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In sponsored search advertising, advertisers bid to be displayed in response to a keyword search. The operational activities associated with participating in an auction, i.e., submitting the bid and the ad copy, customizing bids and ad copies based on various factors (such as the geographical region from which the query originated, the time of day and the season, the characteristics of the searcher), and continuously measuring outcomes, involve considerable effort. We call the costs that arise from such activities *keyword management costs*. To reduce these costs and increase advertisers' participation in keyword auctions, search engines offer an opt-in tool called *broad match with automatic and flexible bidding*, wherein the search engine automatically places bids on behalf of the advertisers and takes over the above activities as well. The bids are based on the search engine's estimates of the advertisers' valuations and, therefore, may be less accurate than the bids the advertisers would have turned in themselves. Using a game-theoretic model, we examine the strategic role of keyword management costs, and of broad match, in sponsored search advertising. We show that because these costs inhibit participation by advertisers in keyword auctions, the search engine has to reduce the reserve price, which reduces the search engine's profits. This motivates the search engine to offer broad match as a tool to reduce keyword management costs. If the accuracy of broad match bids is sufficiently high, advertisers adopt broad match and benefit from the cost reduction, whereas if the accuracy is very low, advertisers do not use it. Interestingly, at moderate levels of bid accuracy, advertisers individually find it attractive to reduce costs by using broad match, but competing advertisers also adopt broad match and the increased competition hurts all advertisers' profits, thus creating a "prisoner's dilemma." When advertisers adopt broad match, search engine profits increase. It therefore seems natural to expect that the search engine will be motivated to improve broad match accuracy. Our analysis shows that the search engine will increase broad match accuracy up to the point where advertisers choose broad match, but that increasing the accuracy any further reduces the search engine's profits.

1. Introduction

Sponsored search advertising is emerging as an indispensable part of a firm's advertising strategy. In the United States, over $18 billion dollars were spent on this advertising medium in 2013, accounting for nearly half of the total digital advertising expenditure (IAB 2014). In sponsored search advertising, multiple advertisers bid in an auction run by an Internet search engine (such as Google, Yahoo!, and Bing) to be displayed in response to a specific keyword searched by consumers. When a consumer searches the keyword, the advertisers' ads are displayed in descending order of their bids, conditional on being higher than a reserve price. Typically, an advertiser pays the search engine based on a second price rule when the advertiser's ad is clicked.

Hundreds of millions of search advertising auctions are run every day, essentially whenever a search engine user searches a keyword. Each auction is run in an automated fashion within milliseconds because the auction is triggered after a keyword is searched. The ordered list of ads must be presented with minimal delay to the user who searched the keyword. Therefore, to take part in a keyword auction, an advertiser must pre-specify the keyword it wants to advertise on along with its bid for the keyword and the ad copy it wants to display. When a user searches a keyword, all advertisers who had specified that keyword are included in the auction along with their respective bids, the auction is instantaneously resolved, and the results are displayed. Note that an advertiser may modify its list of keywords, associated bids, and ad copies at any time.

Running an effective search advertising campaign is an effort intensive task for an advertiser. It is extremely difficult to predict the keywords that consumers will search in the future. Indeed, roughly 20% of the searches Google receives in a day have not been seen in the previous 90 days.[1] Consumers may also inadvertently type a wrong spelling and in some instances not know the correct spelling of the focal keyword. On average, misspelled keywords account for 7% of searches.[2] Consumers could also use variations of the chosen keywords, such as plural and singular forms, and synonyms. Thus, consumers' unpredictable search behavior makes it prohibitive for advertisers to exhaustively list and manage all possible keywords in which they are interested. Furthermore, for each of these keywords, advertisers must submit their bids as well as ad copies. In addition, they must adjust their bids and ad copies based on various factors, including the geographical region from which the query originated, the time of day and the season, and the characteristics of the user who triggered the search. The advertiser must also continuously measure outcomes. We call the operational costs that arise in sponsored search advertising *keyword management costs*.

As the costs of participating in keyword search auctions are nontrivial, search engines provide advertisers with campaign management tools to reduce costs in an effort to further spur the growth of search advertising. A popular campaign management tool that search engines offer is called *broad match*. It is an alternative keyword matching process pioneered by Google in 2003 and subsequently adopted by Microsoft and Yahoo! (who call it advanced match). Under broad match, the search engine runs an advertiser's ads when consumers search not only for the exact keywords specified by the advertiser but also for variations of the keywords, such as synonyms, singular and plural forms, and misspellings. In this paper, we model broad match in its most flexible form, which is *broad match with automatic and flexible bidding*. Under this form the search engine also automatically bids on behalf of an advertiser after assessing the valuation of potentially related keywords. Therefore, broad match reduces keyword management costs for advertisers.[3]

We provide some examples to illustrate how broad match works. If an advertiser chooses the keyword "chocolate" and adopts broad match, then its advertisements may be shown on related searches such as "dark chocolate," "bitter dark chocolate," "white chocolate," and possibly even "cocoa." Yet if the same advertiser adopts the traditional "exact match," its advertisement will be displayed only when consumers search exactly for "chocolate." Suppose that Advertiser 1 chooses broad match for its keyword "dark chocolate," Advertiser 2 specifies that it should be exact matched on "chocolate," and Advertiser 3 specifies that it should be broad matched on "chocolate." Further suppose that a user enters the keyword "chocolate." In this case, all three advertisers will be included in the auction to be run in response to the keyword search. The search engine will place a bid on behalf of Advertisers 1 and 3, whereas Advertiser 2 will place its own bid. Next suppose that another user enters the keyword "dark chocolate." In this case, Advertisers 1 and 3 will be included in the auction and the search engine will place bids on their behalf. Finally, consider a user who enters a misspelled keyword such as "dakr chocolate," "dark choccolate," or "dark chokolatte." Because broad match allows the search engine to bid on behalf of an advertiser on misspellings, the search engine will place broad match bids on behalf of Advertisers 1 and 3. Broad match is also accompanied by tools that advertisers can use to customize ad copies based on geography, time, and user characteristics. For instance, on Google, this can be done by using ad copy parameters, which are then filled out by the search engine based on the above factors. Therefore, the final ad copy presented to the user is a message tailored to the search instance in an automated fashion.[4]

These examples highlight the fact that broad match could help advertisers reduce participation costs and reach a larger set of consumers. Recent industry studies attest to the popularity of broad match compared to exact match. On Google, 56% of clicks are through broad-matched keywords compared to only 33% through exact-matched keywords; on Bing, these numbers are 70% and 20%, respectively (Ballard 2013).[5]

Many advertisers seek the services of search engine marketing firms to develop and manage their advertising campaigns. Several of these firms have developed

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[1] https://support.google.com/adwords/answer/2497828?hl=en.

[2] http://www.acquisio.com/blog/keywords/close-variant-matching-all-what-means-your-keywords.

[3] For detailed descriptions of broad match from Google and advanced match from Yahoo!, see https://support.google.com/adwords/answer/2497828?hl=en and http://help.yahoo.com/l/h1/yahoo/ysm/sps/start/overview_matchtypes.html, respectively.

[4] For more details, see https://support.google.com/adwords/answer/6072565.

[5] The remaining clicks can be attributed to phrase match, which is a matching technique similar to broad match, but which produces narrower matches. Specifically, when an advertiser uses phrase match, a match is triggered only when the search query contains all of the keywords in the phrase specified by the advertiser, i.e., other variations are not considered. For example, if the bidding string is "dark chocolate" and there are three search queries, "bitter chocolate," "bitter dark chocolate," and "dark chocolate," then broad match will match all three queries, phrase match will match only the second and third queries, and exact match will match only the third query. Phrase match is conceptually similar to broad match, only narrower. Therefore, our results on broad match can be expected to directionally extend to phrase match. Thus, we do not explicitly consider phrase match in our paper.

algorithms to generate lists of keywords to bid. Still the challenge of developing an exhaustive list of all of the relevant keywords that will be searched by online users is so daunting that search engine marketing firms also resort to tools such as broad match.

Note that in broad match the search engine bids on related keywords based on its own heuristics for imputing advertisers' valuations for the keywords. On the other hand, despite the complexities of its execution, exact match offers advertisers greater control over their search advertising campaigns. In particular, the bids in exact match are based on advertisers' valuations of the keywords rather than the search engine's estimates of valuations. Therefore, the accuracy of the bids placed in broad match may be lower. Herein lies a challenge for an advertiser: How should they make a trade-off between reduced keyword management cost and reduced bid accuracy? When should an advertiser adopt broad match instead of exact match? It is clear that broad match directly reduces advertisers' costs. Yet, will broad match improve the advertisers overall profits given that competing advertisers are strategic players? Also, will broad match improve the search engine's profits? An important limitation of broad match is that the search engine's bids may not be accurate. With better optimization technology, however, the search engine could improve the accuracy of its bids. How far will the search engine go to make investments in improving bid accuracy?

