# Keyword Search Advertising and Limited Budgets

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In keyword search advertising, many advertisers operate on a limited budget. Yet how limited budgets affect keyword search advertising has not been extensively studied. This paper offers an analysis of the generalized second-price auction with budget constraints. We find that the budget constraint may induce advertisers to raise their bids to the highest possible amount for two different reasons, i.e., to accelerate the elimination of the budget-constrained competitor, and to reduce their own advertising cost. Thus, in contrast to the current literature, our analysis shows that both budget-constrained and unconstrained advertisers could bid more than their own valuation. We further extend the model to consider dynamic bidding and budget-setting decisions.

*Keywords*: keyword search advertising; budget constraint; generalized second-price auction; online advertising; competitive bidding strategy; analytical model
*History*: Received: May 30, 2012; accepted: January 2, 2015. Kannan Srinivasan served as guest editor-in-chief and Ganesh Iyer served as associate editor for this article. Published online in *Articles in Advance* May 8, 2015.

## 1. Introduction

Keyword search advertising has become a major form of online advertising with an annual revenue of $18.4 billion in the United States (Interactive Advertising Bureau 2014). In keyword search advertising, a search query (or keyword) triggers a number of text-based advertisements that are listed according to the outcome of the generalized second-price auction. Given its growing practical importance, this new form of online advertising has drawn the attention of academic researchers (Edelman et al. 2007, Varian 2007, Katona and Sarvary 2010). So far the literature has examined the mechanism of the generalized second-price auction, but without considering the budget constraint of advertisers. However, in reality, advertisers do not have unlimited resources allocated for search advertising. Recognizing this constraint, search engines also explicitly ask advertisers to specify their daily spending limit. Therefore, it is crucial to understand how budget constraints affect keyword search advertising. This paper seeks to address this issue by examining the generalized second-price auction with budget constraints.

In the absence of budget constraints, the extant models of the generalized second-price auction suggest that advertisers will be listed in the order of valuations and that they are likely to bid as much as the marginal profit they could earn from the slot they take in equilibrium (Varian 2007).[1] In reality, however, a lower-valuation advertiser might be listed higher than a higher-valuation advertiser (Jerath et al. 2011). Moreover, advertisers could become aggressive and bid beyond their marginal profit, even close to the higher-ranked competitor's bid (Ganchev et al. 2007). These aberrations motivate us to closely examine the impact of a budget constraint on the advertisers' bidding incentives. Specifically, we seek answers to the following questions: How does a budget constraint alter the advertisers' bidding strategy? More important, what is the process by which a budget constraint affects the bidding behavior? As a consequence of such a change, how will the equilibrium profits and the equilibrium listing order be influenced? Also, how does the resulting equilibrium bid compare with the per-click valuation?

To investigate these issues, we develop a model of the generalized second-price auction in the context of keyword search advertising. In this model, advertisers compete for slots in a one-shot bidding game to generate clicks to their website. They play a bidding game with pre-specified budget constraints. We then extend the model to consider a two-period bidding game and the budget-setting stage prior to the bidding game.

Our analysis first shows that, in the presence of a budget constraint, advertisers are not always listed in the order of valuations. We find that a budget-constrained advertiser may concede a slot to a lower-valuation advertiser if its own budget is small. The reason is, with a limited budget, the advertisement will be listed for a limited period of time and generate

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[1] The generalized second-price auction has been shown to have multiple equilibria. Bidding as much as an advertiser's own marginal profit is equivalent to bidding at the lower bound of these solutions, which was shown to be consistent with the advertisers' best locally

envy-free equilibrium (Edelman et al. 2007) and was also suggested as a reasonable bidding strategy following a compelling argument (Varian 2007).

a limited number of clicks, which makes the advertiser an inferior competitor.

Given the listing order, we also find that the presence of a budget-constrained advertiser may motivate *all* lower-ranked advertisers to shift their bids to the highest possible amount that they can bid in equilibrium. Given that, in the generalized second-price auction, an advertiser's bid determines the cost of its higher-ranked competitor, this highest bid is set such that it decreases their respective higher-ranked competitor's profits to the level of the profits the competitor would earn by moving down to the next slot. This implies that due to the budget constraint of *one* advertiser, *multiple* advertisers listed below can experience a profit squeeze, earning zero marginal profit in their current slot.

We identify two different mechanisms that drive the above result. First, advertisers increase their bids to raise the cost of their higher-ranked competitor, but they do so only when the competitor is budget-constrained. To see this, note that in the absence of a budget constraint, increasing the bid and thus the cost of the competitor has no bearing on the advertiser's own profits. However, if a higher-ranked advertiser is budget-constrained, raising the cost of this competitor can accelerate the depletion of its budget. After the competitor exhausts its budget and thus drops out of the auction, all advertisers listed below can move up by a slot, collect more clicks at the same per-click price, and earn additional profits. Given this, the advertiser listed immediately below a budget-constrained advertiser is motivated to increase its own bid because its bid directly affects the cost of the budget-constrained competitor. Interestingly, the budget constraint can also motivate distant advertisers in even lower slots to bid more. This is because an equilibrium bid in the generalized second-price auction is affected by the lower-ranked advertiser's bid, and any bid increase of distant advertisers can ripple through the list of higher-ranked advertisers. This way, even distant advertisers can contribute to the earlier elimination of the budget-constrained competitor, which in turn helps them to improve their own profits. Thus, they will raise their bids as high as possible in equilibrium.

Second, when the competitor is budget-constrained, an advertiser may reduce its own cost by bidding more. This will be the case when an advertiser (call it Advertiser *i*), takes the slot right next to a budget-constrained competitor. As shown above, due to this higher-ranked competitor's budget constraint, another competitor below Advertiser *i* will raise its bid until Advertiser *i*'s profits decrease to the level of profits it would earn at the next slot. Advertiser *i*, by increasing its own bid, can improve this hypothetical next-slot profit because its bid prompts the higher-ranked competitor's elimination from the auction. Therefore,

when Advertiser *i* increases its own bid and thus its own hypothetical next-slot profit, the competitor below Advertiser *i* is left with less room to squeeze Advertiser *i*'s profits. This results in a lower bid of the next-ranked competitor, leading to a smaller cost for Advertiser *i*. Thus, Advertiser *i* can increase its bid to reduce its own cost.

To assess the equilibrium implications of a budget constraint, we examine the equilibrium bids of the budget-constrained advertiser as well as those of others. We find that advertisers listed below a budget-constrained advertiser may set their bids higher than the valuation when the slots are sufficiently heterogeneous in terms of click-through rates (CTRs). To see this, first note that these advertisers can increase their bids until their respective higher-ranked competitor earns no more profits at the current slot than the next slot. Because greater heterogeneity among slots implies a greater difference in profits across slots, advertisers have more room to increase their bids, even beyond their valuation. Interestingly, even the budget-constrained advertiser can bid at least as high as its own valuation if it takes the last slot. Denote such an advertiser by Advertiser *N*. Because of Advertiser *N*'s budget constraint, the next-ranked advertiser in equilibrium sets its bid such that Advertiser *N* does not earn any more profits than those of losing the last slot, in which case it obtains zero profits. This implies that the next-ranked competitor bids exactly the valuation of Advertiser *N*. To win over this competitor, Advertiser *N* bids at least as high as the competitor's bid, which is its own valuation.

We also extend the model to examine two additional issues. First, our analysis of the dynamic bidding decisions shows that the high-valuation advertiser may give up taking the top slot in one period, not only when the budget is sufficiently small but also when it is sufficiently large. We also examine the endogenous budget-setting decisions and find that the low-valuation advertiser may voluntarily choose to be budget-constrained whereas the high-valuation advertiser always chooses a budget large enough to cover the entire period.

## 1.1. Related Literature

This paper studies the role of the budget constraint in keyword search advertising and thus builds on the literature on keyword search advertising. Edelman et al. (2007) and Varian (2007) pioneered theoretical research on search advertising by developing an equilibrium concept specific to this context. Since then, both theoretical and empirical researches have investigated issues on auction mechanisms (Balachander et al. 2009; Zhu and Wilbur 2011; Jerath and Sayedi 2012; Amaldoss et al. 2015a, b), consumer search (Athey and Ellison 2011, Chen and He 2011), listing

order (Katona and Sarvary 2010, Jerath et al. 2011, Xu et al. 2011), keyword choice (Desai et al. 2014, Rutz and Bucklin 2011, Rutz et al. 2011), measurement and evaluation (Rutz et al. 2012), position effects (Agarwal et al. 2011), interaction with organic results (Yang and Ghose 2010), interaction with other media (Joo et al. 2013), advertising design (Rutz and Trusov 2011), dynamic bidding patterns (Zhang and Feng 2011), and click fraud (Wilbur and Zhu 2009). Surprisingly, most of these studies have abstracted away from the budget-constraint, with only a few exceptions (e.g., Amaldoss et al. 2015a, Sayedi et al. 2014, Shin 2010, Wilbur and Zhu 2009). However, in reality, most advertisers are subject to the budget constraint and may alter their strategies due to their limited budgets. Thus, this work focuses on the role that the budget constraint plays in keyword search advertising. In particular, we show that advertisers may shift their bids from the lowest to the highest possible amount when faced with a budget-constrained competitor. This helps us understand the aggressive bidding behavior that is often observed in practice. In a related but independent work, Lu et al. (2015) also study the impact of advertisers' budget constraints in keyword search auctions. They find that under some conditions, an advertiser's budget constraint can increase that advertiser's profit and the search engine's profit. While their focus is on the implications of the bid-raising strategy, especially on the profits of the advertiser and the search engine, we center our focus on uncovering the advertisers' motivations for bid-raising strategy as well as its implications on the equilibrium bid.

