1. Let matrix $\mathbf{B}$ defined as $\mathbf{B} = \begin{pmatrix} 1 & 5 & 0 \\ 1 & 0 & 5 \\ 1 & 0 & 5 \end{pmatrix}$. Are the column vectors of $\mathbf{B}$ linearly independent?

   ( ) Yes. (X) No.

2. Let matrix $\mathbf{B}$ defined as $\mathbf{B} = \begin{pmatrix} 1 & 5 & 0 \\ 1 & 0 & 5 \\ 1 & 0 & 5 \end{pmatrix}$. What is the rank of $\mathbf{B}$? 2

3. Consider matrix $\mathbf{A}$. Then, $\mathbf{A}$ is symmetric if:

   ( ) $\mathbf{A}^{-1} = \mathbf{A}$
   (X) $\mathbf{A}^T = \mathbf{A}$
   ( ) $\mathbf{A}^2 = \mathbf{A}$
   ( ) none of the above.

4. Consider matrix $\mathbf{A}$. Then, $\mathbf{A}$ is idempotent if... Select all that apply:

   [ ] $\mathbf{A}^{-1} = \mathbf{A}$
   ( ) $\mathbf{A}\mathbf{A}^T = \mathbf{A}$
   [X] $\mathbf{A}^2 = \mathbf{A}$
   ( ) $\mathbf{A}^T = \mathbf{A}$
   [X] $\mathbf{I} - \mathbf{A} = 0$

5. Consider the design matrix $\mathbf{X}_{n \times p}$. Then, $(\mathbf{X}^T\mathbf{X})$ can be inverted when...

   Select all that apply:

   [X] $\mathbf{X}$ is of full rank.
   ( ) $\mathbf{X}$ is of rank n.
   [X] $\mathbf{X}$ is of rank p.
   ( )$\mathbf{X}$ is singular.
   [X] $\mathbf{X}$'s columns are linearly independent.

6. The normal equations from the least-squares estimation problem $X^T(y - X\hat{\beta}) = 0$ imply that:

   ( ) All columns of matrix $\mathbf{X}$ are independent.
   ( ) All columns of matrix $\mathbf{X}$ are orthogonal to the data vector $\mathbf{y}$.
   (X) All columns of matrix $\mathbf{X}$ are orthogonal to the estimated residual vector $\mathbf{r}$.
   ( ) All columns of matrix $\mathbf{X}$ and vector $\mathbf{y}$ are independent.

7. Select all the statements that are TRUE. (There might be more than one statements:

   [X] $\hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y$
   [X] $\hat{\sigma}^2 = \frac{RSS}{n-p}$
   ( )$\hat{\sigma}^2 = RSS$
   ( )$\hat{\beta}^T\mathbf{X}^T y = 0$
   [X] $\hat{\beta}^T\mathbf{X}^T r = 0$