We begin our analysis by developing a model in which an advertiser incurs a cost for participating in a keyword auction. We find that as keyword management costs increase, advertisers participate less frequently in a keyword auction, and that this also decreases the search engine's revenue. Therefore, the search engine has the incentive to offer tools such as broad match. With this motivation, we incorporate broad match into the model. Here, advertisers face a trade-off between keyword management cost and bid accuracy in deciding whether to adopt exact match (i.e., high keyword management cost with high bid accuracy) or broad match (i.e., negligible keyword management cost with lower bid accuracy). We find that advertisers adopt broad match as long as bid inaccuracy is not too high. A counterintuitive insight we obtain is that the seemingly helpful broad match could hurt advertisers' profits because of a prisoners' dilemma situation among advertisers. Even though each advertiser finds it attractive to use broad match to reduce its costs, the resulting increased competition among advertisers, coupled with a higher reserve price set by the search engine (because of reduced participation costs), hurts advertisers' profits. This situation, however, arises only for moderate levels of broad match bid accuracy and keyword management cost. For high levels of broad match accuracy and

high keyword management cost, it is profitable for competing advertisers to pursue broad match because the direct positive effect of a reasonably accurate bid at reduced cost dominates the indirect strategic effect of increased competition.

As broad match raises the search engine's profits, one may wonder whether the search engine's profits will increase as broad match bid accuracy improves. Interestingly, we find that the search engine will increase broad match accuracy to the point that advertisers choose broad match, but no further. This occurs because, given that advertisers use broad match, higher bid variability can improve the search engine's profits.

After establishing these key insights, we consider several extensions of the model to capture additional features of the market and to assess the robustness of our original findings. First, in the main model, we assume that the valuation of an advertiser for a keyword changes frequently, such that the advertiser must make its match strategy choice before its valuation is realized. In an extension, we consider the case wherein advertisers can choose match strategy after valuations are realized, which would be the case when advertisers' valuations are stable. We find that our key insights stay the same. Second, we allow competing advertisers to have different keyword management costs. In such a situation, we find that broad match hurts the low-cost advertiser because it takes away its competitive advantage, whereas broad match helps the high-cost advertiser. Third, we consider multiple advertisers. We find that if the number of competing advertisers is sufficiently large, then the negative effect of heightened competition becomes so strong that even if the search engine's valuation estimates are as good as those of the advertisers, broad match hurts advertisers; however, they still choose it because of a prisoners' dilemma situation. Fourth, we allow advertisers to specify a maximum bid that search engines cannot exceed while bidding on the advertisers' behalf in broad match; we find that our basic insights continue to hold.

The increasing prevalence of sponsored search advertising has motivated a growing body of theoretical and empirical academic work (Edelman et al. 2007, Varian 2007, Katona and Sarvary 2010, Yang and Ghose 2010, Athey and Ellison 2011, Jerath et al. 2011, Rutz and Bucklin 2011, Yao and Mela 2011, Zhu and Wilbur 2011, Jerath and Sayedi 2015, Hu et al. 2015, Desai et al. 2014, Jerath et al. 2014, Sayedi et al. 2014, Amaldoss et al. 2015). To our knowledge, the above body of work does not model keyword management costs or broad match.

Levin and Milgrom (2010) state that a search engine can use broad match to include a greater number of competitors in its sponsored search auctions through wider targeting and thus increase its revenue. Our

modeling also finds that the broad match increases the search engine's revenue. However, our focus is keyword management costs and automated participation and bidding, rather than targeting, per se. Furthermore, we obtain a number of results on advertisers' strategies and profits under broad match, and on how accurate the search engine should make the bids on the advertisers' behalf. Eliaz and Speigler (2015) construct a two-sided market model of broad match, with customers and firms as the two sides and the search engine providing the broad match technology to match customers (who provide noisy signals of their preferences through keyword searches) with firms (that are selling related products). Their study provides necessary and sufficient conditions under which broad match induces an efficient market equilibrium.

Our work is also related to auction theory that studies bidding costs (also known as participation costs or entry costs). Samuelson (1985) and Stegeman (1996) study the effect of bidding costs on market efficiency. Samuelson (1985) shows that excluding some bidders ex ante could improve the efficiency of the first price auction. Stegeman (1996) shows a similar result for asymmetric equilibria of the second price auction. The paper most relevant to our work is Tan and Yilankaya (2006). Assuming that the bidders are symmetric and that the cumulative distribution of bidders' valuations is concave, they show that a cutoff strategy is the unique equilibrium of a second price auction with bidding cost. This result is applicable in our model. Milgrom (2008) models bidding costs in position auctions to justify conflation through a restrictive bidding language. To our knowledge, these papers do not study bidding cost reduction tools such as broad match, which is our primary focus.

A small body of literature from the search engine industry has investigated the algorithmic aspects of broad match. Researchers at Google consider two bidding languages, *query language* and *keyword language*, under broad match (Evan-Dar et al. 2009). Given the complexity of broad match, they present an approximate algorithm for determining advertisers' bids and calculating the search engine's revenue in each language. Researchers at Yahoo! propose algorithmic techniques to find relevant keywords for advertisers' campaigns (Broder et al. 2008, Radlinski et al. 2008). Singh and Roychowdhury (2013) investigate how an advertising budget could be split among keywords matched when using broad match. In this research, we view broad match as a tool that facilitates the bidding process and reduces bidding costs. Unlike this literature that examines optimization methods and algorithmic techniques, we focus on equilibrium analysis and the managerial implications of broad match.

The rest of this paper is structured as follows. In §2, we develop our basic model and derive preliminary results highlighting the effect of keyword management cost on advertisers' payoffs and the search engine's revenue. In §3, we incorporate broad match into our model. We identify the conditions under which advertisers will adopt broad match instead of exact match and show how broad match affects advertisers' payoffs and search engine revenue. In §4, we consider various extensions to the model. In §5, we summarize the results, discuss managerial implications, and present directions for future research.

2. Role of Keyword Management Costs in Search Advertising

We consider a search advertising market with two risk-neutral advertisers, i.e., Advertiser 1 and Advertiser 2, and one keyword.[6] There is one advertising slot available for the keyword. A search engine sells the slot in a second price auction with reserve price R. In particular, the advertiser with the highest bid wins the slot and pays the maximum of the second highest bid and the reserve price. If the highest bid is smaller than the reserve price, the slot remains unsold.

We assume that Advertiser i has a private value v_i for the slot. Values v_1 and v_2 are independently drawn from the distribution Uniform[0, 1]. It costs an advertiser $c > 0$ to manage its keyword. As discussed earlier, the cost captures the effort involved in submitting bids and ads, tailoring the bids and ads for different geographical regions, over time, and based on user characteristics, and outcome measurements. Note that cost c is an operational cost of participating in the auction, not the cost to learn the valuation. In other words, this cost will be incurred whenever an advertiser participates in the auction, even after the advertiser knows the valuation of the keyword. We assume that c is common knowledge, and that the clicks volume of the keyword is one unit. If an advertiser participates in the auction and wins, then its utility is $v_i - p - c$ where p is the maximum of R and the second highest bid. If the advertiser loses the auction, its utility is $-c$ as it still incurs the cost of participating in the auction. If the advertiser does not participate in the auction, then its utility is zero.

The advertisers and the search engine play the following game. In Stage 1, the search engine sets the reserve price R. In Stage 2, each advertiser learns its private value v_i for the slot. Then, advertisers simultaneously decide if they want to participate in the auction. If they choose to participate, they incur cost c and place a bid for the slot. Finally, in Stage 3, the search engine runs a second price auction with reserve price R, and collects the payment from the

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[6] The results are easily extendable to $n > 1$ independent keywords.

winner of the auction. Note that the cost c is not part of the search engine's revenue.

We solve for the subgame-perfect equilibrium of this game to understand strategic behavior. Below we present two lemmas and then show how keyword management cost can affect keyword search advertising.

LEMMA 1. *For an advertiser i who decides to participate in the auction, it is weakly dominant to truthfully bid its value v_i.*

LEMMA 2. *There exists a threshold value τ such that Advertiser i participates in the auction if and only if its private value v_i is at least τ.*

Note that these lemmas and their proofs are found in Tan and Yilankaya (2006). For completeness, we provide their proofs in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mksc.2015.0919). The intuition driving Lemma 1 is that the keyword management cost c does not directly affect an advertiser's bid amount, but only influences the decision on whether to participate in the auction. Therefore, if an advertiser decides to participate in the auction, it bids truthfully because the auction is a second price auction. Lemma 2 states that an advertiser participates in the auction if and only if its value is sufficiently high.

