2/3-MMS Allocation

- Properties:
  - Scale Invariant
  - Average is upper bound: $\mu_i \leq \frac{v_i(M)}{n}$
  - Allocating one item to an agent and removing them does not harm
  - Ordered instance

There is an order of items s.t.
$v_i(g_1) \geq v_i(g_2) \geq \cdots \geq v_i(g_n) \quad \forall i \in N.$

Claim: If we can design an algorithm to output α-MMS-allocation for ordered instance, then we can use this algorithm to find α-MMS-allocation for any instance.

	g_1	g_2	g_3	g_4
a_1	10	9	20	5
a_2	1	3	10	15
a_3	100	10	90	80

$\Longrightarrow$ convert into an ordered instance

	g_1'	g_2'	g_3'	g_4'
a_1	20	10	9	5
a_2	15	10	3	1
a_3	100	90	80	10

$\downarrow$ Solve this instance. and find α-MMS allocation.

$v_i(A_i') \geq \alpha\mu_i$ $\forall i$.

$\{g_1'\}$	$\{g_2'\}$	$\{g_3', g_4'\}$	
20	10	9	5
15	10	3	1
100	90	80	10

<u>Goal</u>: find $A=(A_1, A_2, A_3)$

s.t. $v_i(A_i) \geq \alpha \cdot \mu_i$. $\Longleftarrow$

a_1 拿 g_1'

a_2 拿 g_2'

----

a_i 拿 B_i

ordered instance

令 a_2 失去了拿到他眼中最大项的机会.

但他依然有 2nd 3rd ---- 的机会.

同理 a_i 失去了 1st 至 i^{th} 的机会.

但依然有 $(i+1)^{st}$ ---- 的机会.

Hence, there must exists $A=(A_1, \dots, A_n)$.

S.t $v_i(A_i) \geq v_i(A_i') \geq \alpha\mu_i$ $\forall i$.

2/3-MMS Allocation

- Scale the valuations so that $v_i(M) = n$
- Call an item j
  - high-valued if there exists an agent i such that $v_i(j) \geq 2/3$
  - medium-valued if for each agent i $v_i(j) < 2/3$ and there is an agent i for which $v_i(j) \geq 1/3$
  - low-valued if for each agent i, $v_i(j) < 1/3$

- What to do with high-value items?

Step 1: If there is a high-valued item j s.t. $v_i(j) \geq \frac{2}{3}$ then allocate j to i $N \leftarrow N \setminus \{i\}$. $M \leftarrow M \setminus \{j\}$.

Step 2: Normalize valuation s.t. $v_i(n) = n$, $\forall i \in N$.

Keep repeating Step 1, 2.

2/3-MMS Allocation

- Scale the valuations so that $v_i(M) = n$
- Call an item j
  - high-valued if there exists an agent i such that $v_i(j) \geq 2/3$
  - medium-valued if for each agent i $v_i(j) < 2/3$ and there is an agent i for which $v_i(j) \geq 1/3$
  - low-valued if for each agent i, $v_i(j) < 1/3$
- What is remaining? only contain medium-valued and low-valued item.

Case 1: Suppose # medium-valued item $\leq n$.

12
⋮
12
12

keep adding low-valued item.
medium-valued item.
13

} until $\geq \frac{2}{3}$

2/3-MMS Allocation

- Scale the valuations so that $v_i(M) = n$
- Call an item j
  - high-valued if there exists an agent i such that $v_i(j) \geq 2/3$
  - medium-valued if for each agent i $v_i(j) < 2/3$ and there is an agent i for which $v_i(j) \geq 1/3$
  - low-valued if for each agent i, $v_i(j) < 1/3$

- What is remaining?
- Allocate 2 medium-valued items

<u>Case 2</u>: Suppose # medium-valued $> n$.

$v_i(g_1) \geq v_i(g_2) \geq \cdots \geq \boxed{v_i(g_n) \geq v_i(g_{n+1})} \,|\, \cdots \geq v_i(g_m)$.

$\geq \frac{1}{3}$ $\quad < \frac{1}{3}$.

Claim: assign $\{g_n, g_{n+1}\}$ to an agent doesn't decrease the MMS value of remaining items.

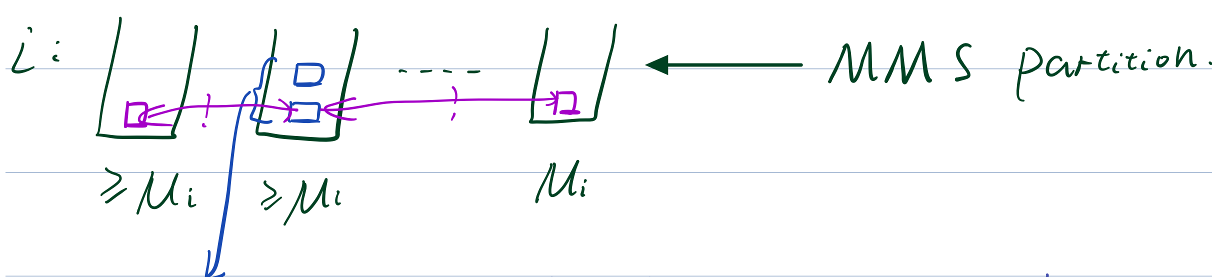
Set medium-valued item $\{g_1, \dots g_n, g_{n+1}, \dots\}$.

Claim: $\exists$ bag B_j in MMS contains <u>two items</u> from S.

$\{g, g'\} \subseteq S$.

$\longrightarrow$ Pigeonhole principle
鸽巢原理.

Claim: Allocating 2 lowest medium-valued items (when <u># medium-valued items</u> $> n$) to some agent and removing them from the instance doesn't decrease the MMS value of remaining items.



there has to be a bundle with ≥ 2 medium-valued items.

将两个最小 medium 放一起最大化 remaining <u>MMS-value</u> (μ_i).

将最小的两 medium-valued 中一个与其他bundle中的 medium-valued item 交换与 降低其它 bundle的value

2/3-MMS Allocation

- Scale the valuations so that $v_i(M) = n$
- Call an item j
  - high-valued if there exists an agent i such that $v_i(j) \geq 2/3$
  - medium-valued if for each agent i $v_i(j) < 2/3$ and there is an agent i for which $v_i(j) \geq 1/3$
  - low-valued if for each agent i, $v_i(j) < 1/3$

- What is remaining?
- Allocate 2 medium-valued items
- What is remaining?

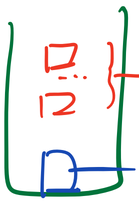
\# bundles of medium-valued items $\leq n$.

12 low-valued
12
medium 12

Keep adding low-valued items until some agent's value $\geq \frac{2}{3}$.

Smart Bag-Filling

$V_i(M)=n \Rightarrow \mu_i \le 1. \ \forall i \in N.$



low-valued items.

medium-valued item.

B

$V_i(B) < 1 \ \ \forall i \in N.$

$V_{i^*}(B) \geq \frac{2}{3}.$

Removing item: $M \setminus B$

$N \leftarrow N \setminus \{i\}$

$V_i(M \setminus B) = V_i(M) - V_i(B) \geq n-1.$

$|N| = n-1$

$B_1, B_2, \dots, B_k.$

$V_i(M \setminus \{B_1 \cup B_2, \dots, B_k\}) \geq n-k.$

$|N| = n-k.$

Example