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In-Store Advertising by Competitors

Dmitri Kuksov,[a] Ashutosh Prasad,[a] Mohammad Zia[a]

[a] University of Texas at Dallas, Richardson, Texas 75080

Contact: dmitri.kuksov@utd.edu (DK); aprasad@utd.edu (AP); m.zia@utd.edu (MZ)

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Abstract. Conventional advice to firms in competitive markets is to raise barriers against competitive poaching of their customers. However, we see instances where a firm enables competitor advertising to its customers. For example, Walmart hosts banner ads for TVs from Sears to customers searching for TVs on Walmart.com, risking a loss of customers in exchange for a commission. This paper explores whether and under what conditions allowing competitor advertising in one's store may be a beneficial strategy. We analyze a duopoly market where customers are heterogeneous in search costs, information, and preferences. We find that hosting a competitor ad for an undifferentiated product can mitigate price competition and boost profits of both firms if the advertising commission is high enough. Otherwise, hosting competitor advertising may decrease the profits of both firms. Thus, there is no conflict of interest between firms in advertising and in setting the ad commission level. Yet the host prefers more efficient ads while the advertiser does not. Furthermore, the equilibrium outcome is asymmetric, with only one store featuring ads of the other. If stores are sufficiently differentiated in marginal costs of the product, the cost disadvantaged store will be the host. We show that the results are robust to displaying price in the ad, to different commission structures, and to customer uncertainty about the commission rate.

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1. Introduction

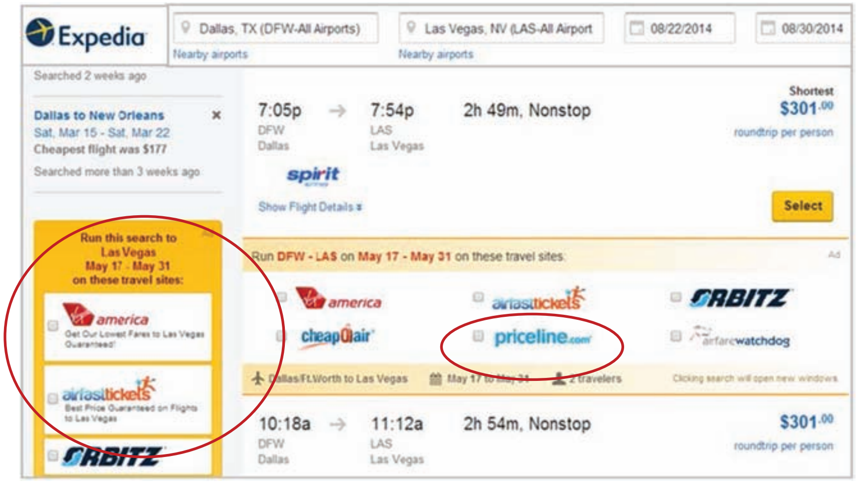
In competitive markets, firms should take steps to reduce competition. The conventional advice is to raise barriers to protect customers from competitive poaching, e.g., to increase product differentiation, improve customer loyalty, and restrict customer search opportunities.[1] However, we see instances that appear to contradict this advice. One such practice is when a firm, who we will call the host store or Store 1, sells its in-store advertising space to a competitor, who we will call the advertiser or Store 2. Rather than obscuring the presence of rivals or obfuscating search, this allows uninformed customers to become aware of substitute offerings in the marketplace and possibly migrate to the competing store. Naturally, the host store may then benefit from a commission for customers it sends to the competitor. Yet if Store 2 is willing to pay a certain commission to gain a customer, would not Store 1 be losing more from foregoing its own sale than it makes in commission?

An example of this type of in-store competitor advertising is when an online shopping website such as *Target.com*, *Expedia.com*, or *Amazon.com* sells ad slots to other websites. While advertisers sometimes offer complementary or at least differentiated products, sometimes these ad spaces are acquired by online retailers who are in direct competition with the host store. For example, Figure 1 (top) shows *Expedia.com* suggesting that the customer who has searched for a flight run the same search on *Priceline.com*, a website owned by a competitor. This is a cost-per-click (CPC) type of online advertising arrangement in which *Expedia* earns a commission (intermediated by an ad-network, such as Google) for every customer who clicks on this ad and is redirected to *Priceline*'s website. As another example, when a customer searches for "TV" on *Target*'s website (Figure 1, middle), *Target.com* displays TV-related sponsored ads in the sidebar that include *Frys.com*, a direct competitor of *Target* in this category. Similarly, *Amazon.com* (Figure 1, bottom) shows related ads from competing retailers to a customer who is shopping for jeans.[2]

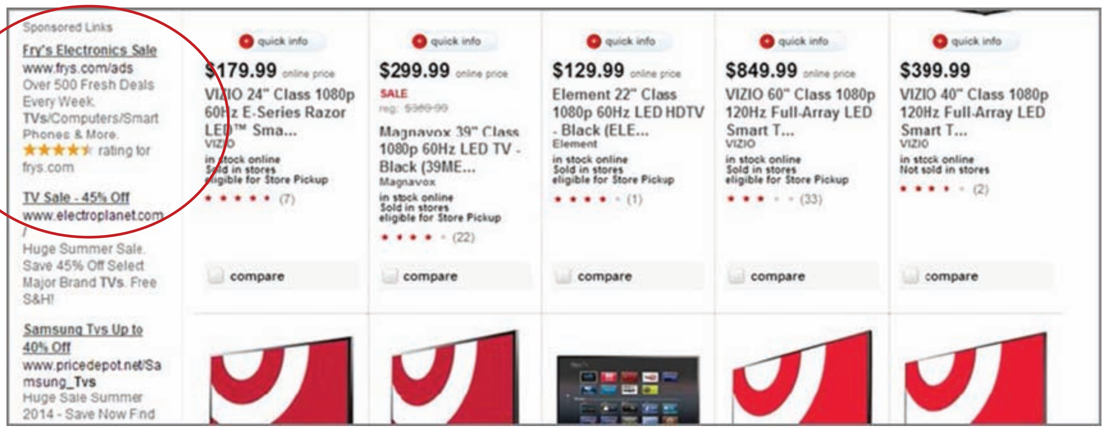
It is unclear a priori whether selling in-store advertising space to a rival can be beneficial for the firm given that the commission is at a level that the competitor is willing to pay. The ad creates awareness and entices possibly uninformed customers to visit the rival store and compare prices. In some instances, these customers will find a better deal at the competing store and end up buying there. Consequently, one

Figure 1. (Color online) Examples of In-Store Competitor Ads

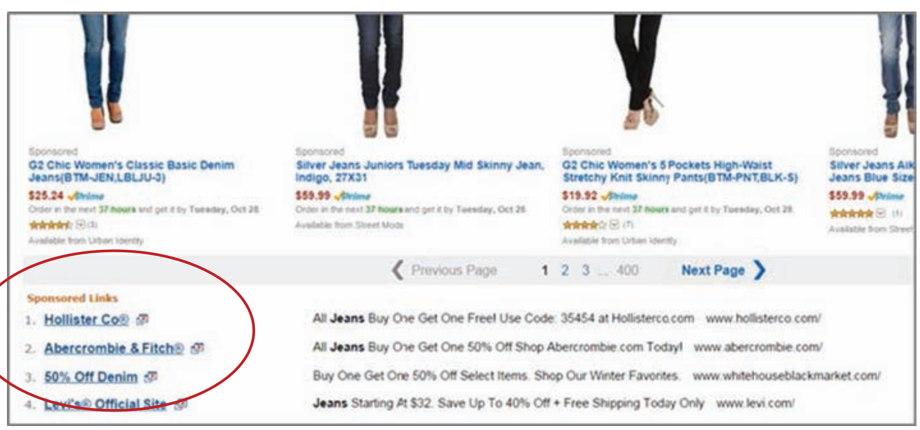
Ads encouraging *expedia* customer to check rates on competitor websites.



Target.com displays *frys.com* ad when customer searches for "*TV*"



Amazon customer searching for "*jeans*" sees competitor Ads



Note. These examples were obtained by conducting searches on the corresponding websites on April 28, 2014.

would expect a firm to find it optimal to sell in-store advertising space to a competitor only if the advertising commission is high enough to compensate for the expected loss of sales to the competition. Still, too high an advertising commission decreases the incentive of the rival firm to buy ad space because the expected earnings from incoming new customers are outweighed by advertising expenses. One may conjecture that since increased customer search should result in higher competition and lower industry prices, there is no way to re-allocate total industry profits through commissions so that both firms benefit. Following this chain of logic, one could conclude that, depending on the commission level, in-store advertising by a competitor will hurt the host firm or its competitor, the advertiser.

Of course, a firm may be interested in allowing other firms' advertising of differentiated or, even better, complementary products. Then advertising by direct competitors represents, so to speak, collateral damage.[3] It may also be that willingness to inform customers about competitors signals low price. It may also be that the resulting increase in price visibility (perhaps, hosting competitor's ads makes it easier for the firms to document each other's prices) could make collusion easier or encourage competitors to differentiate.[4] In addition, agglomeration of ad links may turn a store into a one-stop or first-stop destination, an explanation again relying on ads of differentiated products, since links to differentiated products act as increased assortment in the first store.[5]

In this paper, we examine a more direct and, in a sense, parsimonious explanation of the competitors' willingness to cooperate in advertising by carefully examining the effect of allowing competitor ads for homogenous products. That is, we note that the argument presented above about ads increasing competition is incomplete because the potential increase in consumer search is not the only effect of advertising. While advertising does facilitate consumer search, the commission changes the pricing incentives of both firms. This is because the price that customers see at the host store affects their willingness to search, and thereby affects the ad revenue of the host store. This gives an incentive to the host store to increase price: A customer lost due to a price increase is not only a loss but a commission gained. In turn, the competitor may increase price as well. Thus, we have the opposing effects of increased competition due to more consumer search and reduced competition due to the host retailer's incentive to maximize ad revenue. Which will dominate is not immediately clear.

Analysis of the effects of competitor advertising allows us to answer the following research questions. When is allowing competitor advertising beneficial to the host store and when is it beneficial to the competitor? What commission rate is best for the host store and for the advertising store? What is the optimal ad efficiency, i.e., the extent to which the ad encourages or facilitates customer search, for the two firms? Is there a conflict of interest between the competitors in these decisions (i.e., are the optimal values different)? Finally, how should the display of a competitor's ad affect firms' pricing strategies?

To formally analyze the above questions, we develop a model with two firms selling an undifferentiated product to a consumer population consisting of *Shopper*, *Loyal*, and *Semi-Loyal* segments. The segment of *Shoppers* consists of customers who are informed about both firms and have zero search costs regardless of advertising. The segment of *Loyals* consists of customers who go to their preferred store and only consider buying there.[6] These customers are not affected by advertising. Finally, the segment of *Semi-Loyals* also go to their preferred store, but consider a costly search for a better price at the competitor's store if they see an ad for it. One can think of the *Semi-Loyal* customers as unaware of the other store or having too high a search cost for shopping there without the ad, but the ad reduces their search cost to the point that the decision to search or not becomes worthy of consideration.[7] The presence of *Semi-Loyal* customers allows us to consider the effect of competitor advertising on prices, consumer search, and profits.

1.1. Overview of the Main Results

Our analysis shows that even if stores are symmetric, an asymmetric equilibrium results when one store hosts its rival's ad if the commission rate is high enough. Symmetric cross-advertising is not an equilibrium because a store's profit is higher when it is the only host than when each store shows ads of the other. Thus, either store would withdraw from advertising to become the sole host in the market. We thus discuss the effects of advertising when only one store shows ads of the other.

An interesting result that emerges from our analysis is that the equilibrium profit of each store is U-shaped in the commission rate. When the commission rate is low, the host store (Store 1) decreases prices so that none of its customers find it beneficial to do a costly search and visit the advertiser (Store 2). Then Store 2 focuses more on its *Loyal* customers, increasing prices to extract a higher surplus from them. Thus, when the commission rate is low enough, in-store advertising distorts prices at both stores, but does not change equilibrium profits compared to a no-advertising benchmark. This price distortion benefits the *Shopper* segment and Store 1's *Loyal* segment but hurts Store 2's *Loyal* segment.

When the commission rate is in an intermediate range, competitor advertising results in lower equilibrium profits for both stores. The intuition is the following: As the commission rate increases, the host store has an incentive to allow more of its *Semi-Loyal* customers to search by raising prices. Therefore, Store 1 now mixes between two pricing strategies: (i) charging higher prices and supplementing the profits on sales to its *Loyal* customers with commission on *Semi-Loyal* consumer search, and (ii) charging lower prices to prevent search by *Semi-Loyal* customers. This is an impetus toward higher prices in Store 1. Yet Store 2 now has a higher incentive to price low and attract not only *Shoppers* but also the *Semi-Loyal* customers of

Store 1 (if Store 1's price was above their reservation price). Therefore, Store 2 finds it optimal to price lower with higher probability and Store 1 has to take this into account. Thus, the effect of the increased commission on Store 1's price distribution is that some of the distribution weight is moved up and some down; Store 2's price distribution is moved down. The net effect is that as the commission rate increases in an intermediate range, competition intensifies.

However, if the commission rate is sufficiently high, higher commission starts to increase the host's prices because the incentive to induce consumer search and earn commission starts to outweigh the incentive to fight the advertiser for a larger percentage of searchers. The advertiser can then price higher. In this case, the prices and profits of both firms start to increase in the commission rate.

When the commission rate is sufficiently high, both stores' profits are higher with advertising than without. This is a key result in explaining competitive advertising. Again, the intuition is that the host store is willing to encourage search in exchange for a commission by increasing its prices, thereby increasing the benefit that customers can expect from engaging in the costly search of the rival store. The advertiser, on the other hand, benefits by being able to increase its price as it knows that incoming customers from the host store have observed a high price (otherwise, they would not engage in costly search). Thus, when the commission rate is high enough, the host store benefits from high commissions and the advertiser benefits from higher prices and sales. We find that the host store's profit is always higher than the advertiser's profit.

We also show that there is no conflict of interest between the firms about the level of advertising commission because a marginal change in the commission rate never increases profits of one while decreasing profits of the other.

Another interesting set of results concerns the search cost of *Semi-Loyal* customers. This is managerially relevant, since the search cost of consumers exposed to the ad can be influenced by ad design. By contrast to our results about the optimal commission rate (for both stores), we show that stores can have conflicting incentives when it comes to affecting this search cost. Specifically, if the commission rate is high, so that both firms enjoy increased profit, the host store wants to present customers with the most enticing advertisement while the advertiser's profit is inverted-U-shaped in the search cost. The intuition behind this result is that for the advertiser, a high ad efficiency increases advertising commission costs disproportionately more than its revenue from the new customers.

We examine several extensions. In one, we consider stores with asymmetric marginal costs and find that if the difference in marginal costs is sufficiently large, the host store will tend to be the store with the marginal cost disadvantage. Other extensions show that the profitability of in-store competitor advertising is robust to scenarios where the commission rate is unobserved by customers, the advertising contract is pay-per-action, an intermediary sets the commission terms, product price is displayed in the ad or when a fraction of the *Shopper* segment also clicks the ad link.

1.2. Related Literature

There is an extensive literature on the competitive effects of advertising. While early economists argued that advertising could increase prices due to the advertising costs, Stigler (1961) pointed out that advertising decreases consumer search costs and can therefore increase competition and reduce profits. As Benham (1972) noted, it is important to distinguish between different types of advertising: Persuasive advertising and advertising about product characteristics could increase prices while advertising about price could decrease prices (see also a review in Bagwell 2007). Lal and Sarvary (1999) argued that the decreasing-search-cost effect of the Internet could result in more or less intense competition depending on whether the effect is larger on the information about product characteristics or about prices.[8] Amaldoss and He (2010) show that informative advertising may result in a price increase for horizontally differentiated competing products. Although persuasive advertising may increase prices through increasing customer valuations, Chen et al. (2009) show that persuasive advertising may increase competition when it is directed at the competitor's customer base (this is called combative advertising). In a vertical channel setting, Shaffer and Zettelmeyer (2009) consider in-store displays that act as persuasive advertising and show that a retailer may not wish to allow a manufacturer to do in-store advertising,[9] while Dukes and Liu (2010) show that in-store media used by manufacturers may have a channel-coordination role; Desai (1997) considers how channel contract structure affects advertising decisions. Instead, we consider in-store advertising of direct competitors.