From Lemmas 1 and 2, we can reason that finding an advertiser's bidding strategy reduces to solving for the optimum threshold τ (as a function of c and R). Below this threshold the advertiser will not participate, and above this threshold the advertiser will participate and bid truthfully. After obtaining the threshold value of τ, we can calculate the expected revenue of the search engine (as a function of R and c), and maximize this revenue with respect to the reserve price R to obtain the optimal value of the reserve price (as a function of c). The proposition below characterizes these quantities. The proof of the proposition is available in the appendix.

PROPOSITION 1.
(a) *An advertiser participates in the auction if its valuation is greater than or equal to the threshold τ given by*

$$\tau = \frac{R + \sqrt{R^2 + 4c}}{2}. \tag{1}$$

(b) *The optimum reserve price of the search engine is given by*

$$R^* = \frac{1}{4}\left(3 - \sqrt{1 + 8c}\right). \tag{2}$$

From the above proposition, we obtain the following corollary.

COROLLARY 1. *As the keyword management cost, c, increases, we observe the following:*
(a) *The optimum reserve price decreases.*

(b) *The probability of an advertiser participating in the auction decreases. However, conditional on an advertiser winning the auction, its expected payment decreases. Overall, the advertiser's expected utility decreases.*

(c) *The search engine's expected revenue decreases.*

We can see from Equation (2) that as the keyword management cost increases, the search engine reduces the optimum reserve price to facilitate more competition. After substituting for the optimal reserve price in Equation (1), we can show that τ is an increasing function of c. This suggests that the probability of participating in the auction decreases when cost c increases. This finding, though intuitive, has an interesting implication: As c increases, the competition between the advertisers decreases. Thus, the expected payment of an advertiser, conditional on the advertiser winning the auction, is a decreasing function of c. In other words, as the keyword management cost increases, an advertiser participates in the auction less frequently, which decreases its expected utility. Yet when it does participate, it wins the auction for a lower price, which increases its expected utility. Overall, an advertiser's expected utility decreases with c. Note also that the search engine's expected revenue decreases in keyword management cost c.

The above analysis suggests that if the search engine could reduce keyword management cost, not only would the advertisers' surplus increase, but the search engine's revenue could also increase. Moreover, the search engine could set a higher reserve price. This could explain why search engines are developing a wide range of campaign optimization tools for advertisers.[7] In §3, we study broad match, which is one such widely used tool.

3. Broad Match in Search Advertising

One of the tools most commonly used by advertisers to reduce keyword management costs is broad match. For an advertiser who chooses broad match, the search engine automatically finds new relevant keywords for the advertiser, estimates the advertiser's valuations for those keywords, and accordingly bids on behalf of the advertiser. We model broad match as a tool that reduces the advertiser's keyword management cost to zero. Note, however, that the search engine's bids on behalf of the advertiser may not be accurate. Given these two conflicting effects of broad match, we examine how it affects the advertisers' equilibrium strategies and the search engine's revenue. We allow advertisers to decide whether they want to use broad match. An advertiser who does not use broad match will, by default, use exact match. This implies that the advertiser submits

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[7] For example, see http://adwords.blogspot.com/2007/07/campaign-optimizer-now-available.html.

its own bid and incurs the keyword management cost, and that the search engine uses this exact bid in the auction. Note that the keyword management cost, c, is the operational cost of submitting the bids and the ads, tailoring the bids and the ads based on the geographical source of the query, over time, based on user characteristics, etc., and measuring outcomes. This cost is incurred *if and only if* exact match is used, as under broad match with automatic and flexible bidding the search engine can take over these activities (as discussed earlier in the example).

In the presence of broad match, the decision sequence is as follows. In Stage 1, the search engine decides and announces its reserve price, R. In Stage 2, before realizing their values, advertisers simultaneously decide whether to use broad match or exact match. If an advertiser uses broad match, its keyword management cost reduces to zero. However, the value that the search engine bids on behalf of the advertiser may not be accurate.[8] We assume that if the advertiser's valuation is v, the search engine assesses the valuation to be $v + \epsilon$, where $\epsilon \sim \text{Uniform}[-E, E]$, and bids this value.[9] This is an abstraction of the idea that the search engine estimates a valuation on behalf of the advertiser and, given that it is a second price auction, submits this valuation as the advertiser's bid.[10] The accuracy of the search engine's broad match algorithm decreases as the error E increases. Depending on broad match accuracy and keyword management cost, advertisers decide whether they want to use broad match.

After Stage 2 and before Stage 3, the user searches the keyword and each advertiser learns its own private

Figure 1   Timeline of the Game
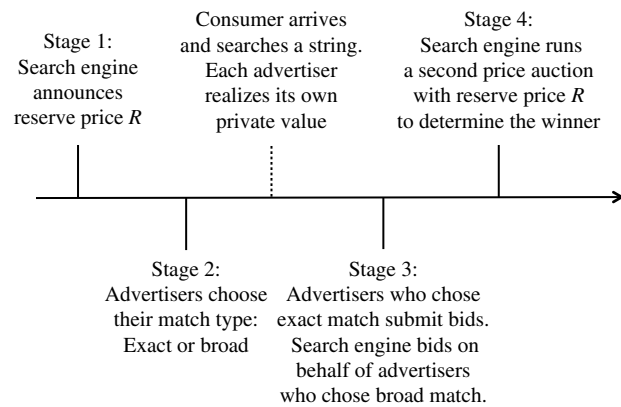
value v_i. In Stage 3, an advertiser who chose exact match in Stage 2 decides whether it wants to participate in the auction, and if so, how much to bid for the keyword. On the other hand, an advertiser who chose broad match in Stage 2 does not have to do anything in Stage 3 as the search engine bids on behalf of the advertiser. Finally, in Stage 4, the search engine runs the second price auction. The timeline of the game is summarized in Figure 1. We solve for the subgame-perfect equilibrium of the game. For simplicity, we assume $E \leq 0.5$.

When a user searches a keyword, the search engine determines the set of competing bidders for the auction related to the keyword (based on their match strategy choices). When the auction is run, the search engine also determines the bids for bidders who are included through broad match. For the bidders who are included through exact match, the search engine uses their own pre-specified bids. The assumption that an advertiser learns its valuation after making its match strategy decision faithfully reflects a common reality: Often advertisers do not know all of the keywords that may be searched and which will be relevant to their ads; these are revealed only after a user keys in the search keyword. In other words, the keywords for which an advertiser's ad is displayed through broad match are unknown to the advertiser. This is a major advantage of using broad match. In such a case an advertiser clearly cannot choose exact match. Even for a keyword known to the advertiser, valuation can change with time because of idiosyncratic factors, related to external events or geographic location, which are extremely difficult to predict. In other words, valuations change too frequently for the advertiser to always make the match strategy choice *after* knowing the valuations. Note that determining match strategy and communicating it to the search engine is expected to be a slower, time consuming process, even if it can be automated. Therefore, in our main analysis here, we assume that an advertiser makes its match strategy decision before learning its valuation. Nevertheless,

[8] One could argue that under exact match also advertisers make bidding decisions based on estimates of valuations that contain errors. However, the search engine is typically expected to have less information than the advertiser on the advertiser's valuation for the keyword. The error in broad match bids arises due to the search engine's lack of information, rather than the advertiser's lack of information. In this context, E can be reinterpreted as the magnitude of the difference in the error rates between the advertiser's estimate and the search engine's estimate. Furthermore, our results hold if the keyword management cost under broad match is positive, as long as it is sufficiently smaller than the corresponding cost under exact match.

[9] For analytical tractability, we assume uniform distributions for valuations and error. We have also conducted our analysis assuming normal distributions for these quantities, for instance, $v \sim N(\mu, \sigma)$ and $\epsilon \sim N(0, E)$. In this case, we do not have closed-form solutions but the model is amenable to numerical analysis. We find that our key insights are robust to this variation.

[10] The technology that the search engine uses to estimate the valuation is such that the support of the distribution from which the bid is selected is from $-E$ to $1 + E$ rather than from 0 to 1. The wider support reflects the inaccuracy in valuation estimation by the search engine. For simplicity, we allow negative valuation estimates by the search engine. However, this will make no difference to the analysis because negative valuations are rendered irrelevant due to the reserve price.

there may be cases in which valuations are stable and do not change frequently. In such cases, it is reasonable to assume that match strategy choice is made after valuations are known. In §4.1, we analyze this scenario and find that our key insights are unchanged.

Note that our main model is based on broad match in its most general form, i.e., *broad match with automatic and flexible bidding* in which the advertisers allow the search engine to bid any amount.[11] Search engines also give advertisers the option to specify a maximum bid under automatic bidding, such that the search engine's bid cannot be higher than this value. We analyze this case in §4.4 and show that our results stay qualitatively the same.

