Taylor Series.

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(x_0)}{i!} (x-x_0)^i$$

we have to truncate.

$$h = x - x_0. \quad f(x_0+h) = \underbrace{\sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!} h^i}_{\substack{\text{truncated} \\ t_n(x)}} + \underbrace{\sum_{i=n+1}^{\infty} \frac{f^{(i)}(x_0)}{i!} h^i}_{\text{error.}}$$

$$|\text{error}| = \left| \frac{f^{(n+1)}(x_0)}{(n+1)!} h^{n+1} + \frac{f^{(n+2)}(x_0)}{(n+2)!} h^{n+2} + \cdots \right|$$

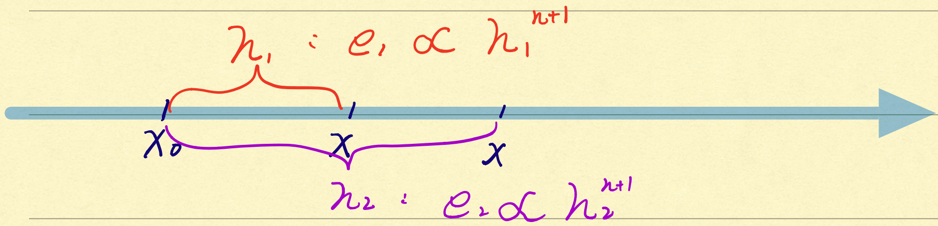
$$\leq |M h^{n+1}| \Rightarrow |\text{error}| = O(h^{n+1})$$

$$f(x) = t_n(x) + R(x)$$

$$R(x) = \sum_{i=n+1}^{\infty} \frac{f^{(i)}(x_0)}{i!} h^i$$

there exists $\xi \in (x_0, x)$ such that $R(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(\xi - x_0)^{n+1}$

$$|\xi - x_0| \leq h \Rightarrow |R(x)| \leq \left| \frac{f^{(n+1)}(\xi)}{(n+1)!} \right| h^{n+1}$$



(set) $\frac{e_1}{e_2} = \left(\frac{h_1}{h_2}\right)^{n+1} \Rightarrow e_2 = \left(\frac{h_2}{h_1}\right)^{n+1} e_1$

L7.5

$f(x) = (3x-15)^2(3x^2-20x+50)^2$ $\quad x^6 \Rightarrow n=6.$

$R_6(x) = T_6(x) - f(x)$

L8.

Linear congruential generator:

Generate a sequence of pseudo-random number. 伪随机数.

given a, c, m, s.

$X_0 = S$, $X_{n+1} = (aX_n + c) \bmod (m)$

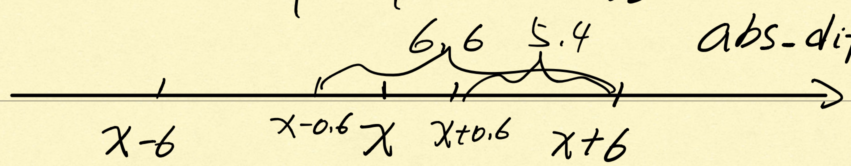
The asymptotic behavior of the error resulting from Monte Carlo methods is $O(\frac{1}{\sqrt{N}})$

N: number of samples.

Example: Sample 100 abs_error = 6

Sample 10000 abs_error = 0.6

abs-diff with 100's $\in [5.4, 6.6]$



H 4.1.

$$7+\frac{8}{1}\times 2-\frac{6}{2}\times 4-\frac{9}{6}\times 8=-1$$
$$-12 \qquad -12$$

$$5+\frac{6\times 2}{1}-\frac{9\times \cancel{4}^{2}}{\cancel{2}}-\frac{5\times \cancel{8}^{4}}{\cancel{6}_{3}}$$

$$17-18-\frac{20}{3}$$

H 4.2.

$$\frac{1}{1+x} \qquad -\frac{1}{(1+x)^2}$$

$$f''(x)=-1+2x-3x^2+4x^3 \cdots$$

H 4.3

$$-7+10x \qquad \frac{10}{2!}=5.$$

H 4.5

$$15x^2+14x+4$$

$$R_2=\left(\frac{h_2}{h_1}\right)^{n+1}R_1$$

H 4.8

$$x-\frac{1}{3!}x^3$$

Hw 4.11.

$4+1=5 \% 5 = 0.$

$1 \% 5 = 1.$

Hw 4.12

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