This paper is also related to the literature on competitive pricing strategies when customers have imperfect price information and heterogeneous search costs (e.g., Varian 1980, Narasimhan 1988, Stahl 1989). The papers by Varian (1980) and Narasimhan (1988) consider two consumer segments: One segment considers buying from one store only while the other buys from the store with the lowest price. In this paper, we consider three segments of customers, i.e., the *Loyal* and *Switcher* segments as in Narasimhan (1988), but also a third segment of *Semi-Loyals* who, when not exposed to the advertising, behave as the *Loyal* segment, but when exposed to advertising, make a rational decision on whether to incur the cost of comparing prices. This third segment essentially behaves as a segment with search costs as in Stahl (1989).

Inasmuch as our paper shows that in-store advertising (given a high enough commission rate) decreases competition, it relates to literature on other information and pricing strategies that reduce competition. For example, price matching guarantees can increase or decrease competition (Chen et al. 2001, Moorthy and Winter 2006, Moorthy and Zhang 2006).

The rest of the paper is organized as follows. Section 2 presents the formal analytical model. Section 3 presents the analysis of this model and derives the results about the effect of advertising and the commission level on firms' profits. Section 4 considers five extensions and variations of the main model to understand the robustness of the main results. Section 5 concludes with some further discussion.

Formal proofs, long algebraic expressions, and the boundaries delineating the regions for each of the equilibrium cases are provided in the appendices.

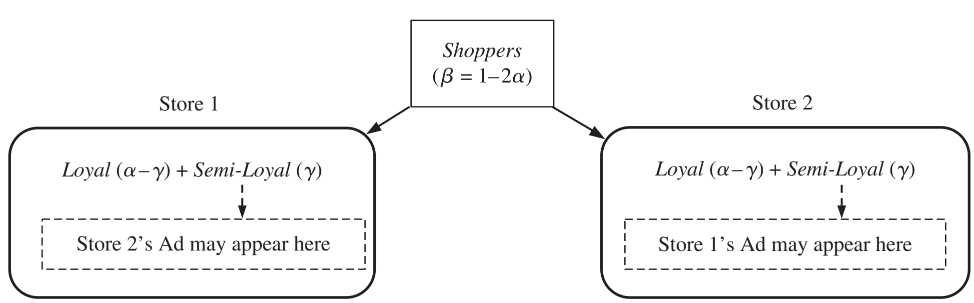
2. Model

Two retail firms sell perfectly-substitutable products at their respective stores, Store 1 and Store 2, and compete on price. We begin by assuming constant and equal marginal costs c. We consider a commission-based advertising wherein a store hosts advertising for the competitor and risks losing customers by making them aware of it. We formalize the advertising and pricing decision process as a three-stage game. In the first stage, stores simultaneously decide whether to allow competitive advertising. In the second stage, stores simultaneously decide whether to advertise if the other store allows it. We assume that the advertiser pays a commission t for every potential customer it receives through the ad link from the host store. For now, we assume that the commission rate is exogenous. This may be reasonable because advertising is often arranged by a third party who may set the commission rate based on the value of advertising in a wider range of industries; however, we later consider stores' incentives to choose the commission rate that maximizes profits. Finally, in the third stage of the game, stores simultaneously set prices.

A unit mass of consumers with valuation v for the product is comprised of three types: *Loyals*, *Semi-Loyals*, and *Shoppers*. Each store has a *Loyal* segment who shops only at that store and buys if the price is below v. *Loyal* consumers are uninfluenced by ads and never search at the other store (perhaps because of their high search cost). Next are the *Semi-Loyal* consumers, who visit their preferred store first and if not exposed to advertising, behave like *Loyal* customers. However, if they see the competing store's advertising, they face a search cost s, which they incur if they visit the other store and learn its price, and then buy at the store with the lower price. Finally, a *Shopper* segment has zero search cost of obtaining prices at both stores and thus buys from the store with the lowest price. In case of a tie in utility (equal prices observed), consumers' decisions are derived to result in equilibrium. The *Loyal* and *Semi-Loyal* segment sizes are $\alpha - \gamma$ and γ per store, respectively, and the total mass of the *Shopper* segment is β, where $\beta = 1 - 2\alpha$. Figure 2 presents a graphical illustration of the market and Table A.1 in the appendix summarizes the main notation.

This segmentation structure captures the reality that some consumers purchase at a particular store because they are uninformed about competing stores or the costs of searching them is too high. The role of competitive advertising then is to inform consumers about the competing store or to reduce the cost of searching there. Search cost heterogeneity exists between the $\alpha - \gamma$ perfectly *Loyal* customers whose behavior is uninfluenced by advertising and who buy at their preferred store, and the γ *Semi-Loyal* customers whose search costs can be reduced by competitive advertising to entice them to search at the rival store if they expect to find a sufficiently better price (search cost heterogeneity also exists between the *Semi-Loyal* and the *Shopper* segment since search is costless for the *Shopper* segment). Another equivalent interpretation is that the *Loyal* segment has a significant store preference (for example, they do not trust the other store), while the *Semi-Loyal* segment's store preference is only due to the search costs or information affected by the presence of an advertisement.

Figure 2. The Market and Customer Segments



3. Analysis

To find the subgame perfect Nash equilibrium of the game defined in Section 2, we analyze three subgames where advertising is done by neither, one or both stores. The former is conceptually the "default" situation from which stores consider and decide whether to allow competitor's advertising or whether to advertise. Thus, we refer to this subgame as the "Benchmark" case. Then we analyze the subgame where advertising occurs in only one store. We call this "Asymmetric Competitor Advertising." Finally, we analyze the subgame where both stores cross-advertise, which we call "Symmetric Competitor Advertising."

We compare profits in these subgames to see the incentive to host (allow) advertising and to advertise (when allowed). If one store decides to host but the other decides to not advertise, the decision to host is irrelevant. Thus the two stages of hosting and advertising decisions lead to only three conceptually different possible subgames in the pricing stage. Comparison of profits between subgames yields the equilibrium of the full game.

3.1. No In-Store Advertising by Competitors ("Benchmark" Subgame)

If in-store competitor advertising is not implemented, the game between the two stores is equivalent to Narasimhan (1988). *Loyal* and *Semi-Loyal* customers combine to behave as *Loyal* to their respective stores (mass α per store), while *Shoppers* (total mass of β) buy at the store with the lowest price. Therefore in equilibrium, both stores use mixed price strategies, drawing their prices from the price distribution $F^B(p)$ defined as follows:

$$F^B(p) = 1 - \frac{\alpha(v-p)}{\beta(p-c)}, \quad \text{for } p \in [p_{0B}, v], \tag{1}$$

$$\text{where} \quad p_{0B} = \frac{\alpha v + \beta c}{\alpha + \beta}, \tag{2}$$

and obtain profit of $\alpha(v-c)$ each. The intuition for this outcome is that each store is guaranteed the demand α from its customers who do not shop around as long as the price is at or below v, and is willing to fight (by reducing price) for the demand from *Shoppers* until the margins lost on non-*Shoppers* equate to the expected profit from *Shoppers*. At the top of the price distribution, each store is guaranteed to have sales to non-*Shoppers* only, while at the bottom of the price distribution, each store is guaranteed to have the demand from its own non-*Shoppers* and all of the *Shoppers*.

3.2. Only One Store Advertises (Asymmetric Competitor Advertising Subgame)

When Store 1 hosts in-store advertising for Store 2, it may lose demand from its *Semi-Loyal* segment to Store 2. Customers in this segment will follow the ad and learn the other store's price if their expected benefit from search covers the cost of search s. Thus, they will visit the advertiser if the host's price is higher than a *reservation price* r that solves the following equation between the expected benefit of search and the cost of search:

$$s = \int_{p_0}^{r} (r-p)\, dF_2(p), \tag{3}$$

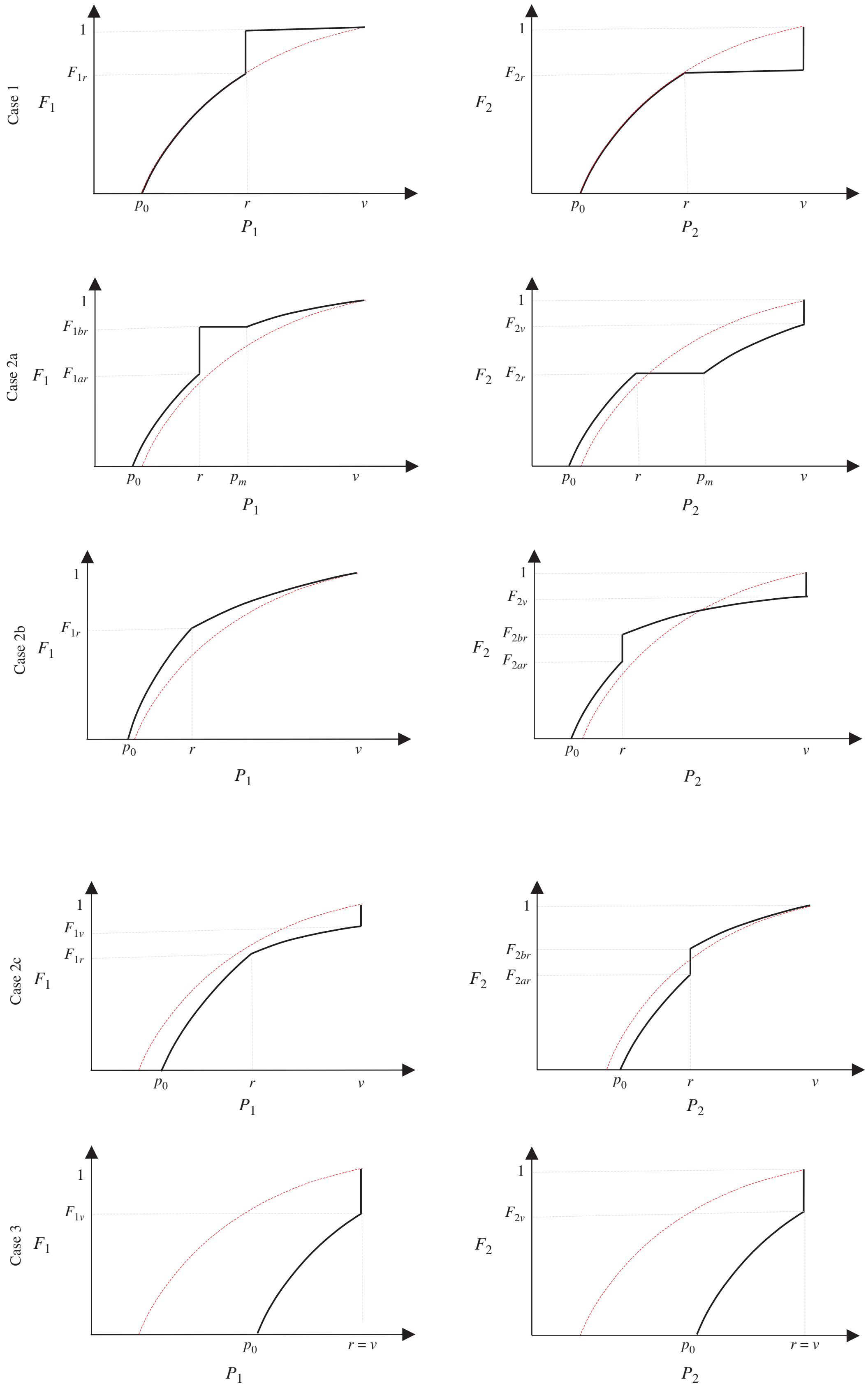
where $F_2(p)$ is Store 2's equilibrium price distribution and p_0 is the minimum price in that distribution.

Because this subgame is asymmetric, the price distributions $F_i(p)$ for each store $i \in \{1, 2\}$ will be asymmetric. Furthermore, there may be mass points and empty intervals in the interior range of the price distributions due to the search behavior of the *Semi-Loyal* customers. For example, if Store 1's price is slightly below r, *Semi-Loyal* customers do not search. However, if Store 1's price is slightly above r then they all search at Store 2, and Store 2 pays a commission of $t\gamma$ to Store 1. This discontinuity in the behavior of the *Semi-Loyal* segment and, thus, in the profit functions at $p_1 = r$, causes mass points and empty spaces in price distributions. Although we provide formal proofs in the appendix, to understand the outcomes, it is useful to intuitively discuss the incentives that stores face in setting the price, the equilibrium equations, and how price distributions are affected by the presence of the ad in Store 1 and by the level of the commission Store 2 pays Store 1 per potential consumer it receives. For ease of discussion, we divide the consideration into three cases discussed below. Figure 3 illustrates the price distributions and Figure 4 depicts the equilibrium regions for each case.

3.2.1. Case 1: Low Commission Rate (t) and High Search Cost (s).[10] This case most resembles the benchmark case. Consider high search cost s for *Semi-Loyal* customers, such that they search only if they see that Store 1's price p_1 is very close to their valuation v (of course, if search costs are even higher, this case is no different from the benchmark model for any commission rate).[11] If the commission rate is low enough, and considering Store 2's price distribution as exogenous for the moment, Store 1 would prefer not to price above the reservation price r of the *Semi-Loyal* segment and to move its distribution of prices to lie in the range from r down to the minimum price p_0 it previously charged (see Figure 3, Case 1).

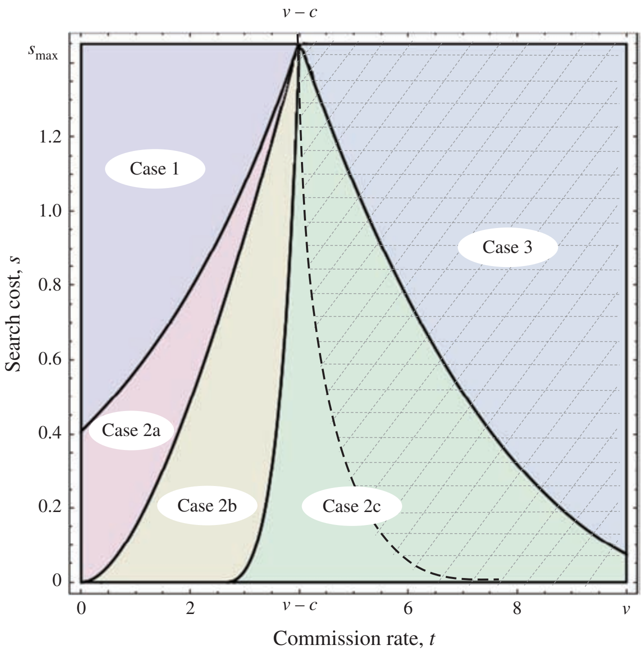
Store 2, anticipating the above adjustment by Store 1, should then adjust its price strategy as follows: First, there is no reason for it to charge prices strictly in the range from v down to r, because a price *just* below v captures the same demand. At the same time, it could make sense to price below r to fight for the *Shopper* segment but only if the chances of "winning" that segment justify the lower margin on its captive (*Loyal*) customers. The latter trade-off is exactly the same that

Figure 3. (Color online) Five Pricing Cases in Asymmetric Subgame



Note. Dashed line is the benchmark.

Figure 4. (Color online) Cases in Asymmetric Subgame



Note. Advertising is profitable for both stores in the Meshed region ($\alpha = 0.3$, $\gamma = 0.2$, $v = 10$, $c = 6$).

Store 2 faced in the benchmark case (because Store 1's pricing strategy ensures that its *Semi-Loyal* segment does not search).

Thus, the distribution of prices in Store 1 below the reservation price of its *Semi-Loyal* customers and where Store 2 prices with positive density is exactly as in the benchmark subgame. The only possibility for this to be an equilibrium is that Store 1 moves all of the price realizations above the *Semi-Loyal* customers' reservation price to a mass point at that reservation price while Store 2 moves all price realizations at or above that reservation price to a mass point at consumer valuation v. We summarize this equilibrium outcome in the following lemma.

Lemma 1 (Case 1 Characteristics). *When the commission rate t is low enough and search cost s is high enough* (*Case* 1 *in Figure* 4), *the price distributions of both stores in the $p < r$ range are the same as in the benchmark case. In addition, Stores* 1 *and* 2 *have mass points of $\alpha(v - r)/(\beta(r - c))$ at, respectively, r and v. Semi-Loyal consumers do not search and stores' profits are the same as in the benchmark case.*[12]

Lemma 1 shows the possibility that the host (Store 1) would have lower prices on average, which is a testable hypothesis. One can speculate that, in practice, lower prices should deliver a larger *Loyal* segment for Store 1 giving an additional incentive to host the ad, although Store 2 may then prefer not to advertise and therefore the advertisement may not materialize in equilibrium. Furthermore, the following cases show that lower average price at the host store is not a robust prediction.

