Two-phase dual simplex method.

nature tableau is not primal feasible.

we want to find a basic feasible solution to begin with.

1. First optimize an artificial obj function using dual simplex
(such as $x_1 + x_2 + \cdots + x_n$)

the final tableau will be primal feasible for any obj function.

2. Use primal simplex method starting from final tableau. Add in orginal obj. row-reduce.

The method above depends on slack vars.

what if we are given constraints in $Ax=b$, $x \geq 0$.

1. First row reduce $Ax=b$. ignoring $x \geq 0$

$\Longrightarrow$ Arrive at a basic solution.

2. Now pick an artificial obj function that gives you a dual feasible tablea (ex: sum of nonbasic vars). Use dual simplex method.

Ex: Find an x. s.t. $\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 5 \\ 2x_1 + 4x_2 + x_3 - 2x_4 = 2 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$

row reduce $\longrightarrow$ $\begin{cases} x_1 + 2x_2 \quad - \frac{5}{7}x_4 = \frac{11}{7} \\ x_3 - \frac{4}{7}x_4 = -\frac{8}{7} \\ x_1, x_2, x_3, x_4 \geq 0. \end{cases}$

Basic vars $(x_1, x_3) = (\frac{11}{7}, -\frac{8}{7})$

Use obj func: $x_2 + x_4$

	x_1	x_2	x_3	x_4	
x_1	1	2	0	$-\frac{5}{7}$	$\frac{11}{7}$
x_3	0	0	1	$-\frac{4}{7}$	$-\frac{8}{7}$
$-z$	0	1	0	1	0

then apply dual simplex method to find a primal feasible tableau.

Adding constraints.

min $x + y$

$$\text{s.t.} \quad -2x - y + s_1 = -6$$
$$-3x - y + s_2 = -7$$
$$-x - 2y + s_3 = -9$$
$$x, y, s_1, s_2, s_3 \geq 0.$$

final tableau $\rightarrow$

	x	y	s_1	s_2	s_3	
x	1	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	1
s_2	0	0	$-\frac{5}{3}$	1	$\frac{1}{3}$	0
y	0	1	$\frac{1}{3}$	0	$-\frac{2}{3}$	4
$-z$	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$	-5

we add an additional constriant $x \geq 2$.

i.e. $-x + s_4 = -2$.

Just add an row into the final tableau

	x	y	s_1	s_2	s_3	s_4	
x	1	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	0	1
s_2	0	0	$-\frac{5}{3}$	1	$\frac{1}{3}$	0	0
y	0	1	$\frac{1}{3}$	0	$-\frac{2}{3}$	0	4
s_4	-1	0	0	0	0	1	-2
$-z$	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	-5

row reduce $\rightarrow$

	x	y	s_1	s_2	s_3	s_4	
x	1	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	0	1
s_2	0	0	$-\frac{5}{3}$	1	$\frac{1}{3}$	0	0
y	0	1	$\frac{1}{3}$	0	$-\frac{2}{3}$	0	4
s_4	0	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	1	-1
$-z$	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	-5

then use dual simplex method.

Shadow costs

LP: max $c^T x$

s.t. $Ax \le b$

$x \ge 0$

we want a small change to either c or b

Can we predict how opt sol changes.

Example: Max $2x+3y$

s.t. $-x+y \le 3$

$x-2y \le 2$

$x+y \le 7$

$x, y \ge 0$

Opt sol is $(x,y) = (2,5)$ opt value is 19.

Let's change $2x+3y \to 2.1x+3y$.

opt sol is still $(2,5)$ opt value is 19.2

$2x+3y \to (2+\delta)x+3y$

obj value at $(2,5)$ is $19+2\delta$

is the lower bound on new opt value

If we change $x+y \le 7 \to x+y \le 7+\delta$.

Prediction: obj val change to $19 \pm \frac{5}{2}\delta$.

$w = \frac{5}{2}$ represents the sensitivity of obj val to changes in corresponding constraint.

Dual vars are often called "shadow cost"

$\frac{5}{2}$ is the cost of a change in $x + y \leq 7$.