Construct an LP with d variables, $2d$ constraints which take 2^d steps.

$$\binom{2d}{d} < 4^d$$

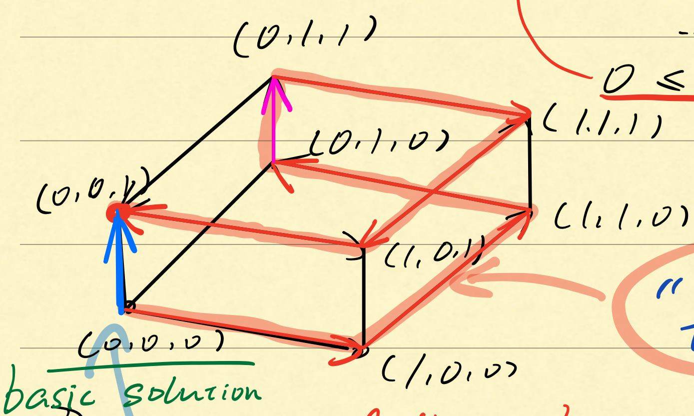
Example: $\max_{x \in \mathbb{R}^d} \; x_d$

s.t. $0 \le x_1 \le 1$

$0 \le x_2 \le 1$

......

$0 \le x_d \le 1$



"terrible" trajectory.

Recursive: follow $d-1$, go to vertex with $x_d = 1$.

follow reverse $d-1$.

We want this!, not this!

Taking terrible trajectory.

Tricking Bland's pivoting rule into taking terrible trajectory.

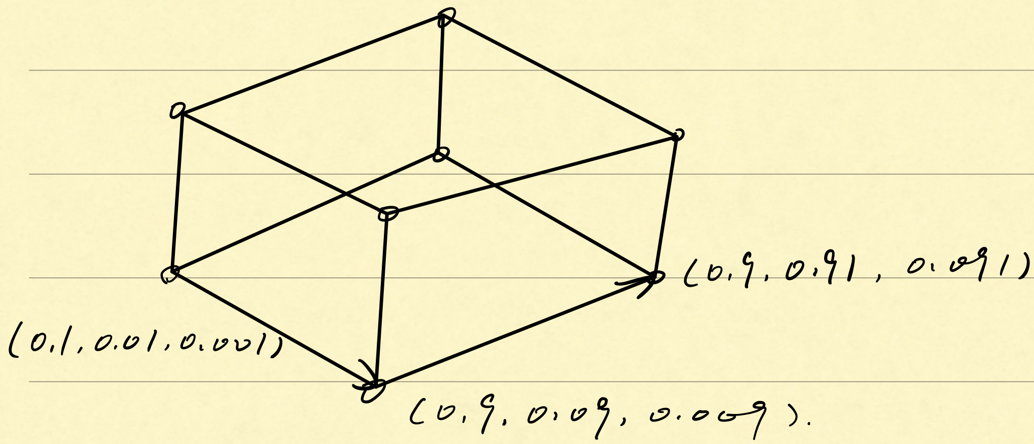
$$\max_{x \in \mathbb{R}^d} \quad x_d$$

$$\text{s.t. } 0.1 \le x_1 \le 1 - 0.1$$

$$0.1x_1 \le x_2 \le 1 - 0.1x_1$$

$$\cdots$$

$$0.1x_{d-1} \le x_d \le 1 - 0.1x_{d-1}$$



$$0.001 < 0.009 < 0.091 < \cdots < 0.999.$$

$$0.1x_{i-1} \le x_i \le 1 - 0.1x_{i-1}$$

$$\Longrightarrow 0.1x_{i-1} - x_i + s_i = 0$$

$$0.1x_{i-1} + x_i + s_i' = 1$$

In initial feasible solution, $x_1, \ldots x_d$ are basic

i.e. $x_i = 1 - 0.1x_{i-1}$ $\quad s_1, \cdots s_d$ basic

$s_1', \cdots s_d'$ nonbasic.

△: In each basic feasible solution: exactly one of S_i or S_i' is basic

Order as: $x_1, x_2, \dots, x_d, S_1, S_1', S_2, S_2', \dots, S_d, S_d'$

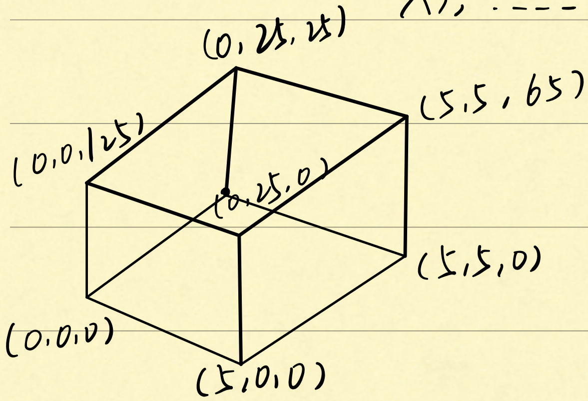
then BPR follows terrible trajectory.

Klee-Minty cube.

tricks the highest reduced cost pivot rule into taking terrible trajectory.

$$\max_{x \in \mathbb{R}^d} \; 2^{d-1}x_1 + 2^{d-2}x_2 + \dots 2^1 x_{d-1} + x_d$$

$$\begin{aligned} \text{s.t.} \quad 5 &\geq x_1 \\ 25 &\geq 4x_1 + x_2 \\ 125 &\geq 8x_1 + 4x_2 + x_3 \\ &\cdots \\ 5^d &\geq 2^d x_1 + 2^{d-1} x_2 + \dots + 4x_{d-1} + x_d. \\ x_1, \dots, x_d &\geq 0. \end{aligned}$$



earlier variables have higher reduced cost, but can not change by much.

Best neighbor rule - consider all possible entering vars and picks one that improves obj. the most.

Complex pivoting rules where an LP with n inequations, d variables, # pivots $\le C^{\sqrt{d}\log n}$

Some constant

Dream a pivot rule whose # pivots $\le \text{poly}(n, d)$.