HW 1, P6 by Sagemath

In [1]:
```
set_random_seed(0)
```

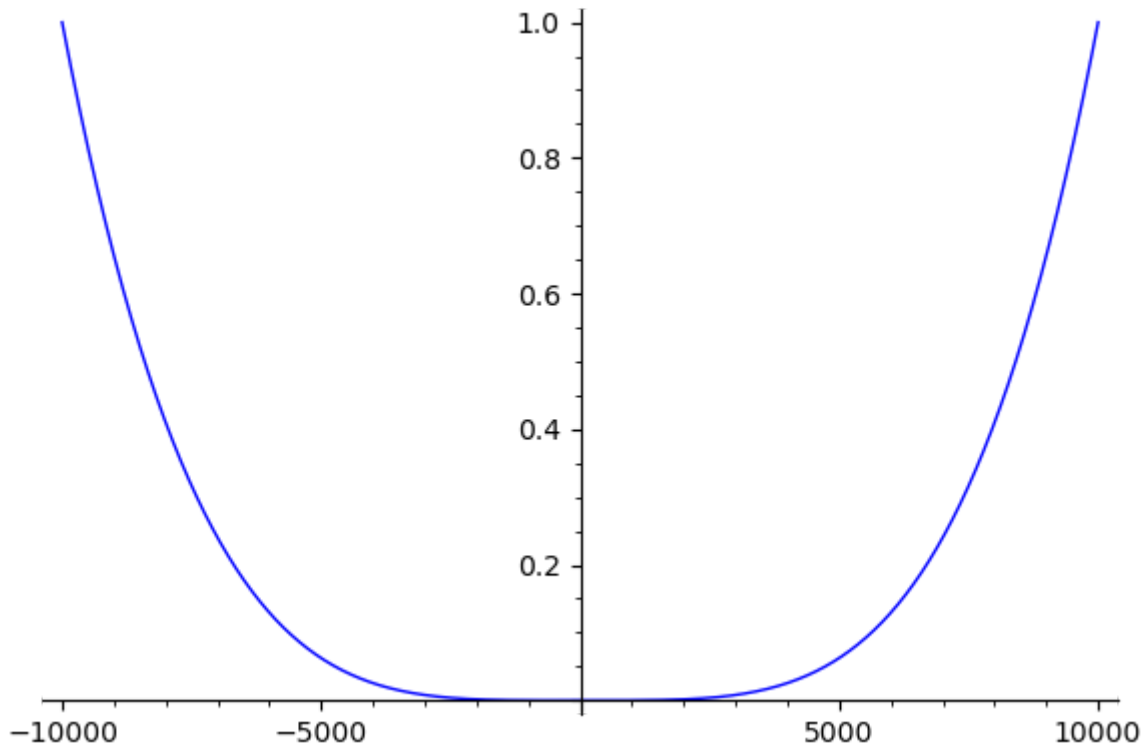
(a)

(i)

In [2]:
```
x=var('x')
f=(1-x**2)**2
```

In [3]:
```
plot(f, x,-10000,10000)
```

Out[3]:



(ii)

