LP tableau is degenerate if some basic variables equal 0 in its basic feasible solution.

Example: $\max\limits_{x_1, x_2 \in \mathbb{R}} \quad x_1 + 4x_2$

s.t. $x_1 + x_2 \le 0$

$x_1 - 3x_2 \le 0$

$-2x_1 + x_2 \le 0$

$x_1, x_2 \ge 0$

All ineq ≥ 0 or ≤ 0

* all moves away from orign are cut off.
* can move, LP is unbounded.

	x_1	x_2	s_1	s_2	s_3	
s_1	1	1	1	0	0	0
s_2	1	-3	0	1	0	0
s_3	-2	1	0	0	1	0
$-z$	1	4	0	0	0	0

Pivot on x_1, remove s_1

	x_1	x_2	s_1	s_2	s_3	
x_1	1	1	1	0	0	0
s_2	0	-4	-1	1	0	0
s_3	0	3	2	0	1	0
$-z$	0	3	-1	0	0	0

Pivot on x_2, remove s_3

	x_1	x_2	s_1	s_2	s_3	
x_1	1	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	0
s_2	0	0	$\frac{5}{3}$	1	$\frac{4}{3}$	0
x_2	0	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0
$-z$	0	0	-3	0	-1	0

$\Longrightarrow$ $(0,0,0,0,0)$ is opt sol

with opt val $z=0$.

Pivoting Rule: Strategy to make decisions on which vars enter/exit basis.

(1) Entering vars: simplex method says

:choose var with reduced cost of correct sign : negative for min, positive for max.

(2) Exiting vars : Simplex methods says choose var with Smallest ratio, that is not negative (often more than one). tie.

*(1) Common to pick entering var with largest abs values (correct sign).

* Can lead to cycling if (2) is not addressed.

Bland's pivoting rule: Give vars an order ($x_1, x_2, x_3, \dots, x_n, s_1, \dots, s_m$). whenever there is a tie for entering/exiting variable always choose earlier variable.

Avoid cycling. Very Slow.

Random pivoting rule: Break all ties randomly.

Statistically avoid cycling. Very Slow.

Lexicographic pivoting: prevent cycling !!!

$1 \gg \epsilon_1 \gg \epsilon_2 \gg \epsilon_3 \; ---- \gg \epsilon_m > 0.$

means "this ineq stays true up to mult either side by factor of any number in our problem."

Add ϵ_i to the RHS of the i^{th} constraint.

Prevent ties from occuring.

For entering choose var w/ largest abs value (correct sign)

Example:

$$
\begin{aligned}
\max_{x_1, x_2 \in \mathbb{R}} \quad & x_1 + 4x_2 \\
\text{s.t.} \quad & x_1 + x_2 \le \epsilon_1 \\
& x_1 - 3x_2 \le \epsilon_2 \\
& -2x_1 + x_2 \le \epsilon_3 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

	x_1	x_2	s_1	s_2	s_3	
s_1	1	1	1	0	0	ϵ_1
s_2	1	-3	0	1	0	ϵ_2
s_3	-2	1	0	0	1	ϵ_3
$-z$	1	4	0	0	0	0

Pivot on x_2 remove s_3

	x_1	x_2	s_1	s_2	s_3	
s_1	3	0	1	0	-1	$\epsilon_1-\epsilon_3$
s_2	-5	0	0	1	3	$\epsilon_2+3\epsilon_3$
x_2	-2	1	0	0	1	ϵ_3
$-z$	9	0	0	0	-4	$-4\epsilon_3$

Pivot on x_1 remove s_1

	x_1	x_2	s_1	s_2	s_3	
x_1	1	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	$(\epsilon_1-\epsilon_3)\frac{1}{3}$
s_2	0	0	$\frac{5}{3}$	1	$\frac{4}{3}$	$(5\epsilon_1+3\epsilon_2+4\epsilon_3)\frac{1}{3}$
x_2	0	1	$\frac{2}{3}$	0	$\frac{1}{3}$	$(2\epsilon_1+\epsilon_3)\frac{1}{3}$
$-z$	0	0	-3	0	-1	$-3\epsilon_1-\epsilon_3$

Opt value is $z = 3\epsilon_1+\epsilon_3 \approx 0$.