MATH 417 - Introduction to Abstract Algebra
Spring 2022

Homework 3
Due Friday February 4

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1. Determine (in a systematic way) the smallest integer x with the property that $x \equiv 4 \pmod 7$ and $x \equiv 9 \pmod{17}$.

2. Write the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 4 & 8 & 5 & 3 & 9 & 2 & 1 \end{pmatrix}.$$

   as a product of disjoint cycles. Determine the smallest positive integer n such that the permutation $\sigma^n = \sigma \circ \sigma \circ \cdots \circ \sigma$ (n times) is the identity.

3. Find a permutation σ of $X = \{1, 2, 3, 4, 5, 6\}$ such that

$$\sigma \circ (1\,2\,6\,5) = (2\,3\,5\,4) \circ \sigma.$$

4. Determine the sign of each of the permutations

$$\sigma = (5\,1\,4)(5\,2\,3\,1)(4\,3\,2)$$
$$\tau = (5\,1)(4\,5\,2)(3\,1\,4)(3\,2)$$

   Conclude that $\sigma \neq \tau$. Provide two witnesses i and j such that $\sigma(i) \neq \tau(i)$ and $\sigma(j) \neq \tau(j)$ to verify your conclusion.