At the heart of such aggressive bidding strategies is the incentive to eliminate the competitor from the auction to achieve long-term gains. The Raising Rivals' Costs (RRC) literature shares a similar idea (Shin 2009). The idea behind the RRC strategy is to put competitors at a disadvantage by raising their costs and to obtain competitive advantage in the market (Salop and Scheffman 1983). We have seen many applications of the RRC strategy in industrial organization theory, including exclusive dealing (Salop and Scheffman 1983, Krattenmaker and Salop 1986), vertical integration (Krattenmaker and Salop 1986, Ordover et al. 1990) and overbuying supply materials (Salop and Scheffman 1987), all as an option of the stronger firm. By contrast, this paper illustrates that, in the context of keyword search advertising, the cost-raising strategy is relevant not only for stronger firms but also for weaker firms.

Based on a similar idea, Pitchik and Schotter (1988) and Benoit and Krishna (2001) have investigated the aggressive bidding behavior in two sequential second-price auctions. They show that the equilibrium bid of the first auction can be high due to their incentive to win the second auction at a lower price. However, in the context of the single-unit second-price auction, this discussion cannot go further than the immediate losing bidder. By contrast, in our setting, the benefit of driving out a certain winner can be shared by multiple lower-ranked bidders. Thus, our research provides a more nuanced discussion on the condition needed for this aggressive bidding incentive to prevail. We also add some new findings that can be shown only in the context of multi-item auctions. For example, in the generalized second-price auction, the profits of multiple bidders can be negatively affected by the budget constraint of one bidder; even the budget-constrained bidder can bid beyond its own valuation. More important, we identify another incentive for an aggressive bid even without any intention to hurt the competitor, i.e., to reduce the bidder's own cost. To our knowledge, such an incentive has not been discussed in any literature and is specific to the generalized second-price auction setting.

In the auction literature, there have been observations of overbidding behavior in various contexts (for example, Coppinger et al. 1980 in the first-price auction; Kagel et al. 1987 in the second-price auction; Che et al. 2011 in the generalized second-price auction). Several explanations have been given for the phenomenon. Chen and Plott (1998) showed that risk-averse bidders may overbid in the first-price auction. Cox et al. (1992) found that the utility of winning can help better explain the overbidding data. In the second-price auction, Kagel et al. (1987) ascribed overbidding to the illusion that it may increase the chance to win. Finally, Morgan et al. (2003) showed that spiteful bidding can rationalize the overbidding in the second-price auction. The bidding pattern that we consider in our paper also involves aggressive bidding behavior, but it does not constitute a behavioral aberration. On the contrary, aggressively bidding under a competitor's budget constraint is a profit-maximizing strategy. In addition, although such a strategy might appear to involve spiteful motivation, we show that it can be rationalized as an act driven by pure self-interest.

The rest of the paper is organized as follows. In §2, we introduce our model. We then provide a theoretical analysis of the model in §3. In §4, we extend the model in two ways and provide analysis for each of these extensions. In §5, we conclude with a summary of the paper, a discussion of its limitations, and directions for future research.

## 2. Model

In this section, we present a model of the generalized second-price auction in the context of keyword search advertising. We first describe the participants and then present the rules of the auction. Later we explain the equilibrium concept used in our model.

## 2.1. Participants

We consider a search engine that auctions off two advertising slots. As in reality, these slots are sequentially listed; thus, the first slot generates more clicks than the second for the same advertiser. To capture this difference, we assume that the CTR of an advertisement in the second slot is discounted by $\delta$ from that in the first slot. Thus, without loss of generality, we represent the slot-specific CTRs of the two slots, respectively, by 1 (for the first slot) and $1 - \delta$ (for the second slot), where $0 < \delta < 1$.

In addition to the search engine, there are three advertisers in this market, denoted by Firm H, Firm M, and Firm L, who advertise on the search engine results page and thus participate in this position auction.[2] Through advertising, they aim to generate traffic to their website and thus strive to collect more clicks with the same advertising budget.

The three advertisers in our model are heterogeneous in their valuation for a click, which we denote by $v_i$, where $i \in \{H, M, L\}$. While this reflects the fact that some advertisers are better than others at converting clicks into sales, we do not explicitly model this conversion process but treat $v_i$ as an exogenous parameter, following the tradition of search advertising literature (see Varian 2007 and Edelman et al. 2007). Advertisers also vary in their ability to generate clicks. Because some advertisers are more attractive or relevant to consumers, they generate more clicks than others at the same slot. To represent this difference, we denote the advertiser-specific CTR by $r_i$, where $i \in \{H, M, L\}$. Then Advertiser $i$'s CTR at the first slot is given by $r_i$, whereas that at the second slot is $(1 - \delta)r_i$. By normalizing the number of impressions to one, we obtain the number of clicks that Advertiser $i$ generates as $r_i$ at the first slot and $(1 - \delta)r_i$ at the second slot. Given the heterogeneity among advertisers described thus far, we define the CTR-weighted valuation as the product of the CTR and the valuation, i.e., $r_i v_i$, where $i \in \{H, M, L\}$, and assume that the three advertisers are ordered by $r_H v_H > r_M v_M > r_L v_L$.

## 2.2. Generalized Second-Price Auction

In the generalized second-price auction for advertising slots, each advertiser submits its bid for a click $b_i$ based on the valuation for a click $v_i$ ($i = H, M, L$). The search engine then ranks the advertisers by their potential profitability, which is given by the product of their CTR and the bid, $\{r_i b_i\}$. We also call this a CTR-weighted bid. The generalized second-price auction rule suggests that each advertiser is charged per click the minimum amount that allows

them to take the current slot. In particular, if Advertiser $i$ takes Slot $j$, this advertiser has to pay per click, $p_{ij} \equiv (r_{[j+1]}b_{[j+1]})/r_i$, where $[j+1]$ denotes the $(j+1)$th-ranked advertiser in terms of the CTR-weighted bid. Then the profit of Advertiser $i$ when ranked at $j$th position is given by

$$\Pi_{ij} = \begin{cases} r_i(v_i - p_{i1}) = r_i v_i - r_{[2]}b_{[2]} & \text{when } j = 1, \\ (1-\delta)r_i(v_i - p_{i2}) = (1-\delta)(r_i v_i - r_{[3]}b_{[3]}) \\ \qquad\qquad\qquad\qquad\qquad \text{when } j = 2, \\ 0 & \text{when } j = 3. \end{cases} \quad (1)$$

Note that, in this auction, there exists a minimum bid $C_0$ in units of the CTR-weighted bid. If an advertiser drops out of the auction for any reason, there might be no advertiser below Advertiser $i$. In such a case, the payment of Advertiser $i$ is determined by this minimum bid, that is, $r_i p_{ij} = C_0$. We assume $C_0 < r_L v_L$.

Each advertiser may be financially constrained. For this possibility, let $K_i$ ($i = H, M, L$) denote Advertiser $i$'s budget for the auction. We assume that the budgets are specified by factors exogenous to the model. Later in a model extension, we also consider the endogenous choice of the budget by each advertiser (see §4.2). Given $K_i$, Firm $i$'s budget constraint is defined to be binding when $K_i$ is less than the total cost that Firm $i$ would incur if it stayed in the auction for the entire period.[3] Thus, when Firm $i$'s budget constraint is binding, the advertisement of Firm $i$ is shown at Slot $j$ only for a fraction of the period ($K_i/(r_i p_{ij})$ at the first slot and $K_i/((1 - \delta)r_i p_{ij})$ at the second slot), after which the lower-ranked advertisers' advertisements move up by one slot.[4] When the budget constraint is not binding, since the budget is greater than the potential payment, the advertisement will be shown for the entire period. Finally, when both Firm $i$ and Firm $i'$ have binding budget constraints, Firm $i$ is said to be more budget-constrained if Firm $i$'s advertisement would be shown for a shorter period than that of Firm $i'$ in their respective slots.

## 2.3. Equilibrium Concept

We model the bidding game among advertisers as a one-shot, complete information game. This game structure reflects the long-term stabilized state of

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[2] In this paper, advertisers are interchangeably referred to as firms or bidders.

[3] Because the cost varies with the slot a firm takes, the binding budget constraint is defined for a pair of a firm and a slot. However, whenever the slot can be explicitly or implicitly fixed, we define the budget constraint of a firm without specifying a slot.

