416 Final study guide

Final covers §1.1 ∼ §7.2 from the textbook (cumulative, also see Test 1 and 2 study guides).
Exclusion: §1.7, §2.6, §2.7, §3.3, §4.4, §6.6 ∼ §6.11, §7.3, §7.4

You should be able to do the following.

Chapter 6, §6.2 ∼ §6.5

- Using Gram-Schmidt process orthogonalize a given basis, and find an orthonormal basis.
- Find the orthogonal projection to a subspace.
- Given a subspace of an inner product space what it means for a subspace to be the orthogonal complement of the subspace.
- Compute the orthogonal complement of a given subspace.
- Know what it means for a linear operator on an inner product space to be an isometry.
- Given a linear operator on an inner product space compute its adjoint linear operator.
- Given a linear operator on an inner product space determine whether it is normal, self-adjoint or neither.
- Prove that some properties of a normal operator on a complex inner product space (Theorem 6.15).
- Understand the equivalent condition for a complex matrix to be unitarily (orthogonally for real matrices) equivalent to a diagonal matrix (Theorem 6.19, 6.20).
- Understand the equivalent condition for a linear operator on V(a finite dimensional real inner product space) to satisfy that there exists an orthonormal basis for V consisting of eigenvectors (Theorem 6.17).
- Understand the equivalent condition for a linear operator on V(a finite dimensional real inner product space) to satisfy that there exists an orthonormal basis for V consisting of eigenvectors whose eigenvalues have absolute value 1 (Theorem 6.18, Corollary 1).
- Understand the equivalent condition for a linear operator on V(a finite dimensional complex inner product space) to satisfy that there exists an orthonormal basis for V consisting of eigenvectors whose eigenvalues have absolute value 1 (Theorem 6.18, , Corollary 2).

Chapter 7, §7.1, §7.2

- Understand what a Jordan block and a Jordan canonical form mean.
- Understand the statement: "Given a linear operator T on a vector space V if the characteristic polynomial of T splits, then there exists a basis β for V such that $[T]_\beta$ is in a Jordan canonical form."
- Given a linear operator on a finite dimensional vector space (or a matrix), compute the basis of each of the generalized eigenspace and its cycle structure, then find a Jordan Canonical Form.
- Find a Jordan Canonical Basis which forms the columns of a matrix Q so that $Q^{-1}AQ = J$.
- Know what a generalized eigenspace means.
- Prove that each generalized eigenspace of a linear operator T is T-invariant.