Goal: find the maximizer / <u>minimizer</u> x^* that maximizes / minimizes the <u>objective function</u> $f(x): \mathbb{R}^n \to \mathbb{R}$

$f'(x^*) = 0$, $f''(x^*) > 0 \Rightarrow$ minimum

$f''(x^*) < 0 \Rightarrow$ maximum

Gradient-free methods

Evaluate $f(x)$

Gradient (first-derivative) methods

Evaluate $f(x), f'(x)$

Second-derivative methods

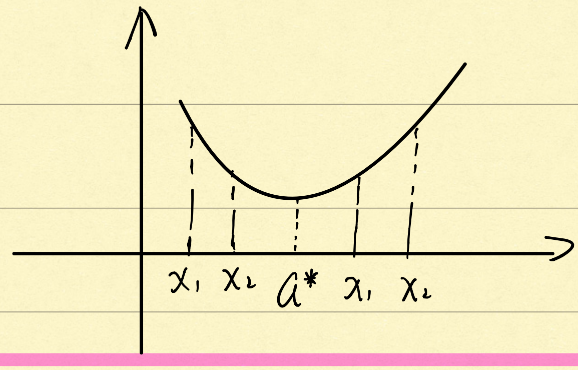
Evaluate $f(x), f'(x), f''(x)$

Golden Section Search (1D)

A function $f: R \to R$ is unimodel 单峰 on an interval $[a, b]$

There is a unique $x^* \in [a, b]$ such that $f(x^*)$ is the minimum in $[a, b]$

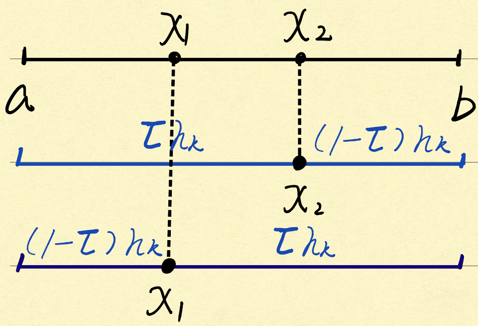
For any $x_1, x_2 \in [a, b]$ with $x_1 < x_2$



$x_2 < x^* \longrightarrow f(x_1) > f(x_2)$

$x_1 > x^* \longrightarrow f(x_1) < f(x_2)$.

condition



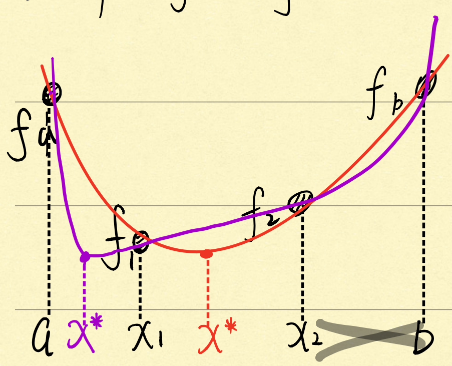
Propose $x_1 = a + (1-\tau) h_k$

$x_2 = a + \tau h_k$

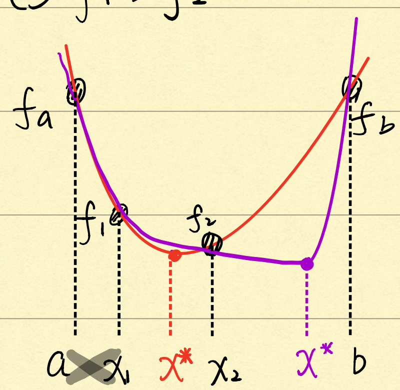
$h_k = b - a$

$x_1 < x_2$

① if $f_1 < f_2$



② $f_1 > f_2$



$x^* \in [a, x_2]$

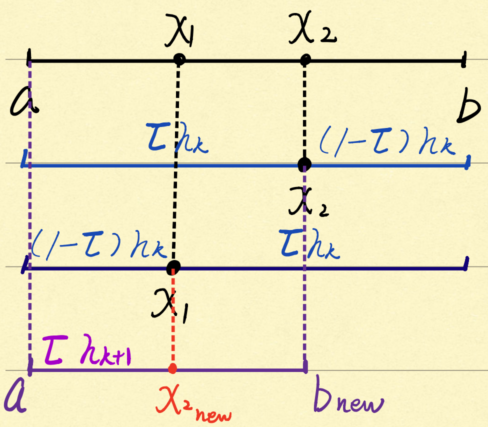
$x^* \in [x_1, b]$

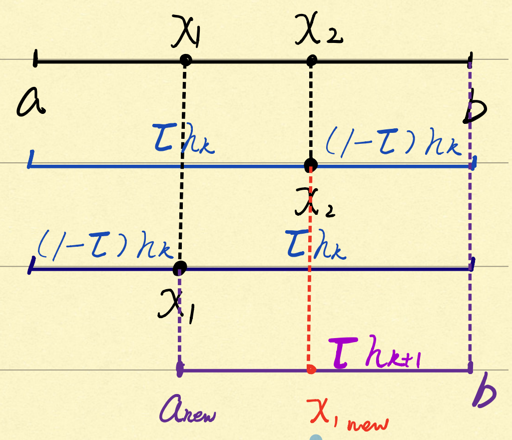
$b_{new} = x_2$

$a_{new} = x_1$

$h_{k+1} = \tau h_k$

Let old x_1 be new x_2 / old x_2 be new x_1.





$$\tau h_{k+1} = \tau^2 h_k = (1-\tau) h_k$$

$$\Rightarrow \tau = 0.618.$$

Golden Section Search

- Derivative free method! only evaluate $f(x)$
- Slow convergence:

$$\lim_{k\to\infty} \frac{|e_{k+1}|}{|e_k|} = 0.618 \quad r = 1 \ (\textit{linear convergence})$$

- Only one function evaluation per iteration $f(a_{new})$ or $f(b_{new})$.