(P) max $C^T X$ s.t. $AX \le b$, $X \ge 0$

(D) min $U^T b$ s.t. $U^T A \ge C^T$, $U \ge 0$

X^* is primal opt U^* is dual opt.

SD $\Rightarrow$ $Z^* = C^T X^* = U^{*T} b$.

$\delta \in \mathbb{R}$ is sufficient small (abs value).

① $C_i \to C_i + \delta$

expect X^* remain opt $Z^* \to Z^* + \delta x_i^*$ (change)

lower bound

② $b_i \to b_i + \delta$

expect X^* remain opt $Z^* \to Z^* + \delta u_i^*$ (change)

upper bound.

How much Small?

Max $5x_1 - x_2 + 7x_3$

s.t. $x_1 + x_2 + x_3 \le 5$

$3x_1 + 4x_3 \le 7$

$2x_1 + x_2 + 3x_3 \le 5$

	X_1	X_2	X_3	S_1	S_2	S_3	
S_1	0	2	0	1	-1	1	3
X_1	1	-4	0	0	3	-4	1
X_3	0	3	1	0	-2	3	1
$-Z$	0	-2	0	0	-1	-1	-12

$x^* = (1, 0, 1)$ $u^* = (0, 1, 1)$

$c = (5, -1, 7)$ $b = (5, 7, 5)$

① what happens when $b_1 = 5 \to 5+\delta$?

change $\delta u_1^* = \delta \cdot 0 = 0$

not change

	X_1	X_2	X_3	S_1	S_2	S_3	
S_1	0	2	0	1	-1	1	$3+\delta \geq 0$
X_1	1	-4	0	0	3	-4	1
X_3	0	3	1	0	-2	3	1
$-z$	0	-2	0	0	-1	-1	-12

not change

valid for $\delta \in [-3, +\infty)$

② $c_2 = -1 \to -1+\delta$?

change $\delta x_2^* = \delta \cdot 0 = 0$

not change

	X_1	X_2	X_3	S_1	S_2	S_3	
S_1	0	2	0	1	-1	1	3
X_1	1	-4	0	0	3	-4	1
X_3	0	3	1	0	-2	3	1
$-z$	0	$-2+\delta$	0	0	-1	-1	-12

≤ 0 not change

valid for $\delta \in (-\infty, 2]$

$\delta \in (-\infty, 2]$.

③ $c_1 = 5 \to 5+\delta$?

change $\delta x_1^* = \delta$

	X_1	X_2	X_3	S_1	S_2	S_3	
S_1	0	2	0	1	-1	1	3
X_1	1	-4	0	0	3	-4	1
X_3	0	3	1	0	-2	3	1
$-z$	δ	-2	0	0	-1	-1	-12

$\Downarrow$

	X_1	X_2	X_3	S_1	S_2	S_3	
S_1	0	2	0	1	-1	1	3
X_1	1	-4	0	0	3	-4	1
X_3	0	3	1	0	-2	3	1
$-z$	0	$-2+4\delta$	0	0	$-1-\delta$	$-1+\delta$	$-12-\delta$

≤ 0 ≤ 0 ≤ 0

Valid for $\delta \in [-\frac{1}{3}, \frac{1}{4}]$

$\delta \in [-\frac{1}{3}, \frac{1}{4}]$

④ $b_2 = 7 \rightarrow 7+\delta$?

change $\delta u_2^* = \delta$

Valid for $\delta \in [-\frac{1}{3}, \frac{1}{2}]$

	X_1	X_2	X_3	S_1	S_2	S_3	
S_1	0	2	0	1	-1	1	$3-\delta \geq 0$
X_1	1	-4	0	0	3	-4	$1+3\delta \geq 0$
X_3	0	3	1	0	-2	3	$1-2\delta \geq 0$
$-Z$	0	-2	0	0	-1	-1	$-12-\delta$

$\delta \in [-\frac{1}{3}, \frac{1}{2}]$