[4] In reality, search engines provide an option to spread the display of the advertisement throughout the entire period. Thus, at any point in the period, the advertisement is shown with a probability $K_i/(r_i p_{ij})$ (at the first slot) or $K_i/((1 - \delta)r_i p_{ij})$ (at the second slot). In either case, by observing the frequency of the display of the competitor's advertisement, each advertiser can correctly infer the advertising budget of the competitor.

the bidding game and thus captures the essence of the competition for the slots while abstracting away from the dynamic aspect that is less essential to the decision problem (see Katona and Sarvary 2010, and Jerath et al. 2011 for similar assumptions; and also Varian 2007 and Edelman and Schwarz 2010 for similar arguments on this assumption). However, to examine the impact of the dynamic decisions on the bidding equilibrium, in §4.1, we offer a model extension wherein we allow for changes in bids across the two periods. In these games, we derive a Symmetric Nash Equilibrium where no firm has an incentive to switch its slot with any other firm (Varian 2007). Because this equilibrium concept results in multiple equilibria, we further refine it by focusing on the Pareto-dominant equilibrium, which is derived as the lower bound of the equilibrium bid for every firm. In the literature (Varian 2007) the lower bound solution is viewed as the compelling equilibrium.

## 3. Analysis

In this section, we provide an analysis of our model with a focus on the impact of the budget constraint in keyword search advertising. For this purpose, we first establish the benchmark by examining a case in which no advertiser has a limited budget. Then we investigate the impact of limited budgets by considering the budget constraint of Firm H, Firm M, and both, in order.

### 3.1. No Budget Constraint

Suppose no firm has a binding budget constraint and the equilibrium listing order is given as H-M-L. Then the Symmetric Nash Equilibrium implies that Firm H cannot earn more profits by moving down to the second slot. Using the profits given in (1), we have

$$\Pi_{H1} = r_H v_H - r_M b_M \geq \Pi_{H2} = (1-\delta)(r_H v_H - r_L b_L). \quad (2)$$

At the same time, Firm M should not earn more profits by switching its slot with Firm H or Firm L. This leads to the following conditions:

$$\Pi_{M2} = (1-\delta)(r_M v_M - r_L b_L) \geq \Pi_{M1} = r_M v_M - r_M b_M, \quad (3)$$

$$\Pi_{M2} = (1-\delta)(r_M v_M - r_L b_L) \geq \Pi_{M3} = 0. \quad (4)$$

Note that when Firm M deviates to the first slot, its payment is given by $r_M b_M$, and not by $r_H b_H$. This is because the deviation from the Symmetric Nash Equilibrium involves switching positions while keeping the payment the same on each position (see Varian 2007 for more clarification). Finally, in equilibrium, Firm L cannot earn more profits by moving up to the second slot

$$\Pi_{L3} = 0 \geq \Pi_{L2} = (1-\delta)(r_L v_L - r_L b_L). \quad (5)$$

These inequalities imply that Firm M and Firm L's equilibrium bids are bounded as follows:

$$\delta r_M v_M + (1-\delta) r_L b_L \leq r_M b_M \leq \delta r_H v_H + (1-\delta) r_L b_L, \quad (6)$$

$$r_L v_L \leq r_L b_L \leq r_M v_M. \quad (7)$$

In addition, the ranking determination rule of the auction implies

$$r_H b_H \geq r_M b_M. \quad (8)$$

While any combination of bids satisfying (6)–(8) makes every firm stay at the current slot and thus constitutes a Symmetric Nash Equilibrium, we focus on the following Pareto-dominant equilibrium (i.e., the lower-bound solution):

$$b_L^* = v_L, \quad (9)$$

$$b_M^* = \delta v_M + (1-\delta)\frac{r_L v_L}{r_M}, \quad (10)$$

$$b_H^* = \delta\frac{r_M v_M}{r_H} + (1-\delta)\frac{r_L v_L}{r_H}. \quad (11)$$

Note that the equilibrium bids of Firm M and Firm L given in (9) and (10) are obtained by equating their own profits in the current slot to the deviation profits in the upper slot; that is, by equating the two sides of inequalities given in (3) and (5). This implies that both firms choose to bid the marginal profit they would earn by moving up by one slot. Because the bid is based on their willingness to pay for the higher slot, this is a reasonable bidding strategy for advertisers to follow (see Jerath et al. 2011 for a similar interpretation).

Finally, note that the equilibrium listing order without any binding budget constraint is always given as H-M-L.[5] This is because a firm with a higher CTR-weighted valuation can generate greater additional profits from the additional clicks obtained from the higher slot compared to the lower slot. Then, according to the aforementioned bidding strategy of the Pareto-dominant equilibrium, the firm with a higher CTR-weighted valuation will bid higher and thus win the higher slot (see also Varian 2007). We next consider the cases with binding budget constraints.

### 3.2. Firm H's Budget Constraint

Suppose Firm H's budget constraint is binding at the first slot whereas the other firms are not budget-constrained (i.e., $(1-\delta)r_L v_L < K_H < \delta r_M v_M + (1-\delta)r_L v_L$

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[5] To see this, replace H, M, and L, respectively by [1], [2], and [3], in inequalities given in (2)–(5). Then, adding (2) and (3) side by side results in $r_{[1]}v_{[1]} \geq r_{[2]}v_{[2]}$ while adding (4) and (5) side by side leads to $r_{[2]}v_{[2]} \geq r_{[3]}v_{[3]}$. Because $r_H v_H > r_M v_M > r_L v_L$, these suggest that in equilibrium, Firm H takes the first slot, Firm M takes the second slot, and Firm L does not take any slot.

and $K_i \geq \delta r_M v_M + (1-\delta) r_L v_L$, $i = M, L$). As shown in the previous section, the equilibrium listing order among advertisers without binding budget constraints is determined by the order of CTR-weighted valuations. Because Firm H's budget constraint is binding only at the first slot, the listing orders other than H-M-L or M-H-L cannot be observed in equilibrium. In fact, by contrast with the previous case, both H-M-L and M-H-L can be observed in equilibrium. More specifically, M-H-L becomes an equilibrium listing order if Firm H is heavily budget-constrained at the first slot, i.e., if $K_H$ is small enough (see Claim A1 in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mksc.2015.0915) for the proof). This is because Firm H's budget constraint shortens the period that Firm H stays in the auction at the first slot and thus decreases the actual number of clicks that Firm H could obtain. This in turn decreases the *effective* CTR-weighted valuation for Firm H at the first slot, causing Firm H to lose the first slot to Firm M. Thus, the equilibrium listing order in this case is not always consistent with the order of CTR-weighted valuations. However, moving forward, we focus on the order of H-M-L for the purpose of the illustration, relegating the analysis of the other case to the online appendix.

When Firm H takes the first slot, it participates in the auction while its budget lasts, but drops out afterwards. Thus, for the $K_H/(r_M b_M)$ fraction of the period, Firm H takes the first slot while Firm M gets the second. For the rest (i.e., $1 - K_H/(r_M b_M)$ of the period), only Firm M and Firm L participate and take the first and second positions, respectively. When Firm L takes the second slot, since there is no other advertiser below it, the payment is given by the minimum bid $C_0$. In this case, the incentive compatibility conditions are given as follows:

$$\Pi_{H1} = \frac{K_H}{r_M b_M} r_H v_H - K_H$$
$$\geq \Pi_{H2} = (1-\delta)(r_H v_H - r_L b_L), \tag{12}$$

$$\Pi_{M2} = \left\{ \frac{K_H}{r_M b_M}(1-\delta) + \left(1 - \frac{K_H}{r_M b_M}\right) \right\}(r_M v_M - r_L b_L)$$
$$\geq \Pi_{M1} = r_M v_M - r_M b_M, \tag{13}$$

$$\Pi_{M2} = \left\{ \frac{K_H}{r_M b_M}(1-\delta) + \left(1 - \frac{K_H}{r_M b_M}\right) \right\}(r_M v_M - r_L b_L)$$
$$\geq \Pi_{M3} = \left(1 - \frac{K_H}{r_M b_M}\right)(1-\delta)(r_M v_M - C_0), \tag{14}$$

$$\Pi_{L3} = \left(1 - \frac{K_H}{r_M b_M}\right)(1-\delta)(r_L v_L - C_0)$$
$$\geq \Pi_{L2} = \left\{ \frac{K_H}{r_M b_M}(1-\delta) + \left(1 - \frac{K_H}{r_M b_M}\right) \right\}(r_L v_L - r_L b_L). \tag{15}$$

Note that when Firm M switches its position with Firm L in deviation, Firm L pays $r_M b_M$ and thus Firm H is still budget-constrained and drops out after a $K_H/(r_M b_M)$ fraction of the period (see the right-hand side of (14) and (15)). By simple inspection, it is clear that (12) determines the upper bound while (13) gives the lower bound of $b_M$. In this case, however, every bid between these two bounds *cannot* be an equilibrium bid for Firm M. This is because Firm M's profit at the second slot increases with its own bid: $\partial\Pi_{M2}/\partial b_M = (K_H/(r_M b_M^2))\delta(r_M v_M - r_L b_L) > 0$.[6] Thus in equilibrium, Firm M makes its bid *uniquely* at the upper bound solution

$$b_M(b_L) = \frac{r_H v_H K_H}{r_M\{K_H + (1-\delta)(r_H v_H - r_L b_L)\}}. \tag{16}$$

Here, note also that this solution is obtained by equating Firm H's profit in the first slot to that in the second slot, i.e., the two sides of (12). This implies that Firm M chooses to bid such that at the first slot, Firm H cannot earn any more profits than it would earn in the second slot.

Similarly, focusing on Firm L's bid, observe that, given $\Pi_{L3}$ in (15) and $b_M(b_L)$ in (16),

$$\frac{d\Pi_{L3}}{db_L} = \frac{\partial\Pi_{L3}}{\partial b_L} + \frac{\partial\Pi_{L3}}{\partial b_M}\frac{\partial b_M(b_L)}{\partial b_L}$$
$$= 0 + (1-\delta)(r_L v_L - C_0)\frac{K_H}{r_M b_M^2}$$
$$\cdot \frac{(1-\delta) r_H v_H r_L K_H}{r_M\{K_H + (1-\delta)(r_H v_H - r_L b_L)\}^2} > 0, \tag{17}$$

which implies that Firm L's profit (i.e., $\Pi_{L3}$) increases with its own bid. Thus, although (14) and (15), respectively, define the upper bound and lower bound of $b_L$, the equilibrium bid is *uniquely* determined at the upper bound, as the following:

$$b_L(b_M) = \left[\{(1-\delta)C_0 + \delta r_M v_M\}(r_M b_M - K_H) + (1-\delta)r_M v_M K_H\right] \cdot \{r_L(r_M b_M - \delta K_H)\}^{-1}. \tag{18}$$

Again, this solution, since it equates the two sides of (15), suggests that Firm M makes no additional profits at the second rank compared to the third. Finally, given $\Pi_{M2}$ in (13) and $b_L(b_M)$ in (18), we confirm that

$$\frac{d\Pi_{M2}}{db_M} = \frac{\partial\Pi_{M2}}{\partial b_L}\frac{\partial b_L(b_M)}{\partial b_M} + \frac{\partial\Pi_{M2}}{\partial b_M}$$
$$= r_L\left\{ \frac{K_H}{r_M b_M}(1-\delta) + \left(1 - \frac{K_H}{r_M b_M}\right) \right\}$$

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[6] More precisely, we need to check the total derivative $d\Pi_{M2}/db_M$, given that in equilibrium, $b_M$ can also affect $\Pi_{M2}$ through $b_L$. We show later that $d\Pi_{M2}/db_M = (\partial\Pi_{M2}/\partial b_L)(\partial b_L(b_M)/\partial b_M) + \partial\Pi_{M2}/\partial b_M > 0$ indeed holds.

$$\cdot \frac{(1-\delta)^2 r_M (r_M v_M - C_0) K_H}{r_L (r_M b_M - \delta K_H)^2}$$

$$+ \frac{\delta K_H}{r_M b_M^2}(r_M v_M - r_L b_L) > 0 \qquad (19)$$

holds.[7]

This analysis, together with the one in the previous section, leads to the following proposition. All proofs can be found in the appendix.

PROPOSITION 1. *An advertiser's budget constraint motivates lower-ranked advertisers to shift their bids from the lowest to the highest possible amount.*

The proposition shows how a budget constraint may alter the equilibrium bidding strategy of advertisers. In the absence of the budget constraint, advertisers bid the lower bound of all bids satisfying the incentive compatibility conditions, in the Pareto-dominant equilibrium. In the presence of the budget constraint, however, the advertisers listed below the budget-constrained advertiser raise their bids to the upper bound.

To see the intuition for this result, note that when the budget-constrained advertiser exhausts its budget, it drops out of the auction. Whenever this happens, all of the lower-ranked advertisers move up by one slot, collect more clicks at the same per-click price, and thus earn additional profits. These additional profits will be further increased if the budget-constrained advertiser can be removed earlier from the auction. Recall that in the generalized second-price auction, the per-click price is determined by the next highest bid. Hence the time that a budget-constrained advertiser lasts in the auction can be affected by the bid of the adjoining lower-ranked advertiser. This creates an incentive for the immediately lower-ranked advertiser to increase its own bid, with the purpose of increasing the cost of the budget-constrained competitor and thus accelerating the removal of this competitor.

For the same reason, the distant lower-ranked advertisers also have an incentive to increase their bids. To see this, note that in the generalized second-price auction, an equilibrium bid is affected by the lower-ranked advertiser's bid. Thus, the increase in any lower-ranked advertiser's bid ripples through a series of higher-ranked advertisers and eventually raises the cost of the budget-constrained advertiser. Even though the adjoining lower-ranked advertiser may increase its own bid up to its maximum, the bid of any distant lower-ranked advertiser could raise it more by raising that maximum. This implies that even distant advertisers can contribute to the earlier elimination of the budget-constrained competitor, which in turn helps them to improve their own profits. Therefore, all of the lower-ranked advertisers, immediate or distant, are motivated to increase their bids as high as possible to accelerate elimination of a budget-constrained advertiser.[8]

Note that the incentive compatibility conditions imply that the upper bound of each advertiser's bid is determined such that the adjoining higher-ranked advertiser has no incentive to move down to the lower slot. In other words, the equilibrium bid makes the higher-ranked competitor earn as much profit as that obtained at the adjacent lower slot. Because the cost-raising incentive can be found in all advertisers below the budget-constrained advertiser, this implies that not only the budget-constrained advertiser but also other advertisers listed below will lose all of the marginal profits at their current slot.

However, we cannot obtain such results in the absence of a budget-constrained advertiser. This is because an advertiser's bid no longer affects the duration of any other firm's advertisement. Thus, raising the cost of a competitor by bidding more has no bearing on the advertiser's own profits. This implies that the budget constraint is a necessary condition for advertisers to bid at the upper bound of all available bids in equilibrium.

Now, one may wonder whether the equilibrium bid can be even higher than the valuation. Note that in the absence of the budget constraint, it has been shown that the equilibrium bids are not always truthful in the generalized second-price auction (Edelman et al. 2007). Especially, as illustrated in (10), in the Pareto-dominant equilibrium, advertisers typically bid lower than their own valuation. By contrast to this common view, we have the following result.

PROPOSITION 2. *Advertisers may bid more than their valuation when the CTRs of advertising slots are sufficiently different and a competing advertiser is budget-constrained.*

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[7] More formally, suppose $d\Pi_{M2}/db_M \leq 0$. Then Firm M's optimal bid is determined either at the lower bound by (13) or at any constant between the two bounds, but at these solutions, it is easy to see that $\partial b_M(b_L)/\partial b_L \geq 0$ holds and thus, we have $d\Pi_{L3}/db_L = \partial \Pi_{L3}/\partial b_L + (\partial \Pi_{L3}/\partial b_M)(\partial b_M(b_L)/\partial b_L) \geq 0$. This implies that Firm L's optimal bid is given as in (18), i.e., the upper bound, or as any constant between the two bounds. In either case, (19) holds true, but this is a contradiction. Therefore in equilibrium, we always have $d\Pi_{M2}/db_M > 0$. In addition, by (17), $d\Pi_{L3}/db_L > 0$ also holds. Last, the equilibrium bids are derived by simultaneously solving (16) and (18). This derivation as well as the equilibrium condition is reported in the appendix (see the proof of Proposition 2).

[8] To be more precise, this holds for all lower-ranked advertisers who are not budget-constrained. This result is in contrast with the findings in the generalized second-price auctions, where a bidder's budget constraint initiates the cost-raising incentive of the adjoining lower-ranked bidder only (see Pitchik and Schotter 1988 and Benoit and Krishna 2001).

To understand the proposition, consider an advertiser who is ranked lower than a budget-constrained advertiser. As noted earlier, in equilibrium, this advertiser increases its own bid and thus the cost of the higher-ranked competitor until the competitor loses all of the additional benefits of taking the higher slot. As the advertising slots become more heterogeneous in terms of the CTR, these additional benefits increase and thus the equilibrium bid necessary to make the competitor break even at its current slot also increases. Therefore, if the CTRs are sufficiently different, the equilibrium bid will be so high that it can be even greater than the valuation. This result suggests that although the generalized second-price auction does not always induce truthful bidding, it could achieve a better outcome for the search engine in the presence of a budget-constrained advertiser. In particular, the search engine could improve its own profitability by properly designing the distribution of the CTRs across the advertising slots.

So far we have focused on the case wherein Firm H is budget-constrained. In §3.3, we examine the impact of the budget constraint of another advertiser, Firm M.