3.2.2. Case 2: Intermediate Commission Rate (t).

A higher commission rate allows Store 1 to be less concerned about search by *Semi-Loyal* consumers. This is an impetus for higher prices. As the commission rate increases, Store 1 will have an incentive to sometimes raise prices above the reservation price and allow *Semi-Loyal* consumers to search (see Figure 3, Case 2a). An increase in prices at Store 1 and the additional inflow of consumers are advantageous for Store 2. This gives us an intuition that the commission could allow the stores to tacitly collude on higher prices.

However, as more consumers search, both stores also have a greater incentive to compete for the searchers by reducing prices. Therefore, while Store 1 moves some density of the price distribution up to take advantage of the commissions, some density of price distributions of each store may also be moved down. Moreover, if customers expect some prices to be lower, the benefit of search increases and, therefore, the reservation price of *Semi-Loyal* consumers decreases. When the commission rate is not too high, the end result is that although some part of the price distribution of Store 1 is reallocated up, another part is reallocated down. By contrast, the price distribution of Store 2, relative to its distribution in Case 1 above, is shifted down over the entire range. This is because high prices at Store 1 give Store 2 an incentive not to have a mass point at the top but rather to lower it to fight for the *Shopper* segment and for Store 1's *Semi-Loyal* segment. As prices are shifted down from the top of the price distribution, stores have higher incentive to further undercut and shift price distribution down at the lower part of the distribution as well. The net result is increased competition and lower profits, summarized in the following lemma.

Lemma 2A. *When the commission rate is in the low-intermediate range* (*Case* 2a *in Figure* 4), *stores' profits are less than in the benchmark case and are decreasing in the commission rate.*

When the commission rate increases further, the incentive for Store 1 to increase prices becomes stronger. At some commission level (Case 2b in Figure 4), Store 1's distribution of prices has a positive density for all prices above its *Semi-Loyal* consumer reservation price (see Figure 3, Case 2b). In this case, the positive effect of a marginal increase in the commission rate on Store 1's profit exactly offsets the negative effect due to the intensified fight for searchers. Store 2, however, still suffers from higher commissions since at the highest price it has to pay commission fees with positive probability without a chance of selling to the incoming consumer traffic.

Lemma 2B. *When the commission rate is in the mid-intermediate range* (*Case* 2b *in Figure* 4), *Store* 1*'s profit is lower than in the benchmark case and does not change with the commission rate. Store* 2*'s profit is lower than in the benchmark case and decreases in the commission rate.*

Finally, when the commission rate increases even further, Store 2 reacts by also moving the price distribution up, and profits start to increase for both stores (Case 2c in Figure 3). Interestingly, although the effects of the commission on the profits of the two stores are unequal, the commission level at which profits start to increase in the commission rate is the same for both stores. Furthermore, when the commission rate is high enough, *both stores' profits can be higher than in the benchmark case*. The result is noted in the following lemma.

Lemma 2C. *When the commission rate is in the high-intermediate range* (*Case* 2c *in Figure* 4), *both store profits are increasing in the commission rate. With a high enough commission rate, both stores' profits can be higher than in the benchmark case.*

If the commission rate is sufficiently high, Store 1 is eager to send all its *Semi-Loyal* customers to Store 2 and receive profits from the commission alone. As discussed in Section 3.2.3, this outcome is actually not achievable for any commission rate, because as Store 1 attempts to price above r, Store 2 increases prices as well and discourages consumer search. Thus, $r = v$ becomes a binding constraint.

3.2.3. Case 3: High Commission Rate (t). As discussed in Section 3.2.2, as the commission rate increases, Store 1 first has an incentive to allow *Semi-Loyal* consumers to search and then increasingly prices at levels to encourage search. The result was that initially the distribution of prices widened, but then, narrowed. As the narrowing continues and price distributions condense to the upper limit (v), the incentive for consumer search declines, and therefore, the *Semi-Loyal* consumer reservation price increases. When the commission rate reaches a certain level, the reservation price becomes equal to v. This defines the case under consideration in this section.

As t increases beyond this level, Store 1's incentive to allow and encourage consumer search increases, and therefore it prices at v with higher probability (i.e., its distribution of prices has a larger mass point at v). If all of Store 1's *Semi-Loyal* customers searched when $p_1 = v$, Store 2 would consider the following: First, Store 1's prices are higher. Second, more high valuation consumers are coming to it (since Store 1's *Semi-Loyal* consumers now only come to Store 2 when $p_1 = v$; after arriving at Store 2, they effectively behave as customers *Loyal* to Store 2). Therefore, Store 2 would optimally increase prices as well. Yet that means that Store 1's *Semi-Loyal* consumers' incentive to search decreases, and therefore, they should not necessarily be searching even when they observe $p_1 = v$. The only possible equilibrium is that Store 1's *Semi-Loyal* consumers randomize their search when facing $p_1 = v$. Thus, a fraction k of *Semi-Loyal* consumers search when facing $p_1 = v$. In equilibrium, as the commission rate increases and Store 1 tries to increase search by pricing at the highest level, the fraction searching decreases. We summarize the equilibrium characteristics of this case in Lemma 3.

Lemma 3 (Case 3 Characteristics). *When the commission rate is high enough* (*Case* 3 *in Figure* 4), *the reservation price* r *is equal to* v *and both distributions have mass points at* $r = v$ (*see Figure* 3, *Case* 3). *Moreover, at price* $p_1 = r = v$, *a fraction* $k = ((\alpha+\beta)p_0 - \alpha v - c\beta)/((t - v + c)\gamma)$ *of Semi-Loyal customers search. The host's profit* $\pi_1 = (\alpha+\beta)\cdot(p_0 - c)$ *is above its profit in the benchmark case and does not depend on the commission rate. The advertiser's profit is also above the benchmark, increases with commission rate, and asymptotically approaches* $\bar{\pi}_2 = \pi_1 - ((v-c)/\beta)((\alpha+\beta)p_0 - \alpha v - c\beta)^2$.[13]

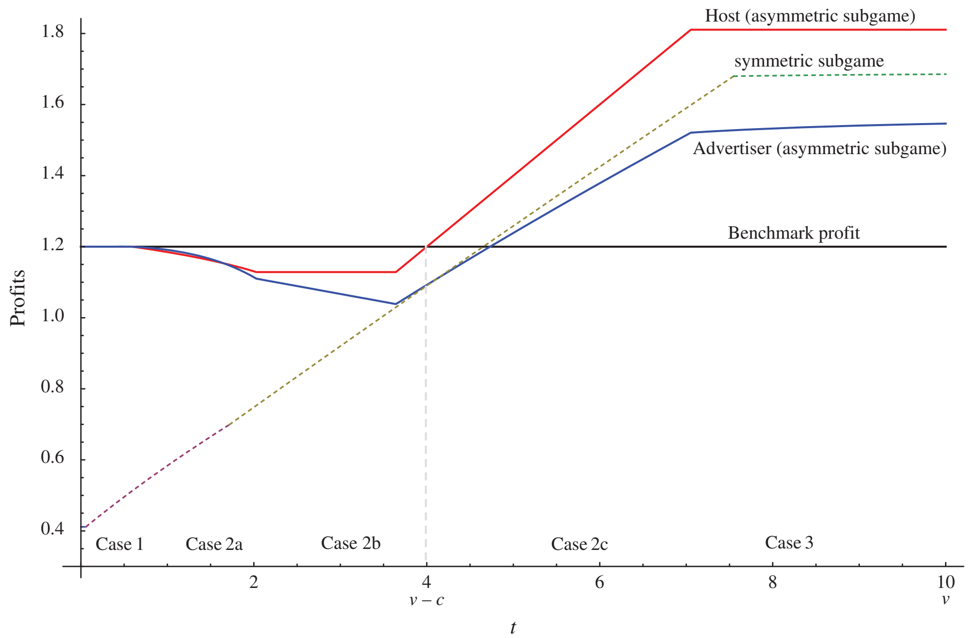
One may expect that when the commission rates are so high that Store 1 would want to send all *Semi-Loyal* customers to Store 2, it would benefit from as high a commission as possible while Store 2 would suffer from it. However, in the Case 3 equilibrium, it turns out that Store 1's profit is flat in the commission rate while Store 2's profit asymptotically increases in the commission rate. The intuition is that advertising is profitable because in equilibrium, few *Semi-Loyals* actually go to the advertiser's website (and the higher the commission the fewer *Semi-Loyals* search in equilibrium), so the loss (from paying commission to Store 1) is relatively small, but the overall price increase allows for higher profits on all the other customers who buy from Store 2.

3.2.4. Summary of Asymmetric Competitor Advertising Subgame. Combining the above cases, we obtain the following proposition that summarizes the profitability of asymmetric competitive advertising. The meshed area in Figure 4 shows the commission rates and search costs for which asymmetric competitor advertising is industry-profitable. Figure 5 depicts the change in profits as a function of the commission rate, relative to the benchmark model.

Proposition 1 (Asymmetric Competitor Advertising). *When a store hosts the ad of its competitor, profits are U-shaped in the commission rate and first* (*weakly*) *decrease and then* (*weakly*) *increase for both stores. For low commission rates, profits for each store are at or below Benchmark profits. For high commission rates, profits for each store are above Benchmark profits. Thus, there is no conflict of interest between stores in adjusting the commission rate.*

This proposition offers several insights: First, whether in-store advertising will increase competition depends, nonmonotonically, on the commission rate. Although the anti-competitive effect of commissions may be intuitive, it does not hold for small commission rates. Likewise, although it seems reasonable that advertising without commission ($t = 0$) should increase competition, it does not necessarily do so. This

Figure 5. (Color online) Profits as a Function of Commission Rate



Note. $\alpha = 0.3$, $\gamma = 0.2$, $s = 0.5$, $v = 10$, $c = 6$.

is because stores are indifferent between choosing a price from a range of prices, and they can adjust price distributions so as to negate the potential customer search. The commission, on the other hand, may be enough incentive for Store 1 to facilitate search, and more intense search is ultimately detrimental to profits unless the anti-competitive effect of commissions is high enough.

Second, one may observe that since the ranges of commissions where profits increase or decrease are exactly the same for the two stores, the competing stores do *not* have a conflict of interest in setting the commission rate. This is a surprising result because the most direct effect of the commission is the monetary transfer from one store to the other. The intuition for why the effect on profits is aligned is that the effect of the commission on the level of competition and pricing is the same for both firms and this effect outweighs the direct effect of monetary transfer.

3.3. Both Stores Advertise (Symmetric Competitor Advertising Subgame)

We now examine the subgame where both stores act as host and advertiser for each other. The stores and advertising arrangements are symmetric, as are segment sizes, search costs, and commission rates. Given this, we consider a symmetric equilibrium in prices.

The analysis is similar to the asymmetric advertising subgame with three cases in equilibrium corresponding to different values of the commission rate and search cost. In Case 1, the commission rate is low and stores decrease their prices below the *Semi-Loyal* segment reservation price (denoted r_s) to prevent search. Therefore, profits drop to $\alpha(r_s - c)$ compared to $\alpha(v - c)$ in the benchmark case. As the commission rate increases, stores' incentive to charge prices above reservation price increases (Cases 2a and 2b). In this situation, a store both sends its own *Semi-Loyal* customers, and receives its rival's *Semi-Loyal* customers, with positive probability. In this case, profits are increasing in commission rate and become larger than in the benchmark case if the commission rate is high enough. Finally, as the commission rate increases further, the (symmetric) price distributions have a mass point at the top price, and the average price is strictly higher than in the benchmark case.

The following proposition summarizes the equilibrium outcomes under symmetric advertising.

Proposition 2 (Symmetric Competitor Advertising). *When each store symmetrically hosts and advertises, profits are (weakly) increasing in the commission rate and asymptotically approach a finite upper bound. Profits are below Benchmark profits for low commission rates and exceed the Benchmark profits for high commission rates.*

Proposition 2 suggests that reciprocity of in-store advertising does not erase its profitability. If both stores host in-store advertising of their competitors, their profits will still be higher than benchmark for high enough commission rates.

3.4. Equilibrium Hosting and Advertising Choices

Comparing the three subgames, we find that the host's profit in the asymmetric subgame is *always* higher than a store's profit in the symmetric subgame (e.g., Figure 5). This implies that symmetric advertising cannot be an equilibrium outcome because stores would deviate by not offering to host in the first stage or not agreeing to become an advertiser in the second stage if offered. The subgame perfect Nash equilibrium outcome is described in the following proposition.

Proposition 3 (Optimality of Competitor Advertising). *If the commission rate is low, competitor advertising does not increase profits over benchmark and thus no advertising is the equilibrium. If the commission rate is high enough, asymmetric competitor advertising is the equilibrium outcome. If either store has the power to set the commission rate, it will be set at a high level and competitor advertising will occur.*[14]

So far, we have considered the commission rate to be exogenous. This may be a realistic assumption if advertising is implemented by a third party and the per-click fees are generally set based on the supply and demand for advertising or if the host sets the advertising rate to attract advertising of unrelated products (but does not prohibit direct competitor advertising). Turning to the retailer choice of the commission rate, Proposition 3 implies that if either store were to set the commission rate, it would prefer it at the highest possible level. Practically, of course, very high commission rates could induce some effects not considered in our model. For example, it may invite such adverse behavior as click fraud (when Store 1's employees pretend to be customers and go to Store 2), recordkeeping discrepancies or contract-enforcement issues. The commission should then be set at the level below when these issues become significant. Note that once the commission rate is in Case 3, the marginal benefit of increasing it is diminishing for the advertiser and is zero for the host. This suggests that, practically speaking, the boundary between Cases 2 and 3 could be an upper limit on the commission rate. Three additional reasons that too high a commission rate can be suboptimal are given in Section 4. In Section 4.2 we show that an intermediary who sets the commission will not increase it unboundedly because too high a level diminishes consumers search resulting in lower profit for the intermediary. In Section 4.3 we find that marginal cost asymmetry results in decreasing profits for the advertiser at high commission rates. In Section 4.6, we find that if a fraction of the *Shopper* segment also uses the ad link, it acts as a lump sum transfer from the advertiser to the host without any additional benefit for the advertiser. Thus, too high a commission rate will eventually hurt the advertiser's profit.

3.5. Optimal Level of Consumer Search Cost (Ad Efficiency)

Semi-Loyal customers' search cost s can be related to advertising implementation decisions such as ease of following or clicking the ad and its design. In Figure 1, for example, the Expedia ad suggests that the customer "Run DFW-LAS on May 17–May 31 on these travel sites," and travel information is prefilled so that the customer is spared some effort. The location and design of the ad can influence customers actual or expected search cost. Customers might not want to bring their cursor to the bottom of the page and click on a sponsored text link compared to the larger and easier-to-access ad of *Frys.com* on the sidebar of the *Target.com* website. Additionally, showing good ratings of the advertiser, range of the price (e.g., "from \$441"), and call-to-action phrases such as "Save Money," "Great Discount!," and "Book your flight now!" may reduce the psychological search cost. Since the advertiser store and host store can affect advertising efficiency, a relevant question is what search cost they each would prefer.

Manipulation of the search cost may affect the stores' pricing strategies, and intensify or soften price competition. To understand these effects, note that the reservation price Equation (3) equates search cost to the area under the store's price distribution function up to the reservation price (because $s = \int_{p_0}^{r}(r-p)\,dF_2(p) = \int_{p_0}^{r} F_2(p)\,dp$). Thus, an increase in s engenders two effects: First, given a price distribution, the reservation price r must increase, hence *Semi-Loyals* search only after observing relatively higher prices. Second, given the reservation price, the price distribution changes, decreasing the minimum price or decreasing the likelihood of charging prices higher than the reservation price (shifting down the distribution). Recall that in our model, at a low level of commission rate, advertising the competitor could intensify price competition for *Shoppers* and negatively affect stores' profits. In this situation, an increase in search cost may help alleviate competition through a higher reservation price. On the other hand, when the commission rate is high, advertising the competitor is beneficial for both stores and helps to soften price competition. In this case, increasing search cost may not be desirable for stores. In fact, stores may want to encourage customers to search since it enables them to increase average prices. However, as the next proposition shows, stores might not be in full agreement on the optimal level of search cost when the commission rate is high.

Proposition 4. *When asymmetric competitor advertising is implemented, the host's profit is weakly decreasing in search cost while the advertiser's profit is inverse-U-shaped.*[15]

Proposition 4 clarifies how search cost influences profits. In Case 1, stores' profits are not influenced by search cost and remain at the benchmark level. An increase in search cost will move up the reservation price and decrease *price disparity* across stores without changing profits. When the commission increases

(Case 2), increased competition between *Shopper* and *Semi-Loyal* segments decreases stores' profits through decreasing average prices; an increase in *Semi-Loyal* consumers' search cost can help both stores soften competition. This is because a higher search cost increases the reservation price and hence decreases search, which in turn alleviates stores' profit loss.

