Rounding error: $\frac{1}{3} \to 0.333.$

Truncation error: $\sin\theta \approx \theta$ when θ is small.
(approximate)

x : true value , $\hat{x}$: approx

$\hat{x} = x + \Delta x.$

Absolute value: $e_a = |\hat{x} - x|.$

Relative error: $e_r = \dfrac{|\hat{x} - x|}{|x|}$

Significant figures.
/Significant digits (有效数字)

$\hat{x}$ has n significant figures of x if $|x - \hat{x}|$ has zeros in the first n decimal places counting from the leftmost nonzero (leading) digit of x, followed by a digit from 0 to 4.

$\hat{x} = 3.14159$

$|x - \hat{x}| = 0.0000002653 \to \hat{x}$ has 6 sigfigs of x.

2.65×10^{-6}

$\hat{x} = 3.1415$

$|x - \hat{x}| = 0.000092653 \rightarrow \hat{x}$ has 4 sigfigs of x.

0.92653×10^{-4}

$$\text{relative error} = \frac{|x - \hat{x}|}{|x|}$$

rule-of-thumb: relative error $\leq 10 \cdot 10^{-n}$
$= 10^{-n+1}$

$x = 3.14159265358$

$\hat{x} = 3.14$

$u = 0.00159265358 \qquad n = 3.$

$x \cdot 10^{-2} = 0.0314159 26 \cdots$

$x = a_1 . a_2 a_3 a_4 \cdots - a_k. \qquad a_1 \neq 0.$

$\hat{x} = a_1 . a_2 \cdots a_i$

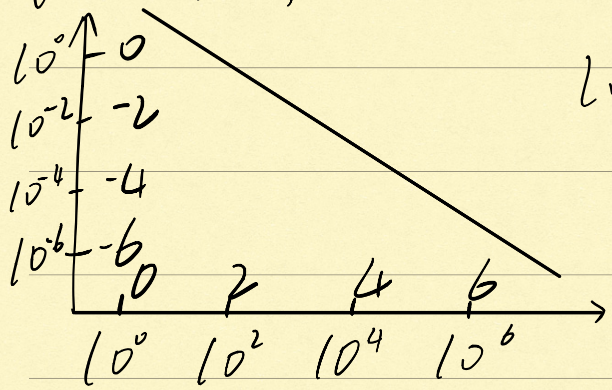
$u = \underbrace{0.0 \cdots 0}_{\text{number} = i} a_{i+1} \cdots a_k \qquad n = i.$

$x \cdot 10^{i-1} = \underbrace{0.00 \cdots 0}_{\text{num} = i-1} a_1 a_2 \cdots a_k.$

plt.loglog(x, y).

$y = a x^b$

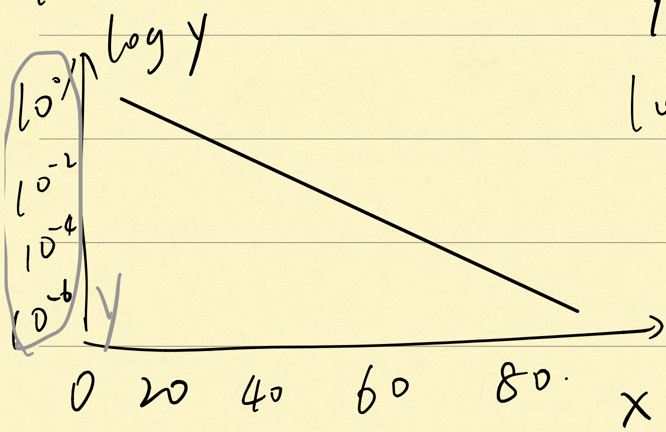
$\log y = \log a + b \log x.$



plt.semilogy(x, y)