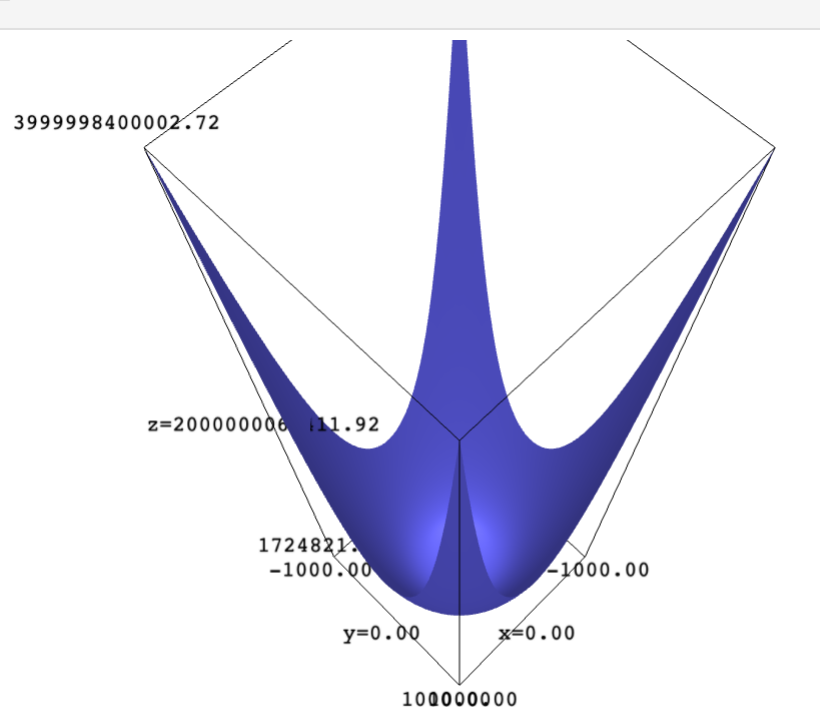
In [4]:
```
A=matrix(2,2,[1,0.6,0.6,1])
var('x1','x2')
```

Out[4]: (x1, x2)

In [5]:
```
f1=lambda x1,x2: ((A-vector([x1,x2]).column()*vector([x1,x2]).row()).norm('frob'))**2
```

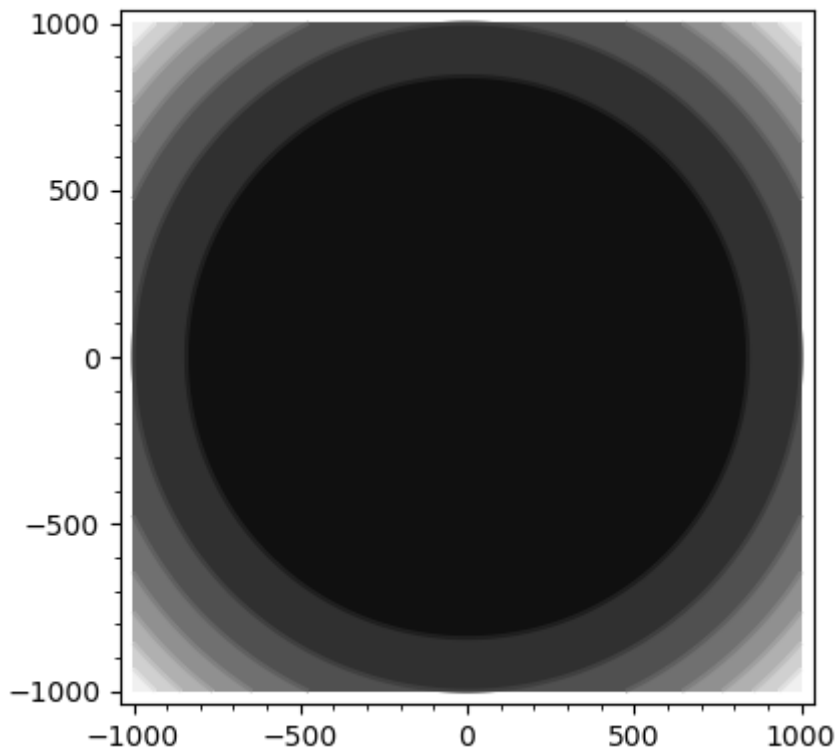
In [6]:
```
plot3d(f1, (x1,-1000,1000), (x2,-1000,1000), aspect_ratio=[1,1,1])
```

Out[6]:



In [7]:
```
contour_plot(f1, (-1000,1000), (-1000,1000))
```

Out[7]:



(iii)

