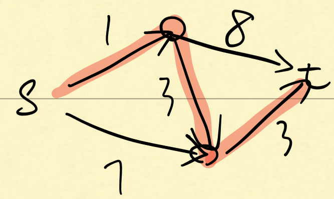
Network (N, A)

minimize the sum of weights on arcs in path
w_{ij}.

$$x_{ij} = \begin{cases} 1 & \text{, if part of path} \\ 0 & \text{, not} \end{cases}$$



$\min \sum w_{ij} x_{ij}$.

$$\text{s.t.} \sum_{(s,j)\in A} x_{sj} - \sum_{(i,s)\in A} x_{is} = 1 \quad (\#\text{ leaving } s - \#\text{ entering } s = 1.)$$

$$\sum_{(i,t)\in A} x_{it} - \sum_{(t,j)\in A} x_{tj} = 1 \quad (\#\text{ entering } t - \#\text{ leaving } t = 1).$$

$$\sum_{(p,j)\in A} x_{pj} - \sum_{(i,p)\in A} x_{ip} = 0 \quad p \in N,\ p \neq s, t.$$

dual: $\max\ u_s - u_t$.

$$u_i - u_j \leq w_{ij}. \quad (i,j) \in A$$

Given initial u^*

(DRP): $\max\ v_s - v_t$

$$\text{s.t.}\quad v_i - v_j \leq 0. \quad (i,j) \in J$$

the set that satisfies $u_i^* - u_j^* = w_{ij}$.

$$v_i \leq 1 \quad \text{for all } i.$$

$$v_t = 0$$

(assume no path from s to t using only edges in J. otherwise $v_s - v_t = 0$).

Optimal solution to DRP is

$$V_i = \begin{cases} 1 & \text{if } i \text{ is reachable from } s \text{ using arcs in } J. \\ 0 & \text{if } t \text{ is reachable from } i \text{ using arcs in } J. \\ 1 & \text{otherwise.} \end{cases}$$

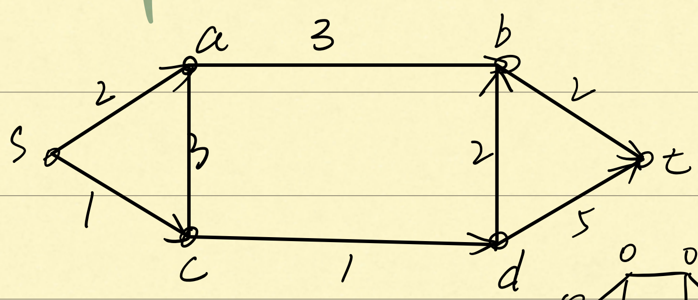
Then $t = \min\limits_{(i,j)\notin J} \{ W_{ij} - (u_i - u_j) \}$

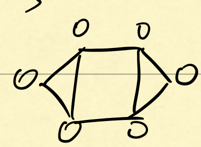
s.t. $V_i - V_j > 0$

$U = U + tV.$ $\Longrightarrow$ repeat

find the nodes reachable from s
find the nodes reach t
} in J

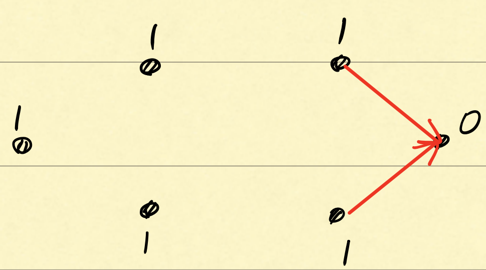
Example:





Initial $U = (0,0,0,0,0,0)$. $\Longrightarrow$ $J = \phi$
no constraints are tight.

$\Longrightarrow$



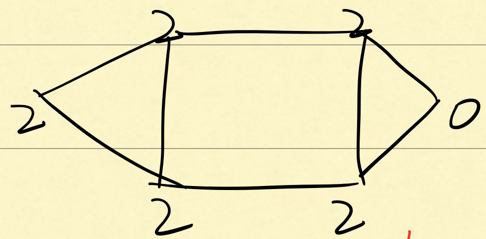
$V = (1,1,1,1,1,\underset{t}{0})$

only (b,t), (d,t) satisfy $V_i - V_j > 0$. i.e $(i,j) \notin J$.

