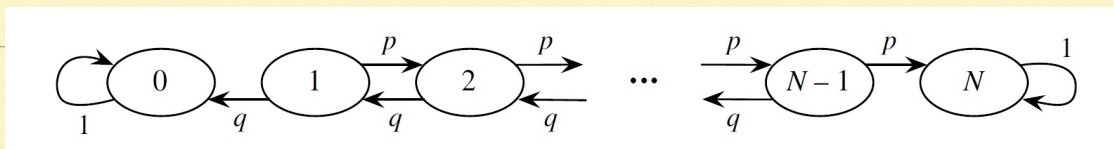two Gamblers, A, B, start with i and
$N-i$ dollars respectively.
A sequence of bets for $\$1$.
each round A has prob p of winning.
prob $q = 1-p$ of losing.
Let X_n be the wealth of A at time n.

The Prob of A wins?

Set P_i : prob of A wins the game.

given A starts with i dollars.

W : the event A wins.

$P_i = p \cdot P(W \mid A$ starts at i and wins round 1$)$

$+ q \cdot P(W \mid A$ starts at i and loses round 1$)$

$$= p \cdot P(W \mid A \text{ starts at } i+1)$$

$$+ q \cdot P(W \mid A \text{ starts at } i-1)$$

$$= p \cdot P_{i+1} + q \cdot P_{i-1}. \qquad \text{difference equation.}$$

$$P_i = \begin{cases} a + b\left(\frac{q}{p}\right)^i, & p \neq q \\ a + bi, & p = q \end{cases}$$

if $p = q = \frac{1}{2}$, $P_i = \frac{i}{N}$

otherwise, $P_i = \dfrac{1 - \left(\frac{q}{p}\right)^i}{1 - \left(\frac{q}{p}\right)^N}$

if $p = 0.49$ $q = 0.51$ $i = 100 = N - i$

$$\Rightarrow P_i = 1.8\%.$$