Pick a point $\hat{\theta} = 0_{200\times 1} = [0, 0, \dots 0]^T$

Pick two vectors $u = [1, 0, 0 \dots 0]^T$ and $v = [0, 1, 0 \dots 0]^T$

In [8]:
```
u=vector([0]*200)
u[0]=1
v=vector([0]*200)
v[1]=1
```

In [9]:
```
u.column()*v.row()
```

Out[9]:
```
200 x 200 dense matrix over Integer Ring (use the '.str()' method to see the entries)
```

In [10]:
```
B=Matrix(RR, 200, lambda i,j: normalvariate(0, 1))
A=B*B.transpose()
```

$$(x_1 u)(x_2 v)^T = [t_{ij}]_{i,j},\ t_{ij} = x_1 x_2 \text{ if } i = 1, j = 2, \text{otherwise } t_{ij} = 0$$

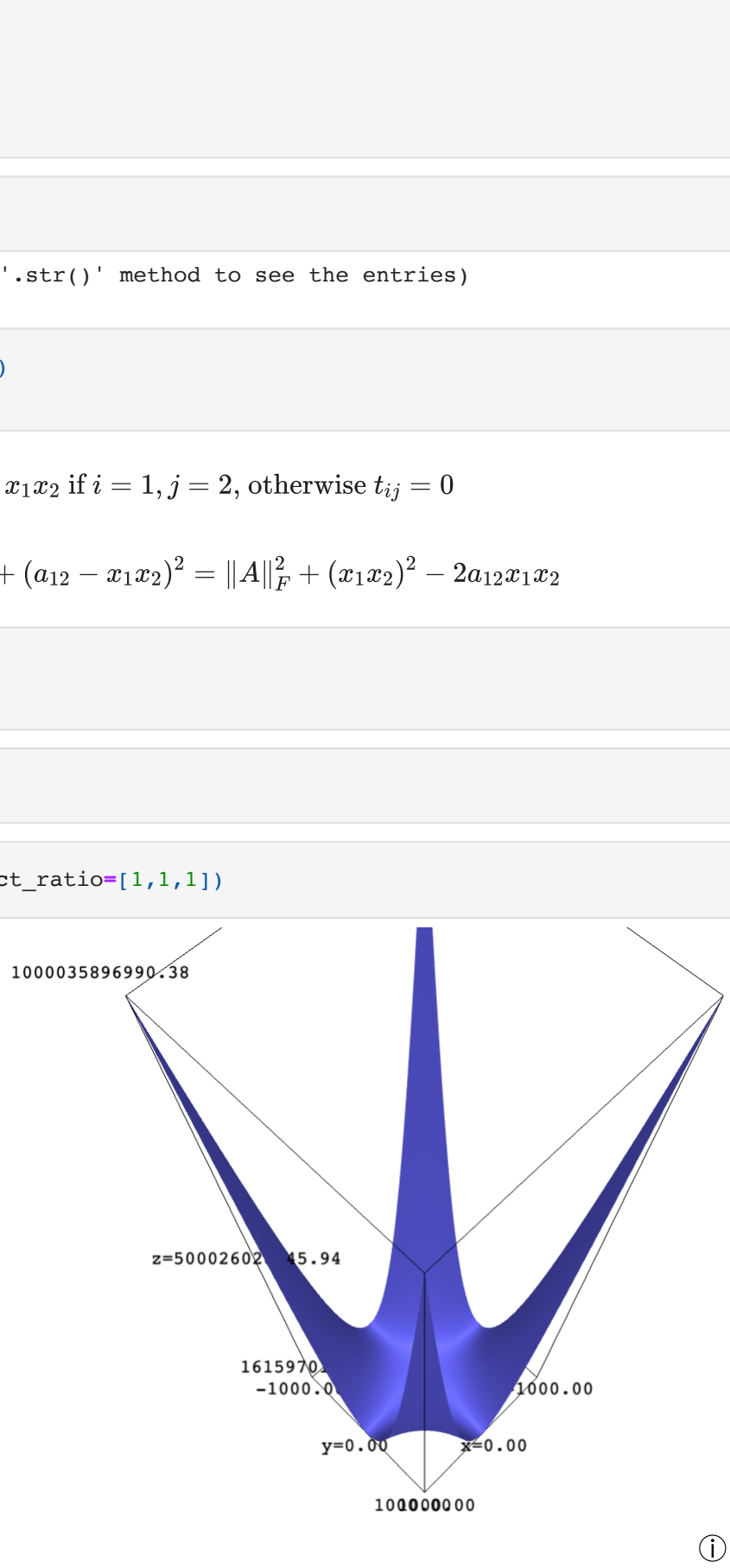
$$\|A - (x_1 u)(x_2 v)^T\|_F^2 = \|A\|_F^2 - a_{12}^2 + (a_{12} - x_1 x_2)^2 = \|A\|_F^2 + (x_1 x_2)^2 - 2a_{12} x_1 x_2$$

