DESIGN ROYALTY PAYMENTS POLICY IN NON-FUNGIBLE TOKEN MARKETS: TACKLING ADVERSE SELECTION WITH SMART CONTRACTS

BY

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ABSTRACT

Asymmetric information in markets often results in inefficiencies by adverse selection, exemplified by the "lemons market" phenomenon. The rapidly emerging non-fungible token (NFT) market faces similar challenges with information asymmetry between NFT creators and buyers. In this paper, we investigate the role of royalty payments and explore how NFT platforms should regulate these payments to mitigate inefficiencies arising from asymmetric information. Our study presents three key findings. First, royalty payments consistently harm NFT creators and the whole market when both creators and buyers hold aligned beliefs. Second, when information asymmetry exists, we demonstrate that royalty payments can enhance market efficiency by retaining high-quality NFT creators while excluding low-quality creators, particularly under regulations where creators set a uniform royalty rate. Third, under regulations where creators can commit to a specific royalty rate, we find that creators may differentiate themselves based on their NFTs' quality, with low-quality NFT creators setting higher prices and lower royalty rates, and high-quality NFT creators opting for lower prices but higher royalty rates. We also establish the conditions for existence and uniqueness of equilibrium under different market conditions. These results provide valuable insights into designing effective NFT marketplace policies that foster a more efficient and equitable market environment.

To my parents, for their unwavering love and support that has kept me strong.
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CHAPTER 1

INTRODUCTION

Asymmetric information in markets often leads to inefficiencies, with buyers unable to differentiate between high-quality and low-quality products, a phenomenon known as the "lemons market" [Akerlof, 1970]. To mitigate this issue, researchers have explored various solutions, such as bundling [Bakos and Brynjolfsson, 1999], signaling [Spence, 1973], and royalty contracts [Beggs, 1992]. In the emerging non-fungible token (NFT) market, asymmetric information presents similar challenges, as buyers may struggle to assess the value and potential future popularity of NFTs accurately. Furthermore, creators' information about the potential popularity of their NFTs may also be inaccurate, complicating the market dynamics. To address these challenges, this paper investigates the role of royalty fees and NFT platform regulations in mitigating the inefficiencies resulting from asymmetric information between creators and buyers.

Prior literature has shown that smart contracts can mitigate informational asymmetry and improve welfare and consumer surplus through enhanced entry and competition [Cong and He, 2019]. [Tirole, 2012] studied the role of government intervention in reducing adverse selection, while [Bhattacharya et al., 2022] explored the connection between auction design and post-auction economic activity in a model of contingent payment auctions. [Beggs, 1992] demonstrated that royalty contracts could improve efficiency and increase the likelihood of a patent being allocated to the right firm in the licensing market.

In this paper, we extend the framework of [Beggs, 1992] to an NFT market where creators set the price and buyers decide whether to accept. [Beggs, 1992] examines a licensing market in which the buyer determines the contract and output level. In this context, the contract serves as a signal for the buyer, who possesses more information, to convey essential details to the seller. However, the royalty payments in Beggs' model function solely as transfer payments from the stage of information asymmetry to the stage where product types and outputs are observable to all parties.

In contrast, our paper explores scenarios where the seller (NFT creator) sets both the price and royalty rate, and the NFT's profitability relies not on the buyer's output level but on the secondary market dynamics among buyers. In this context, the royalty rate can introduce inefficiencies in the secondary market. Our study presents a more generalized form of signaling games in markets with asymmetric information and potential inefficiencies arising from royalty payments. We also elucidate the equilibrium structure in such markets and how welfare is affected by royalty payments from the view of policymakers, offering guidance on designing royalty payments in NFT marketplaces.

Our analysis focuses on the effects of royalty payments under different information structures, emphasizing their significance in addressing information asymmetry from both the buyer and creator perspectives. We consider the NFT creator's ability to collect royalty fees from secondary market transactions due to the existence of smart contracts and discuss the impact of various royalty payment regulations on market efficiency from an NFT marketplace policy design perspective.

In our main model, we assume that NFT creators are more familiar with market conditions and have more information about NFT price fluctuations. Drawing on the lemons market selection [Akerlof, 1970], we identify two main effects of royalty payments in this context: the inefficiency effect, whereby royalty fees can reduce transactions between buyers due to increased costs, potentially leading to lower market efficiency, and the transfer effect, which sees royalty fees transfer profits from the stage where NFT popularity is not revealed to the stage that NFT popularity is revealed, thus alleviating the effect of information asymmetry.

We also examine the implications of different NFT platform regulations on the commitment of creators, such as whether the royalty rate is flexible or fixed. We divide our main model into two situations: 1) creators set the same royalty rate regardless of their own information about NFTs, corresponding to "no royalty payments" (royalty rate equals zero) and "flexible royalty rate" (adjust royalty rates to maximize profits after NFT information is revealed) regulations, and 2) creators set different royalty rates depending on their own information about NFTs, corresponding to "fixed royalty rate" regulations that allow creators to signal their information about the NFT.

In the general model discussion, we find that royalty payments can retain high-quality NFT creators while excluding low-quality creators under regulations where creators set the same royalty rate. Under regulations where creators can commit a royalty rate, we find that if a separate equilibrium is formed, creators must be separated based on their NFTs' qualities, with low-quality NFT creators setting higher prices and lower royalty rates, and high-quality NFT creators setting lower prices but higher royalty rates.

Finally, we discuss a binary example with only two kinds of creators in the market: one producing low-quality NFTs and the other producing high-quality NFTs. We prove the existence of different equilibria under various conditions.

In the extension part of the paper, we explore the effects of royalty payments when buyers possess more familiarity with the market situation. We demonstrate that royalty payments continue to alleviate the inefficiency caused by information asymmetry through transfer effects, enabling creators to cater to a larger market.

In conclusion, this paper contributes to the existing literature by investigating the impact of royalty fees and NFT platform regulations on market efficiency under different information structures. Our findings highlight the potential benefits of royalty payments in addressing information asymmetry between NFT creators and buyers. By considering various NFT platform regulations, we provide valuable insights into how NFT marketplaces can design effective policies to promote a more efficient and fair market environment.

CHAPTER 2

MODEL SETTINGS

Our analysis begins by considering a scenario in which a creator produces an NFT and has the option to sell it on the platform or outside the platform. The creator derives no value from holding the NFT and benefits only from selling it. Based on previous research ([Lovo and Spaenjers, 2018]), we assume that buyers on the platform may derive emotional dividends from possessing an NFT, which represent the utility that buyers receive from retaining the NFT at the end of a stage, denoted as e. The value of e for buyers may vary over time and depends on the popularity of the NFT, represented by ω. We assume that ω can take on two values, "unpopular" or "popular," denoted as $\omega = \text{UP}$ and $\omega = \text{P}$, respectively. If the NFT is "unpopular," no buyer values it, and $e = 0$. Conversely, if the NFT is "popular," buyers' emotional dividends, e, follow a distribution F with probability density function f and support $[0, 1]$.

Our study focuses on the NFT trading process and is modeled by two parts: the primary market and the secondary market. The primary market involves the creator and the first buyer, and is influenced not only by royalty rates but also by the information asymmetry between them. Specifically, we examine a market with asymmetric information, where either the creator or the buyer may possess more information about the NFT. We investigate how different royalty payment policies affect the creator's entry decision, signal strategy, and the first buyer's willingness to pay in equilibrium. In contrast, the secondary market involves trading between the first buyer and a subsequent buyer, and is primarily influenced by the royalty rate. Our paper aims to analyze the impact of royalty payment policies on the decisions made by both the creator and buyers, as well as on the total market efficiency.

2.1 Structures of Information Asymmetry at the Primary Market

In our modeling of the NFT trading process, we denote the time of the primary market as $t = 1$ and the time of the secondary market of the first buyer and the second buyer as $t = 2$. We assume that the NFT is initially "unpopular" with $\omega_1 = \text{UP}$ at the beginning, i.e., $t = 2$. As the NFT is transited to the first buyer, it can either remain "unpopular" or become "popular" at $t = 2$. The information asymmetry between the agents on the platform is represented by their beliefs about the probability of the NFT becoming popular at $t = 2$, i.e., $\text{P}(\omega_2 = \text{P})$. We consider two distinct settings regarding the information asymmetry between the creators and the first buyer in the primary market. In the first setting, the first buyer possesses no more information than the creator. In contrast, in the second setting, the first buyer possesses more information than the creator.

2.1.1 First buyer possesses no more information than the creator

Our primary analysis focuses on scenarios where the creator possesses equal or greater information than the first buyer. Let's consider an NFT's true popularity state at stage $t = 2$, denoted by ω_2, which remains unknown to both the creator and potential buyer. The prior distribution for an NFT's popularity at $t = 2$, ω_2, is given by $\text{P}(\omega_2 = \text{P}) = \rho_0$ and $\text{P}(\omega_2 = \text{UP}) = 1 - \rho_0$, and is common knowledge among creators and buyers. In addition to the shared prior distribution, an NFT creator is assumed to have access to a private signal about their NFT, which is not available to buyers. Based on this signal, the creator can update their beliefs about the NFT's popularity to $\text{P}(\omega_2 = \text{P}) = \rho$ and $\text{P}(\omega_2 = \text{UP}) = 1 - \rho$, where ρ represents the updated belief about the NFT becoming "popular" at stage $t = 2$. The belief ρ follows a distribution $G \in \Delta[0, 1]$ with a mean value of ρ_0 and a probability mass function (p.m.f.) $g : [0, 1] \to [0, 1]$ (considering only discrete distributions). The distribution G is public information accessible to buyers, while the value of a creator's ρ remains private to herself.

Our framework accommodates various levels of information asymmetry. For example, if the distribution of ρ has a support $\{0, 1\}$, according to Bayes' rule, we have $g(1) = \rho_0$ and $g(0) = 1 - \rho_0$. In this case, the creator has complete information about the NFT's popularity at $t = 2$. On the other hand, if the distribution of ρ has a support $\{\rho_0\}$, Bayes' rule gives $g(\rho_0) = 1$, implying that the additional signal does not update the creator's belief, and both the creator and the first buyer possess the same information, resulting in no information asymmetry. In this setting, we use the term "quality" to refer to ρ, the creator's belief about the likelihood of their NFT becoming popular. For instance, if a creator's belief regarding the NFT's probability of becoming popular ρ is high, we describe the quality of that creator's NFT as high.

2.1.2 First buyer has more information than the creator

In the alternative setting, we consider the situation where the first buyer has more information than the creator in the primary market. Similar to the previous setting, the creator has a prior probability of the NFT becoming "popular" at stage $t = 2$, denoted as ρ_0. The information possessed by the first buyer is typically described using a probability distribution $H \in \Delta[0, 1]$ with a mean value of ρ_0 and a probability mass function (p.m.f.) $h : [0, 1] \to [0, 1]$. This distribution represents the first buyer's belief regarding the likelihood of the NFT becoming popular at $t = 2$. In this case, the framework captures varying degrees of information asymmetry in favor of the first buyer. Throughout the analysis of this setting, we use the term "quality" to refer to a first buyer's belief about the NFT's probability of becoming popular. By incorporating different information asymmetry settings between the first buyer and the creator, our analysis provides insights into the role of royalty payments under different information structures in the NFT market.

2.2 Royalty Payments

2.2.1 Royalty fees and commission fees

Royalty payments refer to the fees charged by the NFT creator from the buyer who sells the NFT in secondary trading. Besides royalty fees, the platform may also charge a commission fee from NFT sellers, which we denote as $\alpha \in [0, 1]$, and it is set by the platform. In this study, we consider the commission rate as an exogenous variable to focus on the royalty payment policies' effects. We assume the commission fee only applies to the secondary market, consistent with the practices of major platforms such as "OpenSea". In the primary market, the creator can obtain the entire selling price p_1 from selling the NFT. In secondary trading, the seller (i.e., the first buyer) receives $[1 - (\alpha + \beta)]p_2$ when selling an NFT at a price p_2, and the creator obtains βp_2, while the platform receives αp_2. The creator's decision variable of setting the royalty rate is denoted as $\gamma = \alpha + \beta \in [\alpha, 1]$, given the value of α. The choice of γ influences the creator's payoff from selling the NFT in the primary market, which can be influenced by factors such as the creator's belief about the NFT's quality and the degree of information asymmetry between the creator and the first buyer.

2.2.2 Royalty Payments Regulations

In the context of our model, the platform has the flexibility to regulate royalty payments, which can potentially influence market outcomes in the NFT market. These regulations include the following elements:

- **Existence of royalty fees:** The platform can choose to either allow or disallow the implementation of royalty fees. By introducing royalty fees, creators can potentially receive a share of the revenue from secondary sales of their NFTs, which can influence their pricing and entry strategies.

- **Adjustability of royalty rates:** The platform can decide whether creators are allowed to change the royalty rate during the secondary market trading process. Allowing creators to adjust the royalty rate provides them with more flexibility in adapting to market conditions but may also introduce additional complexity and uncertainty for buyers.

- **Restrictions on royalty rates:** The platform can impose restrictions on the royalty rates that creators can set by enforcing both lower and upper bounds. For instance, OpenSea, one of the largest NFT marketplaces, has set a minimum creator earning of 0.5% (as of February 17, 2023). By imposing such restrictions, the platform can maintain a balance between protecting buyers from excessive fees and ensuring creators receive fair compensation for their work.

In this paper, we discuss three different policies paired with royalty rate value restrictions. First, the "No royalty payments" policy where royalty payment is not allowed on the platform. Second, the "flexible royalty rates" policy where creators can change their royalty rate in the secondary market based on current market conditions of the NFT. Third, the "fixed royalty rates" policy where creators can only commit to a royalty rate at the primary market and cannot change it in the secondary market. The royalty rates set under both the "flexible royalty rates" policy and the "fixed royalty rates" policy should also fulfill the value restrictions set by the platform.

By analyzing the impact of these platform regulations on the trading dynamics and market outcomes, we can better understand the role of royalty fees and other policy interventions in shaping the NFT market. This understanding can help inform platform design and policymaking, ultimately contributing to a more efficient and equitable NFT ecosystem.

2.3 Timeline

In this part, we discuss the timing of the model and introduce a timeline.

At the beginning of the model, the platform releases the regulation rule and its commission rate α. The timing of the model can be divided into two stages: the primary market (stage $t = 1$) and the secondary market (stage $t = 2$).

In the first stage, the creator decides whether to join the platform or choose the outside option based on their information about their NFT. If the creator chooses the outside option, they can sell the NFT by $u(\rho)$. If the creator decides to enter the platform, they set a price p_1 for their NFT and a royalty rate β (equivalent to deciding γ). Then, the first buyer arrives, and if they purchase the NFT, they become the NFT holder. Otherwise, the creator leaves the platform and exchanges the NFT for a profit of $u(\rho)$ in the outside option, where ρ represents the quality of the NFT and the profit function $u(\rho)$ is continuously strictly increasing in ρ (that is $\frac{\partial u(\rho)}{\partial \rho} > 0, \forall \rho \in [0, 1]$).

At the beginning of the second stage, the second buyer arrives. All agents are now aware of the NFT's popularity ω, and at this point, buyers' emotional dividends at stage $t = 2$ are realized only by themselves. If allowed, the creator changes the royalty rate, and the NFT holder (i.e., the first buyer) sets the price. The second buyer then decides whether to buy the NFT.

In summary, the timing of the model can be written as follows:

(0) The platform releases the regulation rule and its commission rate α.

(1.1) The creator realizes their NFT's quality and decides whether to sell the NFT on the platform.

(1.2) If the creator joins the platform, they set royalty rate $\gamma = \alpha + \beta$ and price p_1.

(1.3) The first buyer decides whether to buy the NFT based on the signal $s = p_1, \beta$. (the buyer only has the information about all creators' entry decisions and the signal s received from the creator).

(1.4) If the NFT is bought by the first buyer at a price p_1, the first buyer becomes the NFT owner. Otherwise, the creator leaves the platform and sells the NFT to an outside option.