### 3.3. Firm M's Budget Constraint
In this section, we consider the case wherein Firm M's budget constraint is binding at the second slot. Specifically, suppose $(1-\delta)C_0 < K_M < (1-\delta)r_L v_L$ but $K_i > \delta r_M v_M + (1-\delta)r_L v_L$ $(i = H, L)$. In this case, as we show in the online appendix, the equilibrium listing order is always H-M-L (see Claim A2). Given this order, due to its budget constraint, Firm M stays in the second slot for the $K_M/((1-\delta)r_L b_L)$ period, after which it drops out, and Firm L replaces Firm M in the second slot. Even when deviating to the first slot, Firm M is also budget-constrained and thus participates in the auction only for the $K_M/(r_M b_M)$ of the period; then the top slot is taken by Firm H. Based on this discussion, the incentive compatibility conditions are given as follows:

$$\Pi_{H1} = r_H v_H - \frac{K_M}{(1-\delta)r_L b_L}r_M b_M - \left(1 - \frac{K_M}{(1-\delta)r_L b_L}\right)r_L b_L$$
$$\geq \Pi_{H2} = \left\{\frac{K_M}{r_M b_M}(1-\delta) + \left(1 - \frac{K_M}{r_M b_M}\right)\right\}$$
$$\cdot (r_H v_H - r_L b_L), \quad (20)$$

$$\Pi_{M2} = \frac{K_M}{(1-\delta)r_L b_L}(1-\delta)r_M v_M - K_M$$
$$\geq \Pi_{M1} = \frac{K_M}{r_M b_M}r_M v_M - K_M, \quad (21)$$

$$\Pi_{M2} = \frac{K_M}{(1-\delta)r_L b_L}(1-\delta)r_M v_M - K_M$$
$$\geq \Pi_{M3} = 0, \quad (22)$$

$$\Pi_{L3} = \left(1 - \frac{K_M}{(1-\delta)r_L b_L}\right)(1-\delta)(r_L v_L - C_0)$$
$$\geq \Pi_{L2} = (1-\delta)(r_L v_L - r_L b_L). \quad (23)$$

Whereas (22) and (23), respectively, determine the upper bound and the lower bound of $b_L$, because

$$\frac{d\Pi_{L3}}{db_L} = \frac{\partial \Pi_{L3}}{\partial b_L} + \frac{\partial \Pi_{L3}}{\partial b_M}\frac{\partial b_M}{\partial b_L}$$
$$= \frac{K_M}{r_L b_L^2}(r_L v_L - C_0) + 0 > 0, \quad (24)$$

Firm L's optimal bid amount is determined at the upper bound, defined by (22)

$$b_L^* = \frac{r_M v_M}{r_L}. \quad (25)$$

As before, for this solution, Firm M earns no more profits in the second slot than when it does not take any slot. By contrast, since $d\Pi_{M2}/db_M = \partial\Pi_{M2}/\partial b_M + (\partial\Pi_{M2}/\partial b_L)(\partial b_L/\partial b_M) = 0$, any bid satisfying (20) and (21) can be an equilibrium bid for Firm M. Taking the lower bound solution as the Pareto-dominant equilibrium bid, we have

$$b_M^* = \frac{r_L b_L^*}{r_M} = v_M. \quad (26)$$

Note that, at this set of solutions, all of the incentive compatibility conditions in (20)–(23) always hold. Thus, H-M-L is always an equilibrium listing order.

This analysis confirms the generality of Proposition 1 by showing that even the budget constraint of any mid-ranked advertiser may also induce other advertisers to push their bids upwards. It also confirms that the budget constraint does not alter the bidding behavior of any higher-ranked advertiser. However, as shown in (26), even without any budget-constrained advertiser at a higher slot, an advertiser may still bid high in equilibrium. The following proposition presents such a case.

PROPOSITION 3. *When a budget-constrained advertiser takes the last slot, its equilibrium bid is at least as high as the valuation.*

Recall from the previous section that the (Pareto-dominant) equilibrium bid is determined at the lower bound of all bids satisfying the incentive compatibility condition if no higher-ranked advertiser is budget-constrained. The proposition shows that even this lower-bound solution can be as high as the valuation. To see this, consider an advertiser occupying the last slot, which is also budget-constrained (denote by Advertiser N). This advertiser will earn zero profit when it moves down by one rank since there is no slot below. At the same time, since it is

budget-constrained, according to Proposition 1, the lower-ranked competitor will bid high enough so that Advertiser $N$ earns zero profits (i.e., the profits that accrue at the next rank). This means that the bid of the lower-ranked competitor will be set exactly at the valuation of Advertiser $N$ in equilibrium. Then Advertiser $N$, to win the last slot, has to bid weakly higher than its own valuation. Therefore, an advertiser, even when it does not have any incentive to raise its bid, may be induced to bid at least as high as the valuation. This is because the lower-ranked competitor's cost-raising incentive pushes the focal advertiser's bid upwards as well.

### 3.4. Multiple Firms' Budget Constraints

So far we have considered cases wherein only one firm has a binding budget constraint. In this section, to complete our analysis, we consider multiple firms' budget constraints. Given that this case includes various subcases, we do not derive the equilibrium for every possible case but only present one example of such cases for the purpose of the illustration. In particular, we consider the case wherein both Firm H and Firm M are budget-constrained in the first and second slot, respectively.[9] In this case, as shown in the online appendix, only the following two listing orders can be observed in equilibrium: H-M-L and H-L-M (see Claim A3). Consistent with our earlier findings, this shows that a budget-constrained advertiser (Firm M) can be ranked lower than its lower-valuation but unconstrained competitor (Firm L) in equilibrium. It also implies that an advertiser budget-constrained at both slots (Firm M) can never win over the competitor budget-constrained only at one slot (Firm H).[10] Among the two listing orders, in this section, we focus only on H-M-L, since we are primarily interested in the case wherein multiple firms are budget-constrained in equilibrium.[11]

Now suppose the equilibrium listing order is H-M-L. Further suppose Firm H is more budget-constrained than Firm M in that Firm H drops out earlier; let $l$ denote the duration of the period that Firm M stays in the auction after Firm H drops out.

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[9] This corresponds to $(1-\delta)r_L v_L < K_H < \delta r_M v_M + (1-\delta)r_L v_L$ and $(1-\delta)C_0 < K_M < (1-\delta)r_L v_L$.

[10] This is because at any set of bids, Firm M weakly prefers to take the second slot to the first slot, given that it will exhaust all of its budget at both slots but collect more clicks at the second slot where it pays a lower per-click cost. By contrast, there exists a set of bids at which Firm H is willing to take the first slot while Firm M takes the second slot.

[11] The other equilibrium listing order, H-L-M, involves only one firm budget-constrained in equilibrium and thus the analysis of such a case is essentially identical to that in previous sections. Note also that the rest of the listing orders, M-H-L, M-L-H, L-H-M, and L-M-H, cannot be observed in equilibrium. We provide the details of this analysis in Claim A3 of the online appendix.

Then Firm M takes the second slot for $K_H/(r_M b_M)$ fraction of the period but moves up to the first slot for $l$ fraction. Because Firm M's total advertising cost amounts to its budget, we have

$$\frac{K_H}{r_M b_M}(1-\delta)r_L b_L + l \cdot r_L b_L = K_M, \qquad (27)$$

which is equivalent to

$$l = \frac{K_M}{r_L b_L} - (1-\delta)\frac{K_H}{r_M b_M}. \qquad (28)$$

Then, Firm M's equilibrium profit under the H-M-L order is

$$\Pi_{M2} = \frac{K_H}{r_M b_M}(1-\delta)r_M v_M + l \cdot r_M v_M - K_M$$

$$= \frac{K_M}{r_L b_L}r_M v_M - K_M. \qquad (29)$$

Turning to Firm L, note that even Firm L can take the second slot (for $l$ fraction of the period) and the first slot (for $1 - l - K_H/(r_M b_M)$ fraction) due to dropout of the other firms. Thus, Firm L's profit under the H-M-L order is

$$\Pi_{L3} = l(1-\delta)(r_L v_L - C_0) + \left(1 - l - \frac{K_H}{r_M b_M}\right)(r_L v_L - C_0)$$

$$= \left\{1 - \delta\frac{K_M}{r_L b_L} - (1-\delta+\delta^2)\frac{K_H}{r_M b_M}\right\}(r_L v_L - C_0). \quad (30)$$

Based on these profits, the incentive compatibility conditions are given as follows:[12]

$$\Pi_{H1} = \frac{K_H}{r_M b_M}r_H v_H - K_H$$

$$\geq \Pi_{H2} = \left\{\frac{K_M}{r_M b_M}(1-\delta) + \left(1 - \frac{K_M}{r_M b_M}\right)\right\}$$

$$\cdot (r_H v_H - r_L b_L), \qquad (31)$$

$$\Pi_{M2} = \frac{K_M}{r_L b_L}r_M v_M - K_M$$

$$\geq \Pi_{M1} = \frac{K_M}{r_M b_M}r_M v_M - K_M, \qquad (32)$$

$$\Pi_{M2} = \frac{K_M}{r_L b_L}r_M v_M - K_M$$

$$\geq \Pi_{M3} = \left(1 - \frac{K_H}{r_M b_M}\right)(1-\delta)(r_M v_M - C_0), \quad (33)$$

$$\Pi_{L3} = \left\{1 - \delta\frac{K_M}{r_L b_L} - (1-\delta+\delta^2)\frac{K_H}{r_M b_M}\right\}(r_L v_L - C_0)$$

$$\geq \Pi_{L2} = \left\{\frac{K_H}{r_M b_M}(1-\delta) + \left(1 - \frac{K_H}{r_M b_M}\right)\right\}$$

$$\cdot (r_L v_L - r_L b_L). \qquad (34)$$

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[12] In our derivation of $\Pi_{M3}$, we assume $K_H/(r_M b_M) + K_M/((1-\delta) \cdot r_L b_L) \geq 1$, but this can be satisfied in equilibrium.

