Multinomial Distribution.

Generlization of Binomial (n, p).

n objects $\longrightarrow$ k categories $\{1, 2, \ldots, k\}$.

each object is placed into category j with prob p_j.

where $p_j \geq 0$, $\sum p_j = 1$.

Let X_i = # objects places into category i.

$\sum_{i=1}^{k} X_i = n$.

$X = (X_1, X_2, \ldots, X_k)$ is said to have the multinomial distribution with parameters n and $(p_1, p_2, \ldots p_k)$.

$$X \sim \text{Mult}(n, P).$$

(when $k=2$, $P = (p, 1-p)$, $X \sim \text{Bin}(n, p)$.).

Theorem1: If $X \sim \text{Mult}_k(n, P)$, then the joint P.M.F of X is:

$P(X_1 = n_1, X_2 = n_2, \ldots X_k = n_k)$

$= \dfrac{n!}{n_1! n_2! \cdots n_k!} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$, $\sum_{i=1}^{k} n_i = n$

Theorem 2: $X \sim Mult(n, P) \Rightarrow X_j \sim Bin(n, p_j)$.

$\Rightarrow X_i + X_j \sim Bin(n, p_i + p_j)$.

Theorem 3: $X \sim Mult(n, p)$. for $i \neq j$, $Cov(X_i, X_j) = -n p_i p_j$.

Proof: $Var(X_i + X_j) = Var\, X_i + Var\, X_j + 2\, \underline{Cov(X_i, X_j)}$

$n(1-p_i-p_j)(p_i+p_j) \qquad n(1-p_i)p_i + n(1-p_j)p_j$

$$\Rightarrow Cov(X_i, X_j) = \frac{n\left[(\cancel{p_i+p_j} - (p_i+p_j)^2) - \cancel{(p_i+p_j)} + p_i^2 + p_j^2\right]}{2}$$

$$= -n p_i p_j. < 0$$