$x^T \mathbf{1} = 1.$ $y^T \mathbf{1} = 1.$ since $\sum_i x_i = 1 = \sum_j y_j$

$u_1 = x^T A y$ $u_2 = x^T B y. = y^T B^T x.$

①. $x_i > 0 \Rightarrow (Ay)_i = u = \max\{(Ay)_j \mid \forall j \in \{1, 2, \ldots m\}\}.$

②. Non-degenerated condition:

A two player game is called non-degenerated if no mixed strategy (of support size k) has more than k pure best responses.

A example of degenerate : $A = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

strategy 3^{rd} column has two best responses.

Proposition: In any non-degenerated game, every NE (x^*, y^*) have supports of equal size.

Algorithm: Equilibria by Support enumeration.

Input: non-degenerat game

Output: Nash equilibrium.

① $\forall k = \{1, 2, \ldots \min\{m, n\}\}.$

and each pair (I, J) of k-sized subsets

$\{1,2,\dots,m\}$ $\{1,2,\dots n\}$.

of S_1 and S_2 respectively.

Solve equations: $\begin{cases} \sum_{i\in I} x_i b_{ij} = v & \forall j \in J, \ \sum_i x_i = 1. \\ \sum_{j\in J} a_{ij} y_j = u & \forall i \in I, \ \sum_j y_j = 1. \\ x \geq 0, \ y \geq 0 \end{cases}$

find feasible solutions.

Polytope: affine combination: $z_1, z_2, \dots z_k \in \mathbb{R}^d$.

sum of the coefficients equals to 1.

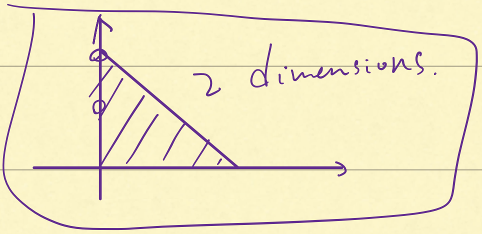
$\sum_{i=1}^{k} \lambda_i z_i$, $\lambda_i \in \mathbb{R}$, $\sum \lambda_i = 1$

convex combination: if λ_i also ≥ 0.

convex set means all convex combinations are in the set.

affinely independent. $z_1, z_2, \dots z_k \in \mathbb{R}^d$ is if no z_i is affine combination of others.

convex set's dimension: d' if and only if it has $d'+1$ affinely independent points and no more.



Polyhedron: $P \subseteq \mathbb{R}^d$ is a set $\{z \in \mathbb{R}^d \mid Cz \leq q\}$
Convex set.
for some matrix C and vector q.
full dimension (d dimensions).

if the Polyhedron is bounded, we call it Polytope.

Face: of P: $\{ z \in P \mid \underbrace{c^T z = q_0}_{\text{hyper plane}} \}$ for some $c \in \mathbb{R}^d$, $q_0 \in \mathbb{R}$.
remaining is $c^T z \leq q_0$.

Vertex: of P is unique element of a zero-dimensional face of P.

Edge: of P $\longleftarrow$ one dimensional face of P.

Facet: of P $\longleftarrow$ face of dimension $d-1$. (i.e. one dimension less).