(2.1) The second buyer arrives. The NFT's popularity ($\omega \in \mathrm{UP}, \mathrm{P}$) is realized by all agents, and at this point, buyers' emotional dividends at stage $t = 2$ are realized only by themselves.

(2.2) The creator may change the royalty rate β if allowed.

(2.3) The NFT holder sets the price p_2.

(2.4) The second buyer decides whether to buy the NFT. If the second buyer doesn't buy the NFT, the NFT holder keeps the NFT.

CHAPTER 3

ANALYSIS FOR SECONDARY MARKET

In this section, we begin by analyzing the efficiency of the secondary market at stage $t = 2$ in the presence of royalty payments. The secondary market is of particular interest since it is affected by royalty payments and can provide insights into the effects of different policies. The analysis of the primary market at stage $t = 1$ will be presented in later sections and can vary under different information asymmetry structures.

Since all agents' profits are zero when the NFT is "unpopular" at stage $t = 2$, we focus on the "popular" NFT in this part. An agent's expected profits at $t = 2$ are directly the product of their belief about the probability of the NFT becoming "popular" at $t = 2$ and the profits we compute in the following context.

3.1 Optimal Pricing

As we set, buyers' emotional dividends e are i.i.d. following distribution F with p.d.f. f and support $[0, 1]$. We denote the first buyer's emotional dividend by e_1 and the second buyer's emotional dividend by e_2. For the first buyer with e_1, his optimal pricing strategy is given by solving the following optimization problem:

$$p_2^*(\gamma, e_1) = \operatorname*{argmax}_{p_2 \in [0,1]} v_1(\gamma, e_1, p_2) = \operatorname*{argmax}_{p_2 \in [0,1]} \{(1-\gamma)\Pr(e_2 > p_2)p_2 + \Pr(e_2 \leq p_2)e_1\}$$

where $v_1(\gamma, e_1, p_2) = (1-\gamma)\Pr(e_2 > p_2)p_2 + \Pr(e_2 \leq p_2)e_1 = (1-\gamma)(1-F(p_2))p_2 + F(p_2)e_1$ represents the profits of the first buyer with emotional dividends e_1 and the price p_2 given γ.

To better understand the behavior of buyers and sellers in the secondary market, we can analyze the optimal pricing strategy of the first buyer with emotional dividend e_1 for a given royalty rate γ. Based on the optimal pricing strategy, we can prove several properties of the optimal price p_2. However, before presenting the lemma, we need to add an assumption that $f'(x) \geq -\frac{2f(x)^2}{1-F(x)}, \forall x \in \operatorname{supp}(g)$, which means that there is no dramatic decrease in the distribution of buyers' emotional dividends. Then, we can show the following lemma.

Lemma 1. *Suppose* $f'(x) \geq -\frac{2f(x)^2}{1-F(x)}, \forall x \in \operatorname{supp}(g)$. *The optimal price of a first buyer with* e_1, $p_2^*(\gamma, e_1)$, *is*

(1), increasing in e_1,

(2), increasing in γ,

(3), equal to 1 *for all* $e_1 \geq 1 - \gamma$,

(4), in $(\frac{e_1}{1-\gamma}, 1)$ *for all* $e_1 < 1 - \gamma$, *and*

(5), the unique solution of p_2 *for equation* $f(p_2)(e_1 - (1-\gamma)p_2) + (1 - F(p_2))(1-\gamma) = 0$

(6), $\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2} > 0, \forall p_2 \in (\frac{e_1}{1-\gamma}, p_2^*(\gamma, e_1))$ *and* $\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2} < 0, \forall p_2 \in (p_2^*(\gamma, e_1), 1)$.

Proof. See appendix A.1. □

The lemma shows that the optimal price set by the first buyer is increasing in both e_1 and γ. Buyers with $e_1 > 1 - \gamma$ would set a price that makes the transaction never happen. Additionally, the lemma identifies the range of possible values for the optimal price p_2 based on e_1 and γ. Specifically, the optimal price lies in the interval $(\frac{e_1}{1-\gamma}, 1)$ for all $e_1 < 1 - \gamma$, and it is equal to 1 for all $e_1 \geq 1 - \gamma$. Finally, the lemma shows that the first derivative of the first buyer's expected profit function $v_1(\gamma, e_1, p_2)$ with respect to p_2 is positive for p_2 in $(\frac{e_1}{1-\gamma}, p_2^{(}\gamma, e_1))$, and negative for p_2 in $(p_2^{(}\gamma, e_1), 1)$. These results imply that a higher emotional dividend and a higher royalty rate lead to a higher NFT price and give some properties that can help our following analysis.

3.2 Profits of Buyers

We now investigate how changes in the royalty rate affect the profits of buyers in the secondary market. Specifically, we analyze the profits of both the first buyer (the seller at $t = 2$) and the second buyer (the buyer at $t = 2$), denoted by $V_{1,t=2}(\gamma)$ and $V_{2,t=2}(\gamma)$, respectively.
First Buyer:

$$\begin{aligned}
V_{1,t=2}(\gamma) &= \int_0^1 v_1^*(\gamma, e_1) f(e_1)\ de_1 \\
&= \int_0^1 \left[(1-\gamma)\Pr(e_2 > p_2^*(\gamma, e_1))p_2^*(\gamma, e_1) + \Pr(e_2 \le p_2^*(\gamma, e_1))e_1\right] f(e_1)\ de_1 \\
&= \int_0^{1-\gamma} \left[(1-\gamma)\Pr(e_2 > p_2^*(\gamma, e_1))p_2^*(\gamma, e_1) + \Pr(e_2 \le p_2^*(\gamma, e_1))e_1\right] f(e_1)\ de_1 \\
&\quad + \int_{1-\gamma}^1 e_1 f(e_1)\ de_1
\end{aligned}$$

Second Buyer:

$$\begin{aligned}
V_{2,t=2}(\gamma) &= \int_0^1 f(e_1) \left[\int_{p_2^*(\gamma, e_1)}^1 (e_2 - p_2^*(\gamma, e_1)) f(e_2) de_2\right] de_1 \\
&= \int_0^{1-\gamma} f(e_1) \left[\int_{p_2^*(\gamma, e_1)}^1 (e_2 - p_2^*(\gamma, e_1)) f(e_2) de_2\right] de_1
\end{aligned}$$

To derive the expressions for $V_{1,t=2}(\gamma)$ and $V_{2,t=2}(\gamma)$, we start by considering the optimal pricing strategy of the first buyer, given by $p_2^*(\gamma, e_1)$. Using this strategy, we can express the expected profits of the first buyer as the integral of the profits over all possible emotional dividends e_1, as shown in the equation above. The integral is split into two parts: one for emotional dividends below $1 - \gamma$, which corresponds to cases where the transaction may happen, and one for emotional dividends above $1 - \gamma$, which corresponds to cases where the transaction never happens.

We begin by showing that

$$v_1^*(\gamma, e_1) = v_1(\gamma, e_1, p_2^*(\gamma, e_1)) = (1-\gamma)(1 - F(p_2^*(\gamma, e_1)))p_2^*(\gamma, e_1) + F(p_2^*(\gamma, e_1))e_1$$

, the first buyer's optimal profits with emotional dividend e_1, is decreasing in γ.

Lemma 2. *The first buyer's profits with* e_1, $v_1^*(\gamma, e_1)$, *is decreasing in* γ, $\frac{\partial v_1^*(\gamma,e_1)}{\partial \gamma} < 0, \forall \gamma \in [0, 1)$.

Proof. For any $\gamma_1 < \gamma_2$,

$$\begin{aligned}
v_1^*(\gamma_2, e_1) &= v_1(\gamma_2, e_1, p_2^*(\gamma_2, e_1)) \\
&= (1-\gamma_2)(1 - F(p_2^*(\gamma_2, e_1)))p_2^*(\gamma_2, e_1) + F(p_2^*(\gamma_2, e_1))e_1 \\
&< (1-\gamma_1)(1 - F(p_2^*(\gamma_2, e_1)))p_2^*(\gamma_2, e_1) + F(p_2^*(\gamma_2, e_1))e_1 \\
&\leq (1-\gamma_1)(1 - F(p_2^*(\gamma_1, e_1)))p_2^*(\gamma_1, e_1) + F(p_2^*(\gamma_1, e_1))e_1 = v_1^*(\gamma_1, e_1)
\end{aligned}$$

The last $\leq$ is because $p_2^*(\gamma_1, e_1)$ is the maximizer of $(1-\gamma_1)(1-F(p))p + F(p)e_1$. Hence, $v_1^*(\gamma, e_1)$ is decreasing in γ. □

Using this result, we can prove that both buyers' profits are decreasing in γ.

Proposition 1 (Both buyers' profits are decreasing in γ)**.** $V_{1,t=2}(\gamma)$ *and* $V_{2,t=2}(\gamma)$ *are both decreasing in* γ.

Proof. Since $v_1^*(\gamma, e_1)$ given e_1 is decreasing in γ, $V_{1,t=2}(\gamma) = \int_0^1 v_1^*(\gamma, e_1)de_1$ is also decreasing in γ.

For any $\gamma_1 < \gamma_2$, $p_2^*(\gamma_1, e_1) < p_2^*(\gamma_2, e_1)$ increases.

$$\begin{aligned}
V_{2,t=2}(\gamma_1) &= \int_0^{1-\gamma_1} f(e_1) \left[\int_{p_2^*(\gamma_1,e_1)}^1 (e_2 - p_2^*(\gamma_1, e_1)) f(e_2) de_2 \right] de_1 \\
&\geq \int_0^{1-\gamma_2} f(e_1) \left[\int_{p_2^*(\gamma_1,e_1)}^1 (e_2 - p_2^*(\gamma_1, e_1)) f(e_2) de_2 \right] de_1 \\
&\geq \int_0^{1-\gamma_2} f(e_1) \left[\int_{p_2^*(\gamma_2,e_1)}^1 (e_2 - p_2^*(\gamma_2, e_1)) f(e_2) de_2 \right] de_1 = V_{2,t=2}(\gamma_2)
\end{aligned}$$

Hence, $V_{2,t=2}(\gamma)$ is decreasing in γ. $\square$

The proposition states that $V_{1,t=2}(\gamma)$ and $V_{2,t=2}(\gamma)$ are both decreasing in γ. This result highlights the importance of the commission rate and royalty rate in the market efficiency of the secondary market.

3.3 Profits of Platform and Creator

In the previous section, we discussed the profits of the buyers and the effect of changing royalty rates on their profits. In this part, we shift our focus to the platform and the creator's profits. The commission fee for the platform and the royalty fee for the creators are both based on the expected total transition value at $t = 2$. We derive the expected total transition value, denoted by $\pi_{t=2}(\gamma)$, which is the integral of the probability of the NFT becoming "popular" multiplied by the optimal price set by the first buyer.

$$\begin{aligned} \pi_{t=2}(\gamma) &= \int_0^1 \Pr(e_2 > p_2^*(\gamma, e_1)) p_2^*(\gamma, e_1) f(e_1)\ de_1 \\ &= \int_0^{1-\gamma} [1 - F(p_2^*(\gamma, e_1))] p_2^*(\gamma, e_1) f(e_1)\ de_1 \end{aligned}$$

Proposition 2 (Total Transition Value is decreasing in γ)**.** $\pi_{t=2}(\gamma)$ *is decreasing in* γ *and* $\pi_{t=2}(1) = 0$*.*

Proof. See appendix A.2 $\square$

Proposition 2 shows that the total transition value is decreasing in γ, meaning that higher commission rates and royalty rates lead to lower total transition value.

The profits of the platform and creators are proportional to the total transition value, and we can find the optimal royalty rate for creators conditional on their entry decisions and the NFT's popularity. The lemma states that there exists a royalty rate, denoted by b_2^*, that maximizes the creator's profits from the royalty fee.

Lemma 3. *There exists a royalty rate* $\beta = b_2^* \in (0, 1-\alpha)$ *maximizing creator's profits from royalty fee,* $\beta\pi_{t=2}(\gamma)$.

Proof. We can find that $\beta\pi_{t=2}(\gamma)\Big|_{\beta=0} = 0$, $\beta\pi_{t=2}(\gamma)\Big|_{\beta=1-\alpha} = 0$, and $\beta\pi_{t=2}(\gamma) > 0, \forall\beta \in (0, 1-\alpha)$. According to the continuity of the function $\beta\pi_{t=2}(\gamma)$, we can prove there exists a maximizer $\beta = b_2^* \in (0, 1-\alpha)$. □

It is important to note that if royalty rates are adjustable on the platform, creators should always set their royalty rate to $\mathrm{Proj}_{[\underline{B},\overline{B}]}(b_2^*)$ at $t = 2$, and buyers will expect this as well. In this case, $\underline{B}$ represents the lower bound of the royalty rate set by the platform, $\overline{B}$ denotes the upper bound, and $\mathrm{Proj}_{[\underline{B},\overline{B}]}(b_2^*)$ signifies the projection of b_2^* onto the interval $[\underline{B}, \overline{B}]$. Consequently, under "no royalty payments" and "flexible royalty rate" regulations, the royalty rate cannot serve as a signal for creators to reveal quality information, as it is independent of the NFT's quality.

Overall, our analysis shows that the royalty rate and commission rate affect the profits of all parties in the market. It highlights the trade-off between the platform's and creators' profits and the buyers' willingness to pay.

3.4 Social Surplus

To analyze the overall welfare implications of the platform's pricing strategy, we define the total social surplus as the sum of all agents' profits at that stage. From the previous section, we have obtained expressions for the profits of the first and second buyers, as well as the profits of the platform and creators. Combining these results, we obtain the total social surplus function, $S(\gamma)$.

$$
\begin{aligned}
S(\gamma) =& V_{1,t=2}(\gamma) + (\alpha+\beta)\pi_{t=2}(\gamma) + V_{2,t=2}(\gamma) \\
=& \int_0^1 \left[(1-\gamma)\Pr(e_2 > p_2^*(\gamma, e_1))p_2^*(\gamma, e_1) + \Pr(e_2 \le p_2^*(\gamma, e_1))e_1\right] f(e_1)\ de_1 \\
& + \gamma \int_0^1 \Pr(e_2 > p_2^*(\gamma, e_1))p_2^*(\gamma, e_1)\ de_1 \\
& + \int_0^1 f(e_1) \left[\int_{p_2^*(\gamma, e_1)}^1 (e_2 - p_2^*(\gamma, e_1))f(e_2)de_2\right] de_1 \\
=& \int_0^1 f(e_1) \left[\int_{p_2^*(\gamma, e_1)}^1 e_2 f(e_2)de_2\right] de_1 + \int_0^1 F(p_2^*(\gamma, e_1))e_1 f(e_1)\ de_1 \\
=& \int_0^1 f(e_1) \left[\int_{p_2^*(\gamma, e_1)}^1 e_2 f(e_2)de_2 + F(p_2^*(\gamma, e_1))e_1\right]\ de_1
\end{aligned}
$$

It is worth noting that the total social surplus depends on the optimal pricing strategy of the first buyer, which in turn depends on the distribution of the first buyers' emotional dividends. The total social surplus function can be expressed as a double integral with respect to the distributions of both buyers' emotional dividends.

Proposition 3 (Social Surplus is decreasing in γ)**.** *$S(\gamma)$ is decreasing in γ.*

Proof. Instead of considering the $S(\gamma)$, we consider the $s(\gamma, e_1) = \int_{p_2^*(\gamma,e_1)}^1 e_2 f(e_2)de_2 + F(p_2^*(\gamma, e_1))e_1$. The derivative of the $s(\gamma, e_1)$ about γ is

$$
\begin{aligned}
\frac{\partial s(\gamma, e_1)}{\partial \gamma} &= \left.\frac{\partial \int_{p_2}^1 e_2 f(e_2)de_2 + F(p_2)e_1}{\partial p_2}\right|_{p_2 = p_2^*(\gamma, e_1)} \frac{\partial p_2^*(\gamma, e_1)}{\partial \gamma} \\
&= (e_1 - p_2^*(\gamma, e_1))f(p_2^*(\gamma, e_1))\frac{\partial p_2^*(\gamma, e_1)}{\partial \gamma}
\end{aligned}
$$

As we proved in previous lemma $p_2^*(\gamma, e_1) > \frac{e_1}{1-\gamma} \ge e_1$ and $\frac{\partial p_2^*(\gamma,e_1)}{\partial \gamma} \ge 0$, $\frac{\partial s(\gamma,e_1)}{\partial \gamma} \le 0$. Then, we can prove $S(\gamma) = \int_0^{1-\gamma} f(e_1)s(\gamma, e_1)de_1$ is decreasing in γ. $\square$

This result implies that the platform's optimal pricing strategy reduces the total social surplus when the commission and royalty rates increase. In other words, a higher commission rate and/or royalty rate reduce the overall welfare gains from the transaction. Therefore, the platform must balance the desire for higher revenue with the potential welfare losses associated with higher commission and royalty rates.

