(F)

4 5 6 8 9 10 11 13 14 15.

$\underbrace{4\ 5\ 6\ 8\ 9\ 10\ 11}_{\text{midterm 1.}}$

Midterm 1: Wk 6.

2: Wk 10 or 11.

3: Wk 14 or 15.

<u>Groups.</u> $(G, *)$ set G with binary operation $*$.

$* : G \times G \to G$. s.t.

① $*$ is associative: $a*b*c = a*(b*c)$ $\forall a,b,c \in G$.

② G containing identity e : $e*x = x*e = x$. $\forall x \in G$.

③ Each element $a \in G$ has an inverse $a' \in G$ s.t.

$a*a' = a'*a = e$.

If (4) $*$ is commutative $a*b = b*a$.

Then G is called <u>abelian group</u>.

Example: $(\mathbb{Z}, +)$. $(n\mathbb{Z}, +)$. $(\mathbb{Z}_n, +)$

$(\mathbb{R}^*, \times)$ $(\mathbb{R}_{>0}, \times)$ $(\{\mathbb{R}_{>0}, \mathbb{R}_{<0}\}, \times)$.

$\mathbb{R}$ except 0.

$(S_X, \circ)$. $(E_X, \circ)$ $(\{E_X, O_X\}, \circ)$.

permutation of X.

Even permutation of X

Odd permutation of X.

Theorem: Let G be a group. The left and right cancelation laws hold in G:

① $a * x = a * y \Rightarrow x = y$.

② $x * a = y * a \Rightarrow x = y$.

Pf: Let $a * x = a * y$.

$\exists\ a'$ s.t. $a' * a = e$.

$a' * (a * x) = a' * (a * y)$

$a' * a * x = a' * a * y$.

$e * x = e * y$.

$x = y$.

Similar for the right cancel law.

Theorem: The linear equation $a * x = b$ and $y * a = b$ has unique sol.

Pf: Existence:

Multiply by a': $a' * (a * x) = a' * b$.

$x = a' * b$ is a solution

Uniqueness: if x' is another solution.

$a * x = a * x' = b \Rightarrow x = x'$.

Def Subgroup: A subset $H \subset G$ is a subgroup of G if H with the induced operation is ~~its~~ itself a group.

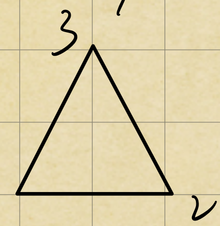
Notation: $H \leq G$.

if $H = G$ H is an improper subgroup.
if $H \subsetneq G$ proper subgroup.

if $H = \{e\}$ then H is a trivial subgroup.
$\{e\} \subsetneq H$, H is a nontrivial subgrop.

Symmetric of a triangle.
$G = \mathrm{Sym}(\Delta)$.

Group of size 6!



(1).
(1 2 3).
(1 3 2)

$\boxed{G = S_X \text{ for } X = \{1, 2, 3\}.}$

(1 2)
(2 3)
(1 3).

is a group.

not abelian

$(\mathbb{Z}_6, +)$ symmetric. abelian.

Invertiable matrices in $M_{2\times 2}(\mathbb{Z}_2)$

$$\left\{ \underset{(1)}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \underset{(12)}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}, \underset{(23)}{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}, \underset{(13)}{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}}, \underset{(123)}{\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}}, \underset{(132)}{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}} \right\}.$$

Ax.

$$x = \underset{1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} \quad \underset{2}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \quad \underset{3}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

Same structure as S_3.