① $Ax = \lambda x = x'$

set $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad A - \lambda I = \begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{bmatrix}$.

$$|A-\lambda I| = \begin{vmatrix} a_{11}+a_{21}-\lambda & a_{12}+a_{22}-\lambda \\ a_{21} & a_{22}-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1-\lambda \\ a_{21} & a_{22}-\lambda \end{vmatrix}$$

$$= (1-\lambda)(a_{22}-\lambda-a_{21}) = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = a_{22}-a_{21} \in [-1, 1]$$

$\Rightarrow$ the eigenvalue of Markov matrix $\in [-1,1]$.

②. more than 1 steady state.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1-\lambda & 1-\lambda \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix}$$

$$= (1-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a_{22}-a_{21}-\lambda & a_{23}-a_{21} \\ 0 & a_{32}-a_{31} & a_{33}-a_{31}-\lambda \end{vmatrix}$$

$$\begin{vmatrix} a_{22}-a_{21}-\lambda & a_{23}-a_{21} \\ a_{32}-a_{31} & a_{33}-a_{31}-\lambda \end{vmatrix}$$

$$= \begin{vmatrix} a_{22}+a_{32}-a_{21}-a_{31}-\lambda & a_{33}+a_{23}-a_{21}-a_{31}-\lambda \\ a_{32}-a_{31} & a_{33}-a_{31}-\lambda \end{vmatrix}$$

$$= \begin{vmatrix} a_{11}-a_{12}-\lambda & a_{11}-a_{13}-\lambda \\ a_{32}-a_{31} & a_{33}-a_{31}-\lambda \end{vmatrix}$$

we can let $a_{12} = a_{13} = a_{11} - 1$

i.e. $a_{11}=1 \quad a_{12}=a_{13}=0$

$\Rightarrow \lambda_2 = 1$

③ $L = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad \frac{\frac{1}{4}}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{3}{4} \cdot L$

$\frac{9}{12}$

$$= \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{10}{12} & \frac{1}{12} & \frac{10}{12} \\ \frac{1}{12} & \frac{10}{12} & \frac{1}{12} \end{bmatrix}$$

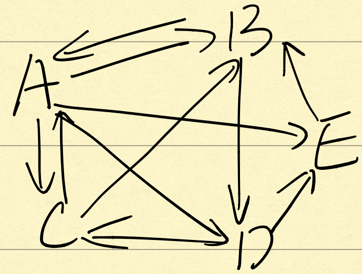
$L = \begin{bmatrix} 0 & 1 & 1 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{12} & \frac{10}{12} & \frac{10}{12} \\ \frac{11}{24} & \frac{1}{12} & \frac{1}{12} \\ \frac{11}{24} & \frac{1}{12} & \frac{1}{12} \end{bmatrix}$$

④

	exl	A	B	exd
exl	0.6	0	0	0
A	0.2	0.4	0.4	0
B	0.1	0.3	0.5	0
exd	0.1	0.3	0.1	1

⑤.



$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{2} & 0 & 1 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$x_3 = x_5$$

$$x_1 = \frac{1}{2}x_2 + \frac{1}{2}x_3$$

$$x_2 = \frac{1}{4}x_1 + \frac{3}{2}x_3$$

$$x_3 = \frac{1}{4}x_1 + \frac{1}{2}x_4$$

$$x_4 = \frac{1}{4}x_1 + \frac{1}{2}x_2$$

$$\left(\frac{10}{7} + \frac{13}{7} + 1 + \frac{9}{7} + 1\right) x_3 = 1.$$

$$\frac{32+14}{7} \quad \frac{46}{7}x_3$$

$$x_3 = \frac{7}{46}$$

$$x_2 = 2x_1 - x_3 = \frac{1}{4}x_1 + \frac{3}{2}x_3$$

$$\frac{7}{4}x_1 = \frac{5}{2}x_3$$

$$x_1 = \frac{5}{2} \times \frac{\cancel{4}\,2}{7} x_3 = \frac{10}{7}x_3$$

$$\frac{10}{7}x_3 = \frac{1}{2}x_2 + \frac{1}{2}x_3$$

$$\frac{20}{7}x_3 = x_2 + x_3$$

$$x_2 = \frac{13}{7}x_3$$

$$x_3 = \frac{10}{28}x_3 + \frac{1}{2}x_4$$

$$\frac{\cancel{18}\,9}{\cancel{28}\,14}x_3 = \frac{1}{2}x_4$$

$$x_4 = \frac{9}{7}x_3$$

⑥. $f(x)=e^{2x}$ $\quad f'(x) = 2e^{2x}$

$$\frac{e^{2(0.1+0.02)} - e^{2(0.1)}}{0.02}$$

$f(x) = 6e^{x}$ $\quad f'(x) = 6e^{x}$

$$\frac{6(e^{0.68} - e^{0.6})}{0.08}$$

⑦. $f(x) = 2x_1^2 - 0.5x_1x_2 + 5x_2^3$

$\frac{\partial f}{\partial x_1} = 4x_1 - 0.5x_2$ $\quad \frac{\partial f}{\partial x_2} = -0.5x_1 + 15x_2^2$