$$(\mathcal{P}) \qquad \begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax \le b,\ 0 \le x \end{array}$$

is the LP

$$(\mathcal{D}) \qquad \begin{array}{ll} \text{minimize} & b^T y \quad y^T b \\ \text{subject to} & A^T y \ge c,\ 0 \le y. \end{array}$$

$y^T A \geq c^T$

Theorem 4.2 (The Strong Duality Theorem) *If either $\mathcal{P}$ or $\mathcal{D}$ has a finite optimal value, then so does the other, the optimal values coincide, and optimal solutions to both $\mathcal{P}$ and $\mathcal{D}$ exist.*

$$(P) \quad \begin{array}{cc|c} A & I & b \\ \hline c^T & 0 & 0 \end{array}$$

Proof: the dual of the dual is the primal

$\Rightarrow$ we can assume primal has a finite optimal value

$\Rightarrow$ there exists a nonsingular record matrix $R \in \mathbb{R}^{n \times n}$

and a vector $y \in \mathbb{R}^m$ such that the optimal

tableau has the form:

不存在最后一行向上加减

$$\underbrace{\begin{bmatrix} R & 0 \\ -y^T & 1 \end{bmatrix}}\begin{bmatrix} A & I & b \\ c^T & 0 & 0 \end{bmatrix} = \begin{bmatrix} RA & RI & Rb \\ c^T - y^T A & -y^T & -y^T b \end{bmatrix}$$

等价于行变换

Since this is opt tableau.

1. $C^T - y^T A \leq 0 \quad -y^T \leq 0$

$\Rightarrow y^T A \geq C^T \,, \quad y \geq 0$

2. $y^T b$ is the primal opt value

this y is feasible in (D)