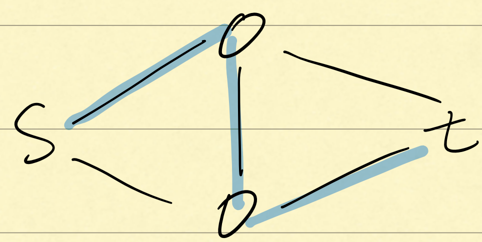
Dijkstra's algorithm.

A network (N, A) with weights $W_{ij} \geq 0$ for each arc $(i, j) \in A$.

goal: shortest <u>s to t</u> path/length



Dijkstra:

① set $l(s) = 0$, $R = \{s\}$.

For all $n \in N - R$: $l(n) = \begin{cases} W_{sn}, & (s,n) \in A \\ \infty, & \text{otherwise.} \end{cases}$

②. select $n \in N - R$ s.t. $l(n) = \min\{l(v) : v \in N - R\}$

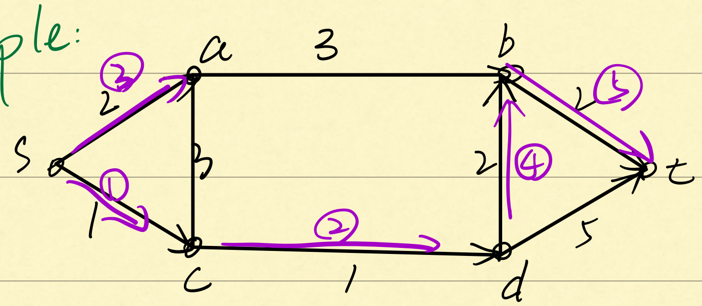
③ For every $v \in N - R$ with $(n, v) \in A$, set $l(v) = \min\{l(v), l(n) + W_{nv}\}$.

④. Set $R = R \cup \{n\}$.

⑤. If $R = N$: terminate

else: go back to step ②

Example:



	R	l(s)	l(a)	l(b)	l(c)	l(d)	l(t)
	S	0	2	∞	1 ($l(s)+w_{sc}$)	∞ ($l(c)+w_{cd}$)	∞
n=c	S,C	0	2	∞	1	2	∞
n=a/d	S,C,d	0	2 ($l(s)+w_{sa}$)	4	1	2	7
n=a	s,c,d,a	0	2	4 ($l(d)+w_{db}$)	1	2	7
n=b	s,c,d,a,b	0	2	4	1	2	6 ($l(b)+w_{bt}$)
n=t	s,c,d,a,b,t	0	2	4	1	2	6

追溯该值最初状态.

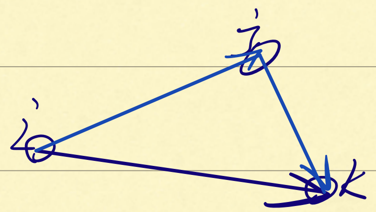
$S \to c \to b \to t$

Floyd-Warshal

$n \times n$ matrix D, entries $d_{ij} = \begin{cases} W_{ij} & \text{if } (i,j) \in A \\ \infty & \text{otherwise.} \end{cases}$

Given D, a triangle operation for fixed node j is:

$d_{ik} = \min\{d_{ik}, d_{ij} + d_{jk}\}$, for all $i, k \in N$ $(i, k \neq j)$.



Theorem: if we perform a triangle operation for all $j \in N$, each d_{ik} becomes equal to the length of shortest path from i to k, assuming $W_{ij} \geq 0$ for all $(i,j) \in A$

Proof: 略 (lecture 34).

Algorithm.

Given D: for $j = 1, \dots, n$:

for $i=1,\dots,n$:

for $k=1,\dots,n$:

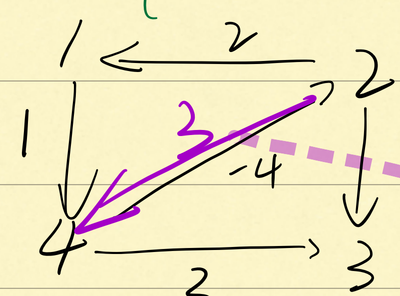
$d_{ik}=\min\{d_{ik}, d_{ij}+d_{jk}\}$

if there are no negative weight cycles, the previous therem still holds.

if there is a negative weight cycle it will cause d_{ii} to become negative: If this occurs shortest path will not exist.

在任何一条路径中加入该点的无数次自循环即可实现无限小。

Example.



$$D=\begin{bmatrix}\infty & \infty & \infty & 1\\ 2 & \infty & 1 & \infty\\ \infty & \infty & \infty & \infty\\ \infty & -4 & 3 & \infty\end{bmatrix}$$

$$j=1:\begin{bmatrix}\infty & \infty & \infty & 1\\ 2 & \infty & 1 & \underline{3}\\ \infty & \infty & \infty & \infty\\ \infty & -4 & 3 & \infty\end{bmatrix}$$

$$j=2: \begin{bmatrix} \infty & \infty & \infty & 1 \\ 2 & \infty & 1 & 3 \\ \infty & \infty & \infty & \infty \\ -2 & -4 & -3 & -1 \end{bmatrix}$$

There is a negative weight cycle.