

HW 1 P6

January 31, 2022

1 HW 1, P6 by Sagemath

```
[1]: set_random_seed(0)
```

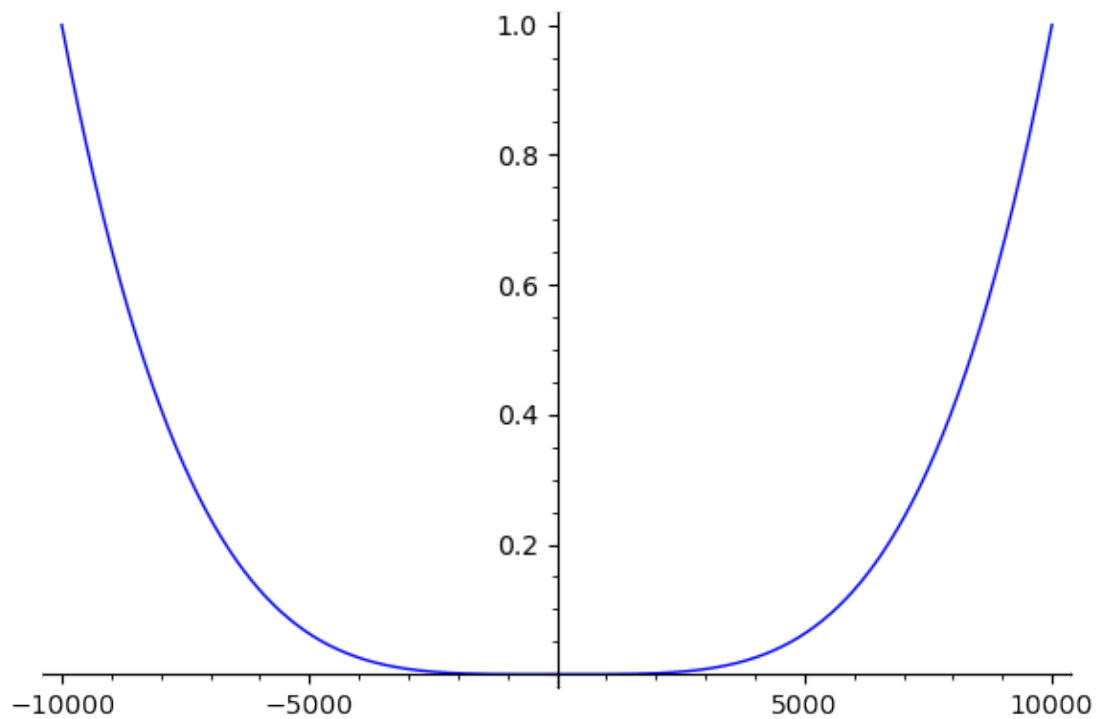
1.1 (a)

1.1.1 (i)

```
[2]: x=var('x')  
f=(1-x**2)**2
```

```
[3]: plot(f, x,-10000,10000)
```

[3]:



1.1.2 (ii)

```
[4]: A=matrix(2,2,[1,0.6,0.6,1])  
     var('x1','x2')
```

```
[4]: (x1, x2)
```

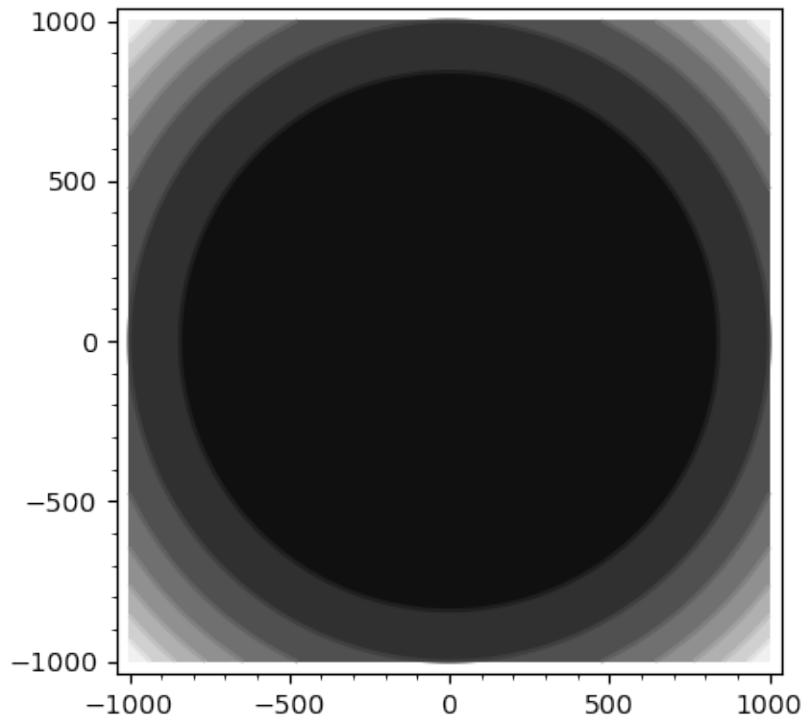
```
[5]: f1=lambda x1,x2: ((A-vector([x1,x2]).column()*vector([x1,x2]).row()).  
    ↪norm('frob'))**2
```

```
[6]: plot3d(f1, (x1,-1000,1000), (x2,-1000,1000), aspect_ratio=[1,1,1])
```

```
[6]: Graphics3d Object
```

```
[7]: contour_plot(f1, (-1000,1000), (-1000,1000))
```

```
[7]:
```



