Theorem 13.2.14 (Thinning)**.** Let $(N(t) : t > 0)$ be a Poisson process with rate λ, and classify each arrival as a type-1 event with probability p and a type-2 event with probability $1-p$, where these classifications are independent of each other and independent of the arrival times. Then the type-1 events form a Poisson process with rate λp, the type-2 events form a Poisson process with rate $\lambda(1-p)$, and these two processes are independent.

Theorem 13.2.16 (Coloring)**.** Let $(N(t) : t > 0)$ be a Poisson process with rate λ, and C be a finite set of "colors", labeled from 1 through c. Suppose that each arrival gets randomly assigned a color from C, with color i having probability p_i. The color assignments are independent of each other and independent of the arrival times. Let $(N_i(t) : t > 0)$ be the color i process, i.e., $N_i(t)$ is the number of arrivals with color i in $(0, t]$. Then $(N_i(t) : t > 0)$ is a Poisson process with rate λp_i, for $i = 1, 2, \ldots, c$, and these c monochromatic processes are independent.

2D.

Definition 13.3.1 (2D Poisson process)**.** Events in the plane $\mathbb{R}^2$ are a *2D Poisson process* with intensity λ if the following conditions hold:

1. The number of events in a region A is distributed $\text{Pois}(\lambda \cdot \text{area}(A))$.
2. The numbers of events in disjoint regions are independent of each other.

As one might guess, conditioning, superposition, and thinning properties apply to 2D Poisson processes. Let $N(A)$ be the number of events in a region A, and let $B \subseteq A$. Given $N(A) = n$, the conditional distribution of $N(B)$ is Binomial:

$$N(B)|N(A) = n \sim \text{Bin}\left(n, \frac{\text{area}(B)}{\text{area}(A)}\right).$$

Poisson processes have numerous extensions, some of which are explored in the exercises. We can allow λ to vary as a function of time or space instead of remaining constant; this is called an *inhomogeneous Poisson process*. We can allow λ to be a random variable; this is called a *Cox process*. Finally, we can allow the rate to increase by λ after each successive arrival; this is called a *Yule process*.
