V_0 $\qquad\qquad$ V_0

V_1 $\qquad\qquad$ $\frac{V_0+V_1}{2}$

$\vdots$

V_n $\qquad\qquad$ $\frac{V_{n-1}+V_n}{2}$

$$\frac{9 \times \cancel{64}^{\,8} + 18 \times \cancel{32}^{\,4}}{8} = 72 + 72 = 144.$$

CSR is Better:

$$2n + 111 < (110)^2$$

$$2n < 109 \times 110 - 1$$

$$n < 55 \times 109 - \frac{1}{2} = 5994.5 \implies n = 5994.$$

$$\frac{\|\Delta y\|}{\|y\|}\frac{\|w\|}{\|\Delta w\|} = \frac{\|A\Delta w\| \cdot \|A^{-1}y\|}{\|y\| \cdot \|\Delta w\|} \leq \|A\|\,\|A^{-1}\|$$

$$\frac{\|\Delta y\|}{\|y\|} \leq \|A\|\,\|A^{-1}\|\,\frac{\|\Delta w\|}{\|w\|}$$

107 $\qquad$ 10^{-4}

$\|Ax\| \leq \|A\|\,\|x\|$ $\qquad$ $\|A\| \geq 30$

60 $\qquad$ 2 $\qquad$ 10

5

$$\underset{2}{\|x\|} \leq \|A^{-1}\| \underset{60}{\|Ax\|} \qquad \|A^{-1}\| \geq \frac{1}{30} \quad \frac{1}{10} \quad \frac{1}{5}$$

inf cond(A)=6

$$ABx = b$$

$$\frac{\|\Delta x\|\,\|b\|}{\|x\|\,\|\Delta b\|} = \frac{\|B^{-1}A^{-1}\Delta b\|\,\|ABx\|}{\|x\|\,\|\Delta b\|} \leq \underbrace{\|A\|\,\|A^{-1}\|}_{6}\ \underbrace{\|B\|\,\|B^{-1}\|}_{16.}$$

$$\frac{\|\Delta x\|}{\|x\|} \leq 6 \times 16 \times 10^{-3}$$

$$\|Ax\|_2 \qquad \frac{\|\Delta x\|}{\|x\|} \leq \|A\|\,\|A^{-1}\| \frac{\|\Delta b\|}{\|b\|}$$

large

least well-conditioned metrix

$$my''(t) + cy'(t) + ky(t) = 0.$$

$$x \in [-1, 1]$$

$$-u''(x) + u'(x) = f(x)$$

$$u(-1) = u(1) = 0.$$

$$Au = b.$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} - D\frac{\partial^2 u}{\partial x^2} = 0$$

$\underline{u(x,t)}$ goal.!

$v \equiv 1.$ $\quad D \equiv 0.1$ $\quad x \in [-1, 1].$

$u(-1,t) \equiv 0.$ $\quad \frac{\partial u}{\partial x}(1,t) = 0.$

$u(x,0) = g(x)$

$u(-1,t) = 0$

$u(1,t) = 0.$

$u(x,0) = u_0 = 0.5$

2.2×10^{-16}