Note that the upper bound and lower bound of $b_M$ are defined by (31) and (32), respectively, while those of $b_L$ are determined by (33) and (34), respectively.

We now examine the bidding strategy of both firms. First, suppose $d\Pi_{M2}/db_M \leq 0$. Then Firm M's optimal bid is determined either at the lower bound by (32): $b_M(b_L) = b_L$ or at any constant between the two bounds, which leads to $\partial b_M(b_L)/\partial b_L \geq 0$. In this case, we have $d\Pi_{L3}/db_L = \partial\Pi_{L3}/\partial b_L + (\partial\Pi_{L3}/\partial b_M)(\partial b_M(b_L)/\partial b_L) > 0$, since (30) suggests both $\partial\Pi_{L3}/\partial b_M > 0$ and $\partial\Pi_{L3}/\partial b_L > 0$. Thus, Firm L's optimal bid is determined at the upper bound by equating the two sides of (33)

$$b_L(b_M)$$

$$= \frac{r_M v_M r_M b_M K_M}{r_L\{r_M b_M K_M + (1-\delta)(r_M v_M - C_0)(r_M b_M - K_H)\}}, \quad (35)$$

which implies

$$\frac{\partial b_L(b_M)}{\partial b_M}$$

$$= -\frac{(1-\delta)r_M v_M(r_M v_M - C_0)K_M K_H}{\{r_M b_M K_M + (1-\delta)(r_M v_M - C_0)(r_M b_M - K_H)\}^2} < 0. \quad (36)$$

Because (29) suggests both $\partial\Pi_{M2}/\partial b_M = 0$ and $\partial\Pi_{M2}/\partial b_L < 0$, this leads to $d\Pi_{M2}/db_M = \partial\Pi_{M2}/\partial b_M + (\partial\Pi_{M2}/\partial b_L)(\partial b_L(b_M)/\partial b_M) > 0$, which is a contradiction. Therefore, we always have $d\Pi_{M2}/db_M > 0$ in equilibrium.

Given $d\Pi_{M2}/db_M > 0$, Firm M bids its upper bound solution given by (31)

$$b_M(b_L) = \frac{r_H v_H K_H + \delta K_M(r_H v_H - r_L b_L)}{r_M(r_H v_H - r_L b_L + K_H)}. \quad (37)$$

Then it is easy to see that

$$\frac{\partial b_M(b_L)}{\partial b_L} = \frac{K_H r_L(r_H v_H - \delta K_M)}{r_M(r_H v_H - r_L b_L + K_H)^2} > 0. \quad (38)$$

This, together with $\partial\Pi_{L3}/\partial b_M > 0$ and $\partial\Pi_{L3}/\partial b_L > 0$, implies that $d\Pi_{L3}/db_L = \partial\Pi_{L3}/\partial b_L + (\partial\Pi_{L3}/\partial b_M)(\partial b_M(b_L)/\partial b_L) > 0$.

This analysis shows that Firm L has an incentive to increase its bid, which is motivated by both Firm H and Firm M's budget constraint (i.e., $(\partial\Pi_{L3}/\partial b_M) \cdot (\partial b_M(b_L)/\partial b_L) > 0$ for the former; $\partial\Pi_{L3}/\partial b_L > 0$ for the latter). This suggests that, as noted earlier, a lower-ranked advertiser, either adjoining or distant, may increase its bid to accelerate elimination of the budget-constrained advertiser. However, the partial derivative of Firm M's profits ($\partial\Pi_{M2}/\partial b_M = 0$) suggests that such an incentive does not apply to Firm M's case. In particular, although Firm M's higher bid may accelerate the elimination of Firm H, Firm H's earlier dropout does not help increase Firm M's own profit. Still,

Firm M bids the highest amount bounded by the incentive compatibility conditions, as suggested by the total derivative: $d\Pi_{M2}/db_M > 0$. On investigating Firm M's motivation in this case, we obtain the following result.

PROPOSITION 4. *Bidding more may reduce an advertiser's own cost.*

The above analysis shows that Firm M has no incentive to raise the cost of Firm H and yet increases its bid. The proposition suggests that this is because bidding more could help Firm M to reduce its own advertising cost. To see how this might occur, consider an advertiser that is listed immediately below a budget-constrained advertiser. For convenience, denote such an advertiser by Advertiser B. We denote the budget-constrained advertiser listed above Advertiser B by Advertiser A, and the advertiser listed below Advertiser B by Advertiser C. Now recall from Proposition 1 that an advertiser's budget constraint induces even the distant lower-ranked advertisers to raise their bids. This implies that, due to Advertiser A's budget constraint, Advertiser C will increase its bid until Advertiser B's profits reach the level of profits that it would earn at the next slot. Note that Advertiser B can improve this hypothetical next-slot profit by increasing its own bid, since an increase in its bid accelerates the elimination of Advertiser A.[13] By doing so, Advertiser B leaves less room for Advertiser C to increase the cost and decrease the profits of Advertiser B. Therefore, Advertiser B, by bidding more, can induce Advertiser C to bid less, thus reducing its own advertising cost.

Note that we observe this phenomenon only from the advertiser that is listed immediately below the budget-constrained competitor. If an advertiser is listed farther away, its bid has no direct influence on the elimination of the competitor and thus, on its own profits in any slot. In this case, the link between its own bid and the lower-ranked competitor's bid cannot be established. Thus, reducing one's own cost by bidding more is only possible for the advertiser adjoining the budget-constrained advertiser.

When the advertiser immediately below the budget-constrained advertiser also has a limited budget, it may not find it profitable to raise the cost of the competitor, despite the competitor's budget constraint. This is because the number of clicks that it could collect is restrained by its own budget constraint and thus

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[13] More precisely, Advertiser C's bid in its deviation to Advertiser B's position affects the elimination of Advertiser A. However, the Symmetric Nash Equilibrium requires that in this deviation, Advertiser C use Advertiser B's bid as its own bid. Thus, it is effectively Advertiser B's bid that determines the duration of Advertiser A in the auction in this hypothetical situation.

does not increase with expedited removal of the competitor. The proposition shows that, in such a situation, the advertiser bids more only to reduce its own cost.

### 3.5. Discussion

In this section, we have examined the impact of the limited budgets in the bidding equilibrium of keyword search advertising by considering the budget constraint of Firm H, Firm M, and both in order. We have shown that the equilibrium listing order need not be consistent with the order of valuations. The budget-constrained firm may choose to take a lower slot than a lower-valuation advertiser that is not budget-constrained. In addition, the budget constraint allows lower-ranked advertisers to benefit from bidding more, either by accelerating the removal of the higher-ranked competitor or by reducing the advertising cost of their own. Consequently, in equilibrium, advertisers shift their bids from the lower bound to the upper bound of all potential bids satisfying the incentive compatibility conditions. These bids, we find, may be greater than the valuation in equilibrium. Moreover, even a budget-constrained advertiser could bid higher than the valuation, if it takes the last slot. These findings are all specific to the generalized second-price auction with budget constraints and go beyond the insights generated from the study of the generalized second-price auction without budget constraint (Edelman et al. 2007, Varian 2007) and that of the sequential second-price auctions (Pitchik and Schotter 1988, Benoit and Krishna 2001).

## 4. Extensions

In this section, we provide two extensions of our main model to further study the role of budget constraint in keyword search advertising. In §4.1, we present a two-period model of a bidding game and examine how the budget constraint affects the dynamic bidding incentive. We then relax the assumption of the exogenous budget and study the strategic choice of advertising budgets in §4.2.[14] Across both extensions, we limit our attention to the interaction between the first two firms, Firm H and Firm M, while treating Firm L's bid as exogenous. For convenience, we use $C_1$ to replace $r_L b_L$. Proofs as well as detailed analyses of this section are deferred to the online appendix.

