事件单位时间发生概率为 $p$,

至发生为止的总时长 $X \sim GE(p)$.

$$P(X=k) = (1-p)^{k-1} p.$$

$$EX = \frac{1}{p}$$

$$Var\,X = \frac{1-p}{p^2}.$$

$$EX = \sum_{k=1}^{\infty} (1-p)^{k-1} p \cdot k = A.$$

$$P(1 + 2(1-p) + 3(1-p)^2 + \cdots + n(1-p)^{n-1}$$

$$(1-p)A = p\left[(1-p) + 2(1-p)^2 + \cdots + n(1-p)^n\right].$$

$$-pA = p\left[-1 - (1-p) - (1-p)^2 - \cdots - (1-p)^{n-1} + n(1-p)^n\right]. \quad (n \to \infty)$$

$$A = -\left(\frac{1\left((1-p)^n - 1\right)}{1-p-1} + n(1-p)^n\right) \quad (n \to \infty)$$

$n(1-p)^n \to 0$

$$= \frac{1}{p}$$

$$EX^2 = \sum_{k=1}^{\infty} (1-p)^{k-1} p \cdot k^2 = B.$$

$$B = \left[1 + (1+2)(1-p) + (2+3)(1-p)^2 + \cdots + (n-1+n)(1-p)^{n-1} - n^2(1-p)^n\right]$$

$n^2(1-p)^n \to 0$

$$(1-p)B - B = -1 - 2(1-p) - 2(1-p)^2$$

$$- \cdots - 2(1-p)^{n-1}$$

$$+ (2n-1)(1-p)^n$$

$$\to 0 \qquad \to 0.$$

$$B = \frac{1}{p}\left(1 + 2\left(\frac{(1-p)\left((1-p)^{n-1} - 1\right)}{1-p+1}\right)\right)$$

$$= \frac{2-p}{p^2}$$

$$\text{Var } X = EX^2 - E^2X = \frac{2-p}{p^2} - \frac{1}{p^2}$$

$$= \frac{1-p}{p^2}$$