3.5 Inefficiency Effect of Royalty Payments

To further analyze the inefficiency effect of royalty payments for creators, we first establish a proposition regarding the maximum profits a creator can achieve through the entire process when the creator and the first buyer have symmetric information about the NFT's probability of becoming popular. The maximum profits a creator can get from a popular NFT with a royalty rate β is equal to the first buyer's profits at $t = 2$ plus the profits from the royalty fee, denoted as

$$\overline{M}(\beta) = V_{1,t=2}(\gamma) + \beta\pi_{t=2}(\gamma)$$

We can state that the maximum profits the creator can extract when the NFT is "popular" at $t = 2$, $\overline{M}(\beta)$, is decreasing in β, which implies that if a creator can capture all of the profits from the first buyer (or their information is symmetric), then it is optimal for the creator not to set a royalty rate. Setting a royalty fee at $t = 2$ can reduce the efficiency of the market.

Proposition 4 (Ineffeciency Effect)**.** *The maximal profits the creator can extract when the NFT is "popular" at* $t = 2$*,* $\overline{M}(\beta) = V_{1,t=2}(\gamma) + \beta\pi_{t=2}(\gamma)$*, is decreasing in* β*, i.e.,* $\frac{\partial \overline{M}(\beta)}{\partial \beta} < 0, \forall \beta \in [0, 1-\alpha)$.

Proof. The maximal profits the creator can extract

$$V_{1,t=2}(\gamma) + \beta\pi_{t=2}(\gamma) = \int_0^1 v_1(\alpha, e_1, p_2^*(\gamma, e_1))f(e_1)\ de_1$$

where $v_1(\alpha, e_1, p_2^*(\gamma, e_1)) = (1-\alpha)\text{Pr}(e_2 > p_2^*(\gamma, e_1))p_2^*(\gamma, e_1) + \text{Pr}(e_2 \leq p_2^*(\gamma, e_1))e_1$.

Since $\alpha < \gamma$ and $p_2^*(\cdot, e_1)$ is increasing function, $p_2^*(\gamma, e_1) > p_2^*(\alpha, e_1)$. Based on the previous lemma we proved, $\frac{\partial v_1(\alpha, e_1, p_2)}{\partial p_2} < 0, \forall p_2 \in (p_2^*(\alpha, e_1), 1)$. Hence,

$\left.\frac{\partial v_1(\alpha,e_1,p_2)}{\partial p_2}\right|_{p_2=p_2^*(\gamma,e_1)} < 0.$

$$\frac{\partial v_1(\alpha, e_1, p_2^*(\gamma, e_1))}{\partial \beta} = \left.\frac{\partial v_1(\alpha, e_1, p_2)}{\partial p_2}\right|_{p_2=p_2^*(\gamma,e_1)} \frac{\partial p_2^*(\gamma, e_1)}{\partial \gamma}\frac{\partial \gamma}{\partial \beta} < 0$$

So $v_1(\alpha, e_1, p_2^*(\gamma, e_1))$ is decreasing in β. Then the $\overline{M}(\beta) = V_{1,t=2}(\gamma) + \beta\pi_{t=2}(\gamma)$ is also decreasing in β. $\square$

When the creator can extract all of the profits from the first buyer through both the selling price and royalty fee, setting a higher royalty fee will lead to a lower selling price at $t = 1$, which increases the selling price at $t = 2$ and then decreases the likelihood of the transaction occurring at $t = 2$. In this case, setting a royalty rate would be detrimental to the creator's profits when they can already capture the maximum amount (or their information is symmetric).

In summary, the inefficiency effect of royalty payments for creators is an important issue to consider. If creators can capture all of the profits from the first buyer or their information is symmetric, then setting a royalty rate may decrease the efficiency of the market and lead to lower profits for the creator. It is important to take into account the potential inefficiency effect when considering the use of royalty payments in the NFT market.

3.6 Simple Example: Uniform Distributions

It is useful to give insights by providing simple examples. To help our process of giving simple examples in the following sections, we give a simple example at stage $t = 2$ firstly.

In this part, we consider the distribution of buyers' emotional dividends e follows uniform distribution $e \sim \text{Unif}(0, 1)$ i.e, $F(e) = e, f(e) = 1, \forall e \in [0, 1]$.

The optimal pricing strategy of a first buyer with e_1 given γ is:

$$p_2^*(\gamma, e_1) = \underset{p_2 \in [0,1]}{\text{argmax}}\{(1-\gamma)(1-p_2)p_2 + p_2 e_1\} = \begin{cases} 1, & e_1 \geq 1-\gamma \\ \frac{1}{2}\left(\frac{e_1}{1-\gamma}+1\right), & e_1 < 1-\gamma \end{cases}$$

The expected profits of the first buyer are:

$$\begin{aligned} V_{1,t=2}(\gamma) &= \int_0^{1-\gamma} (1-\gamma)(1-p_2^*(\gamma,e_1))p_2^*(\gamma,e_1) + p_2^*(\gamma,e_1)e_1 \ de_1 + \int_{1-\gamma}^1 e_1 de_1 \\ &= \frac{(1-\gamma)^2}{12} + \frac{1}{2} \end{aligned}$$

The expected profits of the second buyer are:

$$V_{2,t=2}(\gamma) = \int_0^{1-\gamma} \left[\int_{p_2^*(\gamma,e_1)}^1 [e_2 - p_2^*(\gamma,e_1)]de_2\right] de_1 = \frac{1-\gamma}{24}$$

The expected total transition value is:

$$\pi_{t=2}(\gamma) = \int_0^{1-\gamma} (1-p_2^*(\gamma,e_1))p_2^*(\gamma,e_1)de_1 = \frac{1-\gamma}{6}$$

The expected profits of the platform is $\alpha\pi_{t=2}(\gamma) = \frac{\alpha(1-\gamma)}{6}$.

The expected profits of the creator from royalty fee is

$$\beta\pi_{t=2}(\gamma) = \frac{\beta(1-\gamma)}{6}$$

which is concave and maximized at $\beta = b_2^* = \frac{1-\alpha}{2}$.

The total social surplus is:

$$S(\gamma) = \int_0^{1-\gamma} \left[\int_{p_2^*(\gamma,e_1)}^1 e_2 \ de_2 + p_2^*(\gamma,e_1)e_1\right] de_1 + \int_{1-\gamma}^1 e_1 \ de_1 = \frac{(2\gamma+3)(1-\gamma)}{24} + \frac{1}{2}$$

CHAPTER 4

MAIN MODEL

In this section, we will analyze the main model of our study, which investigates how the asymmetric information affects the behavior of creators and first buyers in the primary market. We assume that creators have more information or equal information compared to buyers, and their belief follows the distribution G, while buyers' beliefs are always ρ_0. Creators possess NFTs with quality following distribution $G \in \Delta[0,1]$ with probability mass function (p.m.f.) $g : [0,1] \to [0,1]$. The support set of g is denoted by $\text{supp}(g) = \{x \in \text{supp}(g) : g(x) \neq 0\}$, which is a discrete set based on our assumption.

When the first buyer purchases an NFT at stage $t = 1$, their profit function at $t = 2$ regarding γ is denoted as $V_{1,t=2}(\gamma)$, which is decreasing in γ. The creator's profit function from royalty payments at $t = 2$ regarding γ is denoted as $\beta\pi_{t=2}(\gamma)$, and the platform's profits are denoted as $\alpha\pi_{t=2}(\gamma)$. Here, $\pi_{t=2}(\gamma)$ represents the expected total transition value at stage $t = 2$, which decreases in γ and is equal to zero when $\gamma = 1$.

4.1 Equilibrium

4.1.1 Signaling and Entry

In this section, we introduce the model settings for the entry decisions and signaling strategies of creators in the NFT trading process. A creator needs to make two sequential decisions: whether to join the market and what signal to release if they choose to participate.

To model the entry behavior of creators, we use $\Theta(\rho)$ to denote the proportion of creators who hold NFTs of quality ρ and participate in the market among all potential creators. We impose the constraint that $\Theta(\rho) \leq g(\rho)$ for all $\rho \in [0,1]$, and $\sum_{x \in \text{supp}(g)} \Theta(x) \leq 1$. Additionally, we use $\overline{\Theta}(\rho)$ to represent the proportion of creators who possess NFTs of quality ρ but do not participate in the market among all creators. The functions $\Theta(\cdot)$ and $\overline{\Theta}$ satisfy the condition $\Theta(\rho) + \overline{\Theta}(\rho) = g(\rho)$ for all $\rho \in [0,1]$. We assume that there are n potential creators, with n being sufficiently large ($n \to \infty$). The marginal difference in entry situation caused by one creator's deviation is denoted by $\epsilon = \frac{1}{n} \to 0^+$.

If a creator joins the market, they release a signal denoted by $s = \{p, \beta\}$, which comprises two components: the price p_1 and royalty rate β. If a transaction occurs under commitment $s = \{p, \beta\}$, the expected profits of a creator holding an NFT with quality ρ are given by:

$$\mathbb{E}\Pi_{\text{creator}}(\rho \mid s) = p + \rho\beta\pi_{t=2}(\beta + \alpha)$$

We define the set of all signals presented in the market as

$$\mathcal{S} = \{s_k = \{p_k, \beta_k\}, i = 1, 2, ..., N\}$$

where N is the number of signal types in the market.

For any signal $s_i = \{p_i, \beta_i\} \in \mathcal{S}$, we use $\theta(s_i, \rho) \in (0,1]$ to denote the distribution of creators who hold NFTs with quality ρ, participate in the platform, and send signal s_i. These signals are subject to the restriction that $\Theta(\rho) = \sum_{s \in \mathcal{S}} \theta(s, \rho) \leq g(\rho)$ for all $\rho \in \text{supp}(g)$. Thus, we can use

$$\{\theta(s, \rho)\}_{s \in \mathcal{S}, \rho \in \text{supp}(g)}$$

to represent any entry decisions and signaling strategies of creators at the same time.

4.1.2 Buyer's Beliefs

In this section, we examine the buyer's decision-making process in response to a given signal s_i, which relies on their updated beliefs. Assuming the buyer is informed about the prior distribution of creator types in the market, denoted by $\{\Theta(\rho)\}_{\rho \in \text{supp}(g)}$, they update their beliefs regarding the creator's type to $\{\hat{\theta}(\rho \mid \Theta, s)\}_{\rho \in \text{supp}(g)}$ upon receiving signal s. The function $\hat{\theta}(\cdot \mid \Theta, s) : [0, 1] \to \Delta[0, 1]$ represents the buyer's updating rule and $\sum_{\rho \in \text{supp}(g)} \hat{\theta}(\rho \mid \Theta, s) = 1$. With beliefs $\{\hat{\theta}(\rho \mid \Theta, s)\}_{\rho \in \text{supp}(g)}$, the expected quality of the NFT in the buyer's belief is

$$\sum_{\rho \in \text{supp}(g)} \rho \hat{\theta}(\rho \mid \Theta, s_i)$$

The buyer's reserve price in creators' entry situation Θ, when receiving signal s_i, is calculated based on their expected profits from purchasing the NFT according to their updated beliefs, as illustrated in the following equation:

$$\psi(\Theta, s_i) = \sum_{\rho \in \text{supp}(g)} \rho \hat{\theta}(\rho \mid \Theta, s_i) V_{1,t=2}(\beta_i + \alpha)$$

Moreover, the set of signals that can motivate the buyer to purchase is defined by:

$$H(\hat{\theta}, \Theta) = \{\{p_i, \beta_i\} : p_i \leq \psi(\Theta, \{p_i, \beta_i\})\}$$

4.1.3 Equilibrium

In this section, we define a Perfect Bayesian Equilibrium for the game under study. We describe creators' actions by $\{\theta(s,\rho)\}_{s,\rho}$, the buyer's belief given s by $\{\hat{\theta}(\rho \mid \Theta, s)\}_{\rho}$, and the buyer's action given $s = \{p, \beta\}$ by $\begin{cases} \text{Purchase}, & p \le \psi(\Theta, s) \\ \text{Not Purchase}, & p > \psi(\Theta, s) \end{cases}$. So, we can use $\left(\{\theta(s,\rho)\}_{s,\rho}, \hat{\theta}\right)$ to represent an equilibrium of the game.

We discuss equilibrium of the game based on two part:

1. **Entry Game**: The game between creators that decides the creators' entry situation Θ.

2. **Signaling Game**: Given the creators' entry situation Θ, the signaling game between creators who enter the market and the buyer.

Definition 1 (Perfect Bayesian Equilibrium of Signaling Game)**.** *Given a creators' entry situation Θ, a set of creators' entry situations and signal strategies, $\{\theta(s,\rho)\}_{s,\rho}$, along with the first buyer's beliefs updating rule, $\hat{\theta}$, form a Perfect Bayesian Equilibrium $\left(\{\theta(s,\rho)\}_{s,\rho}, \hat{\theta}\right)$ if and only if the following conditions are satisfied:*

(1). ***Creators who choose to join the market have no incentive to release other signals****. For a creator holding a NFT with quality ρ_i that enters the market, the signal s_i he releases should be the one that maximizes his profits among all signals, i.e.,*

$$s_i \in \underset{s \in H(\hat{\theta}, \Theta)}{\operatorname{argmax}} \mathbb{E}\Pi_{\text{creator}}(\rho_i \mid s), \quad \forall s_i, \rho_i \text{ such that } \theta(s_i, \rho_i) > 0 \tag{Con. 1}$$

(2). ***The buyer's belief is consistent with the distribution of creators' type on the path:***

$$\hat{\theta}(\rho \mid \Theta, s) = \frac{\theta(s,\rho)}{\sum_{\rho' \in \operatorname{supp}(g)} \theta(s, \rho')}, \quad \forall s, \rho \text{ with } \theta(s,\rho) > 0 \tag{Con. 2}$$

Definition 2 (Equilibrium of Entry Game)**.** *Given the optimal profit function of a type-ρ_i creator from joining a market with creators' entry situation Θ', $\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta') = \text{argmax}_{s \in H(\hat{\theta}, \Theta')} \mathbb{E}\Pi_{\text{creator}}(\rho_i \mid s)$, the entry situation of creators Θ is a Nash equilibrium if and only if the following conditions are satisfied:*

(3). ***Creators who choose to join the market have no incentive to leave.***

$$\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta) \geq u(\rho_i), \quad \forall \rho_i \text{ such that} \Theta(\rho_i) > 0 \tag{Con. 3}$$

(4). ***Creators who choose not to join the market have no incentive to join.***

$$\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta^{+(\rho_i)}) < u(\rho_i), \quad \forall \rho_i \text{ such that } \Theta(\rho_i) = 0 \tag{Con. 4}$$

In this definition, $\Theta^{+(\rho_i)}(\rho) = \begin{cases} \Theta(\rho), & \rho \neq \rho_i \\ \Theta(\rho) + \epsilon, & \rho = \rho_i \end{cases}$ represents the updated entry situation if a creator holding an NFT with quality ρ_i decides to enter the market that has entry situation Θ.

4.1.4 Equilibrium Refinement: Same Price for Same Royalty Rate

In this section, we refine the equilibrium with a proposition that demonstrates the domination of a signal with a higher price over a signal with a lower price when both signals have the same royalty rate and induce the buyer to purchase.