In [11]:
```
a=A[1,2]
Af=A.norm('frob')**2
```

In [12]:
```
f2=lambda x1,x2: Af+(x1*x2)**2-2*a*x1*x2
```

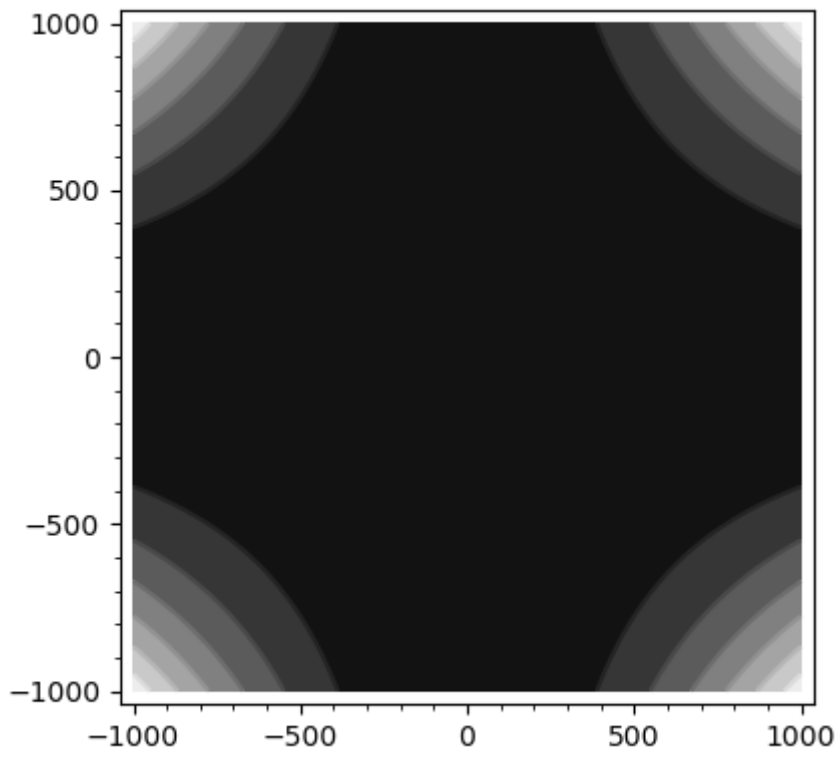
In [13]:
```
plot3d(f2, (x1,-1000,1000), (x2,-1000,1000), aspect_ratio=[1,1,1])
```

Out[13]:



In [14]:
```
contour_plot(f2, (-1000,1000), (-1000,1000))
```

Out[14]:



(b)

$$(n, d, m) = (100, 30, 5)$$

In [15]:
```
n=100
d=30
m=5
```

In [16]:
```
def sigma(x):
    if x<0:
        return exp(x)-1
    else:
        return x
```