Continuing with our analysis, let $T_i \in \{X, B\}$ denote whether Advertiser i chooses exact match (X) or broad match (B) in Stage 2. Furthermore, let EU_{T_1, T_2} denote the expected utility of Advertiser 1 when Advertiser 1 uses T_1-type match and Advertiser 2 uses T_2-type match. For example, $EU_{B, X}$ is the expected utility of Advertiser 1 if Advertiser 1 uses broad match and Advertiser 2 uses exact match. Depending on whether Advertiser i, $i \in \{1, 2\}$, uses exact match or broad match, we have four possible scenarios. Because the two advertisers are symmetric, calculating the expected utility of Advertiser 1 for each of the four cases is enough for our analyses. When an advertiser uses broad match, the probability distribution of its bid is obtained by convolution of the distributions of v and ϵ, and is given by

$$p(x) = \begin{cases} 0 & \text{if } x < -E, \\ \dfrac{E + x}{2E} & \text{if } -E \leq x < E, \\ 1 & \text{if } E \leq x < 1 - E, \\ \dfrac{1 + E - x}{2E} & \text{if } 1 - E \leq x < 1 + E, \\ 0 & \text{if } x \geq 1 + E. \end{cases} \quad (3)$$

Using the above probability distribution function, we calculate advertisers' expected utilities for each of the four possible cases.

Case 1: Both advertisers use broad match. In this case, the expected utility of Advertiser 1 with valuation v is

$$EU_{B, B}(v, E)$$
$$= \int_{\max(R, v-E)}^{v+E} \left(\int_{-E}^{R} (v - R)p(y)\,dy + \int_{R}^{x} (v - y)p(y)\,dy \right.$$
$$\left. + \int_{x}^{1+E} 0\, p(y)\,dy \right) \frac{1}{2E}\,dx. \quad (4)$$

[11] For a description of automatic bidding, see https://support.google.com/adwords/answer/2390311.

Therefore, the expected utility of Advertiser 1 using broad match, conditional on Advertiser 2 using broad match as well, is

$$EU_{B, B}(E) = \int_0^1 EU_{B, B}(v, E)\,dv$$
$$= \frac{1}{30}((-5 + E)E^2 + 5(-1 + R)^2(1 + 2R)), \quad (5)$$

where the expression on the right-hand side (RHS) assumes that $R \in [E, 1 - E]$.

From the search engine's point of view, this case is equivalent to a second price auction in which advertisers' valuations come from the probability distribution function $p(x)$. The optimum reserve price in this case is $R^* = \frac{1}{2}$, confirming our earlier assumption of $R \in [E, 1 - E]$.

Case 2: Advertiser 1 uses exact match but Advertiser 2 uses broad match. In this case, the expected utility derived by Advertiser 1 with valuation v is

$$EU_{X, B}(v, E, c) = \int_{-E}^{R} (v - R)p(y)\,dy + \int_{R}^{v} (v - y)p(y)\,dy$$
$$+ \int_{v}^{1+E} 0\, p(y)\,dy - c. \quad (6)$$

Let τ_B be the value of v at which this expected utility is zero. On assuming $\tau_B \leq 1 - E$, the critical value of τ_B simplifies to

$$\tau_B = \sqrt{2c + R^2}. \quad (7)$$

Therefore, Advertiser 1 participates in the auction if and only if $v \geq \tau_B$, and its expected utility is

$$EU_{X, B}(E, c) = \int_0^{\tau_B} 0\,dv + \int_{\tau_B}^1 EU_{X, B}(v, E, c)\,dv$$
$$= \frac{1}{48}\Big(-E^3 + 16c\big(-3 + 2\sqrt{2c + R^2}\big)$$
$$+ 8\big(1 + R^2\big(-3 + 2\sqrt{2c + R^2}\big)\big)\Big). \quad (8)$$

Case 3: Advertiser 1 uses broad match whereas Advertiser 2 uses exact match. In this case, the expected utility derived by Advertiser 1 with valuation v is

$$EU_{B, X}(v, E, c) = \int_{\max(R, v-E)}^{v+E} \left(\int_0^{\tau_B} (v - R)\,dy \right.$$
$$\left. + \int_{\tau_B}^{x} (v - y)\,dy + \int_x^1 0\,dy \right) \frac{1}{2E}\,dx. \quad (9)$$

The corresponding expected utility of Advertiser 1 is

$$EU_{B, X}(E, c) = \int_0^1 EU_{B, X}(v, E, c)\,dv$$
$$= \frac{1}{6}\big(1 - E^2 + \sqrt{2c + R^2}\big(-2c + 2(-3 + R)R$$
$$+ 3\sqrt{2c + R^2}\big)\big). \quad (10)$$

Table 1 Advertisers' Choice of Broad Match and Exact Match as a Normal-Form Game

	Advertiser 2	
	Broad match (B)	Exact match (X)
Advertiser 1		
Broad match (B)	$(EU_{B,B}, EU_{B,B})$	$(EU_{B,X}, EU_{X,B})$
Exact match (X)	$(EU_{X,B}, EU_{B,X})$	$(EU_{X,X}, EU_{X,X})$

Table 2 Description of Regions in Figure 2

Region	Equilibrium	Broad match for advertisers	Broad match for search engine
A	(B, B)	Helps	Helps
B	(B, B)	Hurts	Helps
C	(B, B) and (X, X)	Hurts	Helps
D	(X, X)	NA	NA

Case 4: *Both advertisers use exact match.* We have already studied this case in §2. Recall that the threshold value, τ_X, under which advertisers do not participate is

$$\tau_X = \tfrac{1}{2}\big(R + \sqrt{4c + R^2}\big). \tag{11}$$

Then the expected utility derived by an advertiser with valuation v is

$$EU_{X,X}(v, c) = \tfrac{1}{4}\big(-2c - R\big(R + \sqrt{4c + R^2}\big) + 2v^2\big), \tag{12}$$

and the corresponding expected utility of the advertiser is given by

$$\begin{aligned} EU_{X,X}(c) &= \int_{\tau_X}^{1} EU_{X,X}(v, c)\, dv \\ &= \tfrac{1}{12}\big(2 + R(-3 + 2R)\big(R + \sqrt{4c + R^2}\big) \\ &\quad + 2c\big(-3 + 3R + \sqrt{4c + R^2}\big)\big). \end{aligned} \tag{13}$$

As discussed earlier, the optimum reserve price of the search engine in this case is $R^* = \tfrac{1}{4}(3 - \sqrt{1 + 8c})$.

Based on the reserve price set by the search engine and the expected utilities above, advertisers decide if they want to adopt broad match or exact match. The game in Stage 2 can be modeled as the normal-form game in Table 1. We find that only (exact match, exact match) and (broad match, broad match) can emerge as equilibria for any reserve price. Interestingly, the advertisers may be helped or hurt by their choice

Figure 2 Advertisers' Equilibrium Strategies as Functions of Keyword Management Cost and Broad Match Error

of broad match. The equilibrium that emerges and the implications for the advertisers depend on the keyword management cost, c, and the broad match error, E. Figure 2 shows advertisers' equilibrium strategies as functions of keyword management cost and broad match error, and Table 2 describes the regions in Figure 2. In the figure, pairs of letters X/B denote equilibrium strategies for Advertisers 1 and 2, respectively, with B and X standing for broad match and exact match; pairs of symbols $\pm$ show how presence of broad match affects advertisers' utilities. In Regions A and B, both advertisers use broad match in equilibrium. In Region D, both advertisers use exact match. In Region C, we have multiple equilibria: Both advertisers use broad match *or* both use exact match. Region A is the only region in which broad match improves the utility of both advertisers. In Region B, advertisers face a prisoners' dilemma situation, i.e., in equilibrium, both advertisers use broad match even though broad match reduces the utility of each advertiser. In Region C, as in Region B, broad match hurts the advertisers (if the (broad match, broad match) equilibrium emerges).

Note that the optimal reserve price is different in each of the four regions. Given that only the symmetric equilibria exist, the only relevant optimal reserve prices are $\tfrac{1}{2}$ (as per Case 1) and $\tfrac{1}{4}(3 - \sqrt{1 + 8c})$ (as per Case 4). In Regions A and B, the search engine sets a reserve price equal to $\tfrac{1}{2}$; at this reserve price, only the (broad match, broad match) equilibrium exists in these two regions. In Region D, at any reserve price, only the (exact match, exact match) equilibrium exists, and the search engine sets the reserve price equal to $\tfrac{1}{4}(3 - \sqrt{1 + 8c})$. In Region C the (broad match, broad match) and the (exact match, exact match) equilibria exist at both the relevant reserve prices.

