MATH 417 - Introduction to Abstract Algebra
Spring 2022

Homework 9
Due Friday April 8

---

1. Find $a, b, c, u, v, w \in \mathbb{Z}_7$ such that

   $$(x^3 + 3x^2 + bx + 1)(x^3 + ax^2 + 3x + c) = x^6 + ux^5 + 2x^4 + vx^3 + 2x^2 + wx + 2$$

   as polynomials in $\mathbb{Z}_7[x]$

2. Determine the number of irreducible polynomials $x^2+bx+c \in \mathbb{Z}_5[x]$. A quadratic polynomial in $\mathbb{Z}_5[x]$ is irreducible if it can not be written as a product of two linear polynomials in $\mathbb{Z}_5[x]$.

3. Let X be the set of bracelets with 6 red beads and 6 blue beads. Find five different bracelets with nontrivial isotropy subgroup. A bracelet has nontrivial isotropy subgroup if some nontrivial rotation of it has the same colors in the same positions.

4. Let $G = S_6$ act on itself by conjugation. The transpositions $\{(12), \ldots, (56)\}$ form a single orbit under conjugation. Let $H = \{g \in G : g(12)g^{-1} = (12)\}$ be the stabilizer of (12).

   a) Give the number of transpositions.

   b) Use part a) to determine the size of H.

   c) Describe the group H as a subgroup of S_6. (Hint: look for a description as a direct product of two subgroups)