$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \cdots + \hat{\beta}_k x_{ik} + \hat{u}_i$$

$$y_i - \bar{y} = \hat{\beta}_1 (x_{i1} - \bar{x}_1) + \cdots + \hat{\beta}_k (x_{ik} - \bar{x}_k) + \hat{u}_i$$

$$\boxed{\frac{y_i - \bar{y}}{\hat{\sigma}_y}} = \boxed{\frac{\hat{\sigma}_1}{\hat{\sigma}_y} \cdot \hat{\beta}_1} \left(\frac{x_{i1} - \bar{x}_1}{\hat{\sigma}_1} \right) + \cdots + \boxed{\frac{\hat{\sigma}_k}{\hat{\sigma}_y}} \cdot \hat{\beta}_k \left(\frac{x_{ik} - \bar{x}_k}{\hat{\sigma}_k} \right) + \boxed{\frac{\hat{u}_i}{\hat{\sigma}_y}}$$

sample standard deviation for y.

$$z_y = \hat{b}_1 \cdot z_1 + \cdots + \hat{b}_k \cdot z_k + \text{error.}$$

$$\hat{b}_j = \frac{\hat{\sigma}_j}{\hat{\sigma}_y} \hat{\beta}_j$$

beta coefficients

or standardized coefficients

Models with interaction:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \cdot x_2 + u$$

$$\frac{\Delta y}{\Delta x_2} = \beta_2 + \beta_3 x_1$$

Reparametrization of interaction effects:

$$y = \alpha_0 + \delta_1 x_1 + \delta_2 x_2 + \beta_3 (x_1 - \mu_1)(x_2 - \mu_2) + u.$$

- **Reparametrization of interaction effects**

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + u$$

Population means; may be replaced by sample means

$$y = \alpha_0 + \delta_1 x_1 + \delta_2 x_2 + \beta_3 (x_1 - \mu_1)(x_2 - \mu_2) + u$$

Effect of x_2 if all variables take on their mean values

- Advantages of reparametrization
  - Easy interpretation of all parameters
  - Standard errors for partial effects at the mean values available
  - If necessary, interaction may be centered at other interesting values

Multiple Regression Analysis: Further Issues

- **Average partial effects** (also called average marginal effect).
- In models with quadratics, interactions, and other nonlinear functional forms, the partial effect depend on the values of one or more explanatory variables
- Average partial effect (APE) is a summary measure to describe the relationship between dependent variable and each explanatory variable
- After computing the partial effect and plugging in the estimated parameters, average the partial effects for each unit across the sample

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_1 \cdot x_2 + u$$

$$\frac{\Delta y}{\Delta x_1} = \beta_1 + 2\beta_3 x_1 + \beta_4 x_2$$

$$APE_{x_1} = \hat{\beta}_1 + 2\hat{\beta}_3 \overline{x_1} + \hat{\beta}_4 \overline{x_2}$$

$$\frac{\Delta y}{\Delta x_2} = \beta_2 + \beta_4 x_1$$

$$APE_{x_2} = \hat{\beta}_2 + \hat{\beta}_4 \overline{x_1}$$