Name : Wenxiao Yang

UIN: 660162495.

Subject: Date:

Problem 1: $P(C=1 \mid M=1)=\frac{1}{2}$ $\quad P(C=1 \mid M=0)=0$ $\quad P(M=1)=\frac{1}{3}$

(a) $P(C_1=0, C_2=0) = P(M=1)P(C_1=0, C_2=0 \mid M=1) + P(C_1=0, C_2=0 \mid M=0) P(M=0)$

$$= \frac{1}{3}\times\frac{1}{2}\times\frac{1}{2} + \frac{2}{3}\times 1 \times 1 = \underline{\underline{\frac{3}{4}}}$$

(b) $P(C_1=0) = \frac{1}{2}\times\frac{1}{3} + 1\times\frac{2}{3} = \frac{5}{6} = P(C_2=0)$.

$P(C_1=0, C_2=0) = \frac{3}{4} \neq \frac{25}{36} = P(C_1=0)\cdot P(C_2=0)$.

Hence it is <u>not</u> independent.

(c) $P(M=1 \mid C_1=0, C_2=0) = \dfrac{P(C_1=0, C_2=0, M=1)}{P(C_1=0, C_2=0)}$

$$= \frac{P(C_1=0, C_2=0 \mid M=1)P(M=1)}{P(C_1=0, C_2=0)}$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{3} \div \frac{3}{4} = \underline{\underline{\frac{1}{9}}}$$

Problem 2:

$X=x$

$\cancel{(a)\ F_{X,Y}(x,y) = P(X\le x, Y\le y) = \int_0^x P(Y\le y \mid X=t)\cdot f_X(t)\,dt}$

$\cancel{= \int_0^x P(Y\le y \mid X=t)\frac{1}{L}\,dt}$

$\cancel{= \int_0^x\int_0^y f(y=s \mid x=t)\frac{1}{L}\,ds\,dt}$

$= \begin{cases} \cancel{\frac{y}{L} + \frac{y}{L}\ln\frac{x}{y}} & \cancel{x\ge y} \\ \cancel{\frac{x}{L}} & \cancel{x<y} \end{cases}$

(a)

$$f_{X,Y}(x,y) = f_{Y|X}(y \mid x)\cdot f_X(x) = \begin{cases} 0 & \text{others} \\ \frac{1}{xL} & 0\le y\le x\le L \end{cases}$$

(b). $f_Y(y) = \int_0^L f_{X,Y}(x,y)\,dx = \int_y^L \frac{1}{xL}\,dx = \begin{cases} \frac{1}{L}\ln\frac{L}{y}, & y\in[0,L] \\ 0, & \text{others} \end{cases}$

(c) $f_{X|Y}(x \mid y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)} = \begin{cases} \dfrac{\frac{1}{xL}}{\frac{1}{L}\ln\frac{L}{y}} = \dfrac{1}{x\cdot\ln\frac{L}{y}} & 0\le y<x\le L \\ 0, & \text{others} \end{cases}$

Subject :   Date:

Problem 3: (a) $P(V \le v) = P(2|U|-1 \le v) = P(|U| \le \frac{v+1}{2})$

$= P(-\frac{v+1}{2} \le U \le \frac{v+1}{2})$

$= \begin{cases} \frac{v+1}{2}, & v \in [-1, 1] \\ 1, & v > 1 \\ 0, & v < -1 \end{cases}$

(b). $Cov(U, V) = E(UV) - E(U)E(V)$

$= E(2U|U| - U) - 0 \cdot E(V)$

$= E(2U|U|) - \underset{=0}{E(U)}$

Since $U|U|$ is odd, we know $E(U|U|) = 0$.

$= 0$

Hence U and V are uncorrelated.

$P(V \le v \mid U = 0) = P(-1 \le v) = \begin{cases} 1, & v \ge -1 \\ 0, & v < -1 \end{cases}$

$\ne P(V \le v)$

Hence U and V are not independent.

Problem 4. (a). $F_X(x) = 1 - e^{-\lambda x}$

$X = F_X^{-1}(U)$

$F_X(x) = y = 1 - e^{-\lambda x}$

$e^{-\lambda x} = 1 - y$

$-\lambda x = \ln(1-y)$

$x = \frac{\ln(1-y)}{-\lambda} = F_X^{-1}(y)$

Hence $\frac{\ln(1-U)}{-\lambda} \sim Exp(\lambda)$.

(b). $X = -\log U_1 U_2 = -\log U_1 - \log U_2$

Since $f_{U_1}(x) = f_{1-U_1}(x)$, $x \in [0,1]$. $\Rightarrow -\log U_1 \sim Exp(1)$

$-\log U_2 \sim Exp(1)$.

$Exp(1)$ is equal to $Gamma(1,1)$.

Hence $X = -\log U_1 - \log U_2 \sim Gamma(2,1)$

$f_X(x) = x e^{-x}$