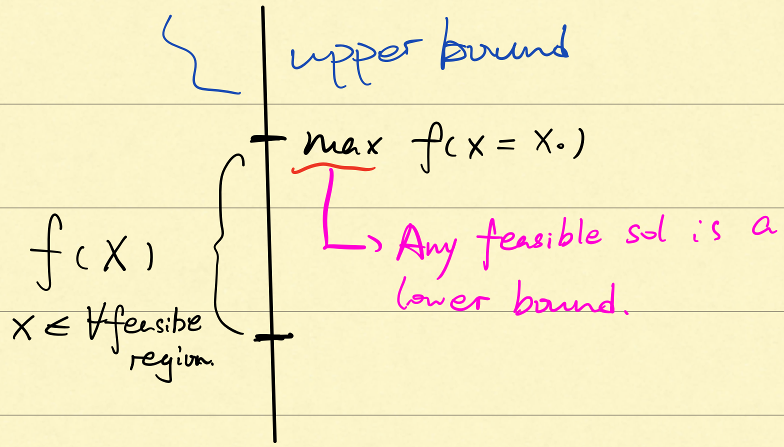
upper bound

lower bound



Example.

$$\max \; x_1 + x_2$$

$$\text{s.t.} \quad 2x_1 + x_2 + 4x_3 \le 3$$
$$x_1 + x_2 - 3x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0.$$

$x_1 + x_2 \le 2x_1 + x_2 + 4x_3 \le 3$

obviously it is upper bound.

$$u_1(2x_1 + x_2 + 4x_3) + u_2(x_1 + x_2 - 3x_3) \le 3u_1 + u_2$$

$$(2u_1 + u_2)x_1 + (u_1 + u_2)x_2 + (4u_1 - 3u_2)x_3 \le 3u_1 + u_2$$

best upper bound :

$$\min \; 3u_1 + u_2$$
$$\text{s.t.} \quad 2u_1 + u_2 \ge 1$$
$$u_1 + u_2 \ge 1.$$
$$4u_1 - 3u_2 \ge 0$$
$$u_1, u_2 \ge 0$$

Weak and strong duality.

Primal LP.

$$\max \; C^T x$$
$$\text{s.t.} \quad Ax \le b \qquad (P)$$
$$x \ge 0$$

A is $m \times n$.

we can do above process for any linear program

$\forall u \in \mathbb{R}^m, u \geq 0 \quad u^T A x \leq u^T b.$

dual LP.

$$\begin{aligned} \min\ & u^T b \\ \text{s.t. } & u^T A \geq c^T \\ & u \geq 0 \end{aligned} \quad (D) \iff \quad \begin{aligned} \min\ & b^T u \\ \text{s.t. } & A^T u \geq c \\ & u \geq 0. \end{aligned}$$

Theorem 12.1 (week duality)

For any $x \in \mathbb{R}^n$ that is feasible for primal LP, and any $u \in \mathbb{R}^m$ that is feasible for dual LP we have $c^T x \leq b^T u$.

Theorem 12.2 (Strong duality)

If either (P) or (D) has an opt solution, then so does the other. the obj values of opt solutions are equal.

Duality for any LP.

change first example:

$$\max \ x_1 + x_2$$
$$\text{s.t.} \ 2x_1 + x_2 + 4x_3 \leq 3$$
$$x_1 + x_2 - 3x_3 \leq 1$$
$$x_1, \cancel{x_2}, x_3 \geq 0.$$

unconstrained vars give us equality("=") constraint in dual LP.

($k x_2 \ ? \ x_2$
only when $k=1$ when can sure "?" is "=")

want lower bounds: the relationship between (P) and (D) is swapped.

The relationship between (P) and (D) is Symmetric: if (D) is the dual of (P), then (P) is the dual of (D).