Row Reduce $Ax = b$.

$$EAx = Eb.$$

$\mathcal{B}$: basic variables.

$$A_{\mathcal{B}}^{-1} A x = A_{\mathcal{B}}^{-1} b$$

$$\max_{x \in \mathbb{R}^n} \ C^T x \qquad (A_{m \times n},\ b \in \mathbb{R}^m,\ c \in \mathbb{R}^n)$$
$$\text{s.t. } Ax = b$$
$$x \geq 0$$

- $A_{\mathcal{B}}$ is invertible ($x_N = 0$, $x_{\mathcal{B}} = A_{\mathcal{B}}^{-1} b$).
- $A_{\mathcal{B}}^{-1} b \geq 0$.

	X_B	X_N	
X_B	I	Q	P
$-Z$	0^T	r^T	$-Z_0$

$\Rightarrow$ $P = A_{\mathcal{B}}^{-1} b.$

$$A' = [A_{\mathcal{B}} \ A_N]$$

$$A_{\mathcal{B}}^{-1} A' = [I \quad \underline{A_{\mathcal{B}}^{-1} A_N}] = [I \quad \underline{Q}]$$

$$Z_0 = C^T x = C_{\mathcal{B}}^T X_{\mathcal{B}} + C_N^T X_N \overset{(X_N = 0)}{=} C_{\mathcal{B}}^T X_{\mathcal{B}} = C_{\mathcal{B}}^T A_{\mathcal{B}}^{-1} b = C_{\mathcal{B}}^T P.$$

To eliminate costs below basic variables:

bottom row: $C^T - C_B^T(A_B^{-1}A') = [0^T, r^T]$

$r^T = C_N^T - C_B^T(A_B^{-1}A_N)$

$\Rightarrow \quad P = A_B^{-1}b, \quad Q = A_B^{-1}A_N$

$r^T = C_N^T - C_B^T A_B^{-1} A_N = C_N^T - C_B^T Q$

$z_0 = C_B^T A_B^{-1} b = C_B^T P$

	X_B	X_N	
X_B	A_B	A_N	b
$-z$	C_B^T	C_N^T	0

$\Rightarrow$

	X_B	X_N	
X_B	I	$\underbrace{A_B^{-1}A_N}_{Q}$	$\underbrace{A_B^{-1}b}_{P}$
$-z$	0^T	$\underbrace{C_N^T - C_B^T(A_B^{-1}A_N)}_{r^T}$	$\underbrace{-C_B^T(A_B^{-1}b)}_{-z_0}$

A_B^{-1} is what we will compute.

think it as a product of E elementary row operations matrices

Revised Simplex method:

① Pricing: (Step determine entering variable.)

Compute entries of r^T one by one until we find one with right sign.
元素

through non-basic variables N one by one.

until find reduced cost of right sign.

Use Bland's pivoting rule: pick first

$r_j = c_j - c_B^T A_B^{-1} A_j$. ($j^{th}$ in A, $(j-|B|)^{th}$ in r, Q)

$Q_j = (A_B^{-1} A_N)_j = A_B^{-1} A_j$

i^{th} in x_B basic variable ratio $\frac{P_i}{Q_j}$

$\Longrightarrow$ Update A_B^{-1}, P

$B \to B'$ $\quad A_B^{-1} b \to A_{B'}^{-1} b \quad A_B^{-1} \to A_{B'}^{-1}$

$\Longrightarrow$ we can make a mini-tableau for x_j.

$$| A_B^{-1} | Q_j | P \Longrightarrow | A_{B'}^{-1} | \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \text{ith row} | P'$$

row-reduce this mini-tableau.

$$\begin{array}{ccc} A_B^{-1} A_N = A_N & A_B = I & P = A_B^{-1} b \\ \begin{matrix} 2 & 3 & 5 & 1 \\ 1 & 1 & 2 & 0 \end{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} 9 \\ 4 \end{matrix} \\ \begin{matrix} 2 & 1 & 5 & -1 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & 0 \\ r^T = c_N^T & c_B^T & z_0 \end{array}$$

$$A_N' \quad A_B'$$

$$\begin{matrix} 2 & 3 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 \end{matrix} \quad \begin{matrix} 5 & 0 \\ 2 & 1 \\ 5 & 0 \end{matrix} \quad \begin{matrix} 9 \\ 4 \\ 0 \end{matrix}$$

$$C_N'^T \qquad C_B'^T$$

$$A_B'^{-1} A_N' \qquad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \quad \begin{pmatrix} \frac{9}{5} \\ \frac{2}{5} \end{pmatrix} P$$

$$\Gamma'^T = C_N'^T - C_B'^T (A_B'^{-1} A_N')$$

$$\begin{matrix} \frac{2}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & -\frac{1}{5} & -\frac{2}{5} & -\frac{2}{5} \end{matrix}$$

2 3 1 1