Lemma 4. *Given $p_1 > p_2$, suppose signals $s_1 = \{p_1, \beta_*\}$ and $s_2 = \{p_2, \beta_*\}$ both induce purchases, then*

$$\mathbb{E}\Pi_{\text{creator}}(\rho \mid s_1) > \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_2), \quad \forall \rho \in \text{supp}(g)$$

Proof. Intuitively, $\mathbb{E}\Pi_{\text{creator}}(\rho \mid s_1) = p_{1,1} + \rho\beta_*\pi_{t=2}(\beta_* + \alpha) > p_{1,2} + \rho\beta_*\pi_{t=2}(\beta_* + \alpha) = \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_2), \forall \rho \in \text{supp}(g)$. □

According to this lemma, when the royalty rate is fixed, a creator will aim to set the highest possible price for their NFT, regardless of its quality. In other words, when fixing the royalty rate β_i, only the highest price that can persuade the buyer to purchase matters, and all rational creators will set this price.

Proposition 5. *Given an equilibrium, there doesn't exist two signals $s_1 = \{p_1, \beta_1\} \in \mathcal{S}$ and $s_2 = \{p_2, \beta_2\} \in \mathcal{S}$ such that $p_1 \neq p_2$ and $\beta_1 = \beta_2$.*

Proof. Consider any $s_1, s_2 \in \mathcal{S}$ with $\beta_1 = \beta_2$. (Let the order being $p_1 \geq p_2$). Assume $p_1 > p_2$. Based on lemma 4, we know

$$\mathbb{E}\Pi_{\text{creator}}(\rho \mid s_1) > \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_2), \forall \rho \in \text{supp}(g) \tag{1}$$

Since $s_2 \in \mathcal{S}$, $\theta(s_2, \rho_2) > 0$ must hold for some $\rho_2 \in \text{supp}(g)$. According to the Con. 1 in definition 1, s_2 with $\theta(s_2, \rho_2) > 0$ must satisfy

$$s_2 \in \underset{s \in H(\hat{\theta}, \Theta)}{\text{argmax}}\, \mathbb{E}\Pi_{\text{creator}}(\rho_2 \mid s)$$

Then, we have $\mathbb{E}\Pi_{\text{creator}}(\rho_2 \mid s_2) \geq \mathbb{E}\Pi_{\text{creator}}(\rho_2 \mid s_1)$, which contradicts Equation (1) above. Therefore, $p_1 = p_2$ is proved. s_1 and s_2 must be the same signal if $\beta_1 = \beta_2$. $\square$

This proposition shows that in a perfect Bayesian equilibrium, two creators cannot commit the same royalty rate but set different prices. Based on this proposition, we can simplify our equilibrium analysis by considering that there is only one unique kind of signal given a specific royalty rate. The price in equilibrium, conditional on the royalty rate β_i, is represented by

$$p^*(\beta_i) = \max\{p : \{p, \beta_i\} \in H(\hat{\theta}, \Theta)\}$$

Moreover, we can assume a creator who enters the market always prefer the signal with higher price at $t = 2$ if there are more than one signals can generate the highest profits for the creator. Based on the proposition 5, the signal with a higher price is equivalent to the signal with a lower royalty rate. This assumption can restrict our attention on the game that has unique mapping between the creator's type and optimal signaling strategy. We use $s^*(\rho)$ to represent the signal that a type-ρ creator sends in equilibrium, which means $\theta(s^*(\rho),\rho)=\Theta(\rho)$.

4.1.5 Off-Path Beliefs Assumption

In the game, we need to consider a "reasonable" beliefs of the buyer. Because there are infinite possible signals that can be sent by creators, the buyer's off-path beliefs play an important role in forming a perfect Bayesian equilibrium. To refine equilibrium, we add an assumption on the buyer's off-path beliefs in any equilibrium.

Assumption 1 (Off-Path Beliefs in Equilibrium)**.** *Consider an equilibrium with* $\left(\{\theta(s,\rho)\}_{s,\rho},\hat{\theta}\right)$*. For an off-path signal* $s_i=\{p_i,\beta_i\}$*, define the set of creator types such that deviating to signal* s_i *can, in the best case (i.e., the transaction happens), give higher profits than then profits in equilibrium as*

$$\mathcal{R}(s_i)=\{\rho\in\operatorname{supp}(g):\mathbb{E}\Pi_{\text{creator}}(\rho\mid s^*(\rho))<\mathbb{E}\Pi_{\text{creator}}(\rho\mid s_i)\}$$

If $\mathcal{R}(s_i)=\emptyset$*, this signal is dominated by other signals for all creators. In this case, we do not need to discuss this signal, as no creator would send it, regardless of the buyer's beliefs. If* $\mathcal{R}(s_i)\neq\emptyset$*, we assume that the buyer's beliefs should reflect the fact that all creators with a type in* $\mathcal{R}(s_i)$ *have the motivation to deviate to the signal, as follows:*

$$\hat{\theta}(\rho\mid\Theta,s_i)=\frac{\Theta(\rho)}{\sum_{\rho'\in\mathcal{R}(s_i)}\Theta(\rho')},\quad\forall\rho\in\mathcal{R}(s_i)$$

According to this assumption, the types that stand to benefit from the deviation must have relative probabilities that are identical to those of their prior probabilities, meaning that beliefs must shift as little as possible. This assumption is covered in detail in the paper [Grossman and Perry, 1986].

In other words, this assumption states that all creators who could potentially obtain higher profits from a signal are considered to have the motivation to deviate to that signal in the buyer's beliefs. There is no creator type that is less likely to deviate in the buyer's beliefs.

4.2 Same Royalty Rate

In this section, we begin our analysis of the situation where all creators have to set the same royalty rate. Specifically, we consider the regulations "no royalty payments" (all creators have to set the royalty rate 0) and "flexible royalty rate" (given the popularity information, all creators will set the same royalty rate to maximize their profits at $t = 2$). We fix $\beta = \beta_* \in [0, 1]$ for all signals, and thus there is only one potential royalty rate signal in the market. Based on the Proposition 5, we know there is only one signal $s_* = \{p^*(\beta_*), \beta_*\}$ in equilibrium. Then, we can use creators' entry situation $\{\Theta(\rho)\}_\rho$ to directly represent the $\{\theta(s_*, \rho)\}_\rho$ in equilibrium,

$$\Theta(\rho) = \theta(s_*, \rho), \quad \forall \rho \in \mathrm{supp}(g)$$

4.2.1 Signaling Game

Then, according to the consistency of the buyer's on-path beliefs and creators' types in equilibrium (Con. 2 in Definition 1), we can know

$$\hat{\theta}(\rho \mid \Theta, s_*) = \frac{\theta(s_*, \rho)}{\sum_{\rho' \in \mathrm{supp}(g)} \theta(s_*, \rho')} = \frac{\Theta(\rho)}{\sum_{\rho' \in \mathrm{supp}(g)} \Theta(\rho')} \qquad \text{(On-Path Belief)}$$

In an equilibrium that has the common signal $s_* = \{p_*, \beta_*\}$, the buyer's off-path beliefs based on the assumption 1 should follow

$$\hat{\theta}(\rho \mid \Theta, \{p, \beta_*\}) = \frac{\Theta(\rho)}{\sum_{\rho' \in \mathrm{supp}(g)} \Theta(\rho')}, \quad \forall p > p_* \qquad \text{(Off-Path Belief)}$$

Lemma 5. *Given the entry situation of creators Θ, the common signal $s_* = \{p_*, \beta_*\}$ in equilibrium when the royalty rate is fixed should fulfill*

$$p_* = \frac{\sum_{\rho \in \mathrm{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \mathrm{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$$

Proof. Suppose $p_* > \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$. Based on the On-Path Belief, the reserve price of the buyer given s_* is

$$\psi(\Theta, s_*) = \sum_{\rho \in \text{supp}(g)} \rho\hat{\theta}(\rho \mid \Theta, s_*) V_{1,t=2}(\beta_* + \alpha) = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$$

So, s_* is not in $H(\hat{\theta}, \Theta)$, the transaction won't happen. We can conclude $p_* \leq \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$.

Suppose $p_* < \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$. Based on the Off-Path Belief, the reserve price of the buyer given $\{\frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_*\}$ is

$$\psi\left(\Theta, \{\frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_*\}\right) = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$$

So, $\{\frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_*\}$ is $H(\hat{\theta}, \Theta)$, all creators have incentive to deviate to releasing signal $\{\frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_*\}$ instead of s_*. So, $p_* < \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$ can't hold in equilibrium.

All in all, $p_* = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$ is the only possible value in equilibrium. □

Based on this lemma, we only need to discuss the potential equilibrium with unique signal

$$s_* = \left\{ p_* = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_* \right\}$$

Proposition 6 (Unique Equilibrium of Signaling Game)**.** *Given the entry situation of creators* Θ *and fixed royalty rate* β_**, there is a unique perfect Bayesian equilibrium of the signaling game such that the buyer's beliefs fulfill On-Path Belief and Off-Path Belief and* $\theta(s_*, \rho) = \Theta(\rho), \forall \rho \in \text{supp}(g)$*, where* $s_* = \{p_*, \beta_*\}$ *and* $p_* = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha)$.

Proof.

1. **Proof for Con. 1:** Based on Off-Path Belief, any signal $s_i = \{p_i, \beta_*\}$ with $p_i > p_*$ leads to reserve price

$$\psi(\Theta, s_i) = \frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha) = p_*$$

which means s_i can't induce transaction, $s_i \notin H(\hat{\theta}, \Theta)$. So, Con. 1 is proved.

2. **Proof for Con. 2:** exactly the On-Path Belief.

□

4.2.2 Entry Game

Based on the unique equilibrium in the Proposition 6, we can get

$$\begin{aligned}
\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta') &= \underset{s\in H(\hat{\theta},\Theta')}{\text{argmax}}\, \mathbb{E}\Pi_{\text{creator}}(\rho_i \mid s) \\
&= \frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta'(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta'(\rho)} V_{1,t=2}(\beta_* + \alpha) + \rho_i\beta_*\pi_{t=2}(\beta_* + \alpha)
\end{aligned}$$

Then, we can get the sufficient and necessary conditions for the outside option $u(\cdot)$ to give the existence of an equilibrium for the entry game between creators.

Proposition 7 (Equilibrium Existence of Entry Game)**.** *Given the optimal profit function of a type-ρ_i creator from joining a market with creators' entry situation Θ', $\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta') = \frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta'(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta'(\rho)} V_{1,t=2}(\beta_* + \alpha) + \rho_i\beta_*\pi_{t=2}(\beta_* + \alpha)$, the entry situation of creators Θ is a Nash equilibrium if and only if the following conditions are satisfied:*

Condition 1:

$$\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha) + \rho_i\beta_*\pi_{t=2}(\beta_* + \alpha) \geq u(\rho_i), \quad \text{for all } \rho_i \in \{y : \Theta(y) > 0\} \tag{Condition 1}$$

Condition 2':

$$\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)+\rho_i\epsilon}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)+\epsilon}V_{1,t=2}(\beta_*+\alpha)+\rho_i\beta_*\pi_{t=2}(\beta_*+\alpha)\le u(\rho_i),$$
$$\text{for all } \rho_i\in\{y:\overline{\Theta}(y)>0\} \tag{Condition 2'}$$

Proof. Directly given by definition 2 □

Lemma 6. *Condition 2' is equivalent to*

$$\mathbb{E}\Pi^*_{\text{creator}}(\rho_i,\Theta)<u(\rho_i),\ \text{for all } \rho_i\in\left\{y:\overline{\Theta}(y)>0, y>\frac{\sum_{x\in\text{supp}(g)}x\Theta(x)}{\sum_{x\in\text{supp}(g)}\Theta(x)}\right\} \tag{Condition 2.1}$$

$$\mathbb{E}\Pi^*_{\text{creator}}(\rho_i,\Theta)\le u(\rho_i),\ \text{for all } \rho_i\in\left\{y:\overline{\Theta}(y)>0, y\le\frac{\sum_{x\in\text{supp}(g)}x\Theta(x)}{\sum_{x\in\text{supp}(g)}\Theta(x)}\right\} \tag{Condition 2.2}$$

Proof. See appendix A.3.3 □

Another important property of the entry game is that all creators whose NFTs have a quality greater than the average quality in the market choose to join the market.

Lemma 7. *In an equilibrium Θ with $\mathbb{E}\Pi^*_{\text{creator}}(\rho_i,\Theta)=\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}V_{1,t=2}(\beta_*+\alpha)+\rho_i\beta_*\pi_{t=2}(\beta_*+\alpha)$, $\Theta(\rho)=g(\rho)$ for all $\rho\in\{y:\Theta(y)>0, y>\frac{\sum_{x\in\text{supp}(g)}x\Theta(x)}{\sum_{x\in\text{supp}(g)}\Theta(x)}\}$.*

Proof. Suppose there is an equilibrium Θ and ρ such that $\Theta(\rho)>0$ and $\rho>\frac{\sum_{x\in\text{supp}(g)}x\Theta(x)}{\sum_{x\in\text{supp}(g)}\Theta(x)}$. We assume $\Theta(\rho)<g(\rho)$, i.e., $\overline{\Theta}(\rho)>0$.

Since $\Theta(\rho)>0$ and $\overline{\Theta}(\rho)>0$, an equilibrium Θ needs to satisfy two conditions:

(1). Condition 1: $\frac{\sum_{x\in\text{supp}(g)}x\Theta(x)}{\sum_{x\in\text{supp}(g)}\Theta(x)}V_{1,t=2}(\gamma)+\rho\beta\pi_{t=2}(\gamma)\ge u(\rho)$

(2). Condition 2.1: $\frac{\sum_{x\in\text{supp}(g)}x\Theta(x)}{\sum_{x\in\text{supp}(g)}\Theta(x)}V_{1,t=2}(\gamma)+\rho\beta\pi_{t=2}(\gamma)<u(\rho)$

where $\gamma = \beta_* + \alpha$.

However, these two conditions can't hold at the same time. Then, we can conclude $\Theta(\rho) = g(\rho)$ if $\Theta(\rho) > 0$ and $\rho > \frac{\sum_{x \in \mathrm{supp}(g)} x\Theta(x)}{\sum_{x \in \mathrm{supp}(g)} \Theta(x)}$ in an equilibrium Θ. □

This lemma states that in an equilibrium, creators holding NFTs with quality higher than the buyer's belief will either all join the platform or all choose the outside option. With this result, we can refine the equilibrium by focusing on the subset of creators who have NFTs with quality lower than or equal to the buyer's belief.

4.2.3 Traditional Lemons Market (No Royalty Payments)

To highlight the impact of royalty payments, we first present a proposition that establishes the result in the absence of royalties, i.e., when $\beta = 0$, under the "no royalty payments" regulation.

Proposition 8 (High quality creators leave the market first without royalty payments)**.** *Given $\beta = 0$, in an equilibrium, we have $\Theta(y) = g(y)$ for all $y \in [0, \rho)$ where $\rho = \mathrm{argmax}\{x : \Theta(x) > 0\}$.*

Proof. Since $\Theta(\rho) > 0$, the Condition 1 in definition need to be satisfied,

$$\frac{\sum_{x \in \mathrm{supp}(g)} x\Theta(x)}{\sum_{x \in \mathrm{supp}(g)} \Theta(x)} V_{1,t=2}(\gamma) \geq u(\rho)$$

Since $u(\cdot)$ is strictly increasing, we can conclude that $\frac{\sum_{x \in \mathrm{supp}(g)} x\Theta(x)}{\sum_{x \in \mathrm{supp}(g)} \Theta(x)} V_{1,t=2}(\gamma) \geq u(y), \forall y \in [0, \rho)$. Hence, all creators holding NFT with quality $y \in [0, \rho)$ join the platform, i.e., $\Theta(y) = g(y), \forall y \in [0, \rho)$. □

This proposition indicates that if a creator holding an NFT with a particular quality joins the platform, all creators holding lower-quality NFTs than the creator must also enter the market. This result is consistent with the traditional lemons market, where creators holding high-quality NFTs are more likely to leave the market than creators holding low-quality NFTs due to information asymmetry.