In Case 3, the *Semi-Loyal* segment's reservation price r is equal to v. Hence, search cost affects stores' profits through a different mechanism. Because reservation price r does not change with search cost, a decrease in s will only increase the minimum price p_0 and make the price distributions tighter. This, in turn, will increase Store 1's profit which, in equilibrium, is related to the minimum price. The marginal effect of a decrease in search cost on Store 2's profit, however, is twofold: When search cost decreases, the fraction of customers who search at $p_1 = v$ increases, which incentivizes Store 1 to price at v with higher probability. As a result, Store 2's expected earnings from *Shoppers* increases. The downside, however, is paying commission fees on a higher incoming fraction of *Semi-Loyals*. Ultimately, these two marginal effects balance at search cost s^* specified in Proposition 4, resulting in an inverse-U-shaped relationship between s and Store 2's profit function. Figure 6 illustrates how profits change with the search cost and the commission rate. The arrows point in the direction of the profit increase with respect to the search cost and the commission rate.

The maximum profits stores can achieve in the asymmetric advertising case across s and t are $\pi_1^* = (\alpha + \beta) \cdot (v - c)$ and $\pi_2^* = (\alpha + \beta/4)(v - c)$. Note that π_1^* is Store 1's profit in Case 3 when the search cost becomes infinitesimally small, and π_1^* is Store 2's profit asymptote in Case 3 at optimal search cost level s^*. Both profits are higher than in the benchmark case, and $\pi_1^* > \pi_2^*$, i.e., the host earns higher profits than the advertiser. Therefore, although there is a conflict of interest in the level of search cost that maximizes the stores' profits, each is always willing to accept the contract offered by the other since both profits are higher with than without the ad.

4. Model Variations and Extensions

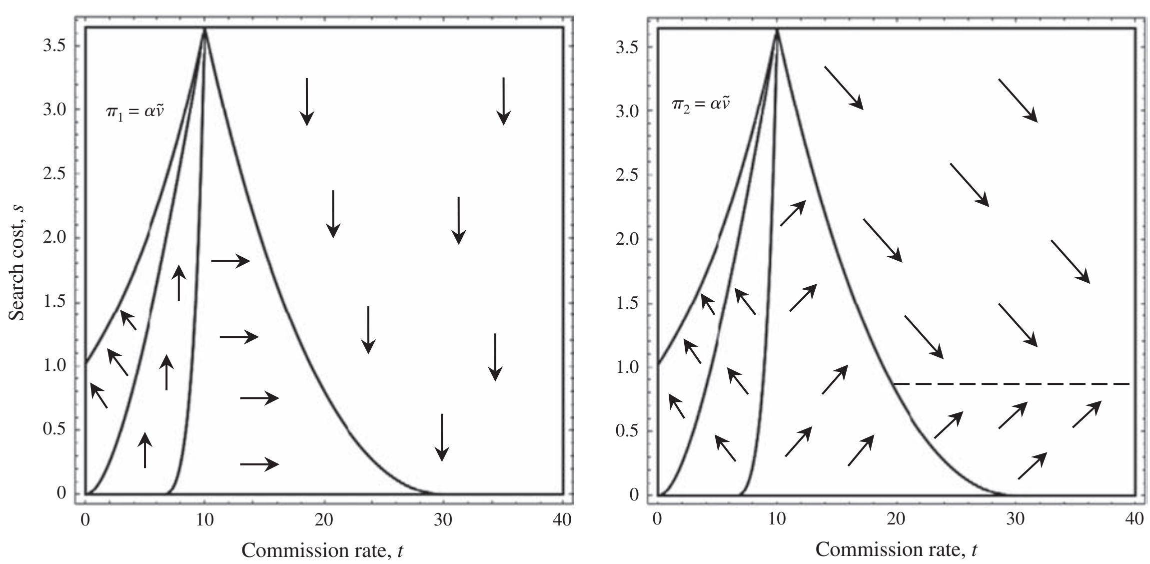
4.1. Commission Rate Unobserved by Customers

So far we assumed that the commission rate is observed by consumers. This section examines the robustness of our results when customers do not observe the commission rate as may be the case in practice.

First, we consider exogenous consumer expectation $\hat{t}$ of t (which might not be equal to the actual commission). Based on this expectation and their search cost, *Semi-Loyal consumers* derive their reservation price $\hat{r}$, and in Case 3 (when $\hat{r} = v$), also $\hat{k}$. We use the "hat" notation to indicate that the values are based on *expectation* of the commission rate. Stores then take into account *Semi-Loyal consumers* search behavior (defined by $\hat{r}$ and $\hat{k}$) to choose the commission rate and their pricing strategies. Let us assume that the commission rate is set at a Pareto optimal level for the firms, i.e., such that no other level can result in a Pareto improvement in equilibrium profits that are the outcome of the advertising and pricing game. To avoid unbounded decisions, assume the feasible range of t stores may choose from is $[0, T]$. All other aspects of the model remain as in the main model (Section 2). The following lemma describes the outcomes.

Lemma 4. *If customers do not observe the commission rate and expect it to be $\hat{t}$, and firms set a Pareto optimal (for the*

Figure 6. Change of Profits with Search Cost and Commission Rate



Notes. π_1 (Left) and π_2 (Right). Arrows show the direction of change with respect to search cost and commission rate.

firms) commission rate, then as long as the search cost or $\hat{t}$ is high enough, the commission rate will be sufficiently high to make advertising (at one store) profitable for both firms; thus, advertising will be observed. When both s and $\hat{t}$ are low, Store 2's profit can at best be equal to its profits without advertising (i.e., the benchmark case).

The difference between this result and the main model is that when the actual commission rate is changed, consumers do not update their belief about it and therefore do not change their search strategy. Rather they follow the search strategy derived in the analysis of the main model for $t = \hat{t}$. As we see from the above lemma, if consumers have a low search cost and expect the commission rate to be low, then with advertising, stores may be unable to earn profits strictly higher than in the benchmark case regardless of how high a commission rate they set. This is the case when *Semi-Loyal* consumers have a relatively low reservation price. Given this reservation price and a high commission level, Store 1's profit $\pi_1 = (\alpha - \gamma)(v - c) + t\gamma$ is increasing in the commission rate. Store 2, however, has an inverse-U-shaped profit function. The maximum of this function never reaches the benchmark profit if the search cost and consumers expectation $\hat{t}$ are both low.

Lemma 4 uses consumer expectation $\hat{t}$ as an exogenous parameter. We now analyze possible outcomes when consumer expectations are consistent with reality, i.e., we look for an equilibrium where the consumer expectation $\hat{t}$ equals the firms' choice of t. Formally, define $\hat{t}$ to be *Semi-Loyal* consumers' belief about t when they observe Store 1 hosting a Store 2 advertisement. (If there is no ad, consumers cannot search and therefore their belief about t is irrelevant.)

Because consumers never observe actual t, this game does not have any subgames. Therefore, we use Perfect Bayesian Equilibrium as a solution concept. Furthermore, similar to Janssen and Shelegia (2015), we focus on equilibria where *Semi-Loyal* consumers use reservation price strategies. The reservation price is now a number $\hat{r}$ that is independent of the actual commission rate. *Semi-Loyals* buy at Store 1 if $p_1 < \hat{r}$ and search if $p_1 > \hat{r}$. The reservation price is based on consumer beliefs $\hat{t}$ about the commission (which technically may change as a function of observed p_1, i.e., $\hat{t} = \hat{t}(p_1)$) and the resulting expected distribution of prices $F_2(\cdot)$ at Store 2. Note that given $\hat{t}(p_1)$, consumers must believe $F_2(\cdot)$ is the price distribution Store 2 would set in equilibrium if $t = \hat{t}(p_1)$. In equilibrium, consumer belief about the commission rate must be correct, i.e., in equilibrium, $t = \hat{t}$. The analysis of this game leads us to the following result.

Proposition 5. *Suppose that stores can set the commission rate at a Pareto optimal (for the firms) level and that customers do not observe this choice (but infer it from equilibrium). Then if search cost s is not too low, in equilibrium, at least one store will host advertising of the competing store.*

Proposition 5 demonstrates the robustness of the main result of Section 3 that stores can benefit from allowing advertising by their rivals by showing that it also holds when customers have imperfect information about commission rates. While the optimal commission rate may depend on the consumer expectation of it, we still find that the two stores do not have opposing interests in setting it because either both prefer the commission rate to be very small (and perhaps, both prefer not to have in-store advertising at the other's store) or both prefer the commission rate to be high. Note that Proposition 5 also implies that if firms have control over the search cost, then, in equilibrium, competitor's advertising will be present in at least one store. If search cost is low, it is also an equilibrium outcome for the commission level to be high and for one store to advertise, although this is no longer a unique equilibrium outcome.

In practice, consumer expectation of the commission rate may be based on knowledge about other markets and thus may not reflect the commission rate in a given contract between the two firms (especially when these firms can agree on the commission rate). Lemma 4 shows that the assumption of consumers observing or inferring the commission rate chosen by the firms is not essential for our results. What we show is that for any consumer expectation of the commission rate (including if customers are unaware of the commissions and assume them to be zero), it is (Pareto) optimal for the firms to agree on the advertising and to set a high commission rate (when the search cost parameter is high enough).

Note that restriction of the choice of advertising and commissions to be Pareto optimal for the two firms is a reasonable assumption. If, for example, one firm makes the other firm a take-it-or-leave-it offer, we would expect the offer to be Pareto optimal regardless of whether it is made by the host or the advertiser. In Section 4.2, we show a similar result when the commission is set by an intermediary.

4.2. Intermediary Setting the Commission

In this section, we consider an intermediary in the market who decides the commission terms at the beginning of the game.[16] In particular, suppose the intermediary makes two decisions: It sets the commission rate t, and it sets its commission share q. The latter is the percentage of the commission that the intermediary keeps. Thus, out of t dollars that the advertiser pays on each click, the intermediary keeps qt and passes $(1 - q)t$ to the host. The intermediary's profit would then be $\pi_f = qt \times E[search]$, where $E[search]$ is the expectation of how many *Semi-Loyal* customers search. As a result, the game will have four stages. In the first stage, the intermediary simultaneously decides the commission t and its share q to maximize its profit. In the second and third stages, stores decide on whether to allow advertising and whether to accept it if offered by the rival. In the fourth stage, stores decide on their pricing strategies and customers decide on searching and/or buying the product.

Given t and q, the analysis of this extended model is very similar to the main model in Section 2, with the same four and five pricing cases, respectively, in symmetric and asymmetric subgames. The difference is that the regions for these cases (Figure 4) are stretched by a factor of $1/(1-q)$ horizontally along the t axis. For example, the commission level $t = v - c$ shifts to $t = (v-c)/(1-q)$. Moreover, the main characteristics of the profit functions remain unchanged. First, profits are U-shaped in the commission rate in the asymmetric subgame and become higher than benchmark given high enough t. Second, profits are weakly increasing in t in the symmetric subgame and eventually become higher than benchmark. Third, the host's profit in the asymmetric subgame exceeds its profit in the symmetric subgame, resulting in an asymmetric equilibrium outcome.

In the first stage of the game, the intermediary sets (q, t) optimally to maximize its profit. The following proposition summarizes this optimal choice.

Proposition 6. *The intermediary maximizes its profit by setting the commission rate at $t^* = (\beta+\gamma)(v-c)/\gamma$ and its commission share at q^* to solve*

$$s = \frac{(\alpha+\beta-q^*(\beta+\gamma))(v-c)}{\beta} \cdot \left\{\frac{q^*(\beta+\gamma)}{\alpha+\beta-q^*(\beta+\gamma)} - \ln\frac{\alpha+\beta}{\alpha+\beta-q^*(\beta+\gamma)}\right\}.$$

At (t^*, q^*), in-store advertising is implemented by only one of the stores, and host and advertiser profits are $(\alpha+\beta-q^*(\beta+\gamma))(v-c)$ and $\alpha(v-c)$, respectively.

Proposition 6 clarifies the role of the intermediary in the in-store competitive advertising contracts. To maximize its profit, the intermediary optimally sets the commission rate at t^* which turns out to be at the boundary of Cases 2c and 3 in the asymmetric advertising subgame. The reason for this is as follows: In Case 2c, search increases with the commission rate because it creates more incentives for Store 1 to send its *Semi-Loyal* customers to Store 2. In Case 3, however, search decreases with commission because $r = v$ and only a fraction k of *Semi-Loyals* search upon observing $p_1 = r = v$, and k is decreasing in t. The proof of Proposition 6 shows that the expected commission paid by the advertiser is decreasing in Case 3 and therefore the intermediary does not want to increase the commission from its level at the boundary of Cases 2c and 3.

Next, consider the commission share q. The intermediary would like to increase q as much as possible, but the advertiser's profit is decreasing in q and falls below benchmark for high enough q. As a result, q is set to give the advertiser just enough incentive to participate. Therefore, in equilibrium, $\pi_2 = \alpha(v-c)$.

In Section 4.1, we showed that our main result is unchanged if customers do not observe the choice of commission rate (Pareto optimally) chosen by stores when the search cost s is high enough. This continues to hold if advertising terms are set by the intermediary, not by the stores. Intuitively, the intermediary's equilibrium choice of commission is in fact at a Pareto optimal level for stores: At any other commission level at least one of the stores is worse off. Therefore, we can expect that customers have rational expectations about this commission level and, in equilibrium, one of the stores hosts the ad for its rival.

4.3. Marginal Cost Differences Across Firms

In the main model, we assumed that stores have equal marginal costs. This section investigates how marginal cost asymmetry between stores can affect their incentives for in-store competitor advertising. Suppose Store 1 and Store 2 have marginal costs c_1 and c_2, respectively, both in $(0, v)$. Moreover, without loss of generality, let $c_1 \geq c_2$ and normalize Store 2's marginal cost to zero ($c_2 = 0$, i.e., consumer valuation and prices in the model reflect how much these values in practice exceed the cost of the lower cost store; the cost of the higher cost store reflects the cost disadvantage).

We begin by revisiting the benchmark (no advertising) subgame to understand how marginal cost asymmetry affects equilibrium pricing and profits. To compensate for its marginal cost, Store 1 increases prices resulting in a mass point, denoted m_1, at the top price v in its price distribution. The price distributions are characterized by the indifference equations

$$\pi_1^B = (\alpha+\beta)(p_0-c_1) = \alpha(p-c_1) + \beta(p-c_1)(1-F_2(p)) = \alpha(v-c_1), \tag{4}$$

$$\pi_2^B = (\alpha+\beta)p_0 = \alpha p + \beta p(1-F_1(p)) = \alpha v + \beta v m_1. \tag{5}$$

Solving these equations, we obtain

$$F_1^B(p) = 1 - \frac{\alpha(v-p)+\beta c_1}{\beta p}, \quad F_2^B(p) = 1 - \frac{\alpha(v-p)}{\beta(p-c_1)}, \quad \text{for } p \in [p_{0B}, v], \tag{6}$$

$$\text{and} \quad p_{0B} = \frac{\alpha v + \beta c_1}{\alpha+\beta}, \quad m_1 = \frac{c_1}{v},$$

$$\pi_1^B = \alpha(v-c_1), \quad \text{and} \quad \pi_2^B = \alpha v + \beta c_1.$$

Intuitively, Store 1's disadvantage in marginal cost decreases its profit while increasing Store 2's profit.

Next, consider in-store advertising. As stores are asymmetric due to $c_1 \geq c_2 = 0$, there are two possible (not equivalent) asymmetric advertising subgames, i.e., Store 1 hosts and Store 2 advertises, and *vice versa*. First, consider Store 1 as host. Because Store 1 has a reduced margin on sales, receiving commission on *Semi-Loyal* customers becomes more attractive. The profitability of in-store advertising expands to lower commission rates, i.e., the region and boundaries of Case 2c in Figure 4 shift left. Now, recall that in the main model, the advertiser's profit asymptotically increased with the commission rate in Case 3 because the marginal benefit of selling to more *Shoppers* outweighed the marginal loss from incoming *Semi-Loyals*. By contrast, if the advertiser has a high enough cost advantage over the host, its earnings from incoming *Semi-Loyals* can be positive since the host is willing to send them at a lower commission rate, and a large commission rate causes a smaller fraction of *Semi-Loyals* to migrate, leading to decreasing profit for the advertiser.

Next, suppose Store 2 hosts the ad for Store 1. Intuitively in this subgame, profits are lower because sales are driven to the less efficient firm. In fact, we can show that if $c1$ is high enough, this subgame will lead to an equilibrium profit for the advertiser below its benchmark profit for any value of commission rate. An implication of this is that when marginal cost heterogeneity is high enough, in the unique equilibrium, Store 1 hosts the ad for Store 2.

