416 Test 2 Study guide

MATH 416: ABSTRACT LINEAR ALGEBRA

Test 2 covers §3.1, 3.2, §4.1 ∼ §4.3, §5.1 ∼ §5.4, §6.1, from the textbook.

You should be able to do the following

Chapter 3, §3.1, §3.2
- Find the corresponding elementary matrix of each of the elementary row operations.
- Write an invertible matrix as a product of elementary matrices.
- Find the rank of a given matrix and linear transformation.
- Find the rank of the product of two matrices and the composition of two linear transformations.
- Using augmented matrix find the inverse of an $n \times n$ matrix for small $n > 2$.

Chapter 4, §4.1 ∼ §4.3
- Using type III elementary row operation find the determinant of a small size matrix $(3 \times 3, 4 \times 4, ...)$.
- Understand the effect of elementary row operations on determinant.
- Using Cramer's rule solve a system of linear equations.

Chapter 5
§5.1 ∼ §5.2
- Determine the eigenvalues and eigenvectors of a given square matrix and a linear operator on a vector space (its matrix representation).
- Determine the eigenspace of each of the eigenvalues.
- Prove the equivalent condition $\det(A - \lambda I) = 0$ for an eigenvalue.
- Test if diagonalizable, and diagonalize if possible.
- Apply diagonalization to find an arbitrary matrix power.

§5.3
- Determine whether a transition matrix has a limit $\lim_{m\to\infty} A^m$, and
- compute the matrix limit $\lim_{m\to\infty} A^m$ if it exists.
- Understand the definitions of probability vector and transition matrix.
- Apply Markov chain to real world problems.

§5.4
Let T be a linear operator on a vector space.
- Prove that a given subspace is T-invariant.
- Find a basis for the T-cyclic subspace generated by a given vector.

Chapter 6, §6.1
- The three main examples of inner products are standard inner product on $\mathbb{F}^n$, Frobenius inner product on the $n \times n$ matrices, and the inner product given by definite integral on the continuous functions.
- Understand the properties of inner products including Cauchy-Schwarz inequality.
- Determine whether an inner product or not.
- Understand why the inner product on $\mathbb{R}^n$ with $n > 0$ is not unique.