Newton's Method

Taylor

find $\underline{\underline{s}}$ to min $f(\overset{\text{initial}}{x}+s)$

$$f(x+s) \approx f(x) + \nabla f(x)^T s + \frac{1}{2} \underbrace{s^T H s}_{\sum_{i=1}^{n}\sum_{j=1}^{n} H_{ij} \cdot s_i s_j} = \hat{f}(s)$$

first order: $\nabla f(x) + Hs = 0$ $\quad$ H is symmetric.

$$\frac{\partial(\frac{1}{2} s^T H s)}{\partial s} = (Hs)^T + (s^T H) = s^T \cdot H^T$$

$$(\nabla f(x)^T + s^T H^T)^T = \nabla f(x) + Hs = 0$$

$$\underset{n\times n}{H(x)}\ \underset{n\times 1}{s} = -\underset{n\times 1}{\nabla f(x)}$$

solve to find Newton step s.

$$H_f(x_k)\, s_k = -\nabla f(x_k)$$

update: $x_{k+1} = x_k + s_k$

- Typical quadratic convergence ☺
- Need second derivatives ☹
- Local convergence (start guess close to solution)
- Works poorly when Hessian is nearly indefinite
- Cost per iteration: $O(n^3)$

will fail if $f(x)$ is linear polynomial.

if $f(x)$ is quadratic polynomial and

H is nonsingular $\Rightarrow$ only one iteration.