$$f : \mathbb{R}^n \to \mathbb{R}$$

$$f(X) = f(x_1, x_2, \ldots\ldots, x_n)$$

$$\text{gradient} : \nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \cdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

$$\text{approx} : (\bar{\nabla} f) = \begin{bmatrix} \frac{f(x+h\delta_1)-f(x)}{h} \\ \frac{f(x+h\delta_2)-f(x)}{h} \\ \cdots\cdots \\ \frac{f(x+h\delta_n)-f(x)}{h} \end{bmatrix}$$

$$\delta_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \delta_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \delta_i = \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \end{array} \to i^{th}$$

$$f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$f(x) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_m) \\ f_2(x_1, x_2, \ldots, x_m) \\ \cdots \\ f_n(x_1, x_2, \ldots, x_m) \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_m} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_m} \end{bmatrix}$$

Example : $f(x_1, x_2) = 4x_1 + 10x_2 + 2x_1x_2$

$$\frac{\partial f(x_1, x_2)}{\partial x_1} \approx \frac{f(1.5, 1) - f(1,1)}{0.5} = 6$$