Proposition 7. *When marginal cost asymmetry is high, then for any commission rate $t > v - c$, in equilibrium, the store with the cost disadvantage will host ads of its competitor, and the advertiser's maximum profit is achieved at the boundary of Case* 2c *and Case* 3.

Proposition 7 resolves the multiplicity of equilibria for which store will host the ad for its rival. The store with higher marginal cost and lower margin benefits more from sending its customers to the competitor in exchange for a commission. Thus, it would tend to be the host, and be less likely to want incoming customers through being the advertiser. When the cost disadvantage is high enough, the outcome is unique. This is also in line with premium brand stores (Sony, Gap, Hilton, Apple, etc.) not hosting ads for lower margin firms (i.e., discount retailers such as Walmart, Amazon, etc.).

4.4. Product Price Displayed in the Ad

In some cases, the price of the product is displayed in the ad. Although this price is not necessarily credible in practice because sometimes it changes after click-through, it is interesting to consider how the results are affected if *Semi-Loyal* consumers can know the price from the ad itself before deciding to follow the link to the other store. In this section, we analyze this variation of the model.

Given that the price is displayed in the ad, *Semi-Loyal* consumers click to the advertiser and purchase the product if they see a low enough ad price. In other words, the comparison now is not between the search cost and the expected benefit of search but between the search cost and the observed benefit of search. That is, *Semi-Loyal* consumers search if and only if $p_1 > p_2 + s$, where p_1 and p_2 are the prices of the host and advertiser store, respectively. This search rule is simpler that the one in the main model.

As the price in such ads usually changes frequently, we keep the assumption that pricing decisions are simultaneous. The indifference equations for each store are as follows. For the host store (Store 1)

$$\pi_1(p) = (\alpha - \gamma)\tilde{p} + \beta\tilde{p}(1 - F_2(p)) + \gamma\tilde{p}(1 - F_2(p - s)) + t\gamma F_2(p - s), \tag{7}$$

where $\tilde{p} \triangleq p - c$. The first term captures the host store profit from its *Loyal* segment. The second term is for *Shoppers*. They purchase at the host only if the price is lower than the rival store price. The last two terms explain *Semi-Loyal* behavior. If the displayed price is higher than the host price minus the search cost, they do not search. This happens with probability $1 - F_2(p - s)$. Otherwise they click, go to, and purchase at the advertiser store, and the host receives the commission (the last term). Similarly, we write the indifference equations for the advertiser as follows:

$$\pi_2(p) = \alpha\tilde{p} + \beta\tilde{p}(1 - F_1(p)) + (\gamma\tilde{p} - \gamma t)(1 - F_1(p + s)). \tag{8}$$

The first term is due to the *Loyal* and *Semi-Loyal* segments of Store 2, who stay and buy at this store. The second term is the expected profit from the *Shopper* segment. Finally, the third term captures the profit from the *Semi-Loyals* of the host store, who come to Store 2 if the ad-price plus the search cost is lower than the host price.

The objective is to find price distribution $F_1(p)$ and $F_2(p)$ that satisfy (7) and (8). These price distributions are in the appendix and are piece-wise functions. When search cost is high enough, the price distribution is as in the benchmark case because *Semi-Loyal* customers are unwilling to search at any price advertised by Store 2. The condition for this is that the minimum price at the advertiser is higher than the maximum price at the host minus the search cost, i.e., the search cost is higher than $v - p_0$. This will result in the condition $s > \beta(v - c)/(\alpha + \beta)$. When search cost decreases from this level, the price distributions will have two "pieces" with a kink at $v - s$. The second piece of the price distribution is recursively obtained from the first piece, using Equations (7) and (8). The number of pieces increases when search cost decreases.

Proposition 8. *When competitor price is displayed in asymmetric competitor advertising*: (i) *The advertiser's profit is never below its profit under no advertising* (*benchmark case*). (ii) *Both stores strictly benefit if $s < \beta(v - c)/(\alpha + \beta)$ and the commission rate is high enough.* (iii) *When $s \geq \beta(v - c)/(\alpha + \beta)$, the equilibrium outcomes are the same as without advertising regardless of the commission level.*

Thus, in equilibrium, in-store advertising still occurs for high enough commission rates. Note that the advertiser's profit can never fall below the benchmark case in this model. In fact, if the advertiser prices at v and shows the price in the ad, the *Semi-Loyals* will not search and the advertiser's profit will at least remain at the benchmark. If the host charges v with positive probability, which happens when the commission rate is high enough, the advertiser will also get business from *Shoppers* and its profit increases over the benchmark. Our analysis shows that for any level of search cost $s < \beta(v-c)/(\alpha+\beta)$, the advertiser and host profits are higher than the benchmark levels if the commission rate is high enough. Yet, unlike in the main model, there could be a conflict of interest between the host and advertiser because the advertiser may wish to advertise even when the commission level is low.

4.5. Pay-Per-Action Advertising Contract

An alternative to the CPC commission structure considered so far is pay-per-action (PPA) where the host receives a commission only when the customer visits the advertising store through the ad *and* makes a purchase. If the customer clicks the link but does not make a purchase, the host receives no commission. The PPA commission can be a fixed amount or proportional to the price paid. The latter is common in the context of referral marketing, e.g., Amazon pays a percentage of the purchase price to host websites in its referral program that send it customers. In this section, we assume that the advertising contract is PPA and investigate how it changes the results of the main model.

In the asymmetric advertising subgame, the host's profit when it charges a price above the *Semi-Loyal* customer reservation price, r, is

$$\pi_1(p) = (\alpha-\gamma)\tilde{p} + \beta\tilde{p}(1-F_2(p)) + \gamma\tilde{p}(1-F_2(p)) + C(p), \quad \text{for } r < p < v,$$

where $\tilde{p} \triangleq p - c$. The first term is the profit from *Loyal* customers, the second term is the expected profit from *Shoppers*, and the last two terms are expected profit and commission $C(p)$ from *Semi-Loyal* customers. When the contract was CPC in the main model, we had $C(p) = t_c\gamma$. When the contract is PPA with a fixed commission, $C(p) = t_F\gamma F_2(p)$, where t_F is the fixed commission paid on each purchase by the advertiser and $F_2(p)$ is the probability that its price is below the host price p, i.e., the probability that *Semi-Loyals* purchase at its store. Finally, when the contract is PPA with a percentage of sale price, $C(p) = t_R\gamma E[p_2 \mid p_2 < p] \times F_2(p) = t_R\gamma\int_{p_0}^{p} p_2\, dF_2(p_2)$, where t_R is the percentage amount and $E[p_2 \mid p_2 < p]$ is the expected price in Store 2 given that it is less than the host store's price.

Proposition 9 (PPA Contracts). *Given asymmetric competitive advertising, fixed PPA and percentage PPA contracts will result in strictly higher than benchmark profit for both stores only if the commission rate is higher than $v-c$ and $(v-c)/v$, respectively.*

An important observation about fixed fee PPA contracts is that the advertiser's profit is never lower than benchmark. This is because the advertiser will pay the commission fee to the host only if it actually sells to the incoming *Semi-Loyals*. The host store, on the other hand, may still earn below-benchmark profit if the commission rate and search cost are both low, similar to the main model. When the commission rate is sufficiently high, both stores' profits are above benchmark level with both types of PPA contracts.

4.6. When Shoppers Also Click on the Ad Link

An assumption in the main model was that *Shoppers* do not use the ad link to navigate between stores. In reality, some customers who can cross-shop at different stores without the link may still use the link if it is there. In this extension, we assume that λ *Shoppers*, where $0 \le \lambda \le \beta$, click on the link.

Because *Shoppers* know prices across stores, advertising does not influence their purchase behavior. Therefore, clicks by *Shoppers* have no effect on stores' pricing strategies. However, *Shopper* clicks transfer a linearly increasing amount λt from the advertiser to the host while the marginal increase in advertiser profit (derived in Case 3 of the main model) goes asymptomatically to zero with the commission rate. Thus, even with small $\lambda > 0$, the advertiser's profit eventually declines with commission rate and the advertiser no longer prefers too high commission rates. The outcome resembles one with click fraud because in both cases no incremental revenue is obtained by the advertiser from clicks. Of course, if λ is high enough, in-store advertising can cause the host's profit to always be above the benchmark and the advertiser's profit to always be below the benchmark of no advertising. Proposition 9 summarizes this result.

Proposition 10. *In the asymmetric competitive advertising subgame, if λ Shoppers click on the link, the advertiser profit is decreasing and the host profit is increasing at high enough commission rates. Specifically, for $\lambda \in (0, \lambda^*]$, the advertiser profit is maximized at the finite commission level t_a^* and is higher than in the benchmark case. For $\lambda \in [\lambda^*, \beta]$ the advertiser profit is below benchmark for any commission level and advertising will not occur in equilibrium.*[17]

This proposition has interesting insights for in-store competitor advertising contracts. First, it reveals why the advertiser's optimal commission rate cannot be infinite in practice: With even a tiny fraction of *Shoppers* using the ad link, the advertiser's profit eventually decreases in commission rate. Second, too many clicks by the *Shopper* segment can erode the advertiser's profit and disincentivize it to participate in the in-store advertising contract. One implication then would be if one of the stores is always the first stop for *Shoppers*, then this store will probably play the role of the advertiser in the in-store advertising contract.

5. Further Discussion and Conclusion

This paper considers whether and under what conditions a retailer would like to host competitor's ads for the same products it carries, as we observe on some Internet retail websites. We model advertising as essentially reducing consumer search costs, so that advertising exposure results in search becoming a consideration for a fraction of consumers whose search cost was too high without advertising exposure (or equivalently, who would be unaware of the competitor in the absence of the ad). One would normally associate reduced search cost in a market of undifferentiated products with lower prices and profits (e.g., Stigler 1961); similarly, increasing advertising costs may result in reduced profits for the advertiser. However, we show that these effects do not necessarily hold.

In particular, Proposition 1 shows that when one firm advertises at the competitor's store, the profits of both stores are U-shaped in the advertising cost (the per-click commission rate). Compared to profits under no advertising, the profits of each firm are lower with advertising when the commission rate is low and higher when the commission rate is high. Note that the effect of advertising commission is nonmonotone: When the commission rate is low, an increase in the commission rate decreases profits of each firm, but when the commission rate is higher, an increase in the commission rate increases profits. Although the effect of the commission rate on profits is monotone (increasing) in the case of symmetric advertising (when each firm advertises at the competitor's store) (see Proposition 2), we show that the symmetric advertising outcome is not an equilibrium. This is because each firm prefers to be the only one to host the competitor's ad (Proposition 3). Of course, if advertising decisions are negotiated, i.e., the result of a cooperative outcome as opposed to each store unilaterally stopping advertising, then advertising by both stores may be observed.

The commission rate is often a function of the general market for advertising and set by a third party. The retailers' decision is then whether to advertise and whether to have clauses in the third-party contracts restricting competitors from advertising on their site. If the commission rate is an outcome of the advertising market across categories, the analysis predicts that we will observe competitor advertising when retail margins are low (when the value of a sale is relatively lower than the ad price per click). This prediction is empirically testable, and one can see some anecdotal evidence supporting it. For example, many discount retailers, such as Walmart, Amazon, eBay, and Target have adopted this type of advertising in their online stores and will occasionally display competitors' ads. Online travel agencies, which usually have a low margin on selling flights and hotels, also use search-based advertising, display their competitors' ads, and encourage their customers to consider other retailers. On the other hand, brand manufacturers that have higher profit margins, such as Sony, Gap, Hilton, Apple, etc., rarely engage in competitor advertising.

At the same time, the model is useful for understanding outcomes when firms have control over the commission rates or over the efficiency of advertising (i.e., how easy it is for customers to follow the advertising link). Firms may be selling a variety of products with different margins, raising the concern that an advertising contract could hurt the firm because the competitor might apply it to different products than originally intended. The no-conflict results about the commission rate and benefit of advertising make the above issue less important than one might suspect. However, conflict in the optimal search cost for customers who might be enticed to search can exist and in a somewhat surprising direction: The advertising host might be too eager (from the advertiser's point of view) to promote the ad (Proposition 4).

We analyzed various checks and extensions of the main model results in Section 4, addressing issues such as: observation of the commission rate, the role of an ad intermediary, type of advertising contract, and price in the advertisement. We conclude that the main model result of an asymmetric equilibrium, where a store hosts its rival's ad at a high enough commission rate, is seemingly robust. Furthermore, when the commission rate is made endogenous in the model, it is often in the interest of the decision maker, whether a store or an intermediary, to set it at the highest level required for advertising in equilibrium. For example, when the commission rate is unobserved by consumers, but they can form rational expectations about it when a store sets it at a Pareto optimal level, then Proposition 5 states that if the search cost is not too low, an asymmetric equilibrium will result where one store hosts the ad for the other store. If an intermediary sets the commission rate, then because the intermediary gets a share of advertising generated, it encourages advertising in equilibrium and also does not set the commission rate so high that the advertiser will balk by pricing at a very high level to limit the traffic of customers following the ad link (Proposition 6). While the main model considers CPC advertising fees, we also formally analyzed PPA ad contracts, both fixed fee and percentage of sales, and find that the result is qualitatively similar (Proposition 9).

Another question is whether the advertising commission in practice corresponds to the high commission rate range in our model. There is some evidence in the popular press that the CPC advertising cost may in fact be higher than the seller's margin. One e-commerce blog (see http://goo.gl/ZMYNGC), for example, notes that "An average CPC of $5.18 (for 'zumba dance dvd' keyword), even at the high end of margin, is an alarming cost." As another example, Google Adwords suggests a bid of $9.41 for the keyword "business cards" (Google states that the suggested bid "is calculated by looking at the cost-per-click (CPC) that advertisers are paying for the keyword"), while the Google Shopping result page for this keyword reveals that the price range for business cards is $2–$15. This suggests that the CPC may be higher than the margins. Furthermore, many customers click an advertising link repeatedly as they decide on their purchase. This effectively multiplies the commission rate by a factor greater than one. Similarly, *Shoppers* may know that Store 1 has an advertising link that can be used as a navigation tool to Store 2 and may use it when they already know that Store 2 has the lower price and therefore they need to get to Store 1 to buy. As a result, the advertising link will incur an additional cost to the advertiser and effectively increase the commission rate. Another example is the referral program by Amazon (Amazon associate). Referral fees can be as high as 10% of the product prices, which could be comparable to Amazon's margin on the product. Moreover, often the product is sold through a third party on Amazon's website and Amazon receives 8% to 15% of the sales. Hence, it seems that the referral commission in some cases could be comparable to what Amazon actually earns. Consider also travel websites such as *hotels.com* where the website operator may receive 10% to 15% margin of undiscounted sales, but as a loyalty program, it offers registered users a price deduction equal to the average price of 10 previous stays on the next stay and needs to pay a 3% credit card fee and may have other expenses. Therefore, operating margins can be quite small.

Although we abstracted from the source of the retailers' product costs in the main model, the model can be easily extended to include endogenous c as coming from the manufacturer's decision on the wholesale price. Within our model, given inelastic consumer demand, the manufacturer would set the highest price bounded only by the retailers' participation constraint (since each retailer has a captive loyal segment, the manufacturer may strictly prefer to have both retailers sell its product unless the fixed costs of selling are too high). Assume that each retailer has a fixed cost $FC > 0$ of selling the product. Then, the maximal wholesale price that insures retail participation in the absence of in-store advertising is $w = FC/\alpha$. If the manufacturer does not consider the possibility of in-store advertising, it will then set this wholesale price and the rest of our model applies with the marginal cost of retailers becoming $c = FC/\alpha$. If the manufacturer expects in-store advertising (since it occurs in equilibrium), it may slightly increase the wholesale price as to equate the advertiser's profit to zero (the store that ends up being the host has a higher profit). The rest of the model would still apply. Another possibility is that the loyal segment's demand is downward sloping (i.e., v could be heterogeneous across loyal consumers). In this case, the wholesale price would have to be lower to prevent the retailers from sometimes pricing too high. Analytical solution of mixed strategies with downward sloping demand would be very complicated, but there is no reason to believe the implications would not be conceptually the same.

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Appendix
To limit the appendix length, we provide a sketch of the proofs of the main model results. Detailed proofs and the proofs of extensions are in the online appendix. Additional variables/notations are defined in each case.