4.2.4 Market with Royalty Payments

To explore the effect of royalty payments on market outcomes, we now turn our attention to the equilibrium properties of the market under the "flexible royalty rate" regulation. In contrast to the previous proposition, we find that the impact of quality on market participation can be altered when royalty payments are introduced. Specifically, we show that in the presence of royalty payments, low-quality creators may be the first to leave the market, while high-quality creators remain. This finding stands in stark contrast to the traditional lemons market, where high-quality sellers are typically the first to exit. We formalize this finding in the following proposition:

Proposition 9 (With royalty payments, low quality creators may leave the market while high quality creators remain.)**.** *Under the "flexible royalty rate" regulation with* $\beta > 0$*, there exists an outside option structure* $u(\cdot)$ *such that an equilibrium with* $\Theta(y) < g(y)$ *for some* $y \in [0, \rho)$ *exists, where* $\rho = \text{argmax}\{x : \Theta(x) > 0\}$*.*

Proof. Based on the definition of equilibrium, we know the ρ with $\Theta(\rho) > 0$ must satisfy

$$\frac{\sum_{x\in\text{supp}(g)} x\Theta(x)}{\sum_{x\in\text{supp}(g)} \Theta(x)} V_{1,t=2}(\gamma) + \rho\beta\pi_{t=2}(\gamma) \geq u(\rho) \qquad \text{(Condition 1)}$$

As we proved in lemma 7, $\Theta(r) = g(r)$ for all $r \in \{y : \Theta(y) > 0, y > \frac{\sum_{x\in\text{supp}(g)} x\Theta(x)}{\sum_{x\in\text{supp}(g)} \Theta(x)}\}$, so $\Theta(y) < g(y)$ can be satisfied only if $y \leq \frac{\sum_{x\in\text{supp}(g)} x\Theta(x)}{\sum_{x\in\text{supp}(g)} \Theta(x)}$. Then, the condition of $\overline{\Theta}(y) > 0$ is

$$\frac{\sum_{x\in\text{supp}(g)} x\Theta(x)}{\sum_{x\in\text{supp}(g)} \Theta(x)} V_{1,t=2}(\gamma) + y\beta\pi_{t=2}(\gamma) \leq u(y) \qquad \text{(Condition 2.2)}$$

As we showed before, $\beta\pi_{t=2}(\gamma) > 0, \forall\beta \in (0, 1)$. There exists strictly increasing $u(\cdot)$ fulfills

$$u(\rho) \leq \frac{\sum_{x\in\text{supp}(g)} x\Theta(x)}{\sum_{x\in\text{supp}(g)} \Theta(x)} V_{1,t=2}(\gamma) + \rho\beta\pi_{t=2}(\gamma)$$
$$u(y) \geq \frac{\sum_{x\in\text{supp}(g)} x\Theta(x)}{\sum_{x\in\text{supp}(g)} \Theta(x)} V_{1,t=2}(\gamma) + y\beta\pi_{t=2}(\gamma)$$

that is, condition 1 and condition 2.2 can be fulfilled at the same time. □

The above proposition highlights a fundamental difference in the impact of quality on market participation in the presence of royalty payments compared to the traditional lemons market. In the absence of royalty payments, creators holding high-quality NFTs leave the market first due to asymmetric information. Buyers are only willing to pay the same price, resulting in creators receiving the same profit regardless of their NFT's quality. However, in the presence of royalty payments, high-quality creators can expect higher returns due to the royalty fees. As a result, a market equilibrium can emerge where high-quality creators remain in the market, while low-quality creators choose to leave.

Overall, the proposition emphasizes the potential for royalty payments to reshape market dynamics and create more favorable conditions for high-quality NFT creators.

4.2.5 Inefficiency Effect and Transfer Effect

For a buyer with updating rule $\hat{\theta}$ given the creators' entry situation Θ, his belief about the NFT's quality when receiving a signal s_i should follow

$$\delta(s_i) = \sum_{x \in \operatorname{supp}(g)} x\hat{\theta}(x \mid \Theta, s_i)$$

Then, the set of signals that can motivate the buyer to purchase can be rewritten in the form:

$$H(\hat{\theta}, \Theta) = \{s_i = \{p_i, \beta_i\} : p_i \leq \delta(s_i) V_{1,t=2}(\beta_i + \alpha)\}$$

We introduce a more general profit function for a creator holding an NFT with quality ρ_j and royalty rate β in a market where buyers have a belief about the NFT's quality of δ. The expected profit function of selling NFT in this situation is given by:

$$\begin{aligned} M_\delta(\beta, \lambda) &= \delta V_{1,t=2}(\gamma) + \rho_j \beta \pi_{t=2}(\gamma) \\ &= \delta \left(V_{1,t=2}(\gamma) + \lambda \beta \pi_{t=2}(\gamma) \right), \text{ where } \lambda = \frac{\rho_j}{\delta} \in (0, \infty) \end{aligned}$$

The parameter λ represents the ratio of the actual quality of the NFT held by the creator to the buyers' belief about the quality of the NFT. If $\lambda < 1$, this indicates that the creator holds an NFT with quality below the buyers' beliefs. Conversely, if $\lambda > 1$, it suggests that the creator holds an NFT with quality higher than the buyers' beliefs.

Specifically, when the buyers' beliefs align with the actual quality of the NFT held by the creator, i.e., $\lambda = 1$, we have $M_\delta(\beta, 1) = \delta\overline{M}(\beta)$. This expression is proportional to the maximal profits $\overline{M}(\beta)$ that we discussed earlier. Hence, we can divide a creator's profits into two composition

$$
\begin{aligned}
M_\delta(\beta,\lambda) = &\underbrace{\delta\overline{M}(0)}_{\text{Profits can be extracted from a buyer with belief } \delta \text{ in the absence of royalty payments}} \\
&+ \underbrace{\delta(\overline{M}(\beta) - \overline{M}(0))}_{\text{Inefficiency Effect}} + \underbrace{\delta(\lambda - 1)\beta\pi_{t=2}(\gamma)}_{\text{Transfer Effect}}
\end{aligned}
$$

The first component represents the profits that the creator can extract from a buyer with belief δ when there are no royalty payments. Setting a royalty rate $\beta > 0$ generates two effects on a creator's profits, which we refer to as:

1. **Inefficiency Effect:** $\delta(\overline{M}(\beta) - \overline{M}(0))$. The second component represents the market inefficiency caused by the royalty rate. As we have proven in a previous proposition 4, $\overline{M}(\beta)$ is a decreasing function, so this effect should be negative for all $\beta > 0$. The royalty rate impedes some transactions from occurring at stage $t = 2$, decreasing the first buyer's total surplus in the whole process. Since the creator may extract the buyer's total surplus when they have symmetric information, the existence of the royalty rate can decrease the creator's profits when the creator and the first buyer have symmetric information.

2. **Transfer Effect:** $\delta(\lambda - 1)\beta\pi_{t=2}(\gamma)$. The third component is generated by the asymmetric information between the creator and the first buyer. When there is no asymmetric information (i.e., $\lambda = \frac{\rho_j}{\delta} = 1$), this effect does not exist. If the NFT quality is higher than the buyers' belief (i.e., $\lambda = \frac{\rho_j}{\delta} > 1$), the creator can charge a higher royalty fee than the buyer's expectation, benefitting the

creator but hurting the buyer. Conversely, if the NFT quality is lower than the buyers' belief (i.e., $\lambda = \frac{\rho_j}{\delta} < 1$), the royalty fee charged by the creator is lower than the buyer's expectation, resulting in royalty payments that hurt the creator but benefit the buyer.

In summary, the effects of royalty payments on a creator's profits depend on the balance between the inefficiency effect and the transfer effect. The inefficiency effect highlights the potential negative impact of royalty fees on market efficiency, while the transfer effect demonstrates how royalty payments can redistribute value between creators and buyers depending on the extent of asymmetric information. By carefully considering these factors, it is possible to create a more efficient NFT market that balances the profits of creators, buyers, and platforms.

Proposition 10. *Given an entry situation Θ of creators and $\delta = \frac{\sum_{x \in \mathrm{supp}(g)} x\Theta(x)}{\sum_{x \in \mathrm{supp}(g)} \Theta(x)}$, for all $\beta \in (0, 1-\alpha)$, setting a royalty rate β increases the profits for creators who hold NFTs with quality $\rho_j > \delta\left(1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right)$ and decreases the profits for creators who hold NFTs with quality $\rho_j < \delta\left(1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right)$.*

Proof. Since $\beta\pi_{t=2}(\gamma) > 0, \forall \beta \in (0, 1-\alpha)$, we can find sum effect of setting royalty rate β,

$$\delta(\overline{M}(\beta) - \overline{M}(0)) + \delta(\lambda - 1)\beta\pi_{t=2}(\gamma) > 0 \Leftrightarrow \lambda > 1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}$$
$$\delta(\overline{M}(\beta) - \overline{M}(0)) + \delta(\lambda - 1)\beta\pi_{t=2}(\gamma) < 0 \Leftrightarrow \lambda < 1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}$$

where $\lambda = \frac{\rho_j}{\delta}$. □

This proposition shows that setting a royalty rate can consistently discourage some creators holding low-quality NFTs from joining the market, while encouraging some creators holding high-quality NFTs to enter the market. This is because the creators' profits are shifted to the second stage, causing a decrease in the selling price during the first stage. For low-quality creators, their earnings generated by the strategic pooling of relatively high-quality creators and information asymmetry are reduced.

However, since the quality of their NFTs is low, the royalty fee they collect in the second stage is also low, resulting in a decrease in their overall earnings from joining the market. For high-quality creators, the losses caused by information asymmetry in the first stage, where they must pool with low-quality creators, are reduced. Moreover, after the popularity information is revealed in the second stage, the expected royalty fees they collect increase proportionally to their quality, leading to a rise in their overall earnings from joining the market. This proposition suggests that a platform may be able to drive low-quality creators out of the market and attract high-quality creators, who might not have initially participated, by implementing royalty payments.

4.3 Royalty Rate as a Signal

In this section, we extend our analysis to the case where creators can commit to different royalty rates, which can serve as a signal to release quality information. We focus on the "fixed royalty rate" regulation in this scenario.

4.3.1 Pooling Equilibrium

Before giving the analysis of potential separate equilibrium, we first analyze the pooling equilibrium where all creators set the same royalty rate. Based on Lemma 5, we know the common signal $s = \{p, \beta\}$ in pooling equilibrium with entry situation Θ must fulfill $p = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta + \alpha)$. Then, we can induce the form of common signal s_* must satisfy

$$s_* = \left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_* \right\}$$

for some $\beta_* \in [0, 1]$

Proposition 11 (Necessary Conditions for Pooling Equilibrium)**.** *Given the entry situation of creators Θ, if there is a pooling perfect Bayesian equilibrium of the signaling game, $\left(\{\theta(s_*, \rho)\}_\rho, \hat{\theta}\right)$, such that the buyer's beliefs fulfill On-Path Belief and Off-Path Belief and $\theta(s_*, \rho) = \Theta(\rho), \forall \rho \in \text{supp}(g)$, the following conditions must be fulfilled for $s_* = \{p_*, \beta_*\}$.*

$$p_* = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha) \tag{Condition 1}$$

$$\beta_* = \min \left\{ \operatorname*{argmax}_{\beta \in [0,1]} \max_{\rho : \Theta(\rho) > 0} \mathbb{E}\Pi_{\text{creator}} \left(\rho \mid \left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta + \alpha), \beta \right\} \right) \right\} \tag{Condition 2}$$

Proof. See Appendix A.4. □

This proposition shows that if there exists a pooling equilibrium, the structure of signals in the equilibrium must satisfy $\left\{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}V_{1,t=2}(b^*_{\text{pool}}+\alpha), b^*_{\text{pool}}\right\}$, where

$$\begin{aligned}
b^*_{\text{pool}} &= \min\left\{\underset{\beta\in[0,1]}{\text{argmax}} \max_{\rho:\Theta(\rho)>0} \mathbb{E}\Pi_{\text{creator}}\left(\rho \mid \left\{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}V_{1,t=2}(\beta+\alpha), \beta\right\}\right)\right\} \\
&= \min\left\{\underset{\beta\in[0,1]}{\text{argmax}}\, M_{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}}\left(\beta, \frac{\bar{\rho}\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}\right)\right\} \\
&\quad \text{where } \bar{\rho} = \max\{\rho : \Theta(\rho) > 0\}
\end{aligned}$$

In a pooling equilibrium, creators with the highest-quality NFTs establish an optimal signal for themselves, which other creators then imitate.

Contrasting with the flexible royalty rate policy, wherein all creators set b^*_2 when the NFT becomes popular at stage 2, a fixed royalty rate policy allows creators to make commitments. In a pooling equilibrium, creators commit to a royalty rate b^*_{pool}, which is lower than b^*_2.

Lemma 8. $0 \leq b^*_{\text{pool}} < b^*_2$.

Proof. If $b^*_{\text{pool}} = 0$, the lemma obviously holds. So, we discuss the situation that $b^*_{\text{pool}} > 0$.

$$\begin{aligned}
b^*_{\text{pool}} &= \min\left\{\underset{\beta\in[0,1]}{\text{argmax}}\, M_{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}}\left(\beta, \frac{\bar{\rho}\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}\right)\right\} \\
&= \min\left\{\underset{\beta\in[0,1]}{\text{argmax}} \frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}V_{1,t=2}(\beta+\alpha) + \bar{\rho}\beta\pi_{t=2}(\beta+\alpha)\right\}
\end{aligned}$$

Firstly, $M_{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}}\left(\beta, \frac{\bar{\rho}\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}\right) < M_{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}}\left(b^*_2, \frac{\bar{\rho}\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}\right)$ for all $\beta > b^*_2$ because $\bar{\rho}\beta\pi_{t=2}(\beta+\alpha)$ is maximized at b^*_2 and $V_{1,t=2}(\beta+\alpha)$ is decreasing in β. So, $b^*_{\text{pool}} \leq b^*_2$.

Secondly, as b^*_{pool} is not a marginal solution, the gradient must be zero

$$\left.\frac{\partial M_{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}}\left(\beta,\frac{\bar{\rho}\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}\right)}{\partial\beta}\right|_{\beta=b^*_{\text{pool}}}=0$$

Because $\left.\frac{\partial M_{\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}}\left(\beta,\frac{\bar{\rho}\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}\right)}{\partial\beta}\right|_{\beta=b^*_2}=\frac{\sum_{\rho\in\text{supp}(g)}\rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)}\Theta(\rho)}\left.\frac{\partial V_{1,t=2}(\beta+\alpha)}{\partial\beta}\right|_{\beta=b^*_2}<0,$
we can conclude $b^*_{\text{pool}}\neq b^*_2$.
Hence, $b^*_{\text{pool}}<b^*_2$ is proved. $\square$

The commitment choice results in a reduced royalty rate when the NFT becomes popular in equilibrium, thereby weakening (decreasing the absolute values of) both the inefficiency effect and the transfer effect caused by the royalty rate.

$$\begin{aligned}M_\delta(b^*_{\text{pool}},\lambda)=&\underbrace{\delta\overline{M}(0)}_{\text{Profits can be extracted from a buyer with belief }\delta\text{ in the absence of royalty payments}}\\&+\underbrace{\delta(\overline{M}(b^*_{\text{pool}})-\overline{M}(0))}_{\text{Inefficiency Effect}}+\underbrace{\delta(\lambda-1)b^*_{\text{pool}}\pi_{t=2}(b^*_{\text{pool}}+\alpha)}_{\text{Transfer Effect}}\end{aligned}$$

We observe that, in a pooling equilibrium, the commitment choice lowers the royalty rate and diminishes the impact of the royalty rate on creators' profits. The effect of offering a commitment choice is akin to setting an upper bound on the royalty rate for creators. Imposing an upper bound is relatively easier to control and can establish a value that optimizes the achievement of various goals which can't be achieved by giving commitment choice. Therefore, this effect of providing a commitment choice in a pooling equilibrium is not very desirable. We will look at the separate equilibrium that can only be achieved by giving creators commitment choices.

