Norm :

$\| x \| = \sqrt{\langle x, x \rangle}$

$\| x \| : \mathbb{R}^n \to \mathbb{R}_0^+$ is called <u>norm</u> if and only if

1. $\|x\| > 0 \iff x \neq 0$

2. $\| \gamma x \| = |\gamma| \|x\|$ for all scalars γ.

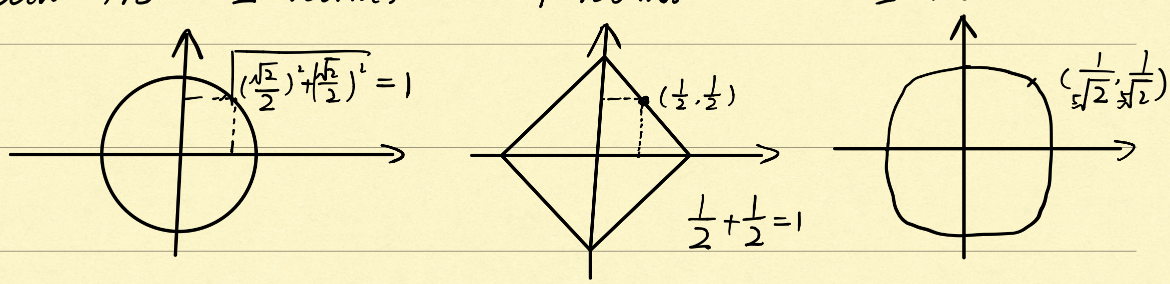
3. $\| x + y \| \leq \| x \| + \| y \|$

Example : p - norms.

$$\left\| \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \right\|_p = \sqrt[p]{|x_1|^p + \cdots + |x_n|^p} \quad (p \geq 1)$$

$p = \infty$: $\| x \|_\infty = \max_i |x_i|$

unit ball in 2-norms 1-norms 5-norms



import numpy.linalg as la.

la.norm (x, 2) => $\| x \|_2$

np.array. flatten( ) : return 1-d array

array([1, 2] [2, 3]) => array ([1, 2, 3. 4])

Normalized x : $normalized\_xs = \frac{x}{\|x\|_p}$

A is a matrix

$\|Ax\| \leqslant \|A\|\|x\|$, $\|AB\| \leqslant \|A\|\,\|B\|$

$$\|A\| = \max_{\|x\|=1} \|Ax\|$$

$\|A\|_F := \sqrt{\sum_{i,j} a_{ij}^2}$ (Frobenius norm)

$\|A\|_1 = \max_j \sum_{i=1}^{n} |A_{ij}|$ $\left[\ \boxed{}\ \right]$

$\|A\|_\infty = \max_i \sum_{j=1}^{n} |A_{ij}|$ $\left[\ \boxed{}\ \right]$

$\|A\|_2 = \max_k \sigma_k$, σ_k is the singular value of A abs(eigenvalue)

$A \cdot A$ 的非负特征值的算术平方根

find the maximum abs

Singular: $Ax=0$ 无穷解 , $Ax=b$ 无穷解或无解

nonsingular: $Ax=0$ 唯一零解. $Ax=b$ 唯一解

(full rank).

$\|A\| = \max\left(\frac{\|Ax\|}{\|x\|}\right) \Rightarrow \|A\| \geqslant \frac{\|Ax\|}{\|x\|}$

$\|Ax\| \leqslant \|A\|\,\|x\|$

$$\|x\| \leq \|A^{-1}\| \, \|Ax\| \implies \|Ax\| \geq \frac{\|x\|}{\|A^{-1}\|}$$