A.1. Derivation of Equilibrium Price Distributions in Asymmetric Subgame
Similar to Narasimhan (1988), one can show that the price distributions have a common minimum price p_0 at which there is no mass point, and may only have mass points at the reservation price r and at the consumer valuation v, since in region $[0, r)$ the *Semi-Loyal* segment can be combined with *Loyals*, and in region $(r, v]$, with *Shoppers*, to reduce consumer

Table A.1. Main Notation

Symbol	Description
α	Size of the *Loyal* segment
β	Size of the *Shopper* segment
γ	Size of the *Semi-Loyal* segment
v	Consumer valuation for the product
t	Commission rate
s	Search cost
c	Marginal cost
π_i	Store i profit, $i \in \{1, 2\}$
$F_i(p)$	Store i price distribution
r	*Semi-Loyal* reservation price
p_0	Lower bound of price distributions
$\tilde{r}$	$r - c$
$\tilde{v}$	$v - c$
$\tilde{p}$	$p - c$
$\tilde{p}_0$	$p_0 - c$

behavior in our model to that shown in Narasimhan (1988). Next we derive equations on p_0 and the equilibrium price distributions in these regions similar to Narasimhan (1988), accounting for the commission transfer when $p_1 > r$ and giving special consideration to pricing and consumer behavior at $p = r$ since this is where the consumer behavior and commission payment changes. Moreover, if $r < v$, then at most one store can have a mass point at v since otherwise deviating to $v - \epsilon$ is profitable for at least one store. A simultaneous mass point at $p_1 = p_2 = r$ may be possible if all *Shoppers* buy from Store 2 because then Store 2 does not need to undercut; Store 1 may not wish to undercut Store 2 because if it did, it would lose the commission. As indicated below, solving the equilibrium equations leads to both stores possibly having a mass point at $p = r$, but only when $r = v$ (Case 3). We now turn back to the equations on equilibrium pricing in $[0, r)$ and $(r, RMv]$ (note that the first region is never empty as it cannot be optimal for *Semi-Loyal* consumers to search at the minimum price for any $s > 0$, but the last range is empty if $r = v$).

For $p < r$, since *Semi-Loyal* consumers do not search, Store 1's profit is $\alpha\tilde{p} + \beta\tilde{p}(1 - F_2^a(p))$, where $F_2^a(p)$ is Store 2's price distribution for $p < v$. For $p > r$, Store 1's *Semi-Loyals* search and it receives the commission. Hence, $\pi_1 = (\alpha - \gamma)\tilde{p} + t\gamma + (\beta + \gamma)\tilde{p}(1 - F_2^b(p))$, where $F_2^b(p)$ is Store 2's price distribution for $p > r$. Approaching the maximum price v, the last equation becomes, $\pi_1 = (\alpha - \gamma)\tilde{v} + t\gamma + (\beta + \gamma)\tilde{v} m_{2v}$, where m_{2v} is the mass point (which can be zero) of Store 2 at v. As Store 1 is indifferent between these prices

$$\begin{aligned}\pi_1 &= (\alpha + \beta)\tilde{p}_0 = \alpha\tilde{p} + \beta\tilde{p}(1 - F_2^a(p)) \\ &= (\alpha - \gamma)\tilde{p} + t\gamma + (\beta + \gamma)\tilde{p}(1 - F_2^b(p)) \\ &= (\alpha - \gamma)\tilde{v} + t\gamma + (\beta + \gamma)\tilde{v} m_{2v}.\end{aligned} \tag{A.1}$$

For Store 2, noting that Store 1's *Semi-Loyals* visit Store 2 if $p_1 > r$, we have $\pi_2 = \alpha\tilde{p} + \beta\tilde{p}(1 - F_1^a(p)) + (\gamma\tilde{p} - t\gamma)(1 - F_1(r))$, where $F_1(r) \in [F_2^a(r), F_2^b(r)]$, the exact value defined by Store 1's *Semi-Loyal* consumer behavior at $p_1 = r$. If Store 2 prices at $p_2 > r$, it pays the commission if $p_1 > r$ and will sell to incoming *Semi-Loyals* only if $p_2 < p_1$, i.e., $\pi_2(p) = \alpha\tilde{p} + (\beta + \gamma) \cdot \tilde{p}(1 - F_1^b(p)) - t\gamma(1 - F_1(r))$. Finally, approaching $p_2 = v$, we obtain Store 2 profit of $\pi_2 = \alpha\tilde{v} + (\beta + \gamma)\tilde{v}\, m_{1v} - t\gamma(1 - F_1(r))$. It is then clear from this and the corresponding equation on Store 1's profit that both stores cannot have a mass point at v (when $r < v$). Store 2's indifference equations are

$$\begin{aligned}\pi_2 &= (\alpha + \beta)\tilde{p}_0 + (\gamma\tilde{p}_0 - t\gamma)(1 - F_1(r)) \\ &= \alpha\tilde{p} + \beta\tilde{p}(1 - F_1^a(p)) + (\gamma\tilde{p} - t\gamma)(1 - F_1(r)) \\ &= \alpha\tilde{p} + (\beta + \gamma)\tilde{p}(1 - F_1^b(p)) - t\gamma(1 - F_1(r)) \\ &= \alpha\tilde{v} + (\beta + \gamma)\tilde{v}\, m_{1v} - t\gamma(1 - F_1(r)).\end{aligned} \tag{A.2}$$

Finally, the equation of the equilibrium decision of Store 1's *Semi-Loyal* customers on whether to search equates the cost and the benefit of search at r and is

$$s = \int_{p_0}^{r} (r - p)\, dF_2^a(p) = \int_{p_0}^{r} F_2^a(p)\, dp. \tag{A.3}$$

By calculating $F_2^a(p)$ from (8) and plugging into (A.2), we obtain

$$s = \frac{\alpha + \beta}{\beta}\tilde{p}_0\left\{\frac{\tilde{r}}{\tilde{p}_0} - \ln\frac{\tilde{r}}{\tilde{p}_0} - 1\right\}. \tag{A.4}$$

Given reservation price r, (8) and (A.1) jointly characterize the price distributions. It remains to consider Store 1's *Semi-Loyal* consumer behavior at $p_1 = r$: If both stores have a mass point at r, where do consumers who saw both prices buy (if both stores do not have a mass point at the same price, prices cannot be equal with a positive probability and consumer behavior in a zero-probability event is irrelevant for stores). We note that $F_i^a(r) < F_i^b(r)$ (for Store $i = 1$ or 2) indicates a mass point at r, while $F_i^a(r) > F_i^b(r)$ indicates a hole for the range of prices above the reservation price until $F_i^a(r) = F_i^b(p)$ price (Gangwar et al. 2013 also find mass point and empty space in the middle of price distributions). Clearly, if both stores have a mass point at the same price (which as we noted above, is only possible at r), equilibrium requires all consumers who saw both prices must buy at Store 2. Otherwise, Store 2 strictly benefits from reducing the price from that mass point to just below it. Yet for Store 1's mass point at $p_1 = r$, we must consider how *Semi-Loyals* search in equilibrium. Denote their probability of search for $p_1 = r$ by $k \in [0, 1]$. Then the probability that Store 1's *Semi-Loyals* search (relevant for the Store 2's indifference equations above) is $F_1(r) = F_2^a(r) + k(F_2^b(r) - F_2^a(r))$.

Before proceeding with examining all possible solutions to the above indifference equations, to reduce the number of cases to consider, we rule out $k > 0$ when Store 1 has a mass point at $r < v$. The argument is as follows: First, if Store 1 has a mass point at r, Store 2 must have an empty range in its price distribution above r because it is optimal to undercut $p_1 = r$ from prices slightly above r to increase its expected demand from *Shoppers*. If $k > 0$, a positive mass of *Semi-Loyal* consumer searches. Store 1 could prevent this without sacrificing margins by reducing price infinitesimally below r. Because it chooses not to do so, its profits are not negatively affected by k being positive (the commission must exactly offset the profits from the sales lost). Therefore, it should also be indifferent to having all *Semi-Loyals* search by pricing just above r. Yet then it would *strictly* benefit by raising the price to the end of the empty range of Store 2's price distribution, as it would be increasing its margin without sacrificing sales. Thus, it must not be indifferent to *Semi-Loyal* consumer search and may price at r only if *Semi-Loyal* search probability at $p_1 = r$ is 0. Note that this argument does not hold at $r = v$ because if Store 1 were to increase prices, it would lose demand due to consumers unwilling to buy above v. Thus $r = v$ (Case 3) must be separately considered.

To fully characterize the distributions, we consider all possibilities of mass points at r and v, and empty spaces in the ranges $[p_0, r)$ and (r, v), and Store 1's *Semi-Loyal* search behavior k when both stores have a mass point at r. There are five cases that are possible in equilibrium. We denoted them by Case 1, Cases 2a, 2b, 2c, and Case 3; they cover the parameter space without overlap. This implies that parameters uniquely identify each case. The distribution and the profit transitions between cases are continuous and the solutions are unique.

A.2. Proof of Lemma 1 (Case 1)

In Case 1, Store 1 charges prices $p < r$ so that *Semi-Loyals* do not search. Let us define $F_{1r} \triangleq F_1(r^-)$ and $F_{2r} \triangleq F_2(r^-)$, as in Figure 3 of Case 1. As Store 1 is indifferent between pricing at p_0, at reservation price r, or any price between them, $\pi_1(p_0) = \pi_1(p) = \pi_1(r^-)$. At price p_0, Store 1 sells to its *Loyal* and *Semi-Loyals* and all *Shoppers*. Thus, $\pi_1(p_0) = (\alpha+\beta)\tilde{p}_0$ where $\tilde{p}_0 \triangleq p_0 - c$. Similarly, $\pi_1(r^-) = \alpha\tilde{r} + \beta\tilde{r}(1-F_{2r})$ where $\tilde{r} \triangleq r - c$. For price p between p_0 and r, $\pi_1(p) = \alpha\tilde{p} + \beta\tilde{p}(1-F_2(p))$ where $\tilde{p} \triangleq p - c$.

Likewise, Store 2 is indifferent between pricing at $p_2 = p_0$, r^-, and v, i.e., $\pi_2(p_0) = \pi_2(p) = \pi_2(r^-) = \pi_2(v)$. We calculate $\pi_2(p_0) = (\alpha+\beta)\tilde{p}_0$, $\pi_2(r^-) = \alpha\tilde{r} + \beta\tilde{r}(1-F_{1r})$ and $\pi_2(v) = \alpha\tilde{v}$ where $\tilde{v} \triangleq v - c$. For price p between p_0 and r, we get $\pi_2(p) = \alpha\tilde{p} + \beta\tilde{p}(1-F_1(p))$.

Solving these sets of indifference equations together, we obtain

$$\tilde{p}_0 = \frac{\alpha\tilde{v}}{\alpha+\beta},$$
$$F_{1r} = F_{2r} = 1 - \frac{\alpha(\tilde{v}-\tilde{r})}{\beta\tilde{r}},$$
$$F_1(p) = F_2(p) = 1 - \frac{\alpha(\tilde{v}-\tilde{p})}{\beta\tilde{p}}.$$

The reservation price r is endogenous and is calculated from Equation (3)

$$\begin{aligned} s &= \int_{p_0}^{r} (r-p)\,dF_2(p) = \int_{p_0}^{r} F_2(p)\,dp \\ &= \int_{p_0}^{r} \left(1 - \frac{\alpha(\tilde{v}-\tilde{p})}{\beta\tilde{p}}\right) dp = \frac{\alpha\tilde{v}}{\beta}\left\{\frac{\tilde{r}}{\tilde{p}_0} - \ln\frac{\tilde{r}}{\tilde{p}_0} - 1\right\}. \end{aligned}$$

To our knowledge, this has a unique solution for r (see Online Appendix Section O1.1.1).

Next, we characterize the range of parameters for which Case 1 holds. The equilibrium in Case 1 breaks down if Store 1 finds it optimal to price above reservation price r to receive a commission. Thus, for Case 1 to hold, the commission rate should be low enough. If Store 1 deviates to price $p_1 = v^-$, it receives the commission and sells to *Shoppers* and *Semi-Loyals* with probability $1 - F_{2r}$. Thus, its profit will be $\pi_1(v^-) = (\alpha-\gamma)\tilde{v} + t\gamma + (\beta+\gamma)\tilde{v}(1-F_{2r})$. At the boundary of Case 1 and Case 2b, this deviation profit is equal to Store 1's profit in Case 1, $(\alpha-\gamma)\tilde{v} + t\gamma + (\beta+\gamma)\tilde{v}(1-F_{2r}) = \alpha\tilde{v}$. We calculate F_{2r} from this equation and plug it into indifference and search equations to get the boundary of Case 1 and Case 2a

$$\text{B12:}\quad s(t) = \frac{\alpha\tilde{v}}{\beta}\left\{\frac{\beta(\beta\tilde{v}+\gamma t)}{(\alpha\beta+\alpha\gamma+\beta\gamma)\tilde{v}-\beta\gamma t} - \ln\frac{(\alpha+\beta)(\beta+\gamma)\tilde{v}}{(\alpha\beta+\alpha\gamma+\beta\gamma)\tilde{v}-\beta\gamma t}\right\}.$$

This completes the characterization of pricing distributions of Case 1.

A.3. Proof of Lemma 2A (Case 2a)

Let $F_{1ar} \triangleq F_1(r^-)$, $F_{1br} \triangleq F_1(r^+)$, $F_{2r} \triangleq F_2(r^-)$, and $F_{2v} \triangleq F_2(v^-)$ as in Case 2a of Figure 3. Store 1's indifference equations are $\pi_1(p_0) = \pi_1(r^-) = \pi_1(p_m) = \pi_1(v^-)$. At price $p_1 = p_0$, it sells to its *Loyal* and *Semi-Loyals*, and to all *Shoppers*. Thus, $\pi_1(p_0) = (\alpha+\beta)\tilde{p}_0$. At r^- it sells to *Shoppers* only if Store 2's price is higher than the reservation price. Thus, $\pi_1(r^-) = \alpha\tilde{r} + \beta\tilde{r}(1-F_{2r})$. At $p_1 = p_m$, Store 1 still sells to its *Loyals*, while *Semi-Loyals* search and generate a commission of $t\gamma$. The *Semi-Loyals* and *Shoppers* will purchase from Store 1 if Store 2's price is higher than p_m. Thus,

$$\begin{aligned} \pi_1(p_m) &= (\alpha-\gamma)\tilde{p}_m + t\gamma + (\beta+\gamma)\tilde{p}_m \Pr(P_2 \ge p_m) \\ &= (\alpha-\gamma)\tilde{p}_m + t\gamma + (\beta+\gamma)\tilde{p}_m(1-F_{2r}). \end{aligned}$$

Finally, at $p_1 = v^-$, Store 1 sells to its *Loyals* and receives a commission of $t\gamma$, and since Store 2 has a mass point at v, *Semi-Loyals* and *Shoppers* purchase from Store 1 if $p_2 = v$. Thus,

$$\begin{aligned} \pi_1(v^-) &= (\alpha-\gamma)\tilde{v} + t\gamma + (\beta+\gamma)\tilde{v}\Pr(P_2 = v) \\ &= (\alpha-\gamma)\tilde{v} + t\gamma + (\beta+\gamma)\tilde{v}(1-F_{2v}). \end{aligned}$$

Next, for Store 2, $\pi_2(p_0) = \pi_2(r^-) = \pi_2(p_m) = \pi_2(v^-)$. At $p_2 = p_0$, Store 2 sells to its own customers as well as *Shoppers*. Moreover, if $p_1 > r$, it will sell to incoming *Semi-Loyals* and pay a commission to the host. Thus $\pi_2(p_0) = (\alpha+\beta)\tilde{p}_0 + (\gamma\tilde{p}_0 - t\gamma)\cdot(1-F_{1br})$. At $p_2 = r^-$, Store 2 sells to *Shoppers* and to *Semi-Loyal* customers of the host store who search only if $p_1 > r$. Thus,

$$\begin{aligned} \pi_2(r^-) &= \alpha\tilde{r} + \beta\tilde{r}\Pr(P_1 \ge r) + (\gamma\tilde{r} - t\gamma)\Pr(P_1 > r) \\ &= \alpha\tilde{r} + \beta\tilde{r}(1-F_{1ar}) + (\gamma\tilde{r} - t\gamma)(1-F_{1br}). \end{aligned}$$

At $p_2 = p_m$, Store 2 still sells to *Semi-Loyals* and *Shoppers* if $p_1 > p_m$. It has to pay the commission of $t\gamma$ if $p_1 > r$, hence

$$\begin{aligned} \pi_2(p_m) &= \alpha\tilde{p}_m + (\beta+\gamma)\tilde{p}_m \Pr(P_1 \ge p_m) - t\gamma\Pr(P_1 > r) \\ &= \alpha\tilde{p}_m + ((\beta+\gamma)\tilde{p}_m - t\gamma)(1-F_{1br}). \end{aligned}$$

Finally, at $p_2 = v^-$, Store 2 can never sell to *Shoppers* or *Semi-Loyals*. Yet it still has to pay the commission of $t\gamma$ if the host price is higher than r. Hence, $\pi_2(v^-) = \alpha\tilde{v} - t\gamma(1-F_{1br})$.

