A symmetric $n \times n$ real matrix M is said to be

positive-definite $\iff x^T M x > 0 \quad \forall x \in \mathbb{R}^n \setminus \{0\}$.

$\iff$ all eigenvalues > 0

$x^T P D P^{-1} x > 0 \quad \forall x \in \mathbb{R}^n \setminus \{0\}. \iff D_i > 0 \quad \forall i.$

Positive semi-definite $\iff x^T M x \geq 0 \quad \forall x \in \mathbb{R}^n$.

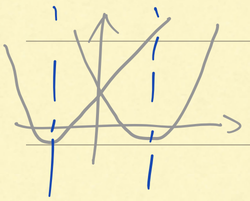
$\iff$ all eigenvalues ≥ 0.

Similarity.

Negative definite $\iff x^T M x < 0 \quad \forall x \in \mathbb{R}^n \setminus \{0\}$

Negative semi-definite $\iff x^T M x \leq 0 \quad \forall x \in \mathbb{R}^n$.

$$\begin{bmatrix} U_{xx} & U_{xL} \\ U_{Lx} & U_{LL} \end{bmatrix} \Rightarrow \begin{bmatrix} U_{xx} - t & U_{xL} \\ U_{Lx} & U_{LL} - t \end{bmatrix} \quad U_{xx}U_{LL} - (U_{xx} + U_{LL})t + t^2 - U_{xL}^2 = 0.$$



positive definite
negative definite
} 代入 $t=0$ $\quad U_{xx}U_{LL} - U_{xL}^2 > 0$

$\frac{U_{xx}+U_{LL}}{2} > 0$

$\frac{U_{xx}+U_{LL}}{2} < 0$.

$$\left.\begin{array}{l}\text{positive semi-definite}\\ \text{negative semi-definite}\end{array}\right\}\begin{array}{l}\text{代入}\\ t=0\end{array}\quad U_{xx}U_{LL}-U_{xL}^2 \geq 0$$

$$\frac{U_{xx}+U_{LL}}{2} \geq 0$$

$$\frac{U_{xx}+U_{LL}}{2} \leq 0.$$