### 4.1. Dynamic Bidding

In our main model, we considered a one-shot bidding game while allowing its result to take effect during the entire period. Although this structure well describes the long-term equilibrium of advertiser

interactions, it does not capture the dynamic incentives of advertisers. Given that, in reality, advertisers are allowed to change their bids in a small amount of time, in this section we model the advertiser's dynamic bidding behavior in the two-period bidding game and examine how they change their bids in response to each other's bid across the two periods. To focus on this issue, we assume that both firms have an identical budget, $K$, and that this budget is constrained only when a firm takes the first slot in both periods (i.e., $\delta r_M v_M + 2(1-\delta)C_1 \leq K < 2\delta r_M v_M + 2(1-\delta)C_1$).[15] We further assume that Firm M's CTR-weighted valuation is low relative to that of Firm H, i.e., $r_M v_M - C_1 < 4\delta(1-\delta)(r_H v_H - C_1)$. Under these conditions, one might expect Firm H to take the first slot in both periods. On examining whether this is indeed the case, we have the following proposition. Note that since we consider a fairly short period of time, the discount factor is assumed to be one.

PROPOSITION 5. *Advertisers may switch positions from period to period if the budget is sufficiently small or sufficiently large, but not if it is of a moderate size.*

To understand the first part of the proposition, recall that in the one-shot bidding game, Firm H loses the first slot to Firm M when Firm H's budget is sufficiently small. This is because Firm H's budget constraint decreases its effective CTR-weighted valuation at the first slot. The same intuition applies to the two-period bidding game. When the budget is sufficiently small; by taking the top slot in both periods, Firm H cannot stay in the auction for the entire period. Then, in one of the two periods, Firm H's effective CTR-weighted valuation at the first slot will decrease below that of Firm M, in which case, Firm H loses the top slot to Firm M.

Surprisingly, the proposition also shows that even when the budget is sufficiently large, Firm H can give up the top slot in one period. To see the reason, note that if Firm H takes the top slot in both periods, it will be budget-constrained at the second period. Then, according to Proposition 1, Firm M will bid as high as possible in the second period to exhaust Firm H's budget earlier. Interestingly, even in the first period, Firm M will bid as high as possible because doing so reduces the residual budget for Firm H to use in the second period and thus further benefits Firm M in the second-period bidding game. Given its objective of reducing the residual budget of Firm H, when the overall budget is larger, Firm M will further increase its first-period bid. This implies that a larger budget merely increases Firm H's cost without

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[14] We thank an anonymous reviewer and the associate editor for suggesting these extensions.

[15] We thank the associate editor for suggesting the specific structure of the game.

adding any benefit. In this case, Firm H would be better off giving up the first slot in one period and thus avoiding unnecessary costs. Therefore in equilibrium, we observe that the two firms dynamically change their bids and switch their positions period by period, with either a sufficiently large or sufficiently small budget.

### 4.2. Endogenous Budget Decisions

In previous sections, we have shown that the budget constraint may have a negative effect on the profitability of the budget-constrained firm. Given this, advertisers may want to avoid having a limited budget in search advertising, either by allocating more funds to spend in the current slot or by decreasing their budget and thus moving down to the lower slot. Therefore, it is an open question whether each firm will set a large or small budget, if they are allowed to do so. In this section, we endogenize the budget decisions of Firm H and Firm M and examine how the negative consequence of the budget constraint affects their budget-setting decisions. In particular, we assume that each firm simultaneously sets its own budget in the first period and then makes a bid in the second period based on these budget decisions. To consider the case wherein Firm M is comparable to Firm H, we further assume $r_M v_M - C_1 \geq 2\delta(r_H v_H - C_1)$. The analysis of this model generates the following result.

PROPOSITION 6. *The low-valuation advertiser may voluntarily choose to become budget-constrained in a budget-setting game.*

Recall that in the absence of a budget constraint, Firm H takes the first slot and Firm M takes the second slot in equilibrium. Also, recall that if Firm H is budget-constrained, Firm M squeezes Firm H's profits to the level of profits that Firm H could earn at the second slot. Because Firm H's profits with a budget constraint are clearly less than its profits with no budget constraint, Firm H has no incentive to voluntarily choose to be budget-constrained.

However, the proposition shows that such an incentive may exist for Firm M. To see this, note that Firm M's default profits, i.e., the profits without a budget constraint, are obtained from the second slot. Then, even if Firm M has a limited budget, as long as its budget can cover the cost of taking the second slot, its profits do not change with its budget. This is, indeed, the case when Firm M takes the second slot in equilibrium. Even when Firm M takes the first slot under its own budget constraint, according to Proposition 1, Firm H squeezes Firm M's profits to the level of profits that Firm M could earn at the second slot, which is again

the same as its default profits.[16] In either case, Firm M is indifferent between being budget-constrained only at the first slot and not being budget-constrained at all. Therefore, we may observe Firm M choose to be budget-constrained at the first slot.

## 5. Conclusion

This paper provides a theoretical examination of the role of budget constraints in keyword search advertising. We propose a model of the generalized second-price auction where firms compete for advertising slots by making bids with limited budgets. Our theoretical investigation of the model helps us better understand the following issues.

1. *How does a budget constraint alter the bidding strategy of advertisers? What are the motivations for such a change?* In the absence of a budget constraint, advertisers may bid as much as the marginal profit they earn from the slot that they take in equilibrium. In the presence of a budget constraint, however, advertisers may shift their bids to the highest possible amount, the point at which the higher-ranked competitor makes zero additional profit in its current position. There are two different economic motivations for this change in the bidding strategy. First, advertisers may raise their bids to accelerate elimination of the budget-constrained competitor from the auction and take a more profitable slot. Note that this incentive holds not only for the adjoining advertiser but also for the distant advertisers. Thus, the budget constraint of one advertiser can drive down the profits of multiple advertisers. Second, advertisers may also raise bids to reduce their own advertising cost. If an advertiser is listed below a budget-constrained advertiser, it can induce the next-ranked advertiser to bid less by increasing its own bid. This is because the next-ranked advertiser's bid is determined by the focal firm's profit at the next slot, which increases with the focal firm's own bid. These analyses show that aggressive bids under a competitor's budget constraint are intended to maximize the advertisers' own profits, not merely to raise the rivals' costs.

2. *What are the consequences of a budget constraint in equilibrium bids?* Given that a budget constraint may motivate some advertisers to raise their bids, the next question is how high the bid will rise in equilibrium. We find that the equilibrium bids can be greater than the valuation when the slots are

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[16] Note that it is possible for Firm M to take the first slot in equilibrium when it is budget-constrained. This is because, in this case, Firm H can also benefit by yielding the first slot to Firm M. This, in turn, is because after Firm M drops out, Firm H can be listed at the top slot with a much lower cost than the cost it would incur by taking the first slot and keeping Firm M below itself for the entire period.

sufficiently heterogeneous. This is because greater heterogeneity of slots increases the additional profits the higher-ranked advertiser earns at its current slot, which gives more room for the competitor to bid higher. As to the budget-constrained advertiser, one might expect that the budget constraint would make the bids more conservative. Surprisingly, even a budget-constrained advertiser may sometimes bid beyond its own valuation. We observe this when the budget-constrained advertiser takes the last slot. This is because the next-ranked advertiser, in squeezing the budget-constrained advertiser's profits to zero, bids the valuation of the budget-constrained advertiser in equilibrium. Then, the budget-constrained advertiser's bid has to be weakly higher than the next-ranked advertiser's bid, that is, its own valuation. Therefore, an advertiser's budget constraint, by inducing a next-ranked competitor to bid more, may raise its own bid beyond its own valuation.

3. *What is the impact of a budget constraint on dynamic bidding decisions?* In the bidding game of keyword search advertising, advertisers are allowed to change their bids in a small amount of time. This leads to the question of whether advertisers will change their bids period by period in the presence of a budget constraint and the competitor's cost-raising possibility. We find that the high-valuation advertiser may give up taking the top slot in one period, not only when the budget is sufficiently small but also when it is sufficiently large. This is because its larger budget is only dissipated as a cost. This, in turn, is because, in the presence of a larger budget, the competitor further increases its first-period bid in an effort to minimize the high-valuation advertiser's second-period budget and thus to obtain a better position in the second-period bidding game.

4. *How do advertisers choose their advertising budgets?* In the presence of a budget constraint, advertisers may engage in cost-raising bidding behavior. Thus, advertisers may prefer a large budget to avoid the competitor's cost-raising bid. If firms are allowed to set their budgets prior to the bidding game, the high-valuation firm, indeed, chooses a budget large enough to cover the entire period. However, the low-valuation firm may voluntarily choose to be budget-constrained because it obtains the lower slot even when unconstrained and thus has nothing to lose with the competitor's cost-raising bid. These results show how firms might behave differently due to the cost-raising incentives induced by a budget constraint.

Budget constraints are commonly observed in keyword search advertising. However, the literature on generalized second-price auctions seldom discusses the implication of the advertisers' budget constraint. Our work fills this gap. However, the paper is not without limitations. First, we use a parsimonious

model with two slots and three advertisers. While we believe our results can be easily extended to more general settings, future research could establish the generality of our findings. Second, while we discuss the dynamic bidding incentive in the two-period model, we do not consider other competitive behaviors such as a bidding war. It might be useful to examine the dynamics of competitive bidding strategies (e.g., Zhang and Feng 2011). Third, we do not treat the search engine as a strategic player. However, the search engine might strategically react to advertisers' budget constraints by adjusting budget-constrained advertisers' participation in the auction as well as their bid amounts (if allowed), or even by inducing alternative listing orders through first-page bid estimates (see for example, Charles et al. 2013, Karande et al. 2013, and Amaldoss et al. 2015a). These possibilities could be investigated in addition to the advertisers' strategic behavior in the presence of limited budgets. Finally, even though there is plenty of evidence for aggressive bidding, we have not yet subjected our specific predictions to an empirical test. Field data or experimental data would enrich our understanding of the phenomenon.

