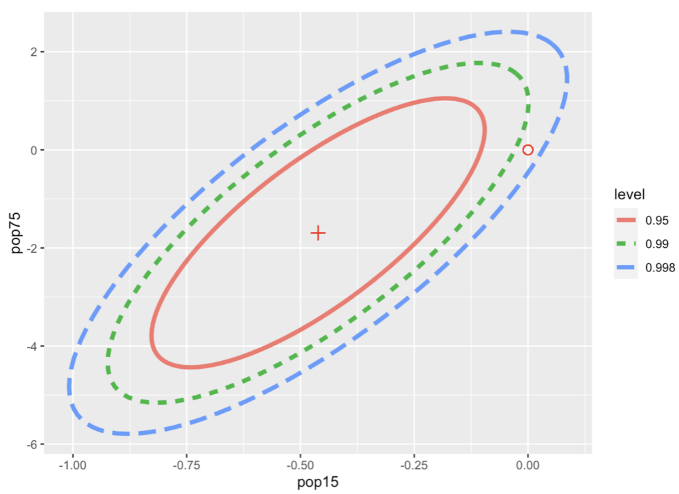
1. "The confidence intervals for individual β_j's are wider than the confidence intervals for the vector β."

   ( ) True. (X) False. ( ) It depends.

   Justification: The vector confidence interval is a family/joint interval for all the betas, so if we extract the interval for a β_j from the vector interval, it will be wider than the interval we would construct for an individual β_j. Of course for the same confidence.

   What might be confusing here is the width of the interval vs. the confidence level. What we discussed is that if we combine many individual 95% CIs then the family confidence will be lower than 95%. But, if we properly construct a family interval with 95% confidence and we compare that with the individual 95% interval for the corresponding parameter then the family interval will be wider than the individual.

2. Consider the savings dataset from the Faraway library. We fit a model where the response is the savings rate (sr) and the predictors include [pop25] (population under 25) and [pop75] (population over 75). The following plot depicts the confidence region for two variables [pop25] and [pop75]:



   For which hypotheses can you determine the outcome based on the plot above?

   ( ) $H_0 : \beta_{pop75} = \beta_{pop25} = 0$ ( ) $H_0 : \beta_{pop75} = 0$ ( ) $H_0 : \beta_{pop25} = 0$ (X) All of the above. ( ) None of the above.

   Justification: We simply need to look at the plot and determine whether the points:
   – (0,0) i.e. both coefficients are equal to zero
   – pop15 = 0, i.e. the $\beta_{pop15} = 0$
   – pop75 = 0, i.e. the $\beta_{pop75} = 0$
   So, we can quickly test all these hypotheses visually. Note that the alternative when testing whether (0,0) is in the region is that at least one of the coefficients is not 0. Note also that the confidence region is a family "interval", so the intervals we extract for pop15 and pop75 are not going to be the same as if we constructed two individual intervals for pop15 and pop75 - in the same spirit as in Q2.

3. The standard error for the estimated mean response $\hat{\mu}^*$ given a new observation x^* is always larger than the predicted value y^* given a new observation x^*.

( ) True. (X) False.
Justification: If we look at the standard error formulas, we will see that the prediction error is always larger than the estimation error.

4. The point estimate for the mean response μ^* given a new observation x^* is equal to:
(X) $(x^*)^T\hat{\beta}$
( )$X^T\hat{\beta}y$
(X) $(x^*)^T(X^TX)^{-1}X^Ty$
(X) the best estimate for y^* at a future x^*.
( ) none of the above
Justification: The first is the formula for the μ^*. The third one is the same formula, if we plug-in the $\hat{\beta}$, the 4th choice is the same thing in words .

5. The standard error of $\hat{\mu}^*$ converges (as the sample size increases) to
( ) σ ( ) $\hat{\sigma}$ ( ) σ^2 ( ) $\hat{\sigma}^2$ (X) 0 ( ) none of the above.
Justification: Recall from the lectures that the limit of the standard error for estimation is zero. (In statistics jargon such estimators are called *consistent*.

6. The variance of $\hat{y}^*$ converges (as the sample size increases) to
( ) σ ( ) $\hat{\sigma}$ (X) σ^2 ( ) $\hat{\sigma}^2$ ( ) 0 ( ) none of the above.
Justification: Recall from the lectures that the limit of the variance for prediction is σ^2.

7. If I want to use Bonferroni correction to construct the confidence interval for $x_1^*, \ldots, x_4^*$, in order for the family confidence level to be 0.95, I should divide $\alpha = 0.05$ by
( ) 1 ( ) 2 (X) 4 ( ) 8 ( ) none of the above.
Justification: 4 is the number of points for which I want to construct the interval.