We see that if broad match is highly accurate, both advertisers benefit from using it if keyword management costs are sufficiently high (Region A). This is because the advertisers save on keyword management costs with only small distortions in their bids. Interestingly, however, we also find that both advertisers may be worse off in the equilibrium wherein both use broad match (Regions B and C). In other words, under certain conditions, broad match creates a prisoners' dilemma situation: Advertisers use broad match in equilibrium even though their utilities decrease because of that choice. Each advertiser uses broad match to
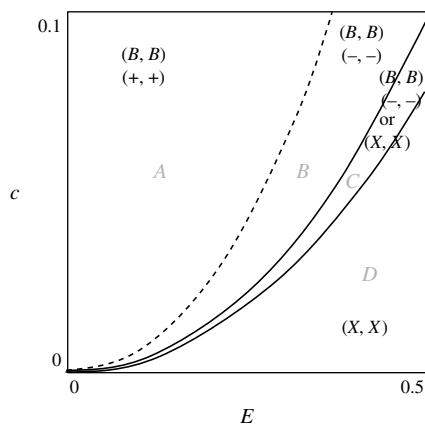
avoid the keyword management cost and to participate more often in the auction. However, when using broad match, two forces are at play that reduce the advertiser's payoff. First, broad match increases competition because it eliminates the keyword management cost, which motivates advertisers to participate more in the auction. Second, broad match increases the optimum reserve price. In other words, in the absence of broad match, as discussed in §2, the search engine must reduce the reserve price to compensate for the cost. Yet there is no need for such an adjustment when both advertisers use broad match. Therefore, in the equilibrium where both advertisers use broad match, their payoffs could actually decrease because of broad match (Regions B and C). Yet, a high level of accuracy in broad match could compensate for these negative forces (Region A).[12]

Finally, note that if keyword management cost is sufficiently small or the broad match error is sufficiently large (Region D), both advertisers adopt exact match. This makes intuitive sense; given the lack of accuracy of broad match bids, it is a better strategy to avoid the errors in bids even though the management cost is incurred for exact match. We obtain the following proposition. The proof of the proposition is provided in the appendix.

PROPOSITION 2. *For a low level of broad match accuracy, both advertisers use exact match (Region D in Figure 2). For a high level of broad match accuracy, both advertisers use broad match, which increases the advertisers' expected utilities (Region A in Figure 2). For a medium level of broad match accuracy, both advertisers use broad match in equilibrium even though broad match reduces the expected utility of both advertisers (Regions B and C in Figure 2). For any level of broad match accuracy, the advertisers use broad match if keyword management cost is sufficiently high.*

Next, we study how the search engine's revenue is affected by broad match accuracy. Recall that the advertisers' incentive to use broad match increases as broad match accuracy increases. Furthermore, broad match increases the search engine's revenue as it increases competition in the keyword auction by removing the keyword management cost. This leads to the question: Would the search engine's revenue increase as broad match accuracy increases? Surprisingly, the answer to this question is no. More specifically, it is in the search engine's best interest to make broad match just accurate enough so that advertisers use broad match, but not make it overly accurate. The intuition

for this result is as follows. A larger broad match error increases the variability in the advertisers' bids. Given this, draws from the right side of the advertisers' bid distribution $p(\cdot)$ increase the search engine's revenue, whereas draws from the left side of the distribution cannot decrease the search engine's revenue when the bids are already lower than the reserve price. In other words, if the variability of the advertisers' bids increases (that is, broad match accuracy decreases), the search engine benefits from higher bids but is partially shielded from reduction in revenue from lower bids. Therefore, in the broad match equilibrium, we could observe an increase in the search engine's revenue as broad match accuracy decreases. We state this in the following proposition. The proof of the proposition is provided in the appendix.

PROPOSITION 3. *Broad match increases the optimum reserve price and the search engine's revenue. However, in an equilibrium in which advertisers use broad match, the search engine's revenue is a decreasing function of broad match accuracy.*

Proposition 3 shows that even if the search engine could make broad match very accurate, it will not do so. In other words, the search engine makes broad match accurate enough so that advertisers use broad match in equilibrium. However, improving broad match accuracy any further hurts the search engine's revenue. This result has the essence of the results found in Ganuza (2004) and Coleff and Garcia (2014).

4. Extensions

In the preceding analysis, we made simplifying assumptions to facilitate the exposition of our key results. We now extend the model in multiple ways to relax some of our assumptions and in the process capture additional features of the keyword advertising market. Details of the analyses for the extensions are provided in the online appendix.

4.1. Stable Advertiser Valuations

In our main model, we assume that advertisers' valuations vary over time and that they choose match strategies before valuations are revealed. In this section, we assume that advertisers know their valuations before choosing match strategies. This reflects situations in which advertisers' valuations for keywords remain stable over time. In terms of the model stages, first, the search engine announces the reserve price. Second, each advertiser realizes its valuation v. Third, given its valuation, v, each advertiser decides whether to use broad match and thereby delegate the bidding authority to the search engine but save on keyword management cost, to use exact match and specify a bid but incur the cost or not to participate in the auction. Fourth, the auction is run.

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[12] In §4.3, we show that if there are more advertisers in the market, even a completely accurate broad match cannot compensate for these negative forces, i.e., advertisers use broad match in equilibrium, but it always reduces their payoffs.

Figure 3   Equilibrium Bidding Strategies of Advertisers as a Function of Keyword Management Cost c and Broad Match Error E
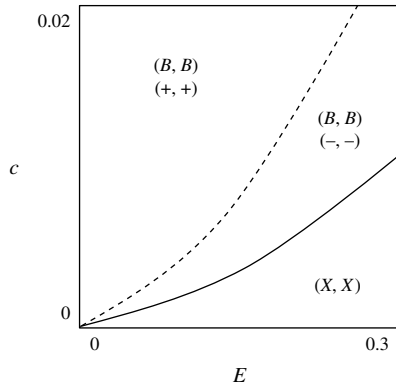
Our analysis shows that an advertiser's choice of broad match versus exact match depends on the relative values of the error E versus the keyword management cost c. For this model, their choice does not depend on the advertiser's valuation. In particular, when participating in the auction both advertisers use broad match in equilibrium, if and only if $c \geq E^2/6$. If $c < E^2/6$, advertisers prefer exact match, but participate only if their value is at least τ where $\tau = (R + \sqrt{R^2 + 4c})/2$ as shown in §2. If $c \geq E^2/6$, advertisers prefer broad match, but participate only if their valuation is above the threshold t (derived in the online appendix).

Figure 3 presents the optimal match strategy of an advertiser as a function of c and E. Note that this figure is qualitatively the same as Figure 2. In particular, if c is sufficiently small and E is sufficiently large, both advertisers use exact match. If c is sufficiently large and E is sufficiently small, both advertisers use, and benefit from, broad match. For medium values of E and c, we have a prisoners' dilemma situation in which both advertisers use broad match, though it lowers their profits.

4.2. Asymmetric Keyword Management Costs

In our main model, we assume that advertisers' keyword management costs are the same. In reality, advertisers could have asymmetric costs because of differences in bid making processes and level of automation. Now we allow for this possibility by assuming that one advertiser is of the high (H) type and the other is of the low (L) type. The high-type advertiser has automated its bidding process with its keyword management cost being c_H, whereas the low-type advertiser uses a less sophisticated bidding process with its cost being c_L. We assume that $c_H < c_L$, which implies that the high-type advertiser is more efficient than the low-type advertiser. In this set-up, let τ_L and τ_H be the threshold values at which the low-type advertiser and the high-type advertiser participate in the auction. We compute these threshold values through an analysis similar to that

in §2. After obtaining the advertisers' bidding strategies, we compute the search engine's expected revenue and the optimal reserve price R.
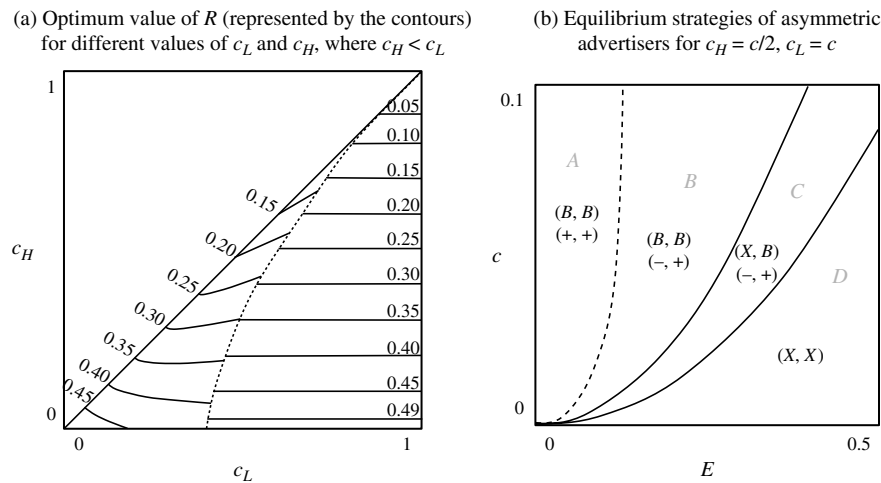
Figure 4(a) presents the optimum value of R for different values of c_L and c_H. Note that the optimum reserve price is a decreasing function of c_H. However, it is a nonmonotone function of c_L. Furthermore, there is a discontinuity in the optimum reserve price when the L-type advertiser drops out of the auction. If c_L is sufficiently high and c_H is sufficiently low, the L-type advertiser does not participate in the auction any more. In this case, the search engine sets the reserve price independent of c_L, and the optimum reserve price, at the point of discontinuity, increases to a higher value (given by $R^* = (1 - c_H)/2$). Moreover, after the discontinuity, the optimum reserve price is constant in c_L; before the discontinuity, the optimum reserve price could be increasing or decreasing in c_L (depending on the value of c_H).