In [17]:
```
Train=Matrix(n,d+1, lambda i,j: normalvariate(0, 1))
```

In [18]:
```
x_train=Train[:,:-1]
y_train=Train[:,-1]
```

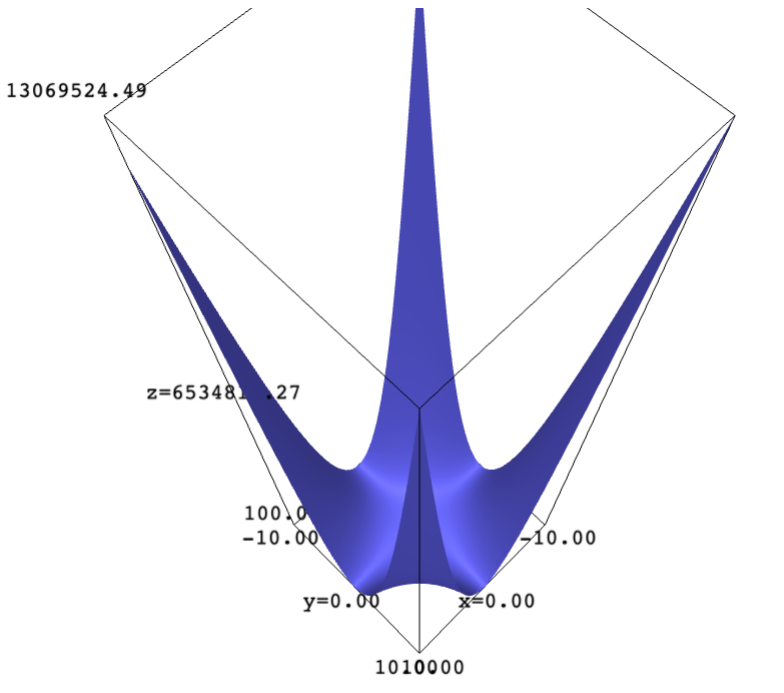
Projection onto a 2-dimensional space: $v = x_1[1, 0, 0, \ldots, 0]^T_{m\times 1}$, $W = x_2[[1, 1, \ldots, 1]_{1\times 30}, 0_{1\times 30}, 0_{1\times 30}, 0_{1\times 30}, 0_{1\times 30}]^T_{m\times d}$

$$\sum_{i=1}^{n} \left(y_i - v^T \sigma\left(W x_i\right)\right)^2 = \sum_{i=1}^{n} \left(y_i - x_1 \sigma(x_2 sum(x_i))\right)^2$$

In [19]:
```
def f3(x1,x2):
    fv=0
    for i in range(n):
        fv+=(y_train[i][0]-x1*sigma(x2*sum(x_train[i])))**2
    return fv
```

In [20]:
```
plot3d(f3, (x1,-10,10), (x2,-10,10), aspect_ratio=[1,1,1])
```

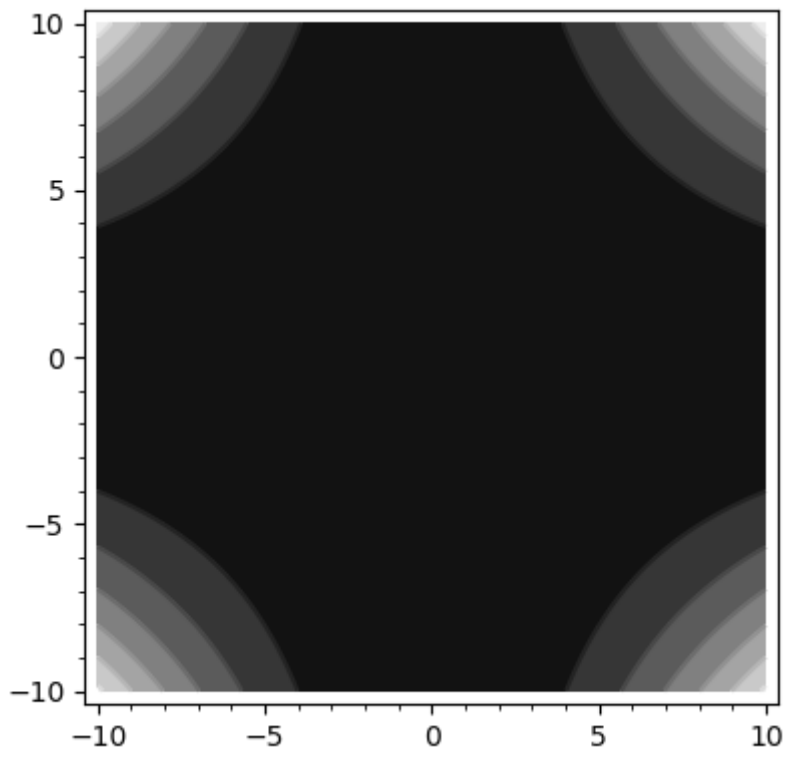
Out[20]:



In [21]:

```
contour_plot(f3, (-10,10), (-10,10))
```

Out[21]:



$$(n, d, m) = (100, 10, 5)$$

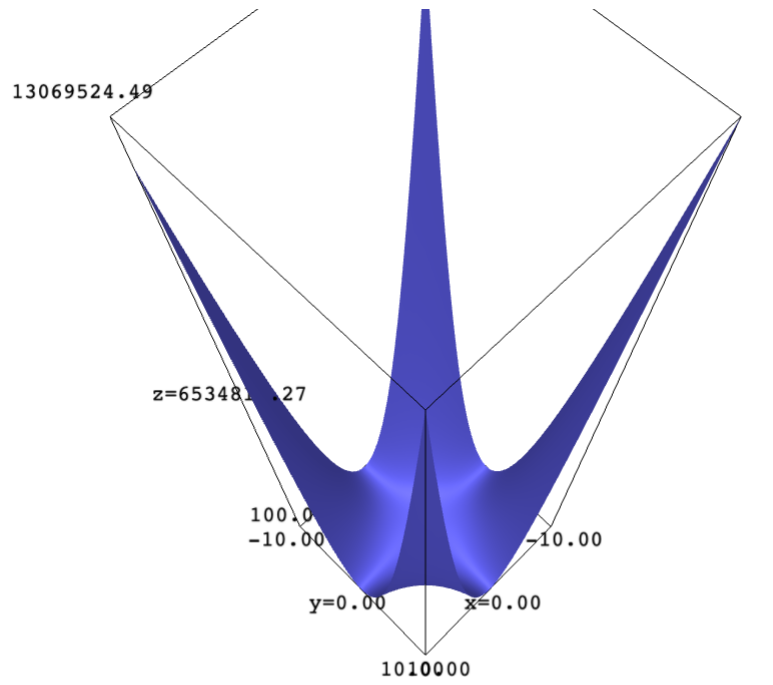
In [22]:

```
n=100
d=10
m=5
```

In [23]:

```
plot3d(f3, (x1,-10,10), (x2,-10,10), aspect_ratio=[1,1,1])
```

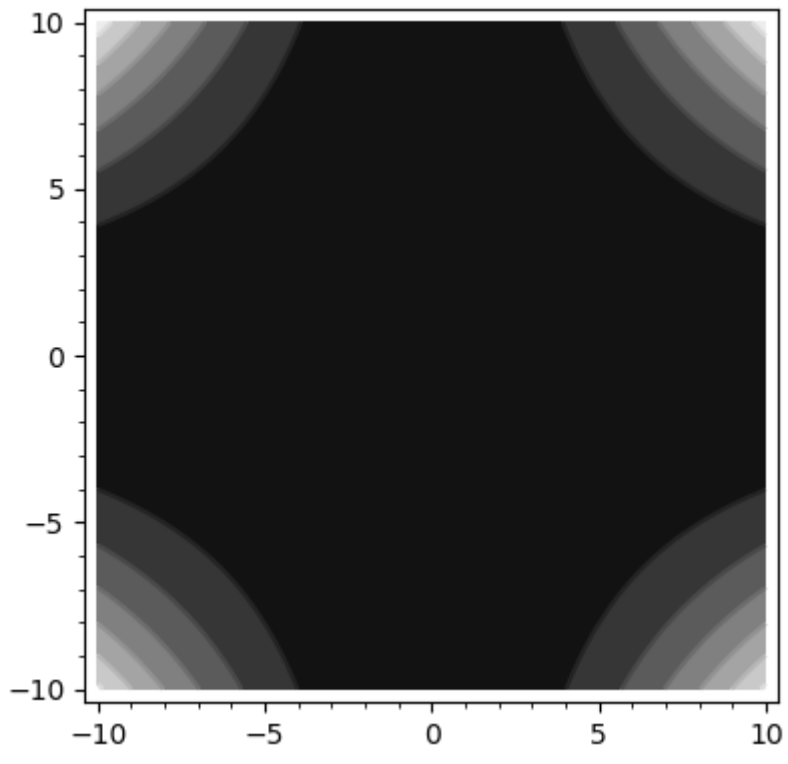
Out[23]:



In [24]:

```
contour_plot(f3, (-10,10), (-10,10))
```

Out[24]:

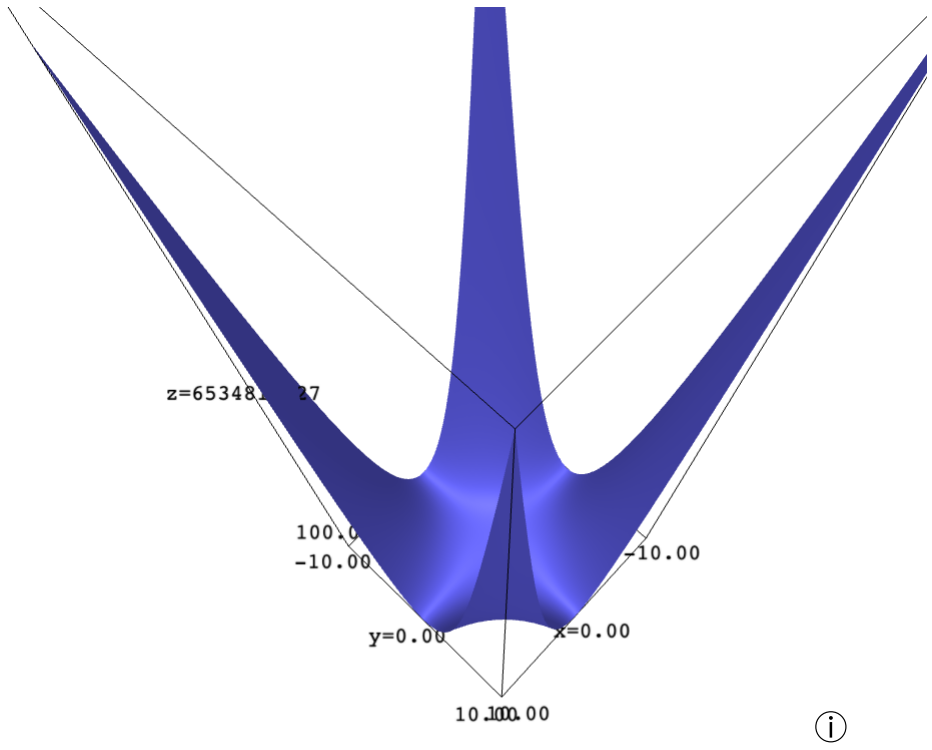


$$(n, d, m) = (100, 3, 5)$$

In [25]:
```
n=100
d=3
m=5
```

In [26]:
```
plot3d(f3, (x1,-10,10), (x2,-10,10), aspect_ratio=[1,1,1])
```

Out[26]:



In [27]:
```
contour_plot(f3, (-10,10), (-10,10))
```

Out[27]: