Assumption MLR.3. (No perfect collinearity)

non of the indep vars is constant and there are no exact linear relationships among them.

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + u.$ X

perfect, exact relationship but not linear.

can be estimated.

MLR.4: $E(u_i | x_{i,1}, \cdots x_{i,k}) = E(u_i) = 0.$

$E(u | x_1, \cdots x_k) = E(u) = 0$

$\Rightarrow Cov(u, x_1) = 0$

$Cov(u, x_k) = 0.$

$$\tilde{\beta}_1 = \frac{\sum (X_{1,i} - \bar{X}_1)(Y_i - \bar{Y})}{\sum (X_{1,i} - \bar{X}_1)^2}$$

$$= \frac{\sum (X_{1,i} - \bar{X}_1) Y_i}{\sum (X_{1,i} - \bar{X}_1)^2}$$

$$= \frac{\sum (X_{1,i} - \bar{X}_1)(\beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + u_i)}{\sum (X_{1,i} - \bar{X}_1)^2}$$

$$= \frac{\beta_1 \sum (X_{1,i} - \bar{X}_1) X_{1,i} + \beta_2 \sum (X_{1,i} - \bar{X}_1) X_{2,i} + \sum (X_{1,i} - \bar{X}_1) u_i}{\sum (X_{1,i} - \bar{X}_1)^2}$$

Since $\sum (X_{1,i} - \bar{X}_1) = 0$

$$\sum (X_{1,i} - \bar{X}_1)^2 = \sum (X_{1,i} - \bar{X}_1) X_{1,i}$$

$$\sum (X_{1,i} - \bar{X}_1)(X_{2,i} - \bar{X}_2) = \sum (X_{1,i} - \bar{X}_1) X_{2,i}$$

$$\tilde{\beta}_1 = \beta_1 + \beta_2 \frac{\sum (X_{1,i} - \bar{X}_1)(X_{2,i} - \bar{X}_2)}{\sum (X_{1,i} - \bar{X}_1)^2} \quad \tilde{\delta}_1$$

$$+ \frac{\sum (X_{1,i} - \bar{X}_1) u_i}{\sum (X_{1,i} - \bar{X}_1)^2}$$

$$\Rightarrow \tilde{\beta}_1 = \beta_1 + \frac{\sum (X_{1,i} - \bar{X}_1) u_i}{\sum (X_{1,i} - \bar{X}_1)^2} + \beta_2 \tilde{\delta}_1$$

$$E(\tilde{\beta}_1) = \beta_1 + \beta_2 E(\tilde{\delta}_1) = 0.$$

$$= \beta_1$$

X_1, X_2 are uncorrelated

if $\beta_2 \tilde{\delta}_1 > 0$ $E(\tilde{\beta}_1) > \beta_1$

overestimate the partial effect of X_1 on Y.

if $\beta_2 \tilde{\delta}_1 < 0$ $E(\tilde{\beta}_1) < \beta_1$

underestimate