumers who have high (or low) reservation prices for one product are also those who have high (or low) reservation prices for the other product. The advantage of MB vis-à-vis PB and PC arguably disappears here as there is no reduction in buyer diversity from bundling (Schmalensee 1984, p. S220).[10] However, this scenario is suited for price skimming if a proportion of the market is myopic. PC-PC and PB-PB are expected to be equally attractive to the seller. By contrast, when $\rho = -1$ (i.e., perfect negative correlation in reservation prices) the consumers are homogeneous in their reservation prices for the bundle even though they value the individual products differently. PB can extract the entire consumer surplus in the first stage itself, and inter-temporal pricing such as with PB-PB is redundant.

4.2 Relaxing the assumption of price commitment

Our main model assumes that the seller commits to second period prices in an open loop manner. Does time inconsistency arise without such commitment? We find that our earlier solutions for PC-PC and PB-PB are *unchanged* by whether there is commitment about second period prices or not. Relaxing the commitment assumption under PC-PB and PB-PC is analytically intractable and here our verification is numerical. This analysis helps reinforce the earlier results of the paper. The derivations are available from the authors.

Our results are robust given the way we have modeled strategic consumers. It is only for these consumers that commitment – or lack of it – makes an impact, because myopic consumers consider only the present period prices and do not wait in anticipation of future price drop. Our strategic consumers form an expectation of future prices and wait to purchase in anticipation of a price drop. Thus our main model already treats strategic consumers as being aware of the committed future price. They behave in accordance with the Coase conjecture. In other words, if the market consisted only of strategic consumers, no price skimming would be possible.

While lower prices in the second period are consistent with waiting behavior by strategic consumers in the first period, an additional point to check is whether the seller could raise prices in the second period when faced with such behavior by strategic customers. We find that prices in fact should decline.

4.3 Allowing for temporal discounting among the strategic consumers and the seller

Many instances of temporal bundling such as retail markdowns of fashion goods are implemented within weeks, if not days, and so the discounting effect is unlikely to be significant. The main model is applicable to such settings.

[10] With perfect positive correlation, PC and PB will yield similar profits as MB, a point noted by Schmalensee (1984, p. S227).

Nevertheless we have set up and analyzed a model in which the seller and the strategic consumers find waiting to be costly (i.e., discount factor δ <1). Given the analytical complexity, we only consider a market of strategic consumers. Such a market results in all customers waiting. The present analysis introduces a discount factor to allow sales in both stages as a version of the previous strategic-myopic dichotomy. The procedure is to identify indifferent customers such that higher valuation customers buy in the first stage and infra-marginal customers buy in the second stage or do not purchase. As in Besanko and Winston (1990), it is assumed that capital markets are perfect, implying that the firm and consumers share a common discount factor, $\phi \in (0, 1)$. The derivations are available from the authors.

Three salient findings:

- Under both PC-PC and PB-PB, having only strategic consumers makes it unappealing for the seller to pursue temporal price discrimination.
- Under PC-PB, the optimal solution is that corresponding to static PC, whereas with PB-PC the optimal solution corresponds to the static PB. Because of the drop in consumers' valuation from discounting, the temporally deferred strategy loses its potency.
- As discounting makes the market's overall valuation drop, the advantage of temporal bundling is diminished relative to traditional (static) MB. While temporal bundling is dominated by MB when only strategic consumers are involved, we speculate that the domain of MB will expand even if a subset of consumers is myopic. While myopic consumers add to the power of temporal bundling, the discounting effect among strategic consumers weakens the argument for price skimming, yielding a net plus to static MB.

5 Discussion

In a departure from the traditional bundling literature that has focused on static bundling and on the ability of mixed bundling to achieve second degree price discrimination, the current study examines the relative attractiveness of alternative temporal bundling strategies to the seller. Pricing implications are also delineated. What follows is a brief discussion of the study's theoretical and managerial implications and select future research directions.

PB-PC, a dynamic form of mixed bundling when seen across two stages, is the best when the market is comprised largely, but not entirely, of strategic consumers. This advantage of PB-PC increases when the marginal costs are lower. The intuition is that PB is more potent under lower marginal costs. The individual products in the second stage are able to target the desirable pockets not tapped in the first stage. When a larger proportion of the market is myopic, temporal pure bundling (PB-PB) is the most profitable when marginal costs are low; temporal pure components (PC-PC) maximizes profits otherwise.

