416 Test 1 Study guide

Test 1 covers §3.4, §1.1 $\sim$ §1.6, §2.1 $\sim$ §2.5 from the textbook.

You should be able to do the following

Chapter 3, §3.4
- Using Gaussian elimination solve a system of linear equations
- Using elementary row operations (§3.1) find Row Echelon Form (REF) and Reduced Row Echelon Form (RREF)
- Using augmented matrix find the solution set of a system of linear equations

Chapter 1, §1.1 $\sim$ §1.6
- The 8 axioms of a vector space. You don't need to memorize, but you should be able to use the criteria for a subspace to prove that a given subspace is a vector space.
- The main examples of vector spaces are $\mathbb{F}^n, P_n(\mathbb{F}), M_{m\times n}(\mathbb{F}), \mathcal{F}(S,\mathbb{F})$ and their subspaces, where $\mathbb{F}=\mathbb{R}$ or $\mathbb{C}$, and S is a nonempty set.
- Find the subspace spanned by a given subset of a vector space.
- Given a set of linearly dependent vectors write one vector as a linear combination of the other vectors.
- Using the criteria for a subspace prove that a given subset of a vector space is a subspace.
- Determine whether a given set of vectors is linearly dependent or linearly independent.
- Using the criterion for linear independence prove that a given set of vectors is linearly independent.
- Know what it means for a given subset of a vector space to be a basis.
- Prove that a given subset of a vector space is a basis.

Chapter 2, §2.1 $\sim$ §2.5
- Prove that a given map between two vector spaces is a linear transformation
- Given a linear transformation T compute $N(T), R(T)$, nullity ($=\dim N(T)$), rank ($=\dim R(T)$), and verify the dimension theorem (also called rank-nullity theorem)
- Given a linear transformation prove that it is one-to-one or onto.
- Given an ordered basis β of a vector space compute the coordinate vector $[v]_\beta$ of a vector v.
- Given a linear map $T: V\to W$ and given ordered bases β,γ of V, W, respectively, compute the matrix representation $[T]_\beta^\gamma$
- Compute the matrix multiplication, addition and scalar multiplication
- Compute the inverse of small size matrices ($2\times 2, 3\times 3, 4\times 4, ...$)
- Given two ordered bases β,β' for a vector space V compute the change of coordinate matrix $Q=[I_V]_{\beta'}^{\beta}$ such that $[v]_\beta = Q[v]_{\beta'}$ for any $v\in V$, and
- $[T]_{\beta'} = Q^{-1}[T]_\beta Q$ for every linear transformation T on V