4.3.2 Necessary Conditions for Separate Equilibrium

Based on our intuition, the ability to set a royalty rate as a signal for NFT quality information may lead to market segmentation, where NFTs are separated based on their quality. We aim to explore the equilibrium where different creators are separated to some extent based on their quality in this section.

To show the desired effect of flexible royalty payments, using some necessary conditions for a separate equilibrium is enough.

Consider a separate equilibrium $\left(\{\theta(s,\rho)\}_{s,\rho},\hat{\theta}\right)$ of the signaling game such that there are $N \geq 2$ different signals in the market. Signals are denoted as $\mathcal{S} = \{s_i, i = 1,...,N\}$. Based on the Con. 1 of definition 1,

$$s_i \in \underset{s\in H(\hat{\theta},\Theta)}{\operatorname{argmax}} \mathbb{E}\Pi_{\text{creator}}(\rho \mid s), \quad \forall \rho \text{ such that } \theta(s_i,\rho) > 0,\ i = 1,...,N$$

Hence, we can conclude $s_i \in H(\hat{\theta},\Theta), i = 1,...,N$ and

$$\mathbb{E}\Pi_{\text{creator}}(\rho \mid s_i) \geq \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_j),\ \text{for all } i,j \in \{1,2,...,N\}, \rho \in \{y : \theta(s_i,y) > 0\} \tag{NC 1}$$

4.3.3 Theorem: Separation Based on Quality

Building on our intuition, setting a high royalty rate and a low first-stage price can serve as a signal for high-quality NFTs, while a low royalty rate and a medium price can be a signal for NFTs with not-so-high quality. We can extend our intuition to this general setting and present the result as a theorem that shows creators are separated based on their NFTs' qualities in separate equilibrium.

Theorem 1 (creators are separated based on quality). *In a separate equilibrium $\left(\{\theta(s,\rho)\}_{s,\rho},\hat{\theta}\right)$ of the signaling game such that there are $N \geq 2$ different signals in the market. Signals are denoted as $\mathcal{S} = \{s_i, i = 1,...,N\}$. The order of signals follows $\beta_1\pi_{t=2}(\beta_1+\alpha) < \beta_2\pi_{t=2}(\beta_2+\alpha) < ... < \beta_N\pi_{t=2}(\beta_N+\alpha)$. Then, following properties of the separate equilibrium hold*

(1). $\text{argmax}\{x : \theta(s_i, x) > 0\} < \text{argmin}\{x : \theta(s_j, x) > 0\}$, $\forall i, j \in \{1, 2, ..., N\}, i < j$.

(2). $p_1 > p_2 > ... > p_N$

Proof. See appendix A.5. $\square$

This important theorem clarifies the underlying mechanisms of the NFT market when creators can commit to a royalty fee. The theorem suggests that if a set of entry decisions $\{\theta(s_i, \rho), i = 1, 2, ..., N\}$ with beliefs $\hat{\theta}$ forms an equilibrium, the resulting market structure is consistent with our intuition, emphasizing the role of royalty fees as a signaling tool. In this equilibrium, creators are grouped according to their NFT quality.

Low-quality creators tend to set higher first-stage prices and lower royalty fees when the NFT becomes popular. This strategy allows them to differentiate from high-quality creators by setting a higher initial price and committing to charge a lower royalty fee, signaling their relatively lower NFT quality.

In contrast, high-quality creators prefer lower first-stage prices and higher royalty fees as the NFT gains popularity. This choice demonstrates their confidence in the quality of their NFTs, expecting greater long-term returns through higher royalty fees after the NFT becomes popular.

The theorem emphasizes the importance of informative signals in the NFT market. By allowing creators to commit to a royalty fee, the market provides an opportunity for them to send "informative" signals that convey valuable information about their NFTs' quality. This mechanism helps buyers make better-informed decisions and can potentially reduce adverse selection problems caused by information asymmetry.

CHAPTER 5

ANALYSIS FOR BINARY EXAMPLE

5.1 Some Assumptions

In this section, we discuss the situation where there are two qualities of NFT, $\text{supp}(g) = \{\rho_l, \rho_h\}$, with $\rho_l < \rho_h$. The proportions of each type of NFT are $g_l = g(\rho_l)$ and $g_h = g(\rho_h)$, where $g_l + g_h = 1$. The profits from the outside option for each type of NFT are u_l and u_h, with $u_l \leq u_h$. To satisfy Bayes' rule, $\rho_0 = g_l\rho_l + g_h\rho_h$.

In this example section, we impose certain restrictions that allow us to focus on the most relevant scenarios and investigate the role of royalty payments in the trading process.

We have already established some properties of $\beta\pi_{t=2}(\gamma)$, a creator's profits from royalty fees when the NFT is popular, namely:

1. $\beta\pi_{t=2}(\gamma) = 0$ when $\beta = 0$ or $\beta = 1 - \alpha$.
2. $\beta\pi_{t=2}(\gamma) > 0$ when $\beta \in (0, 1 - \alpha)$.
3. $\beta\pi_{t=2}(\gamma)$ is maximized at $\beta = b_2^* \in (0, 1-\alpha)$. (which requires $\left.\frac{\partial \beta\pi_{t=2}(\gamma)}{\partial \beta}\right|_{\beta = b_2^*} = 0$)

We have also demonstrated some properties of $V_{1,t=2}(\gamma)$, the first buyer's profits at $t = 2$, namely:

1. $V_{1,t=2}(\gamma)$ is decreasing in β, $\frac{\partial V_{1,t=2}(\gamma)}{\partial \beta} < 0, \forall \beta \in [0, 1 - \alpha)$.
2. $V_{1,t=2}(\gamma) \geq \int_0^1 e_1 f(e_1)\ de_1, \forall \beta \in [0, 1 - \alpha]$, where equality holds if and only if $\beta = 1 - \alpha$.

3. $\overline{M}(\beta) = V_{1,t=2}(\gamma) + \beta\pi_{t=2}(\gamma)$ is decreasing in β, $\frac{\partial \overline{M}(\beta)}{\partial \beta} = \frac{\partial V_{1,t=2}(\gamma)}{\partial \beta} + \frac{\partial \beta\pi_{t=2}(\gamma)}{\partial \beta} < 0, \forall \beta \in [0, 1-\alpha)$.

In this section, to restrict our attention to effects of different royalty payments regulations, we introduce the following assumptions:

Assumption 2. *$\beta\pi_{t=2}(\gamma)$ is a function that is concave in β.*

Under this assumption, we have $\frac{\partial^2 \beta\pi_{t=2}(\gamma)}{\partial \beta^2} < 0$, $\frac{\partial \beta\pi_{t=2}(\gamma)}{\partial \beta} > 0, \forall \beta \in (0, b_2^*)$ and $\frac{\partial \beta\pi_{t=2}(\gamma)}{\partial \beta} < 0, \forall \beta \in (b_2^*, 1-\alpha)$. This assumption implies that the profits the creator derives from royalty fees increase as β increases in $[0, b_2^*)$, while the profits decrease as β increases in $(b_2^*, 1-\alpha)$. Additionally, the rate of increase slows down as β increases in $[0, b_2^*)$, while the rate of decrease accelerates as β increases in $(b_2^*, 1-\alpha)$.

Assumption 3. $\frac{\partial^2 V_{1,t=2}(\gamma)}{\partial \beta^2} > 0$

This assumption suggests that the rate of decrease in the first buyer's profits at $t = 2$ diminishes as β increases in $[0, 1-\alpha)$.

Assumption 4. $\frac{\partial^2 V_{1,t=2}(\gamma)}{\partial \beta^2} < -\frac{\partial^2 \beta\pi_{t=2}(\gamma)}{\partial \beta^2}$, $\forall \beta \in [0, 1-\alpha)$.

This assumption implies that the rate of change in the creator's profits derived from royalty fees is always greater than the rate of change in the decreasing speed of the first buyer's profits at $t = 2$ as β increases.

Lemma 9. *For $\lambda < 1$, $M_\delta(\beta, \lambda)$ is decreasing in $\beta \in [0, b_2^*]$*

Proof. $M_\delta(\beta, \lambda) = \delta\overline{M}(\beta) + \delta(\lambda - 1)\beta\pi_{t=2}(\gamma)$, both components are decreasing in $\beta \in [0, b_2^*]$. □

5.2 Royalty Rate Maximizes Creators' Profits

Based on these assumptions, we can establish a property of the maximizer of a creator's profits function, given buyers' beliefs.

Proposition 12. *With assumptions A1, 3, and 4, given buyers' belief about the NFT's quality being δ and the quality of a creator's NFT being ρ_j (or directly given $\lambda = \frac{\rho_j}{\delta}$),*

(1). the $M_\delta(\beta, \lambda)$ is concave in β for $\lambda > \min_{\beta \in (0,1-\alpha)} \left\{1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta \pi_{t=2}(\gamma)}\right\}$.

And the β that maximizes the profits of the creator $M_\delta(\beta, \lambda)$, b^*_λ, exhibits the following behavior:

(2). When $\lambda < \min_{\beta \in (0,1-\alpha)} \left\{1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta \pi_{t=2}(\gamma)}\right\}$, $b^*_\lambda = 0$.

(3). When $\lambda \geq \min_{\beta \in (0,1-\alpha)} \left\{1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta \pi_{t=2}(\gamma)}\right\}$, b^*_λ exhibits the following properties:

(3.1). b^*_λ is the unique solution of β for $\frac{\partial V_{1,t=2}(\gamma)}{\partial \beta} + \lambda \frac{\partial \beta \pi_{t=2}(\gamma)}{\partial \beta} = 0$

(3.2). b^*_λ is increasing in $\lambda \in [\min_{\beta \in (0,1-\alpha)} \left\{1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta \pi_{t=2}(\gamma)}\right\}, \infty)$.

(3.3). $b^*_\lambda = 0$ when $\lambda = \min_{\beta \in (0,1-\alpha)} \left\{1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta \pi_{t=2}(\gamma)}\right\}$.

(3.4). $b^*_\lambda \to b^*_2$ as $\lambda \to \infty$.

Proof. See appendix A.6. □

This proposition aims to elucidate the structure of the optimal royalty rate in the context of different NFT quality and buyer's belief. It posits that the profit of a creator, whose NFT quality is overestimated by buyers, is maximized when no royalty payments are involved. Conversely, as the creator's NFT becomes increasingly underestimated by buyers, the optimal royalty rate for the creator correspondingly increases. What's more, we can also find the royalty rate maximizes a creator's profits is always lower than the royalty rate that maximizes a creator's buyer from royalty fee at $t = 2$.

By establishing the relationship between the creator's NFT quality, buyer's belief', and the optimal royalty rate, this proposition provides valuable insights into the strategic decisions creators face when determining the royalty rates for their NFTs.

In the following we will discuss how the royalty payments work on shaping market equilibrium and then the platform's profits, giving instructions on how to regulate the royalty payments for the platform.

5.3 Same Royalty Rate

In this section we discuss the "no royalty payments", "flexible royalty rates" regulations with cap (lower bounds and upper bounds), and pooling equilibrium in "fixed royalty rates" regulations.

Given an equilibrium with Θ, the platform's profits are

$$\mathbb{E}\Pi_{\text{platform}}(\Theta,\gamma) = \left[\rho_l\Theta(\rho_l) + \rho_h\Theta(\rho_h)\right]\alpha\pi_{t=2}(\gamma)$$

Creators extract the first buyer's all expected profits in equilibrium:

$$\mathbb{E}\Pi_{\text{buyer1}}(\Theta,\gamma) = 0$$

The second buyer's expected profits are

$$\mathbb{E}\Pi_{\text{buyer2}}(\Theta,\gamma) = \left[\rho_l\Theta(\rho_l) + \rho_h\Theta(\rho_h)\right]V_{2,t=2}(\gamma)$$

The low-quality creators' profits are

$$\mathbb{E}\Pi_{\text{creator}}(\rho_l,\Theta,\gamma) = \max\left\{u_l, \frac{\rho_l\Theta(\rho_l) + \rho_h\Theta(\rho_h)}{\Theta(\rho_l) + \Theta(\rho_h)}V_{1,t=2}(\gamma) + \rho_l\beta\pi_{t=2}(\gamma)\right\}$$

The high-quality creators' profits are

$$\mathbb{E}\Pi_{\text{creator}}(\rho_h,\Theta,\gamma) = \max\left\{u_h, \frac{\rho_l\Theta(\rho_l) + \rho_h\Theta(\rho_h)}{\Theta(\rho_l) + \Theta(\rho_h)}V_{1,t=2}(\gamma) + \rho_h\beta\pi_{t=2}(\gamma)\right\}$$

The total social surplus in the platform is

$$\mathbb{E}S(\Theta,\gamma) = \left[\rho_l\Theta(\rho_l) + \rho_h\Theta(\rho_h)\right]S(\gamma) - \left[u_l\Theta(\rho_l) + u_h\Theta(\rho_h)\right]$$

Based on the lemma 7, we have no need to discuss the situation that partial high-quality creators join the market since it can't happen in equilibrium. In the next, let's find the conditions that can form following equilibrium, (E.1). **None joins the market.** Condition 2' need to be satisfied that $\rho_l\overline{M}(\beta) \leq u(\rho_l)$ and $\rho_h\overline{M}(\beta) \leq u(\rho_h)$.

This equilibrium holds if and only if

$$\overline{M}(\beta) < \min\{\frac{u_l}{\rho_l}, \frac{u_h}{\rho_h}\}$$

In this equilibrium, the platform and buyers' profits are zero. All creators sell their NFTs to outside options.

$$\mathbb{E}\Pi_{\text{platform}} = \mathbb{E}\Pi_{\text{buyer1}} = \mathbb{E}\Pi_{\text{buyer2}} = \mathbb{E}S = 0$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = u_l,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = u_h$$

(E.2). **Only low-quality creators join the market.** That is $\Theta(\rho_l) = g_l, \Theta(\rho_h) = 0$. This equilibrium holds if and only if

(a) Condition 1: $\rho_l\overline{M}(\beta) = \rho_l V_{1,t=2}(\gamma) + \rho_l\beta\pi_{t=2}(\gamma) \geq u_l$

(b) Condition 2.1: $M_{\rho_l}(\beta, \frac{\rho_h}{\rho_l}) = \rho_l V_{1,t=2}(\gamma) + \rho_h\beta\pi_{t=2}(\gamma) < u_h$

The profits and surplus are

$$\begin{aligned}
&\mathbb{E}\Pi_{\text{platform}} = \rho_l g_l \alpha\pi_{t=2}(\gamma) \\
&\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = \rho_l g_l V_{2,t=2}(\gamma) \\
&\mathbb{E}\Pi_{\text{creator}}(\rho_l) = \rho_l V_{1,t=2}(\gamma) + \rho_l\beta\pi_{t=2}(\gamma),\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = u_h \\
&\mathbb{E}S = \rho_l g_l S(\gamma) - u_l g_l
\end{aligned}$$

(E.3). **Only high-quality creators join the market.** That is $\Theta(\rho_l) = 0, \Theta(\rho_h) = g_h$. This equilibrium holds if and only if

(a) Condition 1: $\rho_h\overline{M}(\beta) = \rho_h V_{1,t=2}(\gamma) + \rho_h\beta\pi_{t=2}(\gamma) \geq u_h$

(b) Condition 2.2: $M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h}) = \rho_h V_{1,t=2}(\gamma) + \rho_l\beta\pi_{t=2}(\gamma) \leq u_l$

The profits and surplus are

$$
\begin{aligned}
&\mathbb{E}\Pi_{\text{platform}} = \rho_h g_h \alpha \pi_{t=2}(\gamma) \\
&\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = \rho_h g_h V_{2,t=2}(\gamma) \\
&\mathbb{E}\Pi_{\text{creator}}(\rho_l) = u_l,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = \rho_h V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma) \\
&\mathbb{E}S = \rho_h g_h S(\gamma) - u_h g_h
\end{aligned}
$$

(E.4). **Partial low-quality creators and all high-quality creators join the market.** Suppose the buyers' belief $\delta = \frac{\rho_l \Theta(\rho_l) + \rho_h g_h}{\Theta(\rho_l) + g_h}$.
This equilibrium holds if and only if

(a) Condition 1 and Condition 2.2 for low-quality creator:

$$
\left.
\begin{aligned}
M_\delta(\beta, \tfrac{\rho_l}{\delta}) &= \delta V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma) \geq u_l \\
M_\delta(\beta, \tfrac{\rho_l}{\delta}) &= \delta V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma) \leq u_l
\end{aligned}
\right\} \Rightarrow
$$

$$
M_\delta(\beta, \frac{\rho_l}{\delta}) = \delta V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma) = u_l \qquad \text{(eq.4.1)}
$$

(b) Condition 1 for high quality creator:

$$
M_\delta(\beta, \frac{\rho_h}{\delta}) = \delta V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma) \geq u_h \qquad \text{(eq.4.2')}
$$

Subtracting (eq.4.2') by (eq.4.1):

$$
\beta \pi_{t=2}(\gamma) \geq \frac{u_h - u_l}{\rho_h - \rho_l} \qquad \text{(eq.4.2)}
$$

With $\delta = \frac{\rho_l \Theta(\rho_l) + \rho_h g_h}{\Theta(\rho_l) + g_h}$, we can compute the unique solution of (eq.4.1):

$$
\Theta(\rho_l) = \frac{\rho_h V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma) - u_l}{u_l - \rho_l V_{1,t=2}(\gamma) - \rho_l \beta \pi_{t=2}(\gamma)} g_h = \frac{M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h}) - u_l}{u_l - \rho_l \overline{M}(\beta)}
$$

To let $\Theta(\rho_l)$ fall in $(0, g_l)$:

$$\begin{aligned} u_l \in & \left(\frac{M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h}) + g_l \rho_l \overline{M}(\beta)}{1+g_l}, M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h}) \right) \\ = & \left(\frac{\rho_h + g_l \rho_l}{1+g_l} V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma), \rho_h V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma) \right) \end{aligned}$$

need to be satisfied.