The results are benchmarked against two strategies. First, we show that MB, that is static mixed bundling, is dominated by the temporal bundling strategies except when the market is entirely comprised of strategic consumers. In this extreme case, PB-PC converges to MB. Second, while we find that our alternative strategies of PC-PC, PB-PB and PB-PC are weakly dominated by the general – albeit more cumbersome – MB-MB strategy, this most general strategy yields only a 1.8% profit advantage on average. This might explain why none of our motivating examples from the real world were a form of MB-MB.

Two key pricing results: (a) Relative to static PC or PB, the presence of myopic consumers warrants that the first stage premium under PC-PC and PB-PB be significantly higher in magnitude than the second stage discount; (b) Contrary to conventional wisdom, the bundle price under PB-PC can actually be higher than the sum of the individual product prices.

We discuss managerial implications by revisiting our original motivating examples (section 1). Absent information on the products' marginal costs and the proportion of myopic consumers, we can speculate how our results can be applied. A seller such as Amazon.com arguably incurs lower marginal costs selling eBooks (vs. hardcopies) or downloadable video or music (vs. duplicating and mailing DVD's or CD's). Sequels of blockbuster movies or books (say, the Harry Potter series) arguably have a higher proportion of impatient or myopic consumers than new, less known titles. For classical books or movies, consumers are less likely to be myopic. With these premises, we might make the following recommendations to the sellers:

- In example 1, BluRay DVDs have higher marginal costs relative to stream-ins/ downloads. As both "Argo" and "Life of Pi" won academy awards (e.g., for best picture and best direction respectively), a sizable segment of the market may have eagerly sought each DVD (i.e., significant share of myopic consumers). This confluence of cost and market composition would justify the use of PC-PC.
- Example 2 involves a bundle – a music album in the form of a CD – that is offered by the seller following PB-PB. The album of a popular band such as AC/DC would arguably be sought by a significant segment of myopic consumers. The seller's choice of PB-PB would have been more defensible if the album was sold as an electronic download (lower marginal cost in relation to willingness to pay). If the marginal costs are not very low, the band would need to give PC-PC careful consideration.
- Examples 3 and 4 represent parallels – BluRay DVDs of popular trilogies, with arguably similar demand and marginal cost structures. However, the strategies followed by Walmart are exact opposites: PB-PC for "Toy Story" and PC-PB for "The Lord of the Rings." As per our analysis, PC-PB is weakly dominated whereas PB-PC is profit maximizing when the proportion of strategic consumers is rather large. The studio's approach with Lord of the Rings could be due to rationales beyond the scope of the model, e.g., augmentation – possibly fans repurchase a boxed set that may also include extra footage.

In today's digital age, more products have lower marginal costs compared to their manufactured counterparts. Thus, there should be a greater role for bundling in the initial stages, whether PB-PC or PB-PB (Fig. 7). Yet marginal costs can be higher due to licensing fees or mark-up tied to double marginalization (Bhargava 2012). Such scenarios should favor PC-PC.

In conclusion, we see this study as arguably the first to formally examine when and how the appeal of bundling can be augmented by implementing it temporally. In terms of future research, it would be interesting to relax our assumption of the seller's monopoly. Competition will exert a downward pressure on prices and could potentially dent the ability to price discriminate. Another context for incorporating the influence of strategic actors is when the different products are from different manufacturers who may partner in creating bundles, cobranded products, or co-promote their offerings (see Yalcin et al. 2013). How effective would temporal bundling be under such coopetition? How would the vertical structure of the industry influence the choice of strategy? The discounting in valuations under temporal bundling is another avenue for further exploration. The related discussion in section 4.3 assumes the discount factor is the same for both strategic consumers and the seller. But what if the discount factors are different? Further, what if we have myopic consumers who discount? We have not explored this at it would complicate the analysis analytically and numerically. The challenge is that there are always many ‘demand geometries’ in the second stage arising from the purchase behavior in the first stage, affecting the second stage price and in turn the behavior in the first stage. That said, we urge future work that comprehensively examines the role of discounting in temporal bundling. Rao's (2015) work on online content prices considers products that can be purchased outright or rented. The impact of renting and per-period consumption utility on the choice of alternative temporal strategies is an interesting avenue for further work. On the empirical side, it would be insightful to do a systematic survey of pricing practices (e.g., examining pricing for a sample of 1000 movie videos sold on Amazon) and tie them to the comparative statics predicted by the model. Such extensions will further our understanding of mixed forms of price discrimination involving multiple products.