As mentioned above, the search engine may *increase* the reserve price as the cost of the L-type advertiser increases. The intuition is as follows. Existence of the L-type advertiser forces the optimum reserve price to be lower than it would be for the H-type advertiser only. However, as the L-type advertiser's cost increases, the search engine may benefit from sacrificing the L-type advertiser to extract higher revenue from the H-type advertiser. In other words, from the search engine's perspective, as the keyword management cost of the L-type advertiser increases, the advertiser becomes less valuable for improving the search engine's profits. Therefore, for sufficiently large c_L, the search engine increases the reserve price, which induces the L-type advertiser to leave the auction, but extracts more revenue from the H-type advertiser.

Note that as c_H increases, the L-type advertiser is more likely to be sacrificed. For example, when $c_H = 0$, a change in c_L from 0.30 to 0.31 decreases the optimum reserve price. However, if $c_H = 0.25$, the same change in c_L increases the optimum reserve price. The reason is as follows: When c_H is small (and the reserve price is large), the marginal effect of reserve price on the search engine's revenue from the H-type advertiser is low. Therefore, the search engine could more easily lower the reserve price to accommodate the change in L-type advertiser's keyword management cost. However, as c_H increases, the marginal effect of the reserve price on the search engine's revenue from the H-type advertiser increases. Therefore, for a sufficiently large c_H, an increase in c_L leads to an increase in the optimum reserve price, thus sacrificing the L-type advertiser.

Next, we analyze the effect of broad match on the strategies of the asymmetric advertisers. Details of the analysis are provided in the online appendix. To show the effect of broad match error and keyword management cost on advertisers' behavior, in Figure 4(b) we

Figure 4   Figures for the Case of Asymmetric Keyword Management Costs

(a) Optimum value of R (represented by the contours) for different values of c_L and c_H, where $c_H < c_L$

(b) Equilibrium strategies of asymmetric advertisers for $c_H = c/2$, $c_L = c$

plot the firms' strategies for a specific level of cost asymmetry. Specifically, we draw the figure for $c_L = c$ and $c_H = c/2$; the results are qualitatively the same for other levels of asymmetry. The following proposition characterizes the different outcomes.

PROPOSITION 4. *If broad match accuracy or keyword management cost is sufficiently low, both advertisers use exact match (Region D in Figure 4(b)). If broad match accuracy or keyword management cost is sufficiently high, both advertisers use broad match and benefit from using broad match (Region A in Figure 4(b)). For medium-low values of broad match accuracy and keyword management cost, only the L-type (high cost) advertiser uses broad match (Region C in Figure 4(b)). However, for medium-high values of broad match accuracy and keyword management cost, both advertisers use broad match (Region B in Figure 4(b)). In Regions B and C, the L-type advertiser benefits from broad match, whereas the H-type advertiser is hurt by broad match.*

Note that if c_H/c_L becomes sufficiently small, then Region A in Figure 4(b) shrinks to zero. This implies that, although the H-type advertiser uses broad match if broad match accuracy is sufficiently high, it does not benefit from broad match for any level of broad match accuracy. This is because broad match eliminates the H-type advertiser's cost advantage over the L-type advertiser. Therefore, as the asymmetry between advertisers increases, the H-type advertiser is harmed more by broad match. For a sufficiently large cost asymmetry, even a broad match with zero error cannot compensate the H-type advertiser's loss due to elimination of the cost advantage.

Next, we turn to the search engine's revenue. We obtain the following proposition.

PROPOSITION 5. *In an equilibrium in which one or both advertisers use broad match, the search engine's revenue*

is a decreasing function of broad match accuracy. If c_H is sufficiently small and c_L is sufficiently large, the search engine's revenue is maximized at an accuracy level where only the L-type advertiser uses broad match. Otherwise, the search engine's revenue is maximized at an accuracy level where both advertisers use broad match.

Consistent with Proposition 3, the above proposition shows that, conditional on advertisers' using broad match, the search engine benefits from a larger broad match error. However, unlike Proposition 3, Proposition 5 shows that the search engine does not always try to get both advertisers to use broad match. In particular, if c_H is sufficiently small and c_L is sufficiently large, the search engine only targets the L-type advertiser to use broad match. Intuitively, when c_H is small, broad match accuracy must be high for the H-type advertiser to use broad match. On the other hand, when c_L is large, even at a low broad match accuracy, the L-type advertiser would use broad match. Therefore, when c_H is sufficiently low and c_L is sufficiently high, the search engine abandons the H-type advertiser and sets a relatively low broad match accuracy to extract more profit from the L-type advertiser. Note that this also increases competition for the H-type advertiser.

To summarize, when advertisers have different keyword management costs, broad match takes away the competitive advantage of the low-cost advertiser. In this sense, broad match can "level the playing field" for large and small firms which, in general, can be expected to have low and high keyword management costs, respectively.

4.3. Multiple Advertisers
We generalize the basic model in §2 to allow for $N > 2$ symmetric advertisers. As the number of advertisers increases, τ, the threshold valuation below which an advertiser does not participate in the auction increases.

In other words, some advertisers do not participate in the auction even for a small keyword management cost c because they know that their probability of winning is low. Only advertisers with very high valuations participate. Because of this, even the winner may have a negative payoff if two or more advertisers participate. In particular, when many advertisers compete and their valuations are all very high and close, the winner's payoff $v_i - p$ may not be enough to cover the cost c. Therefore, for a sufficiently large N, an advertiser will only participate if it is almost sure that no other advertiser participates.[13] The valuation threshold for participation, τ, is defined by the following equation:

$$\tau^{N-1}(\tau - R) - c = 0. \tag{14}$$

Because this equation does not have an analytical solution, to analyze the advertisers' strategies, we numerically calculate the solution. The rest of the analysis proceeds as before.

In Proposition 1 we discussed two forces, created by keyword management cost c, which affect the advertisers' utilities. On one hand, as the cost increases, an advertiser must pay a higher cost for participation. Therefore, increasing c could negatively affect the advertisers' utilities. On the other hand, if the cost increases, fewer advertisers participate in the auction. This softens the competition and could positively affect advertisers' utilities. Interestingly, if N is sufficiently large (specifically, if $N > 5$), the "softening competition" effect of keyword management cost may become the dominant force. In particular, when N is sufficiently large and the cost, c, is sufficiently small, advertisers' expected utilities may increase as c increases.

Next, we incorporate broad match into the model to understand its impact. We find that broad match decreases advertisers' expected utility even in the extreme case wherein there is no inaccuracy in bids through broad match. This is because, in the presence of many competitors, entry costs typically soften competition. Still because broad match eliminates keyword management costs, it increases competition and reduces profits. Yet all of the advertisers choose broad match in this case due to a prisoners' dilemma situation. Thus broad match increases the search engine's revenue relative to exact match.

4.4. Setting an Upper Bound for Broad Match Bid
In our main analysis, we consider broad match with automatic and flexible bidding, where the search engine may bid any amount on behalf of the advertisers. Search engines also give advertisers the option to specify a

Figure 5   Advertisers' Strategies as Functions of Keyword Management Cost, c, and Bid Error, E, When the Advertisers Can Specify an Upper Bound for Bids Under Broad Match
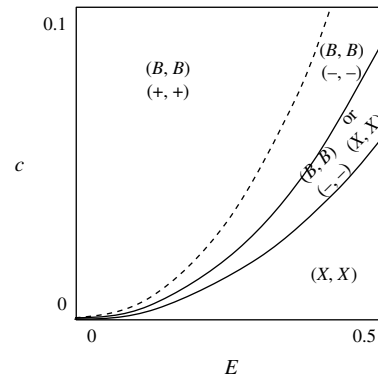
maximum bid under automatic bidding, which implies that the search engine cannot bid higher than this value on the advertiser's behalf (although lower bid values are allowed). This is an option that advertisers can choose to prevent bids that may be too high. In this section, we consider the situation wherein an advertiser can set an upper-bound Ω on how much the search engine can bid on its behalf under automatic bidding in broad match. If the search engine's estimate for an advertiser's valuation is larger than Ω, then the search engine bids Ω on behalf of the advertiser. We solve for the symmetric pure strategy Nash equilibria of the game.

