

Steepest Descent Method.

$f(x): \mathbb{R}^n \rightarrow \mathbb{R}$ at a point x , the function will decrease its value in the direction of steepest

descent: $-\nabla f(x)$

How far? $x_{k+1} = x_k - \alpha_k \nabla f(x_k)$? Line Search.

want to find α_k s.t. $\min_{\alpha_k} f(x_k - \alpha_k \nabla f(x_k))$

$$\text{first order: } \frac{df}{d\alpha_k} = 0 = \frac{\partial f}{\partial x_{k+1}} \cdot \frac{\partial x_{k+1}}{\partial \alpha_k}$$

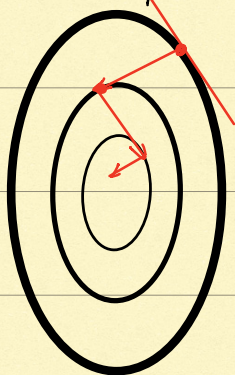
$$= \nabla f(x_{k+1}) (-\nabla f(x_k)) = 0$$

$$\Rightarrow \nabla f(x_{k+1}) \cdot \nabla f(x_k) = 0$$

$\nabla f(x_{k+1})$ is orthogonal to $\nabla f(x_k)$

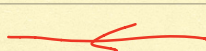
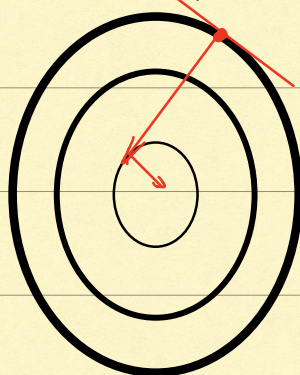
\Rightarrow gives Zig-zag pattern convergence. Linear convergence.

$$39x^2 + y^2$$

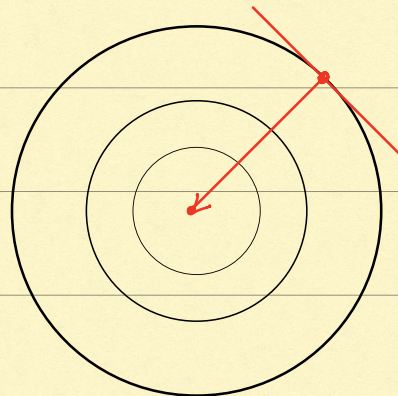


slow

$$21x^2 + y^2$$



$$x^2 + y^2$$



quick