

if today is rainy, tomorrow $\frac{1}{3}$ rainy $\frac{2}{3}$ sunny.

if today is sunny, $\frac{1}{2}$ rainy $\frac{1}{2}$ sunny.

Markov chain: X_0, X_1, \dots

$$\mathcal{X} = \{R, S\}.$$

$$Q = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

if two rainy/sunny day \Rightarrow must be sunny/rainy tomorrow.

New Markov chain: $Y_n = (X_n, X_{n-1}) \quad n \geq 1.$

$$\begin{array}{c} (R, R) \quad (R, S) \quad (S, R) \quad (S, S) \\ \begin{array}{l} (R, R) \\ (R, S) \\ (S, R) \\ (S, S) \end{array} \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \end{array}$$