Feasible region $= \{ x \mid Ax \le b,\ x \ge 0 \}$.

and $b \ge 0$.

Slack vars form a starting basis.

gives an intial feasible solution.

may not be feasible ?

$$\min_{x \in \mathbb{R}^4} \quad x_1 + x_2 - x_3 - x_4$$

$$\text{s.t.} \quad -3x_1 + 2x_2 + x_3 + x_4 = 7.$$

$$2x_1 - x_2 - x_3 - 3x_4 = -1.$$

$$x_1, x_2, x_3, x_4 \ge 0.$$

Two phase:

1. Solve an auxilary problem . to determine if LP is feasible, if it is, we find a basic fensible solution to orig LP.

Liner programming.

2. Use that basic feasible solution to do Simplex method on orig LP.

Auxiliary Problem: Given LP w/ $\begin{matrix} Ax=b \\ x \geq 0 \end{matrix}$.

① for given constraint w/ $A_i x = b_i$ w/ $b_i < 0$. multiply both side by -1.

$$\min_{x \in \mathbb{R}^4} \quad x_1 + x_2 - x_3 - x_4$$

$$\text{s.t.} \quad -3x_1 + 2x_2 + x_3 + x_4 = 7.$$

$$-(2x_1 - x_2 - x_3 - 3x_4) = 1.$$

$$-2x_1 + x_2 + x_3 + 3x_4 = 1.$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

②. Add new nonnegative vars. $x_1^a, x_2^a, \ldots$ Called artifical vars, one for each constraint.

$Ax = b \longrightarrow Ax + I x^a = b$.

$$\min_{x \in \mathbb{R}^4} \quad x_1^a + x_2^a = z^a$$

$$\text{s.t.} \quad -3x_1 + 2x_2 + x_3 + x_4 + x_1^a = 7.$$

$$-2x_1 + x_2 + x_3 + 3x_4 + x_2^a = 1.$$

$$x_1, x_2, x_3, x_4, x_1^a, x_2^a \geq 0.$$

when $x_1^a = x_2^a = 0$, reduces to origunal LP.

If we can find an opt sol to Aux LP, and opt val is 0, we take that opt sol, discard all aux vars. gives basic feasible sol to orig LP.

If opt value is not $0 \Rightarrow$ no feasible sol to orig LP.

	x_1	x_2	x_3	x_4	x_1^a	x_2^a	
x_1^a	-3	2	1	1	1	0	7
x_2^a	-2	1	1	3	0	1	1
② $-z$	1	1	-1	-1	0	0	0
① $-z^a$	0	0	0	0	1	1	0

separate

not compute in the same time.

$\Rightarrow$ get two different solutions.

	x_1	x_2	x_3	x_4	x_1^a	x_2^a	
x_1^a	-3	2	1	1	1	0	7
x_2^a	-2	1	1	3	0	1	1
$-z$	1	1	-1	-1	0	0	0
$-z^a$	5	-3	-2	-4	0	0	-8

① $\min Z^a$: Pivot on X_4, remove X_2^a

	X_1	X_2	X_3	X_4	X_1^a	X_2^a	
X_1^a	$-\frac{7}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	0	1	$-\frac{1}{3}$	$\frac{20}{3}$
X_4	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{1}{3}$
$-Z$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$
$-Z^a$	$\frac{7}{3}$	$-\frac{5}{3}$	$-\frac{2}{3}$	0	0	$\frac{4}{3}$	$-\frac{20}{3}$

pivot on X_2, remove X_4

	X_1	X_2	X_3	X_4	X_1^a	X_2^a	
X_1^a	1	0	-1	-5	1	-2	5
X_2	-2	1	1	3	0	1	1
$-Z$	3	0	-2	-4	0	-1	-1
$-Z^a$	-1	0	1	5	0	3	-5

pivot on X_1 remove X_1^a

	X_1	X_2	X_3	X_4	X_1^a	X_2^a	
X_1	1	0	-1	-5	1	-2	5
X_2	0	1	-1	-7	2	-3	11

$-z$	0	0	1	11	-3	5	-16
$-z^a$	0	0	0	0	1	1	0

$\Longrightarrow$ feasible solution: $(x_1, x_2, x_3, x_4) = (5, 11, 0, 0)$.

	x_1	x_2	x_3	x_4	
x_1	1	0	-1	-5	5
x_2	0	1	-1	-7	11
$-z$	0	0	1	11	-16