MATH 417 - Introduction to Abstract Algebra
Spring 2022

Homework 5
Due Friday February 18

---

1. Fraleigh Section 5 Problem 39: Answer the ten True or False questions.

2. Determine all elements $a \in \mathbb{Z}_{24}$ such that $\langle a \rangle \leq \mathbb{Z}_{24}$ is a subgroup of order 8.

3. The regular 9-gon with vertices 1 to 9 has 18 symmetries that include $\rho = (1\,2\,3\,4\,5\,6\,7\,8\,9)$ and $\sigma = (1\,2)(3\,9)(4\,8)(5\,7)$. Give the disjoint cycle form for the permutations $\sigma\rho, \sigma\rho^2$ and $\rho\sigma, \rho^2\sigma$.

4. Recall that an isomorphism $\phi : G \to G'$ between groups G and G' is a bijection such that $\phi(xy) = \phi(x)\phi(y)$. Let G be a group and, for $g \in G$, let $\phi_g : G \to G$ be the map $\phi_g(x) = gxg^{-1}$.

    a) Show that ϕ_g is a bijection, for all $g \in G$.
    b) Show that ϕ_g is an isomorphism, for all $g \in G$.

5. Let S_6 be the group of permutations of $\{1, 2, 3, 4, 5, 6\}$. [1]

    a) Show that there are as many permutations in S_6 with cycle structure $(1\,2)$ as there are permutations with cycle structure $(1\,2)(3\,4)(5\,6)$.
    b) Show that there are as many permutations in S_6 with cycle structure $(1\,2\,3\,4\,5\,6)$ as there are permutations with cycle structure $(1\,2\,3)(4\,5)$.

6. Find the maximal possible order for an element in S_9.

---

[1] In general, an isomorphism $\phi : S_n \to S_n$ is of the form ϕ_g as defined in the previous problem. The group S_6 is an exception. The map that sends $(1\,2) \mapsto (1\,2)(3\,4)(5\,6)$ and $(1\,2\,3\,4\,5\,6) \mapsto (1\,2\,3)(4\,5)$ gives an isomorphism that does not preserve the cycle structure.