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Appendix 1

Proofs of propositions 1, 2 and 3

Proof of proposition 1 The maximization problem is,

$$\max_{P,\delta}\ 2(P-c)(1-P)\alpha + 2(P-\delta-c)[\alpha\delta + (1-\alpha)(1-P+\delta)].$$

The objective function is separable in the two products and it is globally concave. The necessary conditions for maximum yield:

$$P = (1+c)\Big/2 + \delta(2-\alpha)\Big/2,$$
$$\delta = ((2-\alpha)P-(1+c)+\alpha)\Big/2.$$

Inserting the first into the second, we get $\delta = (1-c)/(4-\alpha)$. The optimal profit expression can be obtained by substituting these back into the objective function.

The prices in the two stages can be put in a simplified form as $P_1 = \frac{1+c}{2} + \frac{(2-\alpha)(1-c)}{2(4-\alpha)}$ for the regular price and $P_2 = \frac{1+c}{2} - \frac{\alpha(1-c)}{2(4-\alpha)}$ after substituting for the discount.

Proof of proposition 2

Step 1: Demand and profit derivations for the three cases:

Case (a): $P_{B1} > P_{B2} \geq 1$. From the α myopic consumers, demand in the first and second stages are $\alpha(2-P_{B1})^2/2$ and $\alpha(2-P_{B2})^2/2 - \alpha(2-P_{B1})^2/2$ respectively, at bundle prices of P_{B1} and P_{B2} respectively (Fig. 2(a) (iii)). From the $1-\alpha$ strategic consumers, purchases only occurs in the second stage at the discounted price P_{B2} and demand is $(1-\alpha)(2-P_{B2})^2/2$ (Fig. 2(b) (iii)). Thus, the profit function is,

$$\max_{P_{B1},\, P_{B2}}\ \alpha(P_{B1}-2c)\frac{(2-P_{B1})^2}{2} + (P_{B2}-2c)\left((1-\alpha)\frac{(2-P_{B2})^2}{2} + \alpha\left(\frac{(2-P_{B2})^2}{2} - \frac{(2-P_{B1})^2}{2}\right)\right).$$

Case (b): $P_{B1} \geq 1 \geq P_{B2}$. The demands from the myopic consumers at the bundle prices of P_{B1} and P_{B2} are $\alpha(2-P_{B1})^2/2$ and $\alpha\left(1-P_{B2}^2/2\right) - \alpha(2-P_{B1})^2/2$ in the first and second stages, respectively (Fig. 2(a) (ii)). The strategic segment purchases in the second stage and generates demand $(1-\alpha)\left(1-P_{B2}^2/2\right)$ (Fig. 2(b) (ii)). Thus, the profit function is,

$$\max_{P_{B1},\, P_{B2}}\ \alpha(P_{B1}-2c)\frac{(2-P_{B1})^2}{2} + (P_{B2}-2c)\left((1-\alpha)\left(1-\frac{P_{B2}^2}{2}\right) + \alpha\left(1-\frac{P_{B2}^2}{2} - \frac{(2-P_{B1})^2}{2}\right)\right).$$

Case (c): $1 \geq P_{B1} > P_{B2}$. Myopic consumer demands are $\alpha\left(1-P_{B1}^2/2\right)$ at price P_{B1} in the first stage and $\alpha\left(P_{B1}^2 - P_{B2}^2\right)/2$ at price P_{B2} in the second stage (Fig. 2(a) (i)). The strategic consumers have demand $(1-\alpha)\left(1-P_{B2}^2/2\right)$ in the second stage (Fig. 2(b) (i)). Thus, the profit function is,

$$\max_{P_{B1},\, P_{B2}}\ \alpha(P_{B1}-2c)\left(1-\frac{P_{B1}^2}{2}\right) + (P_{B2}-2c)\left((1-\alpha)\left(1-\frac{P_{B2}^2}{2}\right) + \alpha\left(\frac{P_{B1}^2}{2} - \frac{P_{B2}^2}{2}\right)\right).$$

Step 2: The next step is to examine the derived maximization problems.
First consider Case (b): $P_{B1} \geq 1 \geq P_{B2}$. Its objective simplifies to:

$$\max_{P_{B1},\ P_{B2}}\ \alpha(P_{B1}-P_{B2})\frac{(2-P_{B1})^2}{2}+(P_{B2}-2c)\left(1-\frac{P_{B2}^2}{2}\right).$$