We find that the equilibrium value of the upper bound is given by $\Omega = 1 - E/2$. Although the search engine's revenue decreases when advertisers set an upper-bound for broad match bids, broad match equilibrium still exists and it increases the search engine's revenue. On computing advertisers' expected utilities under broad match, we find that an advertiser's net utility is higher in this case than in the basic case presented in §2. Figure 5 shows advertisers' equilibrium strategies as a function of E and c. On comparing Figures 2 and 5, we can readily see that our results stay qualitatively the same when an advertiser can specify a maximum bid under broad match.

Some important features of sponsored search marketing also motivate other extensions. First, search engines typically use an advertiser-specific quality score to calculate an effective bid for an advertiser. This is obtained by multiplying the actual submitted bid by the quality score. We extend our model to allow for different bid multipliers for different advertisers. Second, we extend our analysis to multiple advertising slots. Third, when an advertiser uses broad match, the search engine can bid on its behalf. However, the search engine could misuse this ability and overbid on behalf of the advertiser. Note that there is no evidence that search engines are actually doing this. Also, our previous analysis with an upper bound on the

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[13] Note that this observation is applicable when only one advertising slot is available and only exact match is feasible. When there are multiple advertising slots, the number of participants will not decrease below the number of available slots.

bid effectively limits this behavior. Nevertheless, we examine this issue for its theoretical interest. We find that, in all three of these extensions, the key insights from the main model continue to hold. Analyses for these extensions are available in the online appendix.

5. Conclusions

In this paper, we study the strategic implications of keyword management costs and of broad match, a tool offered by search engines to reduce advertisers' costs, in sponsored search advertising. Our theoretical analysis offers useful insights on several issues of managerial significance.

We find that due to keyword management costs, fewer advertisers participate in the search advertising auction. This reduces competition among advertisers, which in turn reduces the amount an advertiser pays to the search engine conditional on winning. Therefore, search engines have an incentive to reduce the advertisers' keyword management costs. To this end, they offer tools such as broad match. Whereas broad match reduces advertisers' costs, the downside for them is that the bids that the search engine places on the advertisers' behalf may be inaccurate. Interestingly, for moderate levels of broad match error, a prisoners' dilemma arises, and competing advertisers choose broad match even though this lowers their profits. However, advertisers find broad match to be more profitable than exact match when broad match accuracy is high and keyword management cost is also high. Of course, if broad match accuracy is very low, advertisers will not use broad match.

This leads to the question of whether the search engine will be motivated to eliminate the inaccuracy in broad match bids. Interestingly, we find that it is in the search engine's interest to sufficiently increase the broad match level of accuracy so that advertisers adopt broad match. However, when advertisers already adopt broad match in equilibrium, there is no incentive to improve broad match bid accuracy any further. This is because the inaccuracy of broad match induces variation in advertisers' bids, with some bids being lower and others higher than advertisers' own valuations. The search engine can protect itself from the lower bids by stipulating a reserve price, and yet profit from the higher bids.

Extensions and Robustness. We have relaxed several of the simplifying assumptions of the basic model in extensions. Specifically, we extended the basic model to permit advertisers to choose their match strategies after learning their valuations, to allow advertisers to have asymmetric advertising costs, to allow for multiple advertisers, and to let advertisers stipulate an upper bound for broad match bids. The extensions provide certain interesting insights besides confirming the findings of Proposition 2. These results could, in turn, raise new questions.

One interesting question: If the search engine could set the value of broad match error, E, to maximize its revenue, what would be the *endogenous value of E*? On analyzing this issue, we find that, for any value of keyword management cost c, the search engine sets the optimal value of E such that the broad match equilibrium becomes the only equilibrium of the game. In this equilibrium, choosing broad match hurts both advertisers, i.e., a prisoners' dilemma arises. More specifically, with respect to Figure 2, for a given c, the value of E is chosen at the boundary of Regions B and C, on the side of B.

Next, because the search engine knows that its estimate of the advertiser's valuation has an error, and because the search engine knows the error distribution as well as the advertisers' valuation distribution, it can use an updated *Bayes estimate* to bid on behalf of the advertiser. Specifically, the search engine can use the estimate $\hat{x} = E[v \mid x]$ to make the broad match bid. We conduct this analysis and find that our original insights hold under this variation as well.

The benefit of broad match for the advertiser also implies that the search engine could *charge a fee* for providing broad match as a service to advertisers. We find that the search engine can set the optimum broad match fee to extract all of the surplus that advertisers make from using broad match. With respect to Figure 2, Region A disappears (i.e., is replaced by Region B), whereas Regions C and D, and their boundaries, remain unchanged.

In our model, we assume that an advertiser who chooses exact match realizes his valuation for a keyword after the keyword has been searched. We also assume that he can place a bid (or choose to stay out of the auction) at this time. We have made these assumptions for expositional simplicity. However, given that sponsored search auctions are run within milliseconds after a keyword search, one could argue that an *advertiser does not have the opportunity to react* due to the limited time available after a keyword search. In fact, advertisers pre-specify the set of keywords they want to be exact matched (along with the corresponding bids, ad copies, etc.), to the search engine. Because the advertisers pre-identify these keywords, they can also determine keyword valuations based on past performance, sales margins, etc. Thus the advertiser pre-specifies the set of keywords and the corresponding bids under exact match. Because pre-specified automated actions can be executed in the time frame after a keyword is searched and before results are displayed, if a searched keyword matches one of the keywords in the set, the advertiser is included in the auction with the corresponding bid.

Our model and analysis are seamlessly applicable to such a situation with a slight adjustment to the structure

of the game, as described below. Consider a universal set U of keywords in which an advertiser would be interested. In Stage 2 of the game, the advertiser must choose between broad match and exact match. If the advertiser chooses broad match, he will be entered into the auction for whichever keyword is searched from the set U, with the search engine determining the bid. However, if the advertiser chooses exact match in Stage 2, he also provides a set S of keywords to the search engine, along with the corresponding bids, ad copies, etc., for these keywords, with the understanding that only if one of the keywords in the set S is searched, the advertiser is entered into the auction with the corresponding specified bid; otherwise he is not entered into the auction. The advertiser incurs a cost c for every keyword thus specified (though the model focuses on only one keyword that will be subsequently searched, and which may not be in this set). The keywords that the advertiser includes in set S are those with a valuation above a threshold (τ_B derived in §3, Case 2). The rest of the game proceeds in the same way.

This game structure assumes that the advertiser knows the valuations for the keywords before the keyword is searched, which is different from our assumption in §3 that valuation is revealed after a keyword is searched. However, note that the two models are mathematically equivalent. Furthermore, the set of keywords, S, and the corresponding bids, can be re-specified as frequently as desired by the advertiser (e.g., whenever the advertiser realizes that his valuation for a keyword has changed) although he incurs the cost c; this bid will then be used every time the keyword is searched until the next update by the advertiser. This structure continues to capture the aspect that valuations can change more frequently than match strategy choice.[14]

Further Research. Our work is a first step towards studying the impact of keyword management costs and the widely used cost-reduction tool, broad match, on search advertising auctions. There are several opportunities for further research. For example, we do not model budget constraints in this paper. Modeling budget constraints and understanding their effects on advertisers' adoption of broad match could lead to valuable insights. Next, as we mentioned earlier, firms often use marketing agencies to execute search engine

advertising on their behalf. It would be interesting to investigate how the insertion of an intermediary, which may introduce agency considerations, affects our insights. Some marketing agencies also generate exhaustive lists of keywords on which to bid, given a starting seed set of keywords, and also estimate advertisers' valuations for these keywords. One of our key results is that the search engine, when estimating valuations, does not want to improve the accuracy of the estimation beyond a certain point. It would be interesting to understand whether intermediaries have the same or different incentives, and why. Future work can also study other tools provided by search engines to advertisers. One such tool is bid throttling, where the search engine ensures that the limited budget of an advertiser lasts a specified period of time (determined, for example, by the campaign duration) by adjusting the bids and smoothing the spending over time. Finally, another promising avenue for future research would be an empirical study of the effect of keyword management costs, and other cost reduction tools, on advertisers' strategies.

