$$\max_{x \in \mathbb{R}^{|A|}} \sum_{j:(s,j)\in A} x_{sj}$$

$$\text{s.t.} \quad \underline{\sum_{i:(i,k)\in A} x_{ik} - \sum_{j:(k,j)\in A} x_{kj} = 0} \quad k \in N,\ k \neq s,t \qquad u \begin{cases} x_{sj} \ (+1) \\ x_{ij} \ (+1/-1) \ i,j \neq s,t \\ x_{it} \ (-1) \end{cases}$$

$$\underline{x_{ij} \leq c_{ij}} \quad y_{ij} \leftarrow x_{sj} (+1)$$

$\Rightarrow$ Dual LP. dual vars. u_k, y_{ij}.

for every node other than s,t. every arc $(i,j)\in A$

$$\underset{u \in \mathbb{R}^{|N|-2},\ y \in \mathbb{R}^{|A|}}{\text{Minimize}} \sum_{(i,j)\in A} c_{ij} y_{ij}.$$

$$\begin{aligned} \text{s.t.} \quad & u_j + y_{sj} \geq 1 && (x_{sj}) \\ & (y_{sj} \geq 1 - u_j) \\ & -u_i + u_j + y_{ij} \geq 0 && (x_{ij},\ i \neq s,\ j \neq t). \\ & (y_{ij} \geq u_i - u_j) \\ & -u_i \quad + y_{it} \geq 0 && (x_{it}) \\ & (y_{it} \geq u_i) \\ & y \geq 0,\ u \text{ unrestricted.} \end{aligned}$$

introduce "fake vars" $u_s \equiv 1$ $u_t \equiv 0$.

we get: $\min \sum_{(i,j)\in A} c_{ij} y_{ij}$

$$\text{s.t.} \quad y_{ij} \geq u_i - u_j$$

$$u_s = 1$$

$u_t = 0$

$y \geq 0$, u unrestricted.

$y_{ij} = \max\{u_i - u_j, 0\}$.

Assume $u_i = 1$ or 0

Let $S = \{i \in N : u_i = 1\}$, $T = \{i \in N : u_i = 0\}$.

$s \in S$, $t \in T$

$\Rightarrow$ obj func is a sum of c_{ij} where $i \in S, j \in T$.

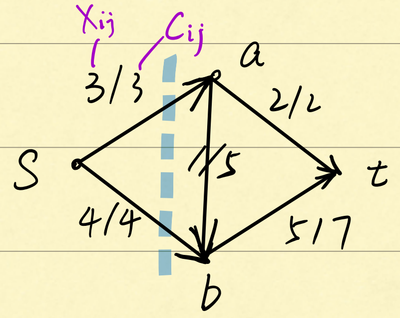
(S, T) is a cut!

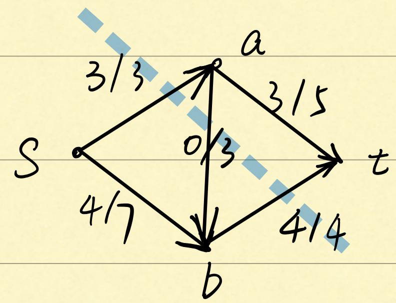
The dual LP is searching for a min cut.

Max-flow/min-cut theorem.

value of a max flow is equal to the capacity of a min cut.

opt sol of the min-cut LP is an integer




a
3/3
3/5
S
0/3
t
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b