The profits and surplus are

$$\begin{aligned} &\mathbb{E}\Pi_{\text{platform}} = \left[\rho_l \Theta(\rho_l) + \rho_h g_h\right] \alpha \pi_{t=2}(\gamma) \\ &\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = \left[\rho_l \Theta(\rho_l) + \rho_h g_h\right] V_{2,t=2}(\gamma) \\ &\mathbb{E}\Pi_{\text{creator}}(\rho_l) = u_l,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = \frac{\rho_l \Theta(\rho_l) + \rho_h g_h}{\Theta(\rho_l) + g_h} V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma) \\ &\mathbb{E}S = \left[\rho_l \Theta(\rho_l) + \rho_h g_h\right] S(\gamma) - \left[u_l \Theta(\rho_l) + u_h g_h\right] \end{aligned}$$

(E.5). **All low-quality creators and all high-quality creators join the market.** The buyers' belief is exactly

$$\rho_0 = \frac{\rho_l g_l + \rho_h g_h}{g_l + g_h}$$

This equilibrium holds if and only if

(a) Condition 1 for low-quality creator:

$$M_{\rho_0}(\beta, \frac{\rho_l}{\rho_0}) = \rho_0 V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma) \geq u_l$$

(b) Condition 1 for high-quality creator:

$$M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0}) = \rho_0 V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma) \geq u_h$$

The profits and surplus are

$$\mathbb{E}\Pi_{\text{platform}} = \rho_0 \alpha \pi_{t=2}(\gamma)$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = \rho_0 V_{2,t=2}(\gamma)$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = \rho_0 V_{1,t=2}(\gamma) + \rho_l \beta \pi_{t=2}(\gamma),\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = \rho_0 V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma)$$
$$\mathbb{E}S = \rho_0 S(\gamma) - (u_l g_l + u_h g_h)$$

5.4 Royalty Rate as a Signal

In this section, we discuss the regulation of "fixed royalty rates" and aim to study the conditions under which a separate equilibrium exists and the form of signals in such an equilibrium.

In the separate equilibrium of the binary case, low-quality creators release signal s_l^* and high-quality creators release signal s_h^*. Creators holding NFTs of the same quality receive the same profits from participating in the market. We consider a situation where creators choose to join the market if they are indifferent between joining and choosing the outside option. Thus, there any possible separate equilibrium must fulfill: $\Theta(\rho_l) = \theta(s_l^*, \rho_l) = g_l, \Theta(\rho_h) = \theta(s_h^*, \rho_h) = g_h$, where $g_l + g_h = 1$.

5.4.1 Beliefs and Equilibrium

Based on assumption 1 for the buyer's off-path beliefs, when receiving an off-path signal s_i, the buyer forms beliefs by comparing creators' profits under s_i to those under on-path signals.

Define the set of signals yielding higher profits for low-quality creators than s_l^* as:

$$\overline{S}(s_l^*) = \{(p_i, \beta_i) \mid p_i + \rho_l \beta_i \pi_{t=2}(\beta_i + \alpha) > p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)\}$$

Define the set of signals yielding higher profits for high-quality creators than s_h^* as:

$$\overline{S}(s_h^*) = \{(p_i, \beta_i) \mid p_i + \rho_h \beta_i \pi_{t=2}(\beta_i + \alpha) > p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)\}$$

The set of creator types for which deviating to signal s_i can, in the best case (i.e., the transaction occurs), yield higher profits than the equilibrium profits is:

$$\mathcal{R}(s_i) = \{\rho \in \text{supp}(g) : \mathbb{E}\Pi_{\text{creator}}(\rho \mid s^*(\rho)) < \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_i)\}$$
$$= \begin{cases} \{\rho_l\}, & \text{if } s_i \in \overline{S}(s_l^*) \backslash \overline{S}(s_h^*) \\ \{\rho_h\}, & \text{if } s_i \in \overline{S}(s_h^*) \backslash \overline{S}(s_l^*) \\ \{\rho_l, \rho_h\}, & \text{if } s_i \in \overline{S}(s_l^*) \cap \overline{S}(s_h^*) \\ \emptyset, & \text{if } s_i \notin \overline{S}(s_l^*) \cup \overline{S}(s_h^*) \end{cases}$$

Then, the buyer's belief given an off-path signal s_i is as follows:

$$\delta(s_i) = \begin{cases} \rho_l, & \text{if } s_i \in \overline{S}(s_l^*) \backslash \overline{S}(s_h^*) \\ \rho_h, & \text{if } s_i \in \overline{S}(s_h^*) \backslash \overline{S}(s_l^*) \\ \rho_0, & \text{if } s_i \in \overline{S}(s_l^*) \cap \overline{S}(s_h^*) \\ \text{no need to discuss}, & \text{if } s_i \notin \overline{S}(s_l^*) \cup \overline{S}(s_h^*) \end{cases} \qquad \text{(Off-Path Updating Rule)}$$

Given a signal s_i, if the profits for a low-quality creator are higher than those under s_l^*, the buyer believes the creator can be of low quality, as such a creator has the incentive to release the signal if the buyer purchases. Similarly, if a signal s_i yields higher profits for a high-quality creator than those under s_h^*, the buyer believes the creator can be high-quality because high-quality creators have incentive to release the signal if the buyer purchases. Therefore, in this situation, the buyer's belief formed by signals should follow the rule above.

Let us denote the set of signals that can lead the first buyer with belief δ to purchase as

$$H(\delta) = \{\{p_i, \beta_i\} \mid p_i \leq \delta V_{1,t=2}(\beta_i + \alpha)\}$$

Lemma 10. *Given the set of signals that can motivate the buyer to purchase, $H(\hat{\theta}, \Theta) = \{s_i = \{p_i, \beta_i\} : p_i \leq \delta(s_i) V_{1,t=2}(\beta_i + \alpha)\}$, the following statements hold:*

(1). $H(\rho_l) \cap \left(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\right) \subseteq H(\hat{\theta}, \Theta)$

(2). $H(\rho_h) \cap \left(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\right) \subseteq H(\hat{\theta}, \Theta)$

(3). $H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) \subseteq H(\hat{\theta}, \Theta)$

Proof. (1): For all $s_i = \{p_i, \beta_i\} \in H(\rho_l) \cap \big(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\big)$. Because $s_i \in \overline{S}(s_l^*) \backslash \overline{S}(s_h^*)$, $\delta(s_i) = \rho_l$. Since $s_i \in H(\rho_l)$, $p_i \le \rho_l V_{1,t=2}(\beta_i + \alpha) = \delta(s_i) V_{1,t=2}(\beta_i + \alpha)$. So, $s_i \in H(\hat{\theta}, \Theta)$. We can conclude $H(\rho_l) \cap \big(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\big) \subseteq H(\hat{\theta}, \Theta)$. (2) and (3) can be proved similarly. □

Building upon this lemma and our previous definition of equilibrium in both signaling and entry games, we can establish the necessary and sufficient conditions for separate equilibrium.

Proposition 13 (Separate Equilibrium Conditions)**.** $\Theta(\rho_l) = \theta(s_l^*, \rho_l) = g_l, \Theta(\rho_h) = \theta(s_h^*, \rho_h) = g_h$ *with the buyer's belief updating rule* $\delta(\cdot)$ *forms a separate equilibrium if and only if the following conditions are satisfied*

(1). When receiving signal s_l^*, the first buyer forms belief ρ_l and purchases, which requires high-quality creators have no incentive to deviate from s_h^* to this signal:

$$\delta(s_l^*) = \rho_l, \;\; s_l^* \in H(\rho_l), \text{ and } s_l^* \notin \overline{S}(s_h^*) \tag{(1)}$$

(2). When receiving signal s_h^*, the first buyer forms belief ρ_h and purchases, which requires low-quality creators have no incentive to deviate from s_l^* to this signal:

$$\delta(s_h^*) = \rho_h, \;\; s_h^* \in H(\rho_h), \text{ and } s_h^* \notin \overline{S}(s_l^*) \tag{(2)}$$

(3). A low-quality creator can get more profits from joining the market than from outside option:

$$p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha) \ge u_l \tag{(3)}$$

(4). A high-quality creator can get more profits from joining the market than from outside option:

$$p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \ge u_h \tag{(4)}$$

(5). Compared to s_l^*, a low-quality creator doesn't have a signal that can get higher

profits for him and can also sell the NFT to the first buyer:

$$H(\rho_l) \cap \left(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\right) = H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset \tag{(5)}$$

(6). Compared to s_h^*, a high-quality creator doesn't have a signal that can get higher profits for him and can also sell the NFT to the first buyer:

$$H(\rho_h) \cap \left(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\right) = H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset \tag{(6)}$$

Proof. See Appendix A.7. □

5.4.2 Structure of Separate Equilibrium

For a royalty rate $\beta_1 > b_2^*$, there always exists $\beta_2 < b_2^*$ such that $\beta_1 \pi_{t=2}(\beta_1 + \alpha) = \beta_2 \pi_{t=2}(\beta_2 + \alpha)$. We don't need to discuss any $\beta > b_2^*$ here, because it is impossible to let either β_l^* or β_h^* fall in $(b_2^*, 1]$.

Lemma 11. *$\beta_l^* \in [0, \beta_2^*]$ and $\beta_h^* \in [0, \beta_2^*]$.*

Proof. See Appendix A.8. □

In this following content, we can restrict our attention on $\beta \in [0, b_2^*]$, where $\beta \pi_{t=2}(\beta + \alpha)$ is increasing in β.

Lemma 12 (Low-qualty creator's price)**.** *For a low-quality creator, his signal $s_l^* = (p_l^*, \beta_l^*)$ with $\beta_l^* > 0$ in separate equilibrium fulfills*

$$p_l^* = \rho_l V_{1,t=2}(\beta_h^* + \alpha)$$

Proof. See Appendix A.9. □

Proposition 14 (Low-quality creator's signal)**.** *In separate equilibrium, low-quality creators release signal $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$.*

Proof. Assume the low-quality creator release a signal $s_l^* = (\rho_l V_{1,t=2}(\beta_1 + \alpha), \beta_1)$ where $\beta_1 > 0$ in separate equilibrium. As we proved in proposition 4, the low-quality creator's profit $\rho_l \overline{M}(\beta)$ is decreasing in β. Hence, $\rho_l \overline{M}(0) > \rho_l \overline{M}(\beta_1)$, which means $\{\rho_l V_{1,t=2}(\alpha), 0\} \in \overline{S}(s_l^*)$. It is also obvious that $\{\rho_l V_{1,t=2}(\alpha), 0\} \in H(\rho_l)$ and $\{\rho_l V_{1,t=2}(\alpha), 0\} \in H(\rho_0)$. Then, we don't need to prove whether $\{\rho_l V_{1,t=2}(\alpha), 0\}$ is in $\overline{S}(s_h^*)$, because we can infer $\{\rho_l V_{1,t=2}(\alpha), 0\} \in H(\rho_l) \cap \big(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\big)$ if $\{\rho_l V_{1,t=2}(\alpha), 0\} \notin \overline{S}(s_h^*)$ and $\{\rho_l V_{1,t=2}(\alpha), 0\} \in H(\rho_0) \cap \big(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\big)$ if $\{\rho_l V_{1,t=2}(\alpha), 0\} \in \overline{S}(s_h^*)$. Either situation contradicts to the (5) of Proposition 13. □

This proposition shows that the low-quality creator chooses zero royalty rate when there is no restriction. We can rewrite the conditions of separate equilibrium in proposition 13 by substituting $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$.

Lemma 13. $H(\rho_l) \cap \overline{S}(s_l^*) = \emptyset$

Proof. For $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$,

$$\begin{aligned}
\mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s_l^*) &= \rho_l \overline{M}(0) \\
&= \max_{\beta} \rho_l \overline{M}(\beta) \\
&= \max_{\beta} \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid \{\rho_l V_{1,t=2}(\beta+\alpha), \beta\}) \\
&= \max_{\{p,\beta\}: p \le \rho_l V_{1,t=2}(\beta+\alpha)} \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid \{p, \beta\})
\end{aligned}$$

Hence, $\forall s_i = \{p_i, \beta_i\} \in \overline{S}(s_l^*)$, $\mathbb{E}\Pi_{\text{creator}}(\rho_l \mid \{p_i, \beta_i\}) > \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s_l^*)$. So, $p > \rho_l V_{1,t=2}(\beta + \alpha)$, i.e, $s_i \notin H(\rho_l)$. We can conclude $H(\rho_l) \cap \overline{S}(s_l^*) = \emptyset$. □

For $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$, $s_l^* \in H(\rho_l)$ in (1) must be satisfied. Because $H(\rho_l) \cap \overline{S}(s_l^*) = \emptyset$, $H(\rho_l) \cap \big(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\big) = \emptyset$ in (5) must be satisfied. So, we don't need to consider these two conditions in the following revised separate equilibrium conditions.

Proposition 15 (Revised Separate Equilibrium Conditions from Proposition 13)**.** $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$, $s_h^* = (p_h^*, \beta_h^*)$, $\Theta(\rho_l) = \theta(s_l^*, \rho_l) = g_l, \Theta(\rho_h) = \theta(s_h^*, \rho_h) = g_h$ *with the buyer's belief updating rule* $\delta(\cdot)$ *forms a separate equilibrium if and only if following conditions are satisfied*

(1). $\delta(s_l^*) = \rho_l$

$s_l^* \notin \overline{S}(s_h^*)$: $\rho_l V_{1,t=2}(\alpha) < p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$

(2). $\delta(s_h^*) = \rho_h$

$s_h^* \in H(\rho_h)$:

$$p_h^* \leq \rho_h V_{1,t=2}(\beta_h^* + \alpha) \tag{(2.1)}$$

$s_h^* \notin \overline{S}(s_l^*)$:

$$p_h^* \leq \rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \tag{(2.2)}$$

(3). $\rho_l V_{1,t=2}(\alpha) \geq u_l$

(4). $p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \geq u_h$

(5). $H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset$

(6). $H(\rho_h) \cap \left(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\right) = \emptyset$

Proposition 15 establishes the conditions sufficient and necessary for a separate equilibrium, providing a more convenient framework for examining the signal of high-quality creators. Then, we will analyze the high-quality creator's signal based on this proposition.