$y = a^{bx}$

$\log y = b \log a \cdot x$

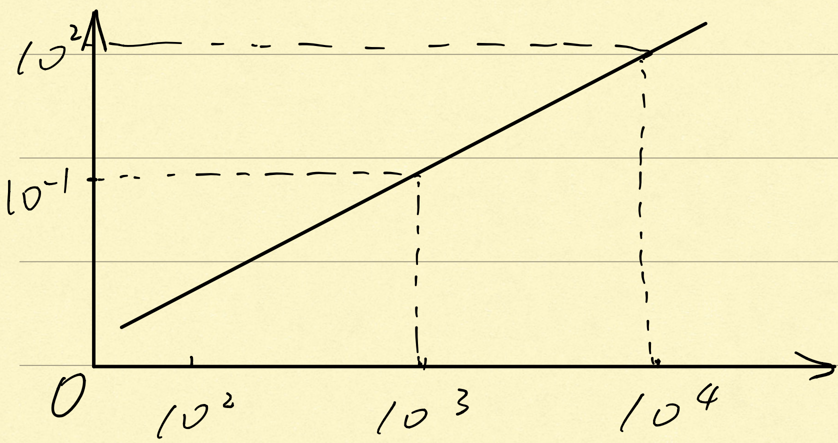


Complexity: Matrix-matrix multiplication.

a matrix $n \times n$, the computational complexity can be represented by: time $= c\, n^a$

Use Log-log. to get constant

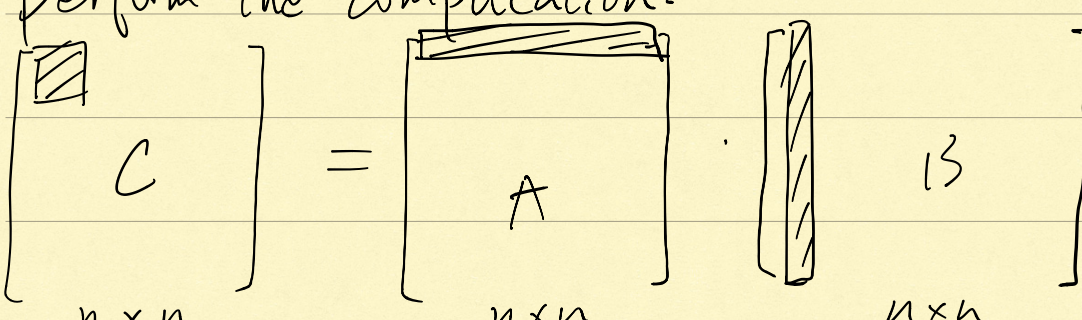
$$\log(\text{time}) = \log c + a \log(n)$$



$$\text{slope} = \frac{\log 10^2 - \log 10^{-1}}{\log 10^4 - \log 10^3} = 3 = a.$$

$$\Longrightarrow \text{time} = c n^3$$

② Count the number of operations needed to perform the computation:



$$C_{11} = u_1 \cdot v_1 \begin{cases} n \text{ multiplication} \\ n \text{ summations} \end{cases}$$

$2n$ operations

$n \times n$ matrix $\Longrightarrow \underbrace{n^2 \cdot (2n)}_{2n^3}$ operations.

Big-O notation.

$f(x) = O(g(x))$ as $x \to \infty$

if and only if

$\exists\, x_0, M,\ |f(x)| \le \underset{\text{constant}}{M} |g(x)| \quad \forall x \ge x_0$

Example: $f(x) = \underbrace{2x^2}_{\text{dominant}} + 27x + 1000 \qquad x \to \infty$

$f(x) \le M(2x^2) \Longrightarrow f(x) = O(g(x))$ as $x \to \infty$

Accuracy: approximating Sin function

$f(x) = \sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \cdots$

Suppose $\hat{f}(x) = x$.

$E = |f(x) - \hat{f}(x)| = |\underline{-\frac{x^3}{6}} + \frac{x^5}{120} - \frac{x^7}{5040} + \cdots|$

$x \to 0 \qquad E \le M(|\frac{x^3}{6}|) \Longrightarrow E = O(x^3)$ as $x \to 0$

Big-O notation (continue)

$f(x) = O(g(x))$ as $x \to a$

if and only if

$\exists M, \delta.\quad |f(x)| \leq M|g(x)| \quad \forall x$ where $0 < |x-a| < \delta$.

Making Prediction. $t = O(n^3)$

$n_1 = 1000 \ — \ t_1 = 10$ seconds

$n_2 = 10000 \longrightarrow \quad \dfrac{t_1}{t_2} = \dfrac{n_1^3}{n_2^3}$

$$t_2 = t_1\left(\frac{n_2}{n_1}\right)^3 = 1000\,t_1 = 10^4 \text{ seconds}$$