$$\|e\| = \left|\frac{\alpha_2}{\alpha_1}\right| \left|\frac{\lambda_2}{\lambda_1}\right|^k \|u_2\|$$

at least a factor of $10^{-n} \longrightarrow x.xxx \times 10^{-(n+1)}$

the eigenvalue of Markov matrix $\in [-1,1]$.

The *Rayleigh Quotient Iteration* improves the convergence rate of the *Shifted Inverse Iteration* using the Rayleigh Quotient as the shift σ_k, i.e.,

$$\boldsymbol{x}_{k+1} = \frac{(\mathbf{A}-\sigma_k\mathbf{I})^{-1}\boldsymbol{x}_k}{\|(\mathbf{A}-\sigma_k\mathbf{I})^{-1}\boldsymbol{x}_k\|}$$

Suppose you are using the Rayleigh Quotient Iteration to estimate the eigenvector of the matrix:

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix}$$

At iteration k, your approximated eigenvector is:

$$\boldsymbol{x}_k = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Determine the shift σ_k (which is given by the **Rayleigh quotient**) corresponding to the eigenvector $\boldsymbol{x}_k$.

$\sigma_k =$ number (rtol=0.001, atol=0.0001) ?

$$\lambda = \frac{x^T A x}{x^T x} = \frac{\begin{bmatrix} 1 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}}{\begin{bmatrix} 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}} = \frac{-16}{10} = \frac{-8}{5}$$