MATH 417 - Introduction to Abstract Algebra
Spring 2022

Homework 8
Due Friday March 25

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1. Find all solutions for each of the following congruences.

   a) $6x \equiv 3 \pmod{12}$

   b) $3x \equiv 6 \pmod{12}$

2. Find the values of $x^6 \pmod{91}$ as x ranges over all integers. Explain your answer.

3. (Fraleigh Exercise 19.26) Let R be a ring that contains at least two elements. Suppose for each nonzero $a \in R$, there exists a unique $b \in R$ such that $aba = a$.

   a) Show that R has no zero divisors.

   b) Show that $bab = b$.

   c) Show that R has unity.

   d) Show that R is a division ring.

4. For each of the given polynomials find all zeros in the indicated ring.

   a) $x^3 + 7x + 4$ in $\mathbb{Z}_{13}$.

   b) $(x+3)(x+1)(x-1)$ in $\mathbb{Z}_{15}$.