STAT 426 – Categorical Data Analysis
Homework 1
Due 5pm of Jan 29, 2020

Below exercises give you a practice in R Markdown while reviewing maximum likelihood estimation, hypothesis testing and the Poisson distribution.

1. **Characteristics of Poisson distribution**

   Let $X_1, X_2, ..., X_n$ be a random sample from a Poisson distribution with mean λ.

   (i) Use mathematical expressions in R Markdown to show what the maximum likelihood estimator $\widehat{\lambda}$ is.

   (ii) What is the exact variance of $\widehat{\lambda}$?

   (iii) Using Fisher Information find the asymptotic variance of $\widehat{\lambda}$. Is it same as in (ii)?

2. Suppose you obtain a random sample from a Poisson distribution that is $(9, 16, 17, 14, 17, 11, 12, 15, 13, 9)$. Find the 95 percent Wald confidence interval for $\widehat{\lambda}$.

3. Using the Wald test statistic and the data from 2., find the p-value for testing $H_0 : \mu = 10 \ \ vs. \ \ H_a : \mu > 10$.

4. Find the p-value of an exact test for the same hypotheses. Hint: If $X_1, X_2, ..., X_n$ is a random sample from a Poisson distribution with mean $\widehat{\lambda}$, then $\sum_{i=1}^{n} X_i$ is Poisson with mean $n\lambda$.

5. Plot the density functions of a Poisson variable with mean 6 and a Binomial variable with $n = 200$ and $p = 3/100$ side-by-side for $x = 0, 1, 2, ..., 10$. What do you observe?