Math 416 Jordan Form (Chapter 7)

(Exclude §1.7, §2.6, §2.7, §3.3, §4.4, §6.6∼ §6.11, §7.3, §7.4)

§7.1, §7.2

1. Classify all types of the Jordan Canonical Form of 2×2 and 3×3 matrices up to similarity.

2. You are given a 4×4 matrix
$$A = \begin{pmatrix} 4 & -2 & 0 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 4 \end{pmatrix}.$$

(1) Compute all the eigenvalues of A. Does the characteristic polynomial of A split over $\mathbb{R}$? $(4-t)^2(2-t)^2$

$A - 4I = \begin{bmatrix} 0 & -2 & 0 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$

(2) Find a basis of each of the generalized eigenspaces.

(3) Find a Jordan canonical basis for A and construct a matrix Q, and then write the corresponding Jordan canonical from J so that $Q^{-1}AQ = J$.

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$(t-2)^4$

$\lambda = 2 \quad m = 4.$

3. You are given a 4×4 matrix

$\begin{bmatrix} -1 & 1 & 1 & 0 \\ -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $A \neq \begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 3 & 1 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$

(1) Compute all the eigenvalues of A. Does the characteristic polynomial of A split?

(2) Find a basis of each of the generalized eigenspaces.

(3) Find a Jordan canonical basis for A and construct a matrix Q, and then write the corresponding Jordan canonical from J so that $Q^{-1}AQ = J$.

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$

4. You are given a 4×4 matrix
$$A = \begin{pmatrix} 1 & 1 & 1 & -1 \\ -1 & 3 & 1 & -1 \\ 1 & -1 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

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(1) Compute all the eigenvalues of A. Does the characteristic polynomial of A split over $\mathbb{R}$?

(2) Find a basis of each of the generalized eigenspaces.

(3) Find a Jordan canonical basis for A and construct a matrix Q, and then write the corresponding Jordan canonical from J so that $Q^{-1}AQ = J$.

5. What if the characteristic polynomial does not split?