$g(x) = E(Y|x)$ r.v. of x.

<u>Definition</u>: The conditional variance of y given x is

$$Var(Y|x) = E((Y - E(y|x))^2 | x)$$

$$= E(w|x), \text{ where } w = (Y - E(y|x))^2$$

$$Var(Y|x) = E(Y^2|x) - E^2(Y|x)$$

$h(x) = Var(Y|x)$ r.v. of X.

Law of total variance.

$$Var\, Y = E(Var(Y|x)) + Var(E(y|x))$$

<u>Proof</u>: $g(x) = E(Y|x)$ $E(g(x)) = E(Y)$

$$E(Var(Y|x)) = E(E(Y^2|x) - E^2(y|x))$$

$$= EY^2 - E(g^2(x))$$

$$Var(E(Y|x)) = Var(g(x)) = E(g^2(x)) - E^2(g(x))$$

$$= E(g^2(x)) - E^2(Y)$$

Hence, $E(Var(Y|x)) + Var(E(Y|x)) = EY^2 - E^2Y = Var\, Y$.