Steepest Descent Method.

find min.

$f(X): \mathbb{R}^n \to \mathbb{R}$ at a point X, the function will decrease its value in the direction of steepest descent: $-\nabla f(X)$

How far? $x_{k+1} = x_k - \alpha_k \nabla f(x_k)$? Line Search.

want to find α_k s.t. $\min_{\alpha_k} f(x_k - \alpha_k \nabla f(x_k))$

first order: $\frac{df}{d\alpha_k} = 0 = \frac{\partial f}{\partial x_{k+1}} \cdot \frac{\partial x_{k+1}}{\partial \alpha_k}$

$= \nabla f(x_{k+1})(-\nabla f(x_k)) = 0$

$\Rightarrow \nabla f(x_{k+1}) \cdot \nabla f(x_k) = 0$

$\nabla f(x_{k+1})$ is orthogonal to $\nabla f(x_k)$

linear convergence.

$\Rightarrow$ gives Zig-zag pattern convergence.