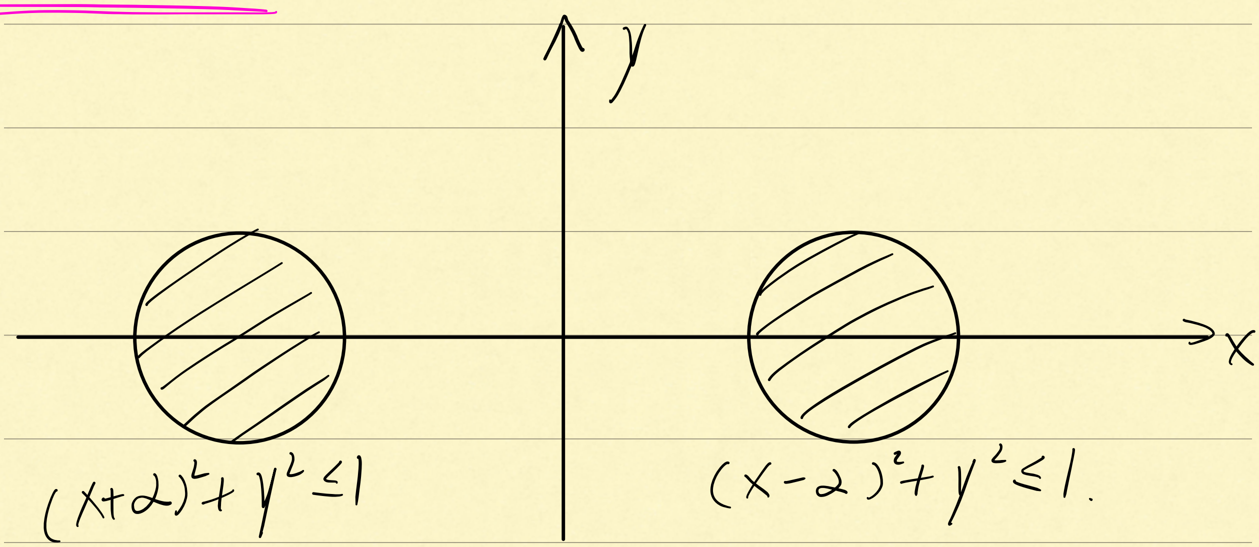
Constraints



You can never draw a set of linear inequations that includes both disks and excludes (0,0).

Proof: Suppose $Ax \le b$ that includes both disks.

$\Rightarrow$ includes points $(-2,0)$ and $(2,0)$.

$\Rightarrow A \begin{bmatrix} -2 \\ 0 \end{bmatrix} \le b$ and $A \begin{bmatrix} 2 \\ 0 \end{bmatrix} \le b$.

$\Rightarrow A \begin{bmatrix} 0 \\ 0 \end{bmatrix} \le b$. $\Rightarrow$ includes (0.0).

Convexity

A subset S in $\mathbb{R}^2$ is convex if whenever

$x, y \in S$, the entire line segment between x and y is in S.

$[x, y] = \{tx + (1-t)y \mid 0 \le t \le 1\}$.

S in convex if $x, y \in S \Rightarrow [x, y] \subseteq S$.

Thm: Feasible region of linear program is convex.

Proof: $AX \le b$.

x, y are in feasible region (S).

$Ax \le b$, $Ay \le b$.

$\Rightarrow A(tx + (1-t)y) \le tb + (1-t)b = b$.

$\Rightarrow [x, y] \subseteq S$.

x, y both satisfy a line inequation. Means both on same side of line. $[x, y]$ is on same side of line.

Fact: Any convex set can be approximated arbitrarily well by linear inequalities.

Standard form: $\underset{x \in \mathbb{R}^n}{\text{maximize}} \quad c^T x$

subject to $Ax \leq b$

$x \geq 0.$

negative $\longrightarrow$ non-negative.

$$\max_{x, y \in \mathbb{R}} \; x - y.$$

s.t. $y \leq 3$

$y \geq 2x - 5$

$x + y \geq 1.$

Replace every variable x by $x^+ - x^-$, where $x^+, x^- \geq 0.$

$\Longrightarrow$ $\max_{x^+, x^-, y^+, y^- \in \mathbb{R}} \; x^+ - x^- - y^+ + y^-$

$y^+ - y^- \leq 3$

$y^+ - y^- \geq 2x^+ - 2x^- - 5$

$x^+ - x^- + y^+ - y^- \geq 1$

$x^+, x^-, y^+, y^- \geq 0.$

(equational form)

$a^T x \leq b \xrightarrow{\sim} a^T x + s = b$ for some $s \geq 0$.

s is called a slack variable : it measures how much slack/flexibility there is to satisfy the inequality constraint.

$$\Longrightarrow \max_{x^+, x^-, y^+, y^-, s_1, s_2, s_3 \in \mathbb{R}} \; x^+ - x^- - y^+ + y^-$$

$$y^+ - y^- + s_1 = 3$$

$$2x^+ - 2x^- - y^+ + y^- + s_2 = 5$$

$$-x^+ + x^- - y^+ + y^- + s_3 = -1$$

$$x^+, x^-, y^+, y^-, s_1, s_2, s_3 \geq 0.$$

General Equational Form

$$\max_{x \in \mathbb{R}^n} \; c^T x$$

$$\text{s.t. } Ax = b$$

$$x \geq 0.$$