The necessary conditions are:

$$\frac{\partial \pi}{\partial P_{B1}}=(2-P_{B1})(2-3P_{B1}+2P_{B2})=0 \Rightarrow P_{B1}=\frac{2}{3}(1+P_{B2}),$$
$$\frac{\partial \pi}{\partial P_{B2}}=2-4\alpha-\alpha P_{B1}^2+4\alpha P_{B1}-3P_{B2}^2+4cP_{B2}=0.$$

Inserting $P_{B1}=2(1+P_{B2})/3$ into the second equation, we get,

$$(27+4\alpha)P_{B2}^2-[36c+16\alpha]P_{B2}-(18-16\alpha)=0.$$

This has the quadratic solution given in the Proposition,

$$P_{B2}=\frac{18c+8\alpha+\sqrt{(18c+8\alpha)^2+(27+4\alpha)(18-16\alpha)}}{27+4\alpha}$$

and only the positive root gives a positive bundle price.

We can find the boundaries between this case and Cases (a) and (c). The former given by $P_{B2}=1$ is given in the next part of the proof. For the latter, put $P_{B1}=1$ which implies from $P_{B1}=2(1+P_{B2})/3$ that $P_{B2}=1/2$. Insert these into the second necessary condition and it gives $5/4-\alpha+2c=0$ which is not possible unless the parameters go out of bounds. Thus Case (c) is never optimal.

Next consider Case (a) $P_{B1}>P_{B2}\geq 1$. The objective can be simplified to:

$$\max_{P_{B1},\ P_{B2}}\ \alpha(P_{B1}-P_{B2})(2-P_{B1})^2\Big/2+(P_{B2}-2c)(2-P_{B2})^2\Big/2$$

The necessary conditions are:

$$\frac{\partial \pi}{\partial P_{B1}}=\alpha(2-P_{B1})\left(1-\frac{3P_{B1}}{2}+P_{B2}\right)=0,$$
$$\frac{\partial \pi}{\partial P_{B2}}=\alpha\frac{(2-P_{B1})^2}{2}+(2-P_{B2})\left(1-\frac{3P_{B2}}{2}+2c\right)=0.$$

Thus, $P_{B1}=2(1+P_{B2})/3$. Inserting it into the second equation, $P_{B2}=\frac{18(1+2c)-8\alpha}{27-4\alpha}$. Since the lowest price possible for this case is given by $P_{B2}=1$, inserting it into the expression for P_{B2} gives the boundary condition, which is $c=0.25+\alpha/9$. Finally, we verify the sufficiency conditions at the solution point.

Proof of proposition 3

Step 1: Demand and profit derivations for the two cases (Refer to Fig. 4):

The analysis of the strategic consumer segment (Fig. 4(b)) when the product prices are lower than the bundle price is the same as for MB: Demand is $(1-\alpha)(1-P)(P_B-P)$ for each product at a margin of $P-c$ and $(1-\alpha)[(1+P-P_B)^2-(2P-P_B)^2/2]$ for the bundle at a margin of P_B-2c. Thus, $(1-\alpha)[2(P-c)(1-P)(P_B-P)+(P_B-2c)((1+P-P_B)^2-(2P-P_B)^2/2)]$ is the profit from strategic consumers when $P_B \geq P$. If the product price is higher than the bundle price, the derivation is the same as for pure bundling: Demand is $(1-\alpha)\left(1-P_B^2/2\right)$ for the bundle at a margin of P_B-2c. Thus, $(1-\alpha)(P_B-2c)\left(1-P_B^2/2\right)$ is the profit from strategic consumers when $P_B \leq P$.

For the segment of myopic consumers, the first stage sales mirrors PC and hence the demand is $\alpha(1-P)$ for each product at a margin of $(P-c)$. The second stage is PB with margin (P_B-2c). Consider the case $P_B \geq P$. The demand for the bundle is $\alpha(2P-P_B)^2/2$. Next, consider the case $P_B \leq P$. The demand for the bundle is $\alpha\left[2(1-P)(P-P_B)+\left(P^2-P_B^2/2\right)\right]$. Thus, the overall profit objectives are:

If $P_B \geq P$,

$$\max_{P_B\in[P,2P];P\in[c,1]}\ (1-\alpha)\left[2(P-c)(1-P)(P_B-P)+(P_B-2c)\left((1+P-P_B)^2-\frac{(2P-P_B)^2}{2}\right)\right]\quad (1)$$
$$+2\alpha(P-c)(1-P)+\alpha(P_B-2c)\frac{(2P-P_B)^2}{2}$$