Two conditions within Proposition 15 determine the upper bound of p_h^* while keeping β_h^* fixed. The first condition, (2.1), is derived from the requirement that $s_h^* \in H(\rho_h)$. The second condition, (2.2), is derived from the requirement that $s_h^* \notin \overline{S}(s_l^*)$. To streamline our analysis, we introduce an assumption: $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha)$. This assumption suggests that a scenario where a low-quality creator can maximize their profit by either convincing the buyer of their low-quality status with a signal of $\{\rho_l V_{1,t=2}(\alpha), 0\}$ or by deceiving the buyer into believing they are a high-quality creator by maximizing royalty fees with a signal of $\{\rho_h V_{1,t=2}(b_2^* + \alpha), b_2^*\}$. The assumption states that the latter option, the low-quality creator releases the signal $\{\rho_h V_{1,t=2}(b_2^* + \alpha), b_2^*\}$ that can lead the buyer mistakenly believes he is high-quality, is more profitable.

Based on this assumption, condition (2.2) emerges as a tighter constraint than (2.1), indicating that we can disregard (2.1) in subsequent discussions without losing any information.

Lemma 14. *Given* $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha)+\rho_l b_2^* \pi_{t=2}(b_2^*+\alpha)$*, in the proposition 15, (2.2) is a tighter constraint for* p_h^* *than (2.1), i.e.,*

$$\rho_l V_{1,t=2}(\alpha) - \rho_l \beta \pi_{t=2}(\beta+\alpha) \leq \rho_h V_{1,t=2}(\beta+\alpha), \quad \forall \beta \in [0, b_2^*]$$

Proof. Let $M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h}) = \rho_h V_{1,t=2}(\beta+\alpha) + \rho_l \beta \pi_{t=2}(\beta+\alpha)$. As we proved in lemma 9, the $M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h})$ is decreasing in β since $\frac{\rho_l}{\rho_h} < 1$. Then, we know $M_{\rho_h}(\beta, \frac{\rho_l}{\rho_h}) \geq M_{\rho_h}(\beta_2^*, \frac{\rho_l}{\rho_h}) = \rho_h V_{1,t=2}(b_2^*+\alpha) + \rho_l b_2^* \pi_{t=2}(b_2^*+\alpha) \geq \rho_l V_{1,t=2}(\alpha), \forall \beta \in [0, b_2^*]$. □

This lemma shows that any signal that satisfies (2.2) also satisfies (2.1) in the proposition 15. That is, we can know that $(p, \beta) \in H(\rho_h)$ if $p \leq \rho_l V_{1,t=2}(\alpha) - \rho_l \beta \pi_{t=2}(\beta+\alpha)$, given $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha) + \rho_l b_2^* \pi_{t=2}(b_2^*+\alpha)$. Then, we don't need to discuss the (2.1) in following discussion.

Lemma 15. *Given* $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha) + \rho_l b_2^* \pi_{t=2}(b_2^*+\alpha)$*,*

$$p_h^* = \rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^*+\alpha)$$

must hold in separate equilibrium.

Proof. See Appendix A.10. □

Proposition 16 (High-quality creator's signal)**.** *Given* $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha) + \rho_l b_2^* \pi_{t=2}(b_2^*+\alpha)$*, high-quality creators release signal*

$$s_h^* = \{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^*+\alpha), b_2^*\}$$

in separate equilibrium.

Proof. See Appendix A.11 □

This proposition 16 shows that, given $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha)+\rho_l b_2^*\pi_{t=2}(b_2^*+\alpha)$, if there is a separate equilibrium, the high-quality creator must release a signal that has the royalty rate that can maximizes the royalty fee at $t=2$. Combined with proposition 14, we know, a separate equilibrium must satisfy $\Theta(\rho_l)=\theta(s_l^*,\rho_l)=g_l, \Theta(\rho_h)=\theta(s_h^*,\rho_h)=g_h$, $s_l^*=\{\rho_l V_{1,t=2}(\alpha),0\}$, and $s_h^*=\{\rho_l V_{1,t=2}(\alpha)-\rho_l b_2^*\pi_{t=2}(b_2^*+\alpha), b_2^*\}$ if separate equilibrium exists.

Proposition 17 (Existence of Separate Equilibrium)**.** *Given*

- $s_l^*=\{\rho_l V_{1,t=2}(\alpha),0\}$ *and* $s_h^*=\{\rho_l V_{1,t=2}(\alpha)-\rho_l b_2^*\pi_{t=2}(b_2^*+\alpha), b_2^*\}$,
- $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha)+\rho_l b_2^*\pi_{t=2}(b_2^*+\alpha)$,
- $\mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s_l^*)=\rho_l V_{1,t=2}(\alpha) \geq u_l$,
- $\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*)=\rho_l V_{1,t=2}(\alpha)+(\rho_h-\rho_l)b_2^*\pi_{t=2}(b_2^*+\alpha) \geq u_h$,
- *and the buyer's belief updating $\delta(\cdot)$ that fulfills $\delta(s_l^*)=\rho_l$, $\delta(s_h^*)=\rho_h$, and Off-Path Updating Rule,*

there exists unique separate equilibrium that $\{\Theta(\rho_l)=\theta(s_l^*,\rho_l)=g_l, \Theta(\rho_h)=\theta(s_h^*,\rho_h)=g_h, \hat{\theta}\}$ if and only if

$$\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) \geq \max_{\beta\in[0,b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0})$$

Proof. See Appendix A.12. □

This proposition supposes, given $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha)+\rho_l b_2^*\pi_{t=2}(b_2^*+\alpha)$ and upper bounds of creators' outside options, there exists unique separate equilibrium if and only if the high-quality creators' profits under signal s_h^* is higher than under any signal that induces buyer's belief to $\delta=\rho_0$.

Given this separate equilibrium that low-quality creators commit signal $s_l^*=\{\rho_l V_{1,t=2}(\alpha),0\}$ and high-quality creators commit signal $s_h^*=\{\rho_l V_{1,t=2}(\alpha)-\rho_l b_2^*\pi_{t=2}(b_2^*+$

$\alpha), b_2^*\}$, the profits and surplus are

$$
\begin{aligned}
&\mathbb{E}\Pi_{\text{platform}} = \alpha \left(\rho_l g_l \pi_{t=2}(\alpha) + \rho_h g_h \pi_{t=2}(b_2^* + \alpha)\right) \\
&\mathbb{E}\Pi_{\text{buyer1}} = g_h \left(\rho_h V_{1,t=2}(b_2^* + \alpha) - \rho_l V_{1,t=2}(\alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha)\right), \\
&\mathbb{E}\Pi_{\text{buyer2}} = \rho_l g_l V_{2,t=2}(\alpha) + \rho_h g_h V_{2,t=2}(b_2^* + \alpha) \\
&\mathbb{E}\Pi_{\text{creator}}(\rho_l) = \rho_l V_{1,t=2}(\alpha),\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = \rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l) b_2^* \pi_{t=2}(b_2^* + \alpha) \\
&\mathbb{E}S = \rho_l g_l S(\alpha) + \rho_h g_h S(b_2^* + \alpha) - (u_l g_l + u_h g_h)
\end{aligned}
$$

5.5 Simulations of Uniform Distributions Example

In this section, we use simulations of a uniform distribution example to help our analysis how different royalty payments policy affect the profits and surplus of every agent including the platform.

According to the section 3.6, we have

$$
\begin{aligned}
&V_{1,t=2}(\gamma) = \frac{(1-\gamma)^2}{12} + \frac{1}{2},\ V_{2,t=2}(\gamma) = \frac{1-\gamma}{24} \\
&\pi_{t=2}(\gamma) = \frac{1-\gamma}{6},\ b_2^* = \frac{1-\alpha}{2},\ S(\gamma) = \frac{(2\gamma+3)(1-\gamma)}{24} + \frac{1}{2}
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0}) &= \rho_0 V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma) \\
&= \rho_0 \left(\frac{(1-\gamma)^2}{12} + \frac{1}{2} \right) + \rho_h \frac{\beta(1-\gamma)}{6} \\
\max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0}) &= \frac{\rho_h^2 (1-\alpha)^2}{12(2\rho_h - \rho_0)} + \frac{\rho_0}{2}
\end{aligned}
$$

The assumptions in Proposition 17 can be written as

$$
\begin{aligned}
&\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha) \\
\Leftrightarrow\ & \rho_l \left(\frac{(1-\alpha)^2}{12} + \frac{1}{2} \right) \leq \rho_h \left(\frac{(1-\alpha)^2}{48} + \frac{1}{2} \right) + \rho_l \frac{(1-\alpha)^2}{24} \\
\Leftrightarrow\ & \rho_l \left(\frac{(1-\alpha)^2}{24} + \frac{1}{2} \right) \leq \rho_h \left(\frac{(1-\alpha)^2}{48} + \frac{1}{2} \right)
\end{aligned}
$$

$$
\rho_l V_{1,t=2}(\alpha) \geq u_l \ \Leftrightarrow\ \rho_l \left(\frac{(1-\alpha)^2}{12} + \frac{1}{2} \right) \geq u_l
$$

$$
\begin{aligned}
\rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l) b_2^* \pi_{t=2}(b_2^* + \alpha) &\geq u_h \\
\Leftrightarrow \ \rho_l \left(\frac{(1-\alpha)^2}{12} + \frac{1}{2} \right) + (\rho_h - \rho_l) \frac{(1-\alpha)^2}{24} &\geq u_h \\
\Leftrightarrow \ \rho_l \left(\frac{(1-\alpha)^2}{24} + \frac{1}{2} \right) + \rho_h \frac{(1-\alpha)^2}{24} &\geq u_h
\end{aligned}
$$

$$
\begin{aligned}
\rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l) b_2^* \pi_{t=2}(b_2^* + \alpha) &\geq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0}) \\
\Leftrightarrow \ \rho_l \left(\frac{(1-\alpha)^2}{24} + \frac{1}{2} \right) + \rho_h \frac{(1-\alpha)^2}{24} &\geq \frac{\rho_h^2 (1-\alpha)^2}{12(2\rho_h - \rho_0)} + \frac{\rho_0}{2} \\
\Leftrightarrow \ \frac{2\rho_h \rho_l - \rho_l \rho_0 - \rho_h \rho_0}{2} (1-\alpha)^2 &\geq 6(2\rho_h \rho_0 - \rho_o^2 - 2\rho_h \rho_l + \rho_l \rho_0)
\end{aligned}
$$

We give some numerical examples to show how the separate equilibrium differs to pooling equilibrium from the view of welfare.

We know the total transaction value at stage $t = 2$, $\pi_{t=2}(\beta + \alpha)$, is decreasing in β given the entry situation of creators. As the platform's profits are proportional to the total transaction value at stage $t = 2$, $\alpha \pi_{t=2}(\beta + \alpha)$, the platform has incentive to restrict the value of royalty rate. According to Proposition 10, higher royalty rate can increase high-quality creators to join the market, changing the entry situation of creators. So, the platform has incentive to tolerate high royalty rate to change the entry situation of creators.

In our numerical examples' analysis, we restrict our analysis on pooling equilibrium that has royalty rate in $[0, b_2^*]$ and separate equilibrium without any bound restrictions.

Example 1. *Commission rate $\alpha = 0.05$. Low-quality creator holds NFT with $\rho_l = 0.8$ with proportion $g_l = 0.96$. High-quality creator holds NFT with $\rho_h = 1$ with proportion $g_h = 0.04$. Low-quality creators have outside option $u_l = 0.4$ and high-quality creators have outside option $u_h = 0.45$.*

1. **Separate Equilibrium** $p_l^* = 0.460167, \beta_l^* = 0$, $p_h^* = 0.430083, \beta_h^* = b_2^* =$

0.475:

$$\mathbb{E}\Pi_{\text{platform}} = 0.0062383$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0.00354875,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.03119167$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.460167,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4676875$$
$$\mathbb{E}S = 0.09944625$$

2. **Pooling Equilibrium** with $\beta = 0$: all low-quality creators and all high-quality creators join the market and $p^* = 0.4647683$

$$\mathbb{E}\Pi_{\text{platform}} = 0.0063967$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.031983$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.4647683,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4647683$$
$$\mathbb{E}S = 0.1011483$$

The outside options of both type creators are relatively low. Pooling equilibrium with $\beta = 0$ can still induce all creators join the market. So, separate equilibrium benefits the first buyer and high-quality creators but hurts all other agents and the total social welfare compared to the pooling equilibrium with $\beta = 0$.

Example 2. *Commission rate $\alpha = 0.05$. Low-quality creator holds NFT with $\rho_l = 0.8$ with proportion $g_l = 0.96$. High-quality creator holds NFT with $\rho_h = 1$ with proportion $g_h = 0.04$. Low-quality creators have outside option $u_l = 0.4$ and high-quality creators have outside option $u_h = 0.465$.*

1. **Separate Equilibrium** $p_l^* = 0.460167, \beta_l^* = 0,\ p_h^* = 0.430083, \beta_h^* = b_2^* = 0.475$:

$$\mathbb{E}\Pi_{\text{platform}} = 0.0062383$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0.00354875,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.03119167$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.460167,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4676875$$
$$\mathbb{E}S = 0.09884625$$

2. **Pooling Equilibrium** with $\beta = 0$: only low-quality creators join the market

and $p^* = 0.460167$

$$\mathbb{E}\Pi_{\text{platform}} = 0.00608$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.0304$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.460167,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.465$$
$$\mathbb{E}S = 0.09424$$

3. **Pooling Equilibrium** with $\beta = 0.01$

   [Equilibrium 1]: only low-quality creators join the market and $p^* = 0.4589067$

$$\mathbb{E}\Pi_{\text{platform}} = 0.006016$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.03008$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.46016,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.465$$
$$\mathbb{E}S = 0.0938496$$

   [Equilibrium 2]: all low-quality creators and all high-quality creators join the market and $p^* = 0.46349573$

$$\mathbb{E}\Pi_{\text{platform}} = 0.0063293$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.0316467$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.464749067,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4650624$$
$$\mathbb{E}S = 0.1001376$$

   In this example, the outside option of low-quality creators are still relatively low, but the outside option of high-quality creators are relatively high. So, in the absence of royalty rate, only low-quality creators will join the market. By increasing royalty rate, high-quality creators can also join the market and forms an equilibrium that has higher social welfare than separate equilibrium.

Example 3. *Commission rate* $\alpha = 0.05$*. Low-quality creator holds NFT with* $\rho_l = 0.8$ *with proportion* $g_l = 0.96$*. High-quality creator holds NFT with* $\rho_h = 1$ *with* *proportion* $g_h = 0.04$*. Low-quality creators have outside option* $u_l = 0.46$ *and high-quality creators have outside option* $u_h = 0.465$*.*

1. **Separate Equilibrium** $p_l^* = 0.460167, \beta_l^* = 0,\ p_h^* = 0.430083, \beta_h^* = b_2^* = 0.475$:

$$\mathbb{E}\Pi_{\text{platform}} = 0.0062383$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0.00354875,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.03119167$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.460167,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4676875$$
$$\mathbb{E}S = 0.04124625$$

2. **Pooling Equilibrium** with $\beta = 0$: only low-quality creators join the market and $p^* = 0.460167$

$$\mathbb{E}\Pi_{\text{platform}} = 0.00608$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.0304$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.460167,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.465$$
$$\mathbb{E}S = 0.03664$$

   In this example, the outside option of high-quality creators are relatively high. So, in the absence of royalty rate, only low-quality creators will join the market.

3. **Pooling Equilibrium** with $\beta = 0.01$

   [Equilibrium 1]: only low-quality creators join the market and $p^* = 0.4589067$

$$\mathbb{E}\Pi_{\text{platform}} = 0.006016$$
$$\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.03008$$
$$\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.46016,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.465$$
$$\mathbb{E}S = 0.0362496$$

   [Equilibrium 2]: all low-quality creators and all high-quality