matrix $A = (a_{ij})_{n\times n}$ $\xrightarrow[\text{row } j.]{\text{remove column } i \text{ \& }}$ M_{ij}

$A_{ij} = (-1)^{i+j} |M_{ij}|$ different order with A.

adjugate matrix: $A^* = \begin{bmatrix} A_{11} & \cdots & A_{n1} \\ \vdots & \ddots & \vdots \\ A_{1n} & \cdots & A_{nn} \end{bmatrix}$

Since $|A| = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} |M_{ij}|$

$= \sum_{i=1}^{n} A_{ij} a_{ij}$ or $\sum_{j=1}^{n} A_{ij} a_{ij}$

$(\sum_{i=1}^{n} A_{ij} a_{ik} = 0,\ k \neq j)$

$$A^* \cdot A = \begin{bmatrix} \sum_{i=1}^{n} A_{i1} \cdot a_{i1} & \cdots & \sum_{i=1}^{n} A_{i1} \cdot a_{in} \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} A_{in} a_{i1} & \cdots & \sum_{i=1}^{n} A_{in} a_{in} \end{bmatrix}$$

$$= \begin{bmatrix} |A| & 0 & \cdots & 0 \\ 0 & & & | \\ \vdots & & & \\ 0 & \cdots & & |A| \end{bmatrix} = |A| \cdot I.$$

$A^* = |A| \cdot A^{-1}$ (if A is inversable)