If $P_B \leq P$,

$$\max_{P_B\in[2c,P];P\in[c,1]}\ (1-\alpha)(P_B-2c)\left(1-\frac{P_B^2}{2}\right)+2\alpha(P-c)(1-P)$$
$$+\alpha(P_B-2c)\left(2(1-P)(P-P_B)+P^2-\frac{P_B^2}{2}\right)\quad (2)$$

Step 2: The boundary between the cases $P_B \geq P$ and $P_B \leq P$ can be expressed analytically by setting $P_B=P$ in the necessary conditions. Consider the case $P_B \leq P$. The necessary conditions yield:

$$\frac{\partial\pi}{\partial P}=2\alpha(1+c-2P)+2\alpha(P_B-2c)(1+P_B-P)=0$$

$$\frac{\partial\pi}{\partial P_B}=(1-\alpha)\left(2cP_B+1-\frac{3P_B^2}{2}\right)+\alpha\left(2(1-P)(P-P_B)+P^2-\frac{P_B^2}{2}\right)+\alpha(P_B-2c)(2P-2-P_B)=0$$

If we set $P_B = P$ in the first of these, we get $P = 1 - c$. If we substitute $P_B = P = 1 - c$ into the second equation, it yields,

$$\alpha\left(14c^2 - 8c\right) = 7c^2 - 10c + 1.$$

We can obtain the condition on the marginal cost for this equation to be satisfied with α in [0, 1] by setting α first to 0 and then to 1 and solving the equation. We find that $c \leq \left(-1 + 2\sqrt{2}\right)/7 \approx 0.261$ for $\alpha < 1$, and $c \geq \left(5 - 3\sqrt{2}\right)/7 \approx 0.108$ for $\alpha > 0$. Within the range [0.108, 0.261], the boundary between the solution cases $P_B < P$ and $P_B > P$ is $\alpha = \frac{7c^2 - 10c + 1}{14c^2 - 8c}$.

Appendix 2

Derivation of demand and profit under MB-MB

In this appendix we outline the derivation of MB–MB demand and profit. We assume and subsequently verify that the second stage component and bundle prices are less than their corresponding first stage prices. Then strategic consumers will postpone purchases to the second stage, and face the standard MB choices. This is also the case in the first stage for myopic consumers.

Thus, in the first stage, the α proportion of myopic consumers contributes,

$$2\alpha(P_1 - c)(1 - P_1)(P_1 - \phi_1) + \alpha(2P_1 - 2c - \phi_1)\left[(1 - P_1 + \phi_1)^2 - \phi_1^2 \Big/ 2\right],$$

where P_1 is the component price and ϕ_1 the bundle discount. The $1 - \alpha$ proportion of strategic consumers in the second stage contributes

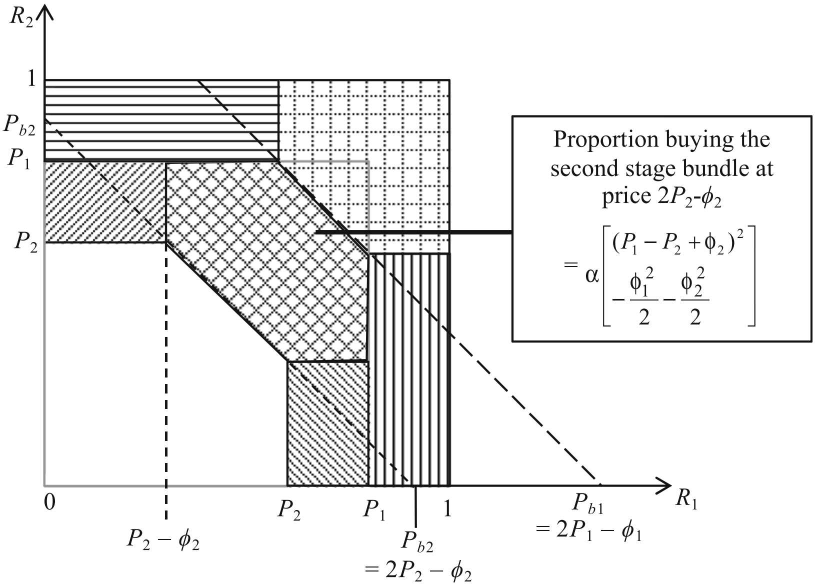
$$2(1 - \alpha)(P_2 - c)(1 - P_2)(P_2 - \phi_2) + (1 - \alpha)(2P_2 - 2c - \phi_2)\left[(1 - P_2 + \phi_2)^2 - \phi_2^2 \Big/ 2\right],$$

where P_2 is the component price and ϕ_2 the bundle discount.

Now consider the second stage demand calculation for myopic consumers. Those consumers who bought the bundle in the first stage have no subsequent impact, but consumers who bought one component in the first period might buy the other component if it is cheap enough in the second stage. They do so only if $P_2 < P_1 - \phi_1$, i.e., it is cheap enough, and their contribution is:

$$\max\left\{0, 2\alpha(P_2 - c)(P_1 - \phi_1 - P_2)(1 - P_1)\right\}.$$

Finally, we consider the segment of myopic consumers who did not buy in the first stage, but buy in the second stage. Two demand patterns occur depending on whether $\Delta \equiv (P_2 - \phi_2) - (P_1 - \phi_1)$ is positive or negative. The case of $\Delta \leq 0$ is shown schematically in the Figure. The other case is analogous.



If $\Delta \leq 0$ then $2\alpha(P_2 - c)(P_1 - P_2)(P_2 - \phi_2) + \alpha(2P_2 - 2c - \phi_2)\left[(P_1 - P_2 + \phi_2)^2 - \frac{\phi_1^2}{2} - \frac{\phi_1^2}{2}\right]$.

If $\Delta \geq 0$ then $2\alpha(P_2 - c)\left[(P_1 - P_2)(P_2 - \phi_2) - \frac{\Delta^2}{2}\right] + \alpha(2P_2 - 2c - \phi_2)\left[\frac{(P_1 - P_2 - \Delta + \phi_2)^2}{2} - \frac{\phi_2^2}{2}\right]$.

Let $P_2 < P_1 - \phi_1$ (i.e., we don't expect much bundle discount in the first stage –we verify the conditions *ex post.*) Then $\Delta \leq 0$. We now proceed by backwards induction. The Stage 2 problem is:

$$\begin{aligned} \max_{\substack{P_2, \phi_2; \\ 1 \geq P_1 - \phi_1 \geq P_2 \geq c; \\ 1 \geq P_2 \geq \phi_2 \geq 0}} \quad & 2(1-\alpha)(P_2 - c)(1 - P_2)(P_2 - \phi_2) + (1-\alpha)(2P_2 - 2c - \phi_2)\left[(1 - P_2 + \phi_2)^2 - \frac{\phi_2^2}{2}\right] \\ & + 2\alpha(P_2 - c)(P_1 - \phi_1 - P_2)(1 - P_1) + 2\alpha(P_2 - c)(P_1 - P_2)(P_2 - \phi_2) \\ & + \alpha(2P_2 - 2c - \phi_2)\left[(P_1 - P_2 + \phi_2)^2 - \frac{\phi_1^2}{2} - \frac{\phi_2^2}{2}\right] \end{aligned}$$

We obtain necessary conditions from this. Then we maximize the first period problem, which is the sum of the profits from the first and second periods, with respect to P_1 and ϕ_1, subject to these necessary conditions. The first stage maximization problem is thus,

$$\begin{aligned}\max_{\substack{P_1,\phi_1;\\ 1\geq P_1\geq c;\\ P_1\geq\phi_1\geq 0}} \; & 2\alpha(P_1-c)(1-P_1)(P_1-\phi_1) + \alpha(2P_1-2c-\phi_1)\left[(1-P_1+\phi_1)^2-\frac{\phi_1^2}{2}\right] \\ & +2(1-\alpha)(P_2-c)(1-P_2)(P_2-\phi_2) \\ & +(1-\alpha)(2P_2-2c-\phi 2)\left[(1-P_2+\phi_2)^2-\frac{\phi_2^2}{2}\right] \\ & +2\alpha(P_2-c)(P_1-\phi_1-P_2)(1-P_1)+2\alpha(P_2-c)(P_1-P_2)(P_2-\phi_2) \\ & +\alpha(2P_2-2c-\phi_2)\left[(P_1-P_2+\phi_2)^2-\frac{\phi_1^2}{2}-\frac{\phi_2^2}{2}\right]\end{aligned}$$

subject to the first order conditions from the previous stage. The Maximize function in Maple was used to obtain the numerical solution of the problem for different values of the parameters.

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