1.1.3 (iii)

Pick a point $\hat{\theta} = 0_{200 \times 1} = [0, 0, \dots, 0]^T$

Pick two vectors $u = [1, 0, 0 \dots 0]^T$ and $v = [0, 1, 0 \dots 0]^T$

```
[8]: u=vector([0]*200)  
     u[0]=1
```

```
v=vector([0]*200)
v[1]=1
```

```
[9]: u.column()*v.row()
```

```
[9]: 200 x 200 dense matrix over Integer Ring (use the '.str()' method to see the
entries)
```

```
[10]: B=Matrix(RR, 200, lambda i,j: normalvariate(0, 1))
A=B*B.transpose()
```

$$(x_1u)(x_2v)^T = [t_{ij}]_{i,j}, \quad t_{ij} = x_1x_2 \text{ if } i = 1, j = 2, \text{ otherwise } t_{ij} = 0$$

$$\|A - (x_1u)(x_2v)^T\|_F^2 = \|A\|_F^2 - a_{12}^2 + (a_{12} - x_1x_2)^2 = \|A\|_F^2 + (x_1x_2)^2 - 2a_{12}x_1x_2$$

```
[11]: a=A[1,2]
Af=A.norm('frob')**2
```

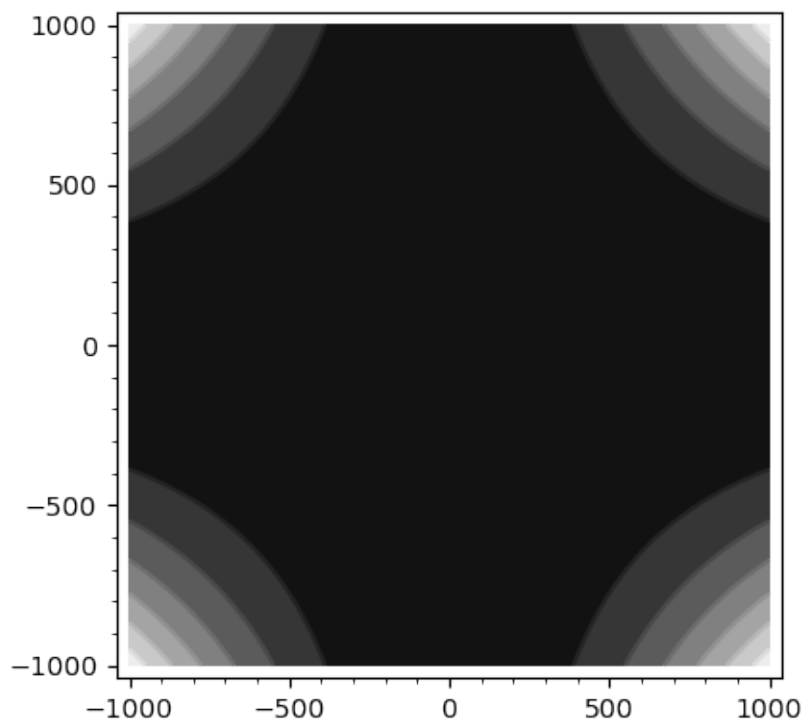
```
[12]: f2=lambda x1,x2: Af+(x1*x2)**2-2*a*x1*x2
```

```
[13]: plot3d(f2, (x1,-1000,1000), (x2,-1000,1000), aspect_ratio=[1,1,1])
```

```
[13]: Graphics3d Object
```

```
[14]: contour_plot(f2, (-1000,1000), (-1000,1000))
```

```
[14]:
```



1.2 (b)

$$(n, d, m) = (500, 30, 5)$$

```
[15]: z=var('z')
      f1=exp(z)-1
      f2=z
      sigma=piecewise([(-infinity,0),f1],[(0,infinity),f2]])
```

```
[16]: sigma(-1)
```

```
[16]: e^(-1) - 1
```

```
[17]: Train=Matrix(500,31, lambda i,j: normalvariate(0, 1))
```

```
[18]: x_train=Train[:, :-1]
      y_train=Train[:, -1]
```

Projection onto a 2-dimensional space: $v = x_1[1, 0, 0, 0, 0]^T$, $W = x_2[[1, 1, \dots, 1]_{1 \times 30}, 0_{1 \times 30}, 0_{1 \times 30}, 0_{1 \times 30}, 0_{1 \times 30}]^T$

$$\sum_{i=1}^n (y_i - v^T \sigma(Wx_i))^2 = \sum_{i=1}^n (y_i - x_1 x_2 \text{sum}(x_i))^2$$

```
[19]: def f3(x1,x2):  
      fv=0  
      for i in range(500):  
          fv+=(y_train[i][0]-x1*x2*sum(x_train[i]))**2  
      return fv
```

```
[20]: plot3d(f3, (x1,-10,10), (x2,-10,10), aspect_ratio=[1,1,1])
```

[20]: Graphics3d Object

```
[21]: contour_plot(f3, (-10,10), (-10,10))
```

[21]:

