MATH 417 - Introduction to Abstract Algebra
Spring 2022

Homework 7
Due Friday March 11

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1. Let $G = D_{12} = \{1, \rho, \ldots, \rho^5, \sigma, \sigma\rho, \ldots, \sigma\rho^5\}$ be the group of symmetries of a regular hexagon and let $H = [G, G] = \{1, \rho^2, \rho^4\}$ be its commutator subgroup.

   a) Give the partition of G into left cosets of H

   b) Determine the structure of the group G/H as abelian group.

2. Determine the structure as abelian group for the following factor groups.

   a) $G = (\mathbb{Z}_8 \times \mathbb{Z}_{12})/\langle(2, 2)\rangle$.

   b) $G = (\mathbb{Z}_8 \times \mathbb{Z}_{12})/\langle(3, 3)\rangle$.

3. Let $N \triangleleft G$ be a normal subgroup. Show that the commutator $[a, b] \in N$ for all $a \in N$ and $b \in G$.

4. Show that the four binary matrices

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

form a ring under matrix addition and multiplication. Decide if the ring is a field.