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Math 482: Linear Programming, Fall 2020

Due Wednesday, September 2, 6PM CST

Homework 1

1. Write a linear program in standard form for the following problem. (Do not solve.)

   A ship is transporting rice and wheat from California to Alaska. It has three cargo holds with the following capacities:

   - The forward cargo hold can carry at most 10,000 tons, and at most 400,000 cubic feet.
   - The middle cargo hold can carry at most 5,000 tons, and at most 250,000 cubic feet.
   - The aft cargo hold can carry at most 12,000 tons, and at most 600,000 cubic feet.

   In addition, for the ship to be balanced, each cargo hold must be filled to the same fraction of its total capacity, with respect to tonnage.

   A ton of wheat takes up 44.7 cubic feet and can be sold at a profit of \$20; a ton of rice takes up 40.9 cubic feet and can be sold at a profit of \$18.

   The goal is to maximize the profit from the ship's cargo.

2. Draw the feasible region of this linear program. Solve it using the naive approach from Lecture 1.

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{maximize}} \quad & x+y \\ \text{subject to} \quad & 6x+5y \le 19, \\ & y \le 4x+9, \\ & 2x-7y \le 15. \end{aligned}$$

3. Convert the linear program below to equational form. Equational form requires that all variables are nonnegative, so you must substitute nonnegative variables for x as we did in Lecture 2.

$$\begin{aligned} \underset{x,y\in\mathbb{R}}{\text{minimize}} \quad & x-2y \\ \text{subject to} \quad & x+y \le 10, \\ & y \ge 0. \end{aligned}$$

4. Use the simplex method to solve the *minimization* problem given in the tableau below:

$$\begin{array}{c|ccccc|c} & x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline x_1 & 1 & 0 & 2 & -1 & 0 & 4 \\ x_2 & 0 & 1 & 2 & 3 & 4 & 9 \\ \hline -z & 2 & 0 & 3 & -1 & 1 & -7 \end{array}$$

5. *(Only 4-credit students need to do this problem.)*

   (a) Rewrite the constraint $|x|+|y| \le 5$ as a combination of linear constraints.

   (b) Show that there is no way to rewrite the constraint $|x|+|y| \ge 5$ as a combination of linear constraints. *Hint: Show that the set of points satisfying this constraint is not convex.*

1. $$\frac{44.7x_1+40.9y_1}{x_1+y_1}=\frac{44.7x_2+40.9y_2}{x_2+y_2}=\frac{44.7x_3+40.9y_3}{x_3+y_3}$$

$$\Rightarrow \frac{y_i}{x_i}=k \quad i=1,2,3$$

And $40(1+k)<44.7+40.9k<50(1+k)$

$$\Rightarrow x_1 \le \frac{400000}{44.7+40.9k} \le \frac{10000}{1+k}$$

$$x_2 \le \frac{5000}{1+k} \le \frac{250000}{44.7+40.9k}$$

$$x_3 \le \frac{12000}{1+k} \le \frac{600000}{44.7+40.9k}$$

So we get standard form:

$$\max_{x_1,x_2,x_3,k\in\mathbb{R}} (20+18k)\sum_{i=1}^{3} x_i$$

subject to:

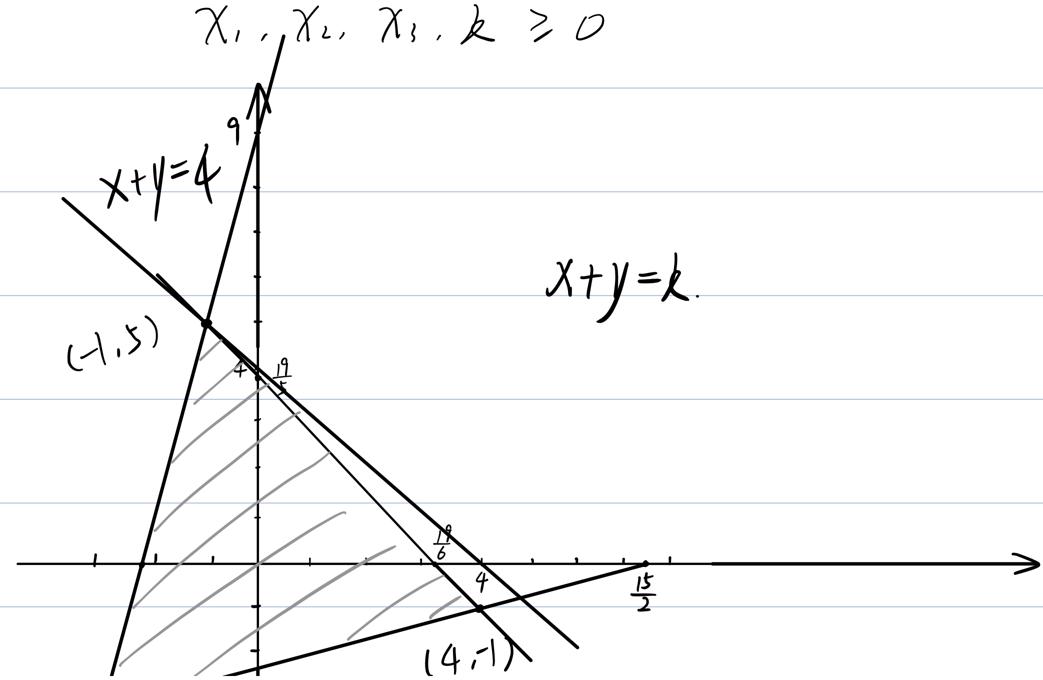
$$x_1 \le \frac{400000}{44.7+40.9k}$$

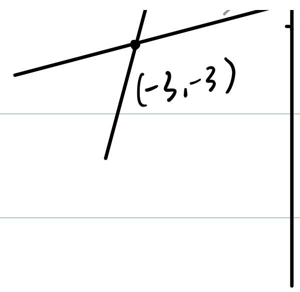
$$x_2 \le \frac{5000}{1+k}$$

$$x_3 \le \frac{12000}{1+k}$$

$$x_1, x_2, x_3, k \ge 0$$

2.





4 is the maximum of $x+y$.

3. replace $x \Rightarrow x^+ - x^-$, $x^+, x^- \geq 0$.

$$\underset{x^+, x^-, y, s \in \mathbb{R}}{\text{minimize}} \quad x^+ - x^- - 2y$$

$$\text{s.t.} \quad x^+ - x^- + y + s = 10$$

$$x^+, x^-, y, s \geq 0.$$

4.

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	2	−1	0	4
x_2	0	1	2	3	4	9
$-z$	2	0	3	−1	1	−7

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	2	-1	0	4
x_2	0	1	2	3	4	9
$-z$	0	0	-1	1	1	-15

$z = 15$

	x_1	x_2	x_3	x_4	x_5	
x_1	-1	1	0	4	4	5
x_2	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	2
$-z$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	-13

$z = 13$

x_3's reduced cost is the only negative one

$\Rightarrow \underline{\min z = 13}$