

## MATH 417 - Introduction to Abstract Algebra

Spring 2022 - Midterm1 - Review questions

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1. Solve the linear equation  $9x \equiv 13 \pmod{70}$  in two steps.
  - a) Determine integers  $s$  and  $t$  such that  $1 = 9s + 70t$ .
  - b) Determine the smallest positive integer solution  $x$ .
2. Let  $A \leq G$  and  $B \leq G$  be subgroups of an abelian group  $(G, +)$ .
  - a) Show that  $A + B = \{a + b : a \in A, b \in B\}$  is a subgroup of  $G$ .
  - b) Show that  $A \cap B$  is a subgroup of  $G$ .
3. Let  $H = 12\mathbb{Z} + 27\mathbb{Z} \subset \mathbb{Z}$  and let  $K = 12\mathbb{Z} \cap 27\mathbb{Z} \subset \mathbb{Z}$ .
  - a) Determine  $h$  such that  $H = h\mathbb{Z}$ .
  - b) Determine  $k$  such that  $K = k\mathbb{Z}$ .
4. Let  $S_3 = \{1, \rho, \rho^2, \sigma, \sigma\rho, \sigma\rho^2\}$ , with  $\rho = (1\ 2\ 3), \sigma = (1\ 2)$ .
  - a) Show that  $\rho\sigma = \sigma\rho^2$ .
  - b) Reduce the permutation  $\rho\sigma\rho^2\sigma\rho\sigma$  to one of the six elements in the set  $S_3$ .
5. Let  $G = \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4$ .
  - a) Determine the number of elements in  $G$  of order 1, order 2, and order 4.
  - b) Determine the number of subgroups of  $G$  of size 4.
6. Give a permutation  $\tau \in S_8$  such that  $\tau(1\ 3\ 5\ 7\ 2\ 4\ 6\ 8) = (1\ 3\ 5\ 7)(2\ 4\ 6\ 8)$ .
7. Give a permutation  $\sigma \in S_8$  such that  $(1\ 3\ 5\ 7) = \sigma(2\ 4\ 6\ 8)\sigma^{-1}$ .
8. List the possible cycle structures for a permutation  $\sigma \in A_5$ .
9. Select all pairs of isomorphic groups among the following three groups.
$$\mathbb{Z}_{14} \times \mathbb{Z}_{24} \times \mathbb{Z}_{35}, \quad \mathbb{Z}_{210} \times \mathbb{Z}_{56}, \quad \mathbb{Z}_6 \times \mathbb{Z}_{40} \times \mathbb{Z}_{49}.$$
10. Prove using only the group axioms that each element in a group  $G$  has a unique inverse in  $G$ .