Newton's Method.

Taylor:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 = \hat{f}$$

$f(x)$: nonlinear

$\hat{f}$: quadratic approximation.

find minimum: $\Rightarrow \hat{f}' = f'(x_0) + f''(x_0)(x - x_0) = 0$

$$\Rightarrow x - x_0 = -\frac{f'(x_0)}{f''(x_0)}$$

$$\Rightarrow \boxed{x = x_0 - \frac{f'(x_0)}{f''(x_0)}}$$

→ Newton step.

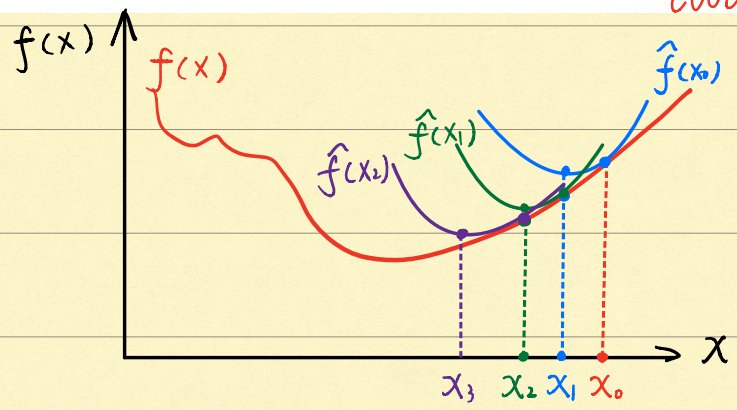
→ Stationary point.

Starting guess: x_0

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

- **Convergence:**
  - Typical quadratic convergence
  - Local convergence (start guess close to solution)
  - May fail to converge, or converge to a maximum or point of inflection

two func evaluation per iteration



if $f(x)$ is quadratic polynomial

no matter what the initial guess is (except x^*),

only need one iteration to get x^*.

x^* may be a minimum, maximum, or saddle point.