## Supplemental Material
Supplemental material to this paper is available at http://dx .doi.org/10.1287/mksc.2015.0915.

## Appendix

**Proof of Proposition 1**
First note that in the Pareto-dominant equilibrium, advertisers bid the lowest possible amount among all bids satisfying the incentive compatibility conditions. Given this, the proposition can be restated as the following two parts: (1) No advertiser has an incentive to bid the highest possible amount unless there is a budget-constrained advertiser; and (2) If there is a budget-constrained advertiser, advertisers listed below the budget-constrained advertiser have an incentive to bid the highest possible amount. Recall that $[j]$ denotes the advertiser taking the $j$th position in the rank of CTR-weighted bids. Whereas the identity of Firm $[j]$ is determined by the listing order, the following proof generally holds for any equilibrium listing order.

*Part* 1. To prove the first statement, suppose no advertiser has a binding budget constraint. First, note that the profits

and incentive compatibility (IC) conditions of this case are identical to those given in §3.1, with H, M, and L, respectively, replaced by [1], [2], and [3]. Then since $\Pi_{[3]3} = 0$, we have $d\Pi_{[3]3}/db_{[3]} = 0$. Next, since $\Pi_{[2]2} = (1-\delta)(r_{[2]}v_{[2]} - r_{[3]}b_{[3]})$, we have $\partial\Pi_{[2]2}/\partial b_{[2]} = 0$. In addition, between the two bounds of $b_{[3]}$, we always have $\partial b_{[3]}/\partial b_{[2]} = 0$ (since $\partial\Pi_{[3]2}/\partial b_{[2]} = 0$ at the lower bound of $b_{[3]}$, which is defined by (5): $0 = \Pi_{[3]2}$; since $\partial\Pi_{[2]2}/\partial b_{[2]} = 0$ at the upper bound of $b_{[3]}$ which is defined by (4): $\Pi_{[2]2} = 0$; and since $b_{[3]}$ is given as a constant in-between). These lead to $d\Pi_{[2]2}/db_{[2]} = \partial\Pi_{[2]2}/\partial b_{[2]} + (\partial\Pi_{[2]2}/\partial b_{[3]})(\partial b_{[3]}/\partial b_{[2]}) = 0$. Therefore, there is no incentive for any bidder to raise the bid.

*Part* 2. To prove the second statement, we consider the following three cases:

  1. when Firm [1] is budget-constrained while others are not,

  2. when Firm [2] is budget-constrained while others are not, and

  3. when Firm [1] and Firm [2] are budget-constrained while Firm [3] is not.

First, the analysis of Case 1 is identical to that in §3.2, with H, M, and L, respectively, replaced by [1], [2], and [3]. Then, by analysis of §3.2, we have $d\Pi_{[2]2}/db_{[2]} > 0$ and $d\Pi_{[3]3}/db_{[3]} > 0$.

Similarly, the analysis of Case 2 is identical to that in §3.3 with H, M, and L, respectively, replaced by [1], [2], and [3]; thus, we have $d\Pi_{[3]3}/db_{[3]} > 0$.

Finally, the analysis of Case 3 is identical to that in §3.4 with H, M, and L, respectively, replaced by [1], [2], and [3]. Thus, we also have $d\Pi_{[2]2}/db_{[2]} > 0$ and $d\Pi_{[3]3}/db_{[3]} > 0$. This proves that an advertiser listed below a budget-constrained advertiser has an incentive to increase its own bid to the maximum possible bid. □

**Proof of Proposition 2**
We prove the proposition by showing a case wherein the statement of the proposition holds. Suppose $\min\{K_M, K_L\} > \delta r_M v_M + (1-\delta)r_L v_L > K_H > (1-\delta)r_L v_L$ holds as in the analysis of §3.2. In this case, by simultaneously solving (16) and (18), we obtain

$$b_M^* = \frac{K_H(X + \sqrt{X^2 - 4\delta r_H v_H Z})}{2r_M Z},$$
$$b_L^* = \frac{Y + \sqrt{X^2 - 4\delta r_H v_H Z}}{2\delta(1-\delta)r_L}, \tag{39}$$

where

$$X \equiv \delta(r_H v_H + K_H) + (1-\delta)$$
$$\cdot \{(1+\delta)r_H v_H + (1-2\delta)r_M v_M - (1-\delta)C_0\}, \tag{40}$$

$$Y \equiv -\delta(r_H v_H - K_H) - (1-\delta)$$
$$\cdot \{(1-\delta)r_H v_H + (1-2\delta)r_M v_M - (1-\delta)C_0\}, \tag{41}$$

$$Z \equiv (1-\delta)\{r_H v_H - \delta r_M v_M - (1-\delta)C_0\} + K_H. \tag{42}$$

By plugging (39) into (13), we have the condition for H-M-L to be an equilibrium listing order, given as follows:

$$\frac{K_H(X + \sqrt{X^2 - 4\delta r_H v_H Z})}{2Z} - r_M v_M$$
$$- \frac{(1-\delta)(r_M v_M - C_0)(X - \sqrt{X^2 - 4\delta r_H v_H Z})}{2\delta r_H v_H} \geq 0. \tag{43}$$

Suppose (43) holds. Note that under the H-M-L listing order, Firm H is budget-constrained at the first slot. Then, based on (39), it is easy to see that $b_M^* > v_M$ is equivalent to

$$\delta > \frac{1}{2r_M v_M\{(r_H v_H - r_M v_M)K_H + (r_M v_M - K_H)(r_M v_M - C_0)\}}$$
$$\cdot \{-r_H v_H W + r_M v_M(W - 2K_H r_M v_M) + 2C_0(K_H - r_M v_M)$$
$$+ ((r_H v_H - r_M v_M)\{r_H v_H W^2 - r_M v_M$$
$$\cdot \{(W - 2K_H r_M v_M)^2 + 4K_H C_0(K_H - r_M v_M)^2\}\})^{1/2}\}, \tag{44}$$

where

$$W \equiv K_H^2 - K_H r_M v_M + (r_M v_M)^2. \tag{45}$$

Note that the above threshold for $\delta$ falls between 0 and 1, since at $\delta = 0$,

$$b_M^* - v_M = -r_M v_M < 0, \tag{46}$$

and at $\delta = 1$,

$$b_M^* - v_M = r_H v_H - r_M v_M > 0. \tag{47}$$

Therefore, when Firm H is budget-constrained, Firm M's equilibrium bid can be greater than its valuation if $\delta$ is large enough (i.e., if the slots are sufficiently heterogeneous). □

**Proof of Proposition 3**
Suppose that a budget-constrained advertiser, e.g., Advertiser $N$, takes the last slot, i.e., Slot 2, and that its budget is binding at this slot. Then, by Proposition 1, the advertiser below Advertiser $N$ will bid at the upper bound that is determined by Advertiser $N$'s IC condition

$$\Pi_{N2} = \Phi(r_N v_N - r_{[N+1]}b_{[N+1]}) = \Pi_{N3} = 0, \tag{48}$$

where $\Phi$ is the CTR of Advertiser $N$ at Slot 2 and $[N+1]$ represents the advertiser ranked next to Advertiser $N$. This implies $r_{[N+1]}b_{[N+1]} = r_N v_N$. Since by the ranking determination rule, $r_N b_N \geq r_{[N+1]}b_{[N+1]}$, it follows that $b_N \geq v_N$. □

**Proof of Proposition 4**
To prove the proposition, it suffices to show a case wherein bidding more reduces an advertiser's own cost. Suppose the following holds, as in §3.4: $(1-\delta)C_0 < K_M < (1-\delta)r_L v_L < K_H < \delta r_M v_M + (1-\delta)r_L v_L < K_L$. Here, consider the H-M-L listing order and further suppose that Firm H is more budget-constrained than Firm M in equilibrium. Then, as shown in the analysis of §3.4, both $d\Pi_{M2}/db_M > 0$ and $d\Pi_{L3}/db_L > 0$ hold. Thus, based on $b_L(b_M)$ derived in (35), we have

$$\frac{\partial b_L(b_M)}{\partial b_M}$$
$$= -\frac{(1-\delta)r_M v_M(r_M v_M - C_0)K_M K_H}{\{r_M b_M K_M + (1-\delta)(r_M v_M - C_0)(r_M b_M - K_H)\}^2} < 0, \tag{49}$$

which implies that $b_L(b_M)$ decreases as $b_M$ increases. Given that Firm M's cost is proportional to $b_L$, Firm M can reduce its own cost by raising its own bid and thus decreasing Firm L's bid $b_L$. □

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