Assumption: there are no zero column in A and B.

game is non-degenerated.

$\bar{P}_1 = \{ (x, v) \in \mathbb{R}^m \times \mathbb{R} \mid x \geq 0, B^T x \leq v \mathbb{1}, x^T \mathbb{1} = 1 \}$.

$\bar{P}_2 = \{ (y, u) \in \mathbb{R}^n \times \mathbb{R} \mid y \geq 0, Ay \leq u \mathbb{1}, y^T \mathbb{1} = 1 \}$.

$x^T B y = (B^T x)^T y \leq \max_i \{ (B^T x)_i \} \leq v$

equals when $y = [0, \cdots 1, \cdots, 0]^T$.

Hence $\max_y \{ x^T B y \} = \max_i \{ (B^T x)_i \} \leq v$: $x^T B y \leq v \ \forall y \iff B^T x \leq v \mathbb{1}$.

$\langle x, By \rangle = \langle B^T x, y \rangle \leq v$.

$x = [x_1, x_2, \dots, x_m]^T \quad x_k = 0$ if $k \in S_1$ is not played.

$(B^T x)_l = v. \Rightarrow l \in S_2$. $\underbrace{y = [0, 0, \cdots, \underset{l^{th}}{1}, \cdots, 0]^T}_{\text{i.e. play } l}$ is best response to x.

The Polyhedron $\bar{P}_1$ lives in $\mathbb{R}^{m+1}$

it has one equality constraint: $\exists (x, v), i$, s.t. $(B^T x)_i = v$.

起码 m 维, 由于 (x, v), $\mathbb{R}^m \times \mathbb{R}$.

$\Rightarrow$ it is m-dimensional Polyhedron.

$\Rightarrow$ in extreme point of $\bar{P}_1$, $\underset{\leq m}{m}$ of the inequality constraints have to be satisfied with equality.

An extreme point (x, v) Corresponds to a situation where $supp(x) = k$, $k \le m$.

these k inequality constraints are satisfied with equality.

$\Rightarrow$

player 2's pure strategies best response to x. $\ge k$.

i.e. strategy gets v.

Because we assume non-degenerated game.

$\Rightarrow$ no mixed strategy has more than k pure best responses

$\Rightarrow$ # player 2's pure strategies best response to x. $\le k$.

i.e. strategy gets v.

i.e. $= k$.

Normalized, so that $u = v = 1$.