Consider

min $x + y$

s.t. $-2x - y + s_1 = -6$

$-3x - y + s_2 = -7$

$-x - 2y + s_3 = -9$

$x, y, s_1, s_2, s_3 \geq 0.$

	x	y	s_1	s_2	s_3	negative
s_1	-2	-1	1	0	0	-6
s_2	-3	-1	0	1	0	-7
s_3	-1	-2	0	0	1	-9
$-z$	1	1	0	0	0	0

min and non-negative

This is not primal feasible

but is dual feasible

find primal feasible sol may be hard

find dual feasible sol will be easier

Dual simplex method

find opt sol by going from dual feasible tableau to dual feasible tableau.

Pick a row with RHS is negative. if we

s_1 -2 -1 1 0 0 $\vdots$ -6

or

Choose a negative entry in that row and make that our new pivot.

	x	y	s_1	s_2	s_3	
s_1	-2	-1	1	0	0	-6
s_2	-3	-1	0	1	0	-7
s_3	-1	-2	0	0	1	-9
$-z$	1	1	0	0	0	0

$\xrightarrow{\text{pivot on } x}$

	x	y	s_1	s_2	s_3	
x	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	3
s_2	0	$\frac{1}{2}$	$-\frac{3}{2}$	1	0	2
s_3	0	$-\frac{3}{2}$	$-\frac{1}{2}$	0	1	-6
$-z$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-3

dual feasible

not primal feasible.

$\downarrow$ pivot on y

	x	y	s_1	s_2	s_3	
y	2	1	-1	0	0	6
s_2	-1	0	-1	1	0	-1
s_3	3	0	-2	0	1	3
$-z$	-1	0	1	0	0	-6

not dual feasible

not primal feasible

Choose entering vars from $x_1, \dots, x_k$. Let $a_1, \dots a_k$ be their coefficient in the pivot row.

	$\underset{x_1}{x}$	$\underset{x_2}{y}$	s_1	s_2	s_3	
s_1	$\underset{a_1}{-2}$	$\underset{a_2}{-1}$	1	0	0	-6

Let $r_1, r_2, \ldots, r_k$ be the reduced costs of $x_1, \ldots x_k$ if we pivot on x_i, new coefficients in pivot row are $\left[\frac{a_1}{a_i} \quad \frac{a_2}{a_i} \;\cdot\cdot\; 1 \;\cdots\; \frac{a_k}{a_i} \right]$

$\Longrightarrow$

new reduced cost $\quad r_j \to r_j - r_i \frac{a_j}{a_i}$

$$\left[r_1 - r_i \frac{a_1}{a_i} \quad r_2 - r_i \frac{a_2}{a_i} \;\cdots\; 0 \;\cdots\; r_k - r_i \frac{a_k}{a_i} \right]$$

we want the signs don't change.

i.e. r_j have the same sign with $r_j - r_i \frac{a_j}{a_i}$

$\Longrightarrow |r_j| \geq |r_i \frac{a_j}{a_i}| \quad \Longrightarrow \quad \left|\frac{r_j}{a_j}\right| \geq \left|\frac{r_i}{a_i}\right|$

So we should choose the x_i with smallest $\left|\frac{r_i}{a_i}\right|$.