1. The variance-covariance matrix of the estimated residual vector $r = y - X\hat{\beta}$ is $Var(r) = \sigma^2(I - H)$, where I is the identity matrix and H is the hat matrix. This equation implies that all estimated residual variances $Var(r_i)$ are equal. True or False?

   ( ) True (X) False

   Justification: The diagonal elements of the hat matrix are not identical, hence the variances of the residuals are not the same.

2. The following output is from the Multiple Linear Regression model fitted to the fat data set from the faraway library. The response variable is the percent body fat calculated using Brozek's equation.

```
Call:
lm(formula = brozek ~ density + age + weight + height + adipos +
    free + neck + chest + abdom + hip + thigh + knee + ankle +
    biceps + forearm + wrist, data = fat)

Residuals:
    Min      1Q  Median      3Q     Max
-5.3449 -0.3154  0.0460  0.2921  8.9773

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.533e+02  1.524e+01  16.613  < 2e-16 ***
density     -2.341e+02  1.314e+01 -17.822  < 2e-16 ***
age          5.733e-03  6.843e-03   0.838 0.403031
weight       1.594e-01  1.636e-02   9.745  < 2e-16 ***
height       1.251e-02  2.354e-02   0.531 0.595649
adipos      -2.339e-01  6.452e-02  -3.626 0.000353 ***
free        -2.301e-01  1.834e-02 -12.545  < 2e-16 ***
neck         1.993e-02  4.995e-02   0.399 0.690285
chest        6.876e-02  2.236e-02   3.075 0.002355 **
abdom        2.381e-02  2.339e-02   1.018 0.309598
hip          1.911e-02  3.120e-02   0.613 0.540799
thigh        6.911e-02  3.134e-02   2.206 0.028387 *
knee         1.163e-02  5.216e-02   0.223 0.823725
ankle        3.348e-03  4.748e-02   0.071 0.943834
biceps      -2.952e-03  3.667e-02  -0.081 0.935897
forearm      9.868e-02  4.248e-02   2.323 0.021023 *
wrist        1.632e-01  1.152e-01   1.416 0.157960
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.9042 on 235 degrees of freedom
Multiple R-squared:  0.9873,    Adjusted R-squared:  0.9864
F-statistic:  1138 on 16 and 235 DF,  p-value: < 2.2e-16
```

   Keeping all remaining variables constant, the average change in body fat per average change in 1 cm increase in leg-related circumference measures (thigh, knee and ankle) is

$$\beta_{leg} = \frac{\beta_{thigh} + \beta_{knee} + \beta_{ankle}}{3}$$

   (a) According to the Gauss-Markov theorem, the best linear unbiased estimate of β_{leg} is (units percent/cm):

   ( ) 0.069 (X) 0.028 ( ) 0.0116 ( ) 0.0033

   Justification: We know that a unbiased estimator for β_{leg} will be

$$\hat{\beta}_{leg} = \frac{\hat{\beta}_{thigh} + \hat{\beta}_{knee} + \hat{\beta}_{ankle}}{3} = \frac{6.911 \cdot 10^{-2} + 1.163 \cdot 10^{-2} + 3.348 \cdot 10^{-3}}{3} = 0.028$$

(b) Based on the output provided above, we
(X) reject ( ) fail to reject ( ) accept
the null hypothesis $H_0 : \beta_1 = \beta_2 = \ldots = \beta_{16} = 0$.
Justification: We just need to look at the p-value that corresponds to the F test, which is $< 2.210^{-6}$. This is less than α=5%, so we reject the null.

(c) The estimated value of σ^2 is (0.818).
Justification: The Residual Standard Error in the output, 0.9042, is the estimator for σ, so we just need to square it.

(d) The percentage of the variation in percentage of body fat explained by the regression model is (98.73).
Justification: This question is asking for R^2.

(e) The number of observations, n, in this data set is (252).
Justification: n is the sample size. From the output, we find that the DF of the error are 235, and we can count 17 parameters estimated, including the intercept.

(f) Based on the information provided above We
( ) accept (X) reject ( ) fail to reject
the null hypothesis $H_0 : \beta_{weight} = 0$.
Justification: We just need to look at the p-value that corresponds to the weight coefficient, which is $< 2.210^{-6}$. This is less than α=5%, so we reject the null.

3. Assume $\mathbf{X}$ to be a matrix of random variables. Then, $\mathbb{E}(\mathbf{AXB} + \mathbf{C})$ is equal to
( ) $\mathbf{A}\mathbb{E}(\mathbf{X})\mathbf{B}^T + \mathbf{C}$
(X) $\mathbf{A}\mathbb{E}(\mathbf{X})\mathbf{B} + \mathbf{C}$
( ) $\mathbf{A}\mathbb{E}(\mathbf{X})\mathbf{B}$
( ) $\mathbf{B}\mathbb{E}(\mathbf{X})\mathbf{A}$
Justification: The expectation is linear. In the scalar case, remember that $\mathbb{E}(3X) = 3\mathbb{E}(X)$. This is the same property here, with the difference that the order of the terms in the multiplication must be preserved, since we deal with matrices.

4. Assume $\mathbf{x}$ to be a vector of random variables. Then, $Var(\mathbf{Ax})$ is equal to
(X) $\mathbf{A}Var(\mathbf{x})\mathbf{A}^T$
( ) $\mathbf{A}\mathbf{A}^T Var(\mathbf{x})$
( ) $\mathbf{A}\mathbf{A}Var(\mathbf{x})$
( ) $Var(\mathbf{x})\mathbf{A}\mathbf{A}^T$
Justification: In the scalar case, we need to square the constant when taken outside the variance. This is how it translates in matrices.

5. Assume $\mathbf{x}, \mathbf{y}$ to be vectors of random variables. Then, $Cov(\mathbf{Ax}, \mathbf{By})$ is equal to
( ) $\mathbf{A}Cov(\mathbf{x}, \mathbf{y})\mathbf{B}$
( ) $\mathbf{A}^T Cov(\mathbf{x}, \mathbf{y})\mathbf{B}^T$

( ) $\mathbf{A}^T Cov(\mathbf{x}, \mathbf{y})\mathbf{B}$
( ) $\mathbf{A}\mathbf{B}^T Cov(\mathbf{x}, \mathbf{y})$
(X) $\mathbf{A} Cov(\mathbf{x}, \mathbf{y})\mathbf{B}^T$ *Justification:* The Covariance property when we factor out constants, translated in matrices. So, the multiplication order needs to be taken into account.