Supplemental Material

Acknowledgments

Appendix

Proof of Proposition 1
To compute the threshold value τ, we focus on Advertiser 1. In equilibrium, Advertiser 2 would participate in the auction if and only if its valuation is at least τ. Note that because there is one keyword in the auction, $1 - \tau$ is the probability of participating in the auction. Furthermore, Advertiser 1 participates in the auction if and only if its expected utility from participating in the auction is greater than or equal to 0. Therefore, it follows that the expected utility of Advertiser 1 from participating in the auction is 0 when its valuation is τ. We use this observation to calculate the equilibrium value of τ. Note that we solve under the assumption that $\tau \geq R$. Otherwise, it is not worthwhile for an advertiser to participate in the auction.

The expected utility (ignoring the keyword management cost) of an advertiser with valuation $v \geq \tau$ who participates in the auction can be calculated as follows. Let the competing advertiser's valuation be denoted by u. For u between 0

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[14] The corresponding adjustment in the game for §4.1 is that in Stage 2, the advertiser must specify a set of keywords, S, along with bids, for which he should be exact matched, and a set S' for which he should *not* be entered into the auction. For the keywords in $U - S - S'$, he will be broad matched (i.e., he will be entered into the auction with automatic bidding by the search engine). By contrast, in the main model in §3, the advertiser must choose to be broad matched with every keyword in U, or exact matched with the keywords he specifies in S. The main model is arguably a more accurate reflection of reality.

and τ, which happens with probability τ, the competing advertiser will stay out of the auction and the focal advertiser will obtain the slot and obtain surplus $v - R$. The expected utility of the focal advertiser in this case is $\tau \times (v - R)$. For u between τ and v, the competing advertiser will bid u, such that the focal advertiser will win the auction and make a payment of u. The expected utility of the focal advertiser in this case is $\int_\tau^v (v - u)\, du = (v - \tau)^2/2$. For u between v and 1, which happens with probability $1 - v$, the competing advertiser will bid higher such that the focal advertiser will lose the auction and obtain 0. The expected utility of the focal advertiser in this case is $(1 - v) \times 0 = 0$. In all of the above cases, the advertiser will incur cost c. Therefore, the expected utility, after incorporating the keyword management cost, is given by

$$\tau \times (v - R) + \frac{(v - \tau)^2}{2} + (1 - v) \times 0 - c. \qquad (15)$$

As noted before, if $v = \tau$, the expected utility is zero. Therefore, we have $\tau(\tau - R) - c = 0$, and obtain

$$\tau = \frac{R + \sqrt{R^2 + 4c}}{2}. \qquad (16)$$

Given τ, the search engine's expected revenue is given by

$$(1 - \tau)^2 \times \left(\tau + \frac{1 - \tau}{3}\right) + 2\tau(1 - \tau) \times R. \qquad (17)$$

The first term in the above expression corresponds to the case wherein both advertisers participate; the second term corresponds to the case wherein only one advertiser participates. Substituting for τ, the search engine's revenue as a function of c and R is

$$-\frac{1}{12}\left(-2 + R + \sqrt{4c + R^2}\right)$$
$$\cdot \left(2 - 4c + R + 4R^2 + \sqrt{4c + R^2} + 4R\sqrt{4c + R^2}\right). \qquad (18)$$

This revenue, as a function of c, is maximized at

$$R^*(c) = \tfrac{1}{4}\left(3 - \sqrt{1 + 8c}\right), \qquad (19)$$

which is a decreasing function of c. Search engine's revenue, with endogenous reserve price R, as a function of c is

$$\frac{1}{96}\left(26 + 10\sqrt{1 + 8c} - \sqrt{10 + 72c - 6\sqrt{1 + 8c}}\right.$$
$$\left. + 3\sqrt{2 + 16c}\sqrt{5 + 36c - 3\sqrt{1 + 8c}} + 8c(-27 + 4\sqrt{1 + 8c})\right), \qquad (20)$$

which is a decreasing function of c.

Conditional on winning, Advertiser i's expected payment is

$$\frac{1}{64v} \cdot \left(10 - 24c - 6\sqrt{1 + 8c} + 3\sqrt{10 + 72c - 6\sqrt{1 + 8c}}\right.$$
$$\left. - \sqrt{2 + 16c}\sqrt{5 + 36c - 3\sqrt{1 + 8c}} + 32v^2\right), \qquad (21)$$

which is a decreasing function of c for any $v > 0$. However, Advertiser i's expected utility (not conditional) is

$$\frac{1}{64}\left(-10 - 40c + 6\sqrt{1 + 8c} - 3\sqrt{10 + 72c - 6\sqrt{1 + 8c}}\right.$$
$$\left. + \sqrt{2 + 16c}\sqrt{5 + 36c - 3\sqrt{1 + 8c}} + 32v^2\right), \qquad (22)$$

which is a decreasing function of c for any $v > 0$. Finally, note that if either of the advertisers participates in the auction, because of a second price auction, the allocation is efficient. Inefficiency happens only if both advertisers decide not to participate in the auction. For endogenous reserve price R^* we have

$$\tau = \tfrac{1}{8}\left(3 - \sqrt{1 + 8c} + \sqrt{10 + 72c - 6\sqrt{1 + 8c}}\right), \qquad (23)$$

which is an increasing function of c. In other words, advertisers participate less frequently as c increases.

Proof of Proposition 2
Note that for any reserve price, $EU_{X,B}$ and $EU_{X,X}$ are decreasing functions of c. On the other hand, $EU_{B,B}$ is constant in c and $EU_{B,X}$ is increasing in c. Therefore, as c grows, independent of the opponent's strategy, each advertiser prefers to use broad match. For sufficiently large c, both advertisers use, and benefit from, broad match in equilibrium. The boundary between Region A and Region B in Figure 2 is defined by the equality $EU_{X,X} = EU_{B,B}$. Note that $EU_{X,X}$ is constant in error E whereas $EU_{B,B}$ is a decreasing function of E. Therefore, advertisers benefit from broad match only if E is sufficiently small.

Similarly, if c is sufficiently small, both advertisers use exact match. Both advertisers using exact match is the unique equilibrium if each advertiser prefers exact match even if its opponent is using broad match. In other words, the boundary between Regions C and D in Figure 2 is defined by $EU_{B,B} = EU_{X,B}$. Finally, both firms using broad match is the unique equilibrium if each advertiser prefers broad match even if the opponent uses exact match. In other words, $EU_{B,X} = EU_{X,X}$ defines the boundary between Regions B and C in Figure 2. Note that $EU_{X,X}$ is constant in E whereas $EU_{B,X}$ is decreasing in E. Therefore, broad match is the unique equilibrium only if E is sufficiently small.

A prisoners' dilemma situation arises if $EU_{B,X} > EU_{X,X}$, both advertisers use broad match in (unique) equilibrium, and $EU_{X,X} > EU_{B,B}$, broad match decreases advertisers expected utilities. These two inequalities hold in Region B in Figure 2.

To show that an asymmetric equilibrium cannot exist, using basic calculus we can verify that the inequality $EU_{X,X} \leq EU_{B,X}$ implies the inequality $EU_{X,B} \leq EU_{B,B}$. In other words, if an advertiser prefers broad match to exact match when the opponent uses exact match, he also prefers broad match to exact match when the opponent uses broad match.

Proof of Proposition 3
Using Myerson's optimal auction framework,[15] we know that the optimum reserve price is the value of R such that $R - (1 - F(R))/f(R) = 0$, where $F(\cdot)$ and $f(\cdot)$ are the cumulative distribution function and probability density function of bidders' valuation distribution, respectively. Therefore, optimum reserve price in a broad match equilibrium is $\frac{1}{2}$, whereas optimum reserve price in an exact match equilibrium is $\frac{1}{4}(3 - \sqrt{1 + 8c})$, which is less than $\frac{1}{2}$ for any $c > 0$. Therefore, broad match increases optimum reserve price.

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[15] Myerson (1981) shows that if $f(z)/(1 - F(z))$ is a monotone nondecreasing function, the optimal reserve price is the value of R where $R - (1 - F(R))/f(R) = 0$.

The search engine's revenue in broad match equilibrium is

$$\frac{1}{60}(25 + 4E^3), \tag{24}$$

which is an increasing function of E. The revenue in an exact match equilibrium is

$$\frac{1}{96}\left(26 + 10\sqrt{1+8c} - \sqrt{10 + 72c - 6\sqrt{1+8c}}\right.$$
$$\left. + 3\sqrt{2+16c}\sqrt{5 + 36c - 3\sqrt{1+8c}} + 8c(-27 + 4\sqrt{1+8c})\right). \tag{25}$$

The revenue in exact match is a decreasing function of c and is maximized at $\frac{5}{12}$ when $c = 0$. On the other hand, revenue in broad match is an increasing function of E and is minimized at $\frac{5}{12}$ when $E = 0$. Therefore, for $E > 0$ or $c > 0$, search engine revenue in broad match equilibrium is always greater than the revenue of exact match equilibrium.

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