For the set $G = \{1, 2, \ldots, 12\}$ define a binary operation $* : G \times G \to G$ that maps an ordered pair $(a, b) \mapsto a * b$ according to the following table.

	b = 1	2	3	4	5	6	7	8	9	10	11	12
a = 1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	1	4	3	7	8	5	6	12	11	10	9
3	3	4	1	2	8	7	6	5	10	9	12	11
4	4	3	2	1	6	5	8	7	11	12	9	10
5	5	6	7	8	9	10	11	12	1	2	3	4
6	6	5	8	7	11	12	9	10	4	3	2	1
7	7	8	5	6	12	11	10	9	2	1	4	3
8	8	7	6	5	10	9	12	11	3	4	1	2
9	9	10	11	12	1	2	3	4	5	6	7	8
10	10	9	12	11	3	4	1	2	8	7	6	5
11	11	12	9	10	4	3	2	1	6	5	8	7
12	12	11	10	9	2	1	4	3	7	8	5	6

1. Collect all pairs (a, b) with $a * b = 7$.

2.. Compute each of the following. What do you observe?

$$(2 * 3) * 7 \qquad (7 * 2) * 3 \qquad (3 * 7) * 2$$
$$7 * (2 * 3) \qquad 3 * (7 * 2) \qquad 2 * (3 * 7)$$

3. Find a subset $H \subset G$ of size four such that $a * b \in H$ for all $a, b \in H$.

4. Find all subsets $H \subset G$ of size three such that $a * b \in H$ for all $a, b \in H$.

5. For $a \in G$ define $\ell_a : G \to G$ as the function $\ell_a(x) = a * x$.
Determine the composition $\ell_4 \circ \ell_5 : G \to G$.

6. For $b \in G$ define $r_b : G \to G$ as the function $r_b(x) = x * b$.
Determine the composition $r_4 \circ r_5 : G \to G$.