$\Longrightarrow$ $t = \min\{2-0, 5-0\} = \underline{2}.$

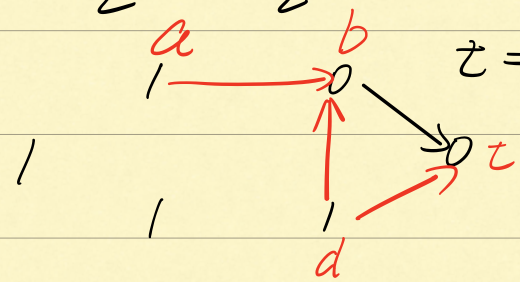
arc (b,t)

$u = (2,2,2,2,2,0) \Rightarrow$



$J = \{(b,t)\}$.

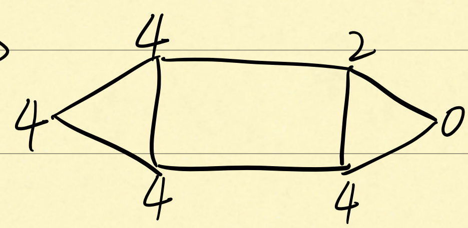
$v = (1,1,1,\underset{b}{0},1,\underset{t}{0})$.



$t = \min\{3-0,\ 5-2,\ 2-0\} = \underline{2}$.

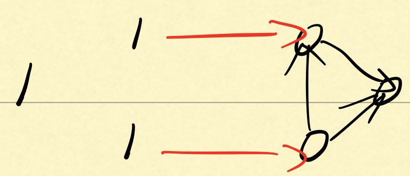
$\underline{\text{arc } (d,b)}$.

$\Rightarrow$



$J = \{(b,t),(d,b)\}$

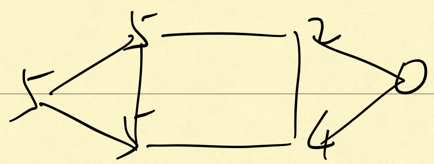
$V = (1\ 1\ 1\ \underset{b}{0}\ \underset{d}{0}\ \underset{t}{0})$



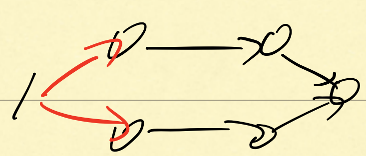
$t = \min\{3-2,\ 1-0\} = 1$.

arc (a,b) (c,d).

$\Rightarrow$



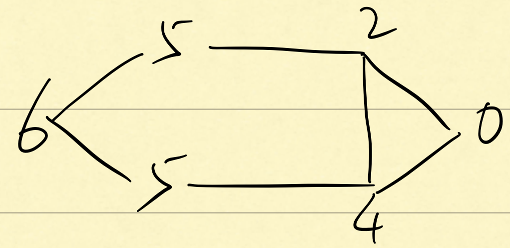
$J = \{(b,t),(d,b),$
$(a,b),(c,d)\}$.



$V = (1,0,0,0,0,0)$

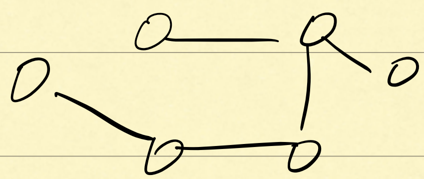
$t = \min\{2-0,\ 1-0\} = 1$.

arc : (s,c).

$\Rightarrow$ 

$J = \{ (s,c), (b,t), (d,b), (a,b), (c,d) \}.$

$V = (0,0,0,0,0,0).$



Done!

$U = (6, 5, 5, 2, 4, 0)$ is the opt solution.

We get the path from s to t by using any path from s to t in J.

$s \to c \to d \to b \to t$