There are $3+3=6$ indifference equations above from both Stores. A seventh equation is the search Equations (A.2). These equations containing seven unknowns, r, p_0, p_m, F_{1ar}, F_{1br}, F_{2r}, and F_{2v} fully describe the price distributions.

Next, we characterize the range of parameters for which Case 2a holds. Note that the price distribution for Case 2a can be continuously deformed into those for Case 2b and Case 1 when $p_m \to r$ and $p_m \to v$, respectively. The boundary of Case 1 and Case 2a is given by Lemma 1. At the boundary with Case 2b, $p_m = r$ and $F_{1ar} = F_{1br}$. Inserting these two conditions into the indifference and search equations, we get the parametric characterization of the boundary of Case 2a and Case 2b (which we call $B2ab$) in the s-t plane shown in Figure 4

$$\text{B2ab:}\quad \begin{cases} s(r) = \dfrac{\tilde{r}(\alpha+\beta)}{\beta(\beta(\alpha+\beta+\gamma)\tilde{r}+\alpha\gamma\tilde{v})}\left\{\beta((\alpha+\beta+\gamma)\tilde{r}-\alpha\tilde{v}) + \alpha(\beta+\gamma)\tilde{v}\ln\dfrac{\beta(\alpha+\beta+\gamma)\tilde{r}+\alpha\gamma\tilde{v}}{\alpha(\beta+\gamma)\tilde{v}}\right\} \\ t(r) = \dfrac{\tilde{r}(\alpha+\beta)((\alpha+\beta+\gamma)\tilde{r}-\alpha\tilde{v})}{(\alpha+\beta)(\beta+\gamma)\tilde{r}+\alpha\gamma(\tilde{v}-\tilde{r})}, \end{cases}$$
$$\text{where } \tilde{r} \in [\alpha\tilde{v}/(\alpha+\beta+\gamma), \tilde{v}].$$

Next, we show that stores' profits (π_1, π_2) are both below the benchmark profit $(\alpha+\beta)\tilde{p}_0^B = \alpha\tilde{v}$ in Case 2a. For Store 1, $\pi_1(p_0) = (\alpha+\beta)\tilde{p}_0 < (\alpha+\beta)\tilde{p}_0^B$ since equating two indifference equations of Store 2, we find $\tilde{p}_0 = (\alpha\tilde{v})/(\alpha+\beta+\gamma(1-F_{1br})) < \tilde{p}_0^B$. For Store 2, we see that $\pi_2(v^-) = \alpha\tilde{v} - t\gamma(1-F_{1br}) < \alpha\tilde{v}$.

Finally, we sketch why both stores' profits are decreasing in t. For Store 1, suppose to the contrary that profit is nondecreasing for some commission rate. Because in Case 1, profit was at benchmark, and in Case 2a below benchmark, this requires an interior turning point at which $\partial\pi_1/\partial t = 0$. Then, from indifference equations, we find that $\partial p_0/\partial t = \partial r/\partial t = \partial p_m/\partial t = \partial F_{1br}/\partial t = 0$, which results in a contradiction $0 = 0 + \gamma$. We then conclude that profits should be monotonically decreasing in this case. See Online Appendix Section O1.1.2 for more details.

A.4. Proof of Lemma 2B (Case 2b)

Let $F_{1r} \triangleq F_1(r)$, $F_{2ra} \triangleq F_2(r^-)$, $F_{2rb} \triangleq F_2(r^+)$, and $F_{2v} \triangleq F_2(v^-)$, as illustrated in Figure 3, Case 2b. Indifference equations for Store 1 are $\pi_1(p_0) = \pi_1(r^-) = \pi_1(r^+) = \pi_1(v^-)$, where $\pi_1(p_0) = (\alpha+\beta)\tilde{p}_0$, $\pi_1(r^-) = \alpha\tilde{r} + \beta\tilde{r}(1-F_{2ra})$, $\pi_1(r^+) = (\alpha-\gamma)\tilde{r} + t\gamma + (\beta+\gamma)\tilde{r}(1-F_{2rb})$ and $\pi_1(v^-) = (\alpha-\gamma)\tilde{v} + t\gamma + (\beta+\gamma)\tilde{v}(1-F_{2v})$. The rationale is similar to the previous case. Similarly, we write indifference equations for Store 2: $\pi_2(p_0) = \pi_2(r) = \pi_2(v^-)$, where $\pi_2(p_0) = (\alpha+\beta)\tilde{p}_0 + (\gamma\tilde{p}_0 - t\gamma)(1-F_{1r})$, $\pi_2(r) = \alpha\tilde{r} + ((\beta+\gamma)\tilde{r} - t\gamma)(1-F_{1r})$, and $\pi_2(v^-) = \alpha\tilde{v} - t\gamma(1-F_{1r})$.

These five indifference equations are solved for five unknowns: p_0, F_{1r}, F_{2ra}, F_{2rb}, and F_{2v}, to yield

$$\tilde{p}_0 = \frac{\alpha(\beta+\gamma)\tilde{r}\tilde{v}}{\beta(\alpha+\beta+\gamma)\tilde{r} + \alpha\gamma\tilde{v}}, \quad F_{1r} = \frac{(\alpha+\beta+\gamma)\tilde{r} - \alpha\tilde{v}}{(\beta+\gamma)\tilde{r}},$$
$$F_{2ra} = \frac{(\alpha+\beta)(\alpha\tilde{v} - (\alpha+\beta+\gamma)\tilde{r})}{\alpha\gamma\tilde{v} - \beta(\alpha+\beta+\gamma)\tilde{r}},$$
$$F_{2rb} = \frac{t\gamma + (\alpha+\beta)\tilde{r}}{(\beta+\gamma)\tilde{r}} + \frac{\alpha(\alpha+\beta)\tilde{v}}{\alpha\gamma\tilde{v} + (\alpha+\beta+\gamma)\beta\tilde{r}},$$
$$F_{2v} = \frac{t\gamma + (\alpha+\beta)\tilde{v}}{(\beta+\gamma)\tilde{v}} - \frac{\alpha(\alpha+\beta)\tilde{r}}{\alpha\gamma\tilde{v} + (\alpha+\beta+\gamma)\tilde{r}}.$$

We also calculate profits

$$\pi_1 = \frac{\alpha(\alpha+\beta)(\beta+\gamma)\tilde{r}\tilde{v}}{\alpha\gamma\tilde{v} + (\alpha+\beta+\gamma)\beta\tilde{r}}, \quad \pi_2 = \frac{\alpha((\beta+\gamma)\tilde{r}\tilde{v} - t\gamma(\tilde{v}-\tilde{r}))}{(\beta+\gamma)\tilde{r}}.$$

Next, we characterize the range of parameters for which Case 2b holds. As there is no mass point at v in Store 2's price distribution, $F_{2v} = 1$ at the boundary of Case 2b and Case 2c, which we call $B2bc$. Using this, we can simplify the indifference equations and parametrically characterize $B2bc$

$$\text{B2bc:} \begin{cases} s(r) = \Big\{\tilde{r}(\alpha+\beta)(\beta((\alpha+\beta+\gamma)\tilde{r} - \alpha\tilde{v}) + \alpha(\beta+\gamma)\tilde{v} \\ \qquad \cdot \ln\dfrac{\beta(\alpha+\beta+\gamma)\tilde{r} + \alpha\gamma\tilde{v}}{\alpha(\beta+\gamma)\tilde{v}}\Big\} \cdot \dfrac{1}{\beta(\beta(\alpha+\beta+\gamma)\tilde{r} + \alpha\gamma\tilde{v})} \\ t(r) = \dfrac{\tilde{r}(\alpha+\beta)((\alpha+\beta+\gamma)\tilde{r} - \alpha\tilde{v})}{\beta(\alpha+\beta+\gamma)\tilde{r} + \alpha\gamma\tilde{v}}, \end{cases}$$
$$\text{where } \tilde{r} \in [\alpha\tilde{v}/(\alpha+\beta+\gamma), \tilde{v}].$$

Expressions for p_0, r, F_{1r}, F_{2ra}, and π_1 do not depend on commission rate, thus since π_1 was strictly below benchmark in Case 2a, it remains so in Case 2b. The expression for π_2 is linearly decreasing in the commission rate, causing it to also remain below benchmark.

A.5. Proof of Lemma 2C (Case 2c)

Let $F_{1r} \triangleq F_1(r)$, $F_{1v} \triangleq F_1(v^-)$, $F_{2ra} \triangleq F_2(r^-)$, and $F_{2rb} \triangleq F_2(r^+)$, as shown in Figure 3, Case 2c. Indifference equations for Store 1 are $\pi_1(p_0) = \pi_1(r^-) = \pi_1(r^+) = \pi_1(v^-)$, where $\pi_1(p_0) = (\alpha+\beta)\tilde{p}_0$, $\pi_1(r^-) = \alpha\tilde{r} + \beta\tilde{r}(1-F_{2ra})$, $\pi_1(r^+) = (\alpha-\gamma)\tilde{r} + t\gamma + (\beta+\gamma)\cdot\tilde{r}(1-F_{2rb})$, $\pi_1(v^-) = (\alpha-\gamma)\tilde{v} + t\gamma$. Similarly, for Store 2: $\pi_2(p_0) = \pi_2(r) = \pi_2(v^-)$, where, $\pi_2(p_0) = (\alpha+\beta)\tilde{p}_0 + (\gamma\tilde{p}_0 - t\gamma)\cdot(1-F_{1r})$, $\pi_2(r) = \alpha\tilde{r} + ((\beta+\gamma)\tilde{r} - t\gamma)(1-F_{1r})$, $\pi_2(v^-) = \alpha\tilde{v} - t\gamma(1-F_{1r}) + (\beta+\gamma)\tilde{v}(1-F_{1v})$. The solutions of the indifference equations are

$$\tilde{p}_0 = \frac{(\alpha-\gamma)\tilde{v} + t\gamma}{(\alpha+\beta)}, \quad F_{2ra} = \frac{\tilde{r}(\alpha+\beta) - \tilde{v}(\alpha-\gamma) - t\gamma}{\tilde{r}\beta},$$
$$F_{2rb} = \frac{\tilde{r}(\alpha+\beta+\gamma) + \tilde{v}(\alpha-\gamma)}{\tilde{r}(\beta+\gamma)},$$
$$F_{1r} = \frac{(\alpha+\beta+\gamma)(\tilde{r}(\alpha+\beta) - \tilde{v}(\alpha-\gamma) - t\gamma)}{\tilde{r}(\alpha+\beta)(\beta+\gamma) - \gamma(\tilde{v}(\alpha-\gamma) + t\gamma)},$$
$$F_{1v} = \frac{(\alpha+\beta+\gamma)((\beta+\gamma)^2\tilde{v}\tilde{r} - (\alpha-\gamma)\gamma\tilde{v}(\tilde{v}-\tilde{r}) - t\beta\gamma\tilde{r} - t\gamma^2\tilde{v})}{\tilde{v}(\beta+\gamma)(\tilde{r}\alpha + \beta(\beta+\gamma) - \gamma(\tilde{v}(\alpha-\gamma) + t\gamma))}.$$

The profits are

$$\pi_1 = (\alpha-\gamma)\tilde{v} + t\gamma,$$
$$\pi_2 = \big[\tilde{r}\big[\beta(\alpha+\beta+\gamma)(\alpha-\gamma)\tilde{v} + t\gamma(\beta + \alpha(\alpha-\gamma) + \gamma(\alpha+\beta))\big] - t(\alpha+\beta)\gamma(\tilde{v}(\alpha-\gamma) + t\gamma)\big]$$
$$\cdot \big[\tilde{r}(\alpha+\beta)(\beta+\gamma) - \gamma(t\gamma + \tilde{v}(\alpha-\gamma))\big]^{-1}.$$

To find the boundary of Case 2c and Case 3, recall that $r = v$ in Case 3. Inserting p_0 and $r = v$ in the search equation gives $B23$

$$\text{B23:} \quad s = \frac{\tilde{v}(\alpha-\gamma) + t\gamma}{\beta}\left\{\frac{\tilde{v}(\alpha+\beta)}{\tilde{v}(\alpha-\gamma) + t\gamma} - \ln\frac{\tilde{v}(\alpha+\beta)}{\tilde{v}(\alpha-\gamma) + t\gamma} - 1\right\}.$$

Finally, we show that π_1 and π_2 are increasing in t. Store 1's profit $\pi_1 = (\alpha-\gamma)\tilde{v} + t\gamma$ is clearly increasing. As to π_2, we first state π_2 and search equation as a function of F_{1r}. Then, we show that F_{1r} is decreasing and π_2 is increasing in commission rate. See Online Appendix Section O1.1.4 for more details.

A.6. Proof of Lemma 3 (Case 3)

We define $F_{1v} \triangleq F_1(v^-)$ and $F_{2v} \triangleq F_2(v^-)$ as illustrated in Figure 3, Case 3. Because both distributions have a mass point at $r = v$, the equilibrium behavior of customers at $r = v$ becomes important. Note that *Shoppers* purchase at Store 2 at $p_1 = p_2 = v$ with probability 1. Otherwise, Store 2 deviates to $v - \epsilon$ to get all of the *Shoppers*. In addition, *Semi-Loyals* search with probability k at $p_1 = v$ and, upon search, purchase at Store 2 as $p_1 = p_2 = v$.

Next, we show the indifference equations. For Store 1, $\pi_1(p_0) = \pi_1(v^-) = \pi_1(v)$. At $p_1 = p_0$, we have $\pi_1(p_0) = (\alpha+\beta)\tilde{p}_0$ as usual. At $p_1 = v^-$, Store 1 keeps all *Semi-Loyal* customers and sells to *Shoppers* if $p_2 = v$, and $\pi_1(v^-) = \alpha\tilde{v} + \beta\tilde{v}(1-F_{2v})$. Finally, at $p_1 = v$, only k fraction of *Semi-Loyals* search and the remaining $(1-k)\gamma$ stay and purchase from Store 1 so that $\pi_1(v) = (\alpha-\gamma)\tilde{v} + (1-k)\gamma\tilde{v} + k\gamma t$.

For Store 2, $\pi_2(p_0) = \pi_2(v^-)$, where $\pi_2(p_0) = (\alpha+\beta)\tilde{p}_0 + (k\gamma\tilde{p}_0 - kt\gamma)(1-F_{1v})$, and $\pi_2(v^-) = \alpha\tilde{v} + ((\beta+k\gamma)\tilde{v} - kt\gamma)\cdot(1-F_{1v})$. These are four equations to be solved for four unknowns: p_0, k, F_{1v}, and F_{2v}. We find

$$k = \frac{(\alpha+\beta)\tilde{p}_0 - \alpha\tilde{v}}{\gamma(t-\tilde{v})}, \quad F_{2v} = 1 - \frac{(\alpha+\beta)\tilde{p}_0 - \alpha\tilde{v}}{\beta\tilde{v}}.$$

We now show that profits in Case 3 are higher than benchmark. Indifference equation of Store 1, $(\alpha+\beta)\tilde{p}_0 = \alpha\tilde{v} + \beta\tilde{v}\cdot(1-F_{2v})$, implies $p_0 > p_0^B$, and therefore $\pi_1 > \alpha\tilde{v}$. Moreover,

p_0 and thus π_1 do not change with commission. Simplifying indifference equations for Store 2, we get

$$\pi_2 = (\alpha+\beta)\tilde{p}_0 - \frac{((\alpha+\beta)\tilde{p}_0 - \alpha\tilde{v})^2(t-\tilde{p}_0)}{\beta\tilde{v}(t-\tilde{v}) + ((\alpha+\beta)\tilde{p}_0 - \alpha\tilde{v})(\tilde{v}-p_0)}. \quad \text{(A.5)}$$

Taking the first derivative with respect to t

$$\frac{\partial\pi_2}{\partial t} = \frac{(\alpha+\beta)(\tilde{v}-\tilde{p}_0)^2((\alpha+\beta)\tilde{p}_0-\alpha\tilde{v})^2}{\{\beta\tilde{v}(t-\tilde{v}) + ((\alpha+\beta)\tilde{p}_0-\alpha\tilde{v})(\tilde{v}-\tilde{p}_0)\}^2}.$$

This is always positive, implying that π_2 is increasing in commission rate. It is then higher than benchmark because it was so at the boundary of Case 2c and Case 3.

Finally, when t increases to infinity, we find Store 2's asymptote

$$\lim_{t\to\infty} \pi_2 = (\alpha+\beta)\tilde{p}_0 - \frac{((\alpha+\beta)\tilde{p}_0-\alpha\tilde{v})^2}{\beta\tilde{v}}.$$

This is less than $\pi_1 = (\alpha+\beta)\tilde{p}_0$. □

A.7. Proof of Proposition 1 (Asymmetric Subgame)

The proof of this proposition follows directly from Lemmas 1, 2A, 2B, 2C, and 3. We showed that Store 1's profit is $\alpha\tilde{v}$ in Case 1, below $\alpha\tilde{v}$ and decreasing in Case 2a, below $\alpha\tilde{v}$ and constant in Case 2b, increasing in Case 2c, and above $\alpha\tilde{v}$ and constant in Case 3. We also showed that Store 2's profit is $\alpha\tilde{v}$ in Case 1, below $\alpha\tilde{v}$ and decreasing in Case 2a, below $\alpha\tilde{v}$ and decreasing in Case 2b, increasing in Case 2c, and above $\alpha\tilde{v}$ and increasing in Case 3. Therefore, both profits are U-shaped and they weakly increase or decrease together and thus there is no conflict of interest in setting commission. □

A.8. Proof of Proposition 2 (Symmetric Subgame)

In the symmetric subgame, there are four different cases in equilibrium, called Case 1, Case 2a, Case 2b, and Case 3. We show that profits in Case 1 are constant and less than $\alpha\tilde{v}$, and in Cases 2a and 2b are increasing with commission rate. Analysis of these are provided in the Online Appendix (see Sections O1.2.1, O1.2.2, and O1.2.3).

In Case 3, $r = v$ and (similar to Case 3 of the asymmetric subgame) a fraction k of *Semi-Loyals* search at $p = r = v$. Define $F_v \triangleq F(v^-)$. Indifference equations are: $\pi(p_0) = \pi(v^-) = \pi(v)$. At $p = p_0$, the focal Store sells to its own α customers and *Shoppers*, earning $(\alpha+\beta)p_0$, and also receives a fraction k of the rival's *Semi-Loyals* with probability $1-F_v$. Therefore, $\pi(p_0) = (\alpha+\beta)\tilde{p}_0 + (k\gamma\tilde{p}_0 - k\gamma t)(1-F_r)$. At $p = v^-$, a store sells to its own α customer and may sell to *Shoppers* and its competitor's *Semi-Loyals* with probability $1-F_v$. Thus, $\pi(v^-) = \alpha\tilde{v} + (\beta\tilde{v} + k\gamma\tilde{v} - k\gamma t)(1-F_v)$. Finally, at $p = v$, a store both sends and receives *Semi-Loyal* customers. It earns $(\alpha-\gamma)\cdot\tilde{v} + (1-k)\gamma\tilde{v}$ from own customers who stay, $\beta\tilde{v}(1-F_v)/2$ from *Shoppers*, $k\gamma t$ commission plus $(k\gamma/2)\cdot\tilde{v}(1-F_v)$ from own *Semi-Loyals* who search but return back to the Store, and $((k\gamma/2)\tilde{v} - k\gamma t)(1-F_v)$ from rival's *Semi-Loyals.* So

$$\begin{aligned}\pi(v) &= (\alpha-\gamma)\tilde{v} + \frac{\beta}{2}\tilde{v}(1-F_v)\\ &\quad + \left\{(1-k)\gamma\tilde{v} + k\gamma t + \frac{k}{2}\gamma\tilde{v}(1-F_v)\right\}\\ &\quad + \left\{\left(\frac{k}{2}\gamma\tilde{v} - k\gamma t\right)(1-F_v)\right\}\\ &= \alpha\tilde{v} + \frac{\beta}{2}\tilde{v} - \left(\frac{\beta}{2}\tilde{v} + k\gamma\tilde{v} - k\gamma t\right)F_v.\end{aligned}$$

Solving indifference equations

$$k = \frac{(1-F_r)\tilde{v}\beta}{2(t-\tilde{v})\gamma}, \quad \tilde{p}_0 = \frac{\tilde{v}(t-\tilde{v}+(t-2tF_r+\tilde{v}F_r^2)\beta)}{t-\tilde{v}+(t+\tilde{v}(F_r-2)F_r)\beta}. \quad \text{(A.6)}$$

In addition, the search equation becomes

$$s = \frac{\tilde{v}\tilde{p}_0F_v}{\tilde{v}-\tilde{p}_0}\left\{\frac{\tilde{v}}{\tilde{p}_0} - \ln\frac{\tilde{v}}{\tilde{p}_0} - 1\right\}. \quad \text{(A.7)}$$

Plugging (A.6) into (A.7) yields an equation for F_v

$$\begin{aligned}s &= \frac{\tilde{v}(t(1+\beta-2\beta F_v) - \tilde{v}(1-\beta F_v^2))}{2\beta(t-\tilde{v})}\\ &\quad\cdot\left(\frac{t(1+\beta)-\tilde{v}(1+(2-F_v)F_v\beta)}{t(1+\beta-2\beta F_v)+\tilde{v}(\beta F_v^2-1)}\right.\\ &\quad\left. - \ln\frac{t(1+\beta)-\tilde{v}(1+(2-F_v)F_v\beta)}{t(1+\beta-2\beta F_v)+\tilde{v}(\beta F_v^2-1)} - 1\right). \quad \text{(A.8)}\end{aligned}$$

Moreover, profits are

$$\pi = \alpha\tilde{v} + \beta\tilde{v}(1-F_v) - \frac{\beta\tilde{v}}{2}(1-F_v)^2, \quad \text{(A.9)}$$

which is higher than benchmark $\alpha\tilde{v}$ since $0 < F_v < 1 \Leftrightarrow \beta(1-F_v) > \beta(1-F_v)^2/2$.

Finally, we outline the proof of why profits are increasing in commission rate. From search Equation (A.7), we obtain $\text{sgn}(\partial F_v/\partial t) = \text{sgn}(\partial p_0/\partial t)$. From indifference equation $\pi(p_0) = \pi(v^-)$, we get $\text{sgn}(\partial F_v/\partial t) = \text{sgn}(\partial p_0/\partial t) = \text{sgn}(\partial k/\partial t)$. From indifference equations $\pi(v^-) = \pi(v)$, we get $\text{sgn}(\partial F_v/\partial t) = \text{sgn}(\partial p_0/\partial t) = \text{sgn}(\partial k/\partial t) = -1$. Thus, p_0, k, and F_v are all decreasing in commission rate. Finally, we take the derivative of profit function (A.9) and get $\partial\pi/\partial t = -\tilde{v}\beta F_v\,\partial F_v/\partial t$. So, profit is increasing in commission rate in Case 3.

A.9. Proof of Proposition 3 (Equilibrium Hosting/Advertising Choice)

In Propositions 1 and 2, we showed that when the commission is low, the advertising contract is not profitable for stores in symmetric and asymmetric subgames and thus the ad is not displayed. When the commission is high, the advertising contract is industry profitable for stores in symmetric and asymmetric subgames. However, we show that the profit in symmetric subgame (π) is lower than the host profit (π_1) in the asymmetric subgame and thus the only possible equilibrium is asymmetric.

In Section A.5, we show that in the region of Case 2c, $\pi_1 = (\alpha-\gamma)\tilde{v} + t\gamma$. Moreover, in Online Appendix Section O1.2.3, we show that $\pi = (\alpha-\gamma)\tilde{v} + t\gamma((1-k)F_{rh} + kF_{rl})$. Therefore, it is seen that $\pi_1 > \pi$ in Case 2 since $0 < (1-k)F_{rh} + kF_{rl} < 1$.

Now we compare these two profits in the region of Case 3. In the asymmetric subgame, we write Store 1's profit as a function of F_{2v}, where, as before, $F_{2v} \triangleq F_2(v^-)$

$$\pi_1 = \alpha\tilde{v} + \beta\tilde{v}(1-F_{2v}), \quad \text{(A.10)}$$

where F_{2v} solves

$$s = \frac{\tilde{v}(1+\beta-2\beta F_{2v})}{2\beta}\left\{\frac{1+\beta}{1+\beta-2\beta F_{2v}} - \ln\frac{1+\beta}{1+\beta-2\beta F_{2v}} - 1\right\}. \quad \text{(A.11)}$$

In the symmetric subgame, the profit is (A.9) where F_v solves (A.8). Recall that π_1 is unchanging in commission rate while π is asymptotically increasing. Therefore, to prove $\pi_1 > \pi$, it suffices to show that $\pi_1 > \bar{\pi}$, where $\bar{\pi}$ is the asymptote of profit in the symmetric subgame. Taking the limit of (A.8) when t goes to infinity

$$s = \frac{\tilde{v}(1+\beta-2\beta F_v)}{2\beta}\left(\frac{1+\beta}{1+\beta-2\beta F_v} - \ln\frac{1+\beta}{1+\beta-2\beta F_v} - 1\right). \tag{A.12}$$

Comparing (A.11) and (A.12), we observe that the same equations solve F_v and F_{2v}. By comparing (A.9) and (A.11), we see that $\pi < \pi_1$ since $F_v = F_{2v}$.

Finally, we need to show $\pi < \pi_1$ in the overlap region of symmetric subgame's Case 2c and asymmetric subgame's Case 3. See Online Appendix Section O1.3 for the proof. Therefore, when the commission rate is set at a high enough level, $\alpha\tilde{v} < \pi < \pi_1$ and advertising will be displayed in one and only one store.

A.10. Proof of Proposition 4 (Effect of Search Cost)

We first take the derivative of search equation in Case 3 ($r = v$) with respect to search cost and obtain

$$\frac{\partial p_0}{\partial s} = \frac{\alpha+\beta}{\beta}\ln\frac{\tilde{p}_0}{\tilde{v}}. \tag{A.13}$$

This is always negative because $\tilde{p}_0 < \tilde{v}$. Hence, $\pi_1 = (\alpha+\beta)\tilde{p}_0$ decreases with s. This implies that the optimal level of search cost for Store 1 is zero. For Store 2, from Equation (A.5) we have

$$\begin{aligned}\frac{d\pi_2}{ds} &= \frac{d\pi_2}{dp_0}\frac{dp_0}{ds}\\ &= \frac{\beta(\alpha+\beta)\tilde{v}(t-\tilde{v}))^2(\tilde{v}-2\tilde{p}_0(\alpha+\beta))}{\{\beta\tilde{v}(t-\tilde{v})+((\alpha+\beta)\tilde{p}_0-\alpha\tilde{v})(\tilde{v}-\tilde{p}_0)\}^2}\times\frac{dp_0}{ds}.\end{aligned} \tag{A.14}$$

Since dp_0/ds is negative, the optimal level of search cost for Store 2 is obtained by equating the numerator to zero, which yields $\tilde{p}_0 = \tilde{v}(\alpha+\beta)/2$. By plugging p_0 into the search equation, we find the optimal level of search cost for Store 2, $s_2^* = \tilde{v}\{\beta - \ln(1+\beta)\}/2\beta$. This is a horizontal line in the $s-t$ plane that intersects with the boundary of Case 2 and Case 3, B23 (see Figure 6, right panel). Therefore, the optimal level of search cost for Store 1 given the commission rate will be on this line or on the boundary B23, whichever is greater.

Proofs of Propositions 5–10.

See Online Appendix Section O2 for proofs related to model variations and extensions.

Endnotes

[1] For example, in the context of search engine advertising where customer poaching may take the form of buying the competitor's keywords, Sayedi et al. (2014) consider responding on traditional media, while Desai et al. (2014) examine defensively buying one's own brand name.

[2] Amazon discontinued its in-store product ads for competing retailers as of October 31, 2015 (http://goo.gl/9rwMRw), but continues to allow sponsored links to competing websites. The discontinuation may be to protect its data from Google (see http://goo.gl/5Ygbb1) or to promote its third-party seller program (whose sponsored ads program is continued), and highlight other influences, not considered in this paper, for adopting in-store advertising of competitors. At the same time, Amazon's decision could be due to competitive drivers that we consider. We thank an anonymous reviewer for alerting us to these developments.

[3] For example, Nault and Tyagi (2001) study horizontal alliances, i.e., alliances among related firms, such as real estate industry brokers, wherein customers of one firm might need to use services in the other firm's area. The alliance allows a member firm to obtain information and services from members in other areas if the customer requires it. The firm recommends a customer whose needs it cannot fulfill to the other firm for a referral fee or royalty.

[4] Villas-Boas (1994) shows that sharing an advertising agency with a competitor may decrease competition; Pancras and Sudhir (2007) consider conditions under which firms may benefit from sharing customer data. Kuksov (2013) argues that firms' response in product differentiation to a reduction in search costs may result in the positive net effect of a reduction in search costs on profits.

[5] Katona and Sarvary (2008) study websites' ad link purchase decisions when websites value consumer traffic and links increase traffic and add to the website value because the content of the links increases the content of the website. Sen et al. (2015) find a significant increase in grocery store purchases when a gas station co-located nearby. Stores may be willing to enter higher price competition in exchange for reaching more customers (e.g., Baye and Morgan 2001 and Iyer and Pazgal 2003 explore seller incentives to cooperate with information intermediaries). For novel products, cooperating with competitors may also promote category growth (see, for example, Lu and Shin 2015).

[6] The presence of *Loyal* and *Shopper* segments results in stores using a mixed strategy pricing in equilibrium (as in, e.g., Narasimhan 1988 and Varian 1980).

[7] The existence of a segment that has a preference for one store but is prone to limited price searching is common in practice and is similar to what Chen et al. (2001) refer to as "opportunistic loyals" and Iyer and Pazgal (2003) call "partial loyals."

[8] Note that even price advertising, when it informs customers about the store carrying a product, may correlate with higher prices (Janssen and Non 2009), although price advertising with signaling motives may indicate lower prices (Simester 1995).

[9] Some persuasive advertising may be viewed as informative about product attributes (Soberman 2004).

[10] The condition on t cannot be satisfied when search cost s is not high enough. Then the equilibrium for low t starts with Case 2.

[11] Note that the consumer reservation price is endogenous, but as customers do not directly observe the distribution of prices and stores set prices simultaneously, the reservation price is determined by the equilibrium strategies and is not affected by specific choices or deviations from equilibrium the stores may entertain.

[12] The *Semi-Loyal* consumer reservation price r is the unique solution of

$$s = \frac{\alpha(v-c)}{\beta}\left\{\frac{r-p_{0B}}{p_{0B}-c} - \ln\frac{r-c}{p_{0B}-c}\right\}.$$

[13] The minimum price p_0 uniquely solves

$$s = \frac{(\alpha+\beta)(p_0-c)}{\beta}\left\{\frac{v-p_0}{p_0-c} - \ln\frac{v-c}{p_0-c}\right\}.$$

[14] To completely specify the equilibrium choices in the decision to host stage, in a trembling hand perfect equilibrium, neither store chooses to be the host when the commission rate is low and both choose to host when the commission rate is high. Alternatively, if there is a small cost of hosting, only one chooses to host ads.

[15] The advertiser's profit is maximized at

$$s^* = \max\left\{\frac{(v-c)(\alpha-\gamma)+t\gamma}{\beta}\left(\frac{(v-c)(\alpha+\beta)}{(v-c)(\alpha-\gamma)+t\gamma} - \ln\frac{(v-c)(\alpha+\beta)}{(v-c)(\alpha-\gamma)+t\gamma} - 1\right), \frac{v-c}{2\beta}(\beta-\ln(1+\beta))\right\}.$$

[16] Google's DoubleClick is an example of such an intermediary. Although it employs an auction mechanism to set the CPC, Google has considerable control over the commission outcome as it decides on the auction format, reserve price, and the quality score adjustments. In addition, Google sets the percentage of the commission it gets. For example, within our model, the auction would reduce to the reserve price, since only one advertiser is present.

[17] Here

$$\lambda^* \triangleq \frac{\gamma(\alpha+\beta)(\tilde{v}-\tilde{p}_0)((\alpha+\beta)\tilde{p}_0-\alpha\tilde{v})}{(\beta\tilde{v}+\gamma(\tilde{v}-\tilde{p}_0))((\alpha+\beta)\tilde{p}_0-(\alpha-\gamma)\tilde{v})},$$

$$t_a^* \triangleq \tilde{v} + ((\alpha+\beta)\tilde{p}_0-\alpha\tilde{v})\max\left\{\frac{1}{\gamma}, \frac{(\sqrt{\alpha+\beta}-\sqrt{\lambda})(\tilde{v}-\tilde{p}_0)}{\beta\sqrt{\lambda}\tilde{v}}\right\}$$

and p_0 is the minimum price in Case 3 of the asymmetric game. In addition, $\tilde{p}_0 = p_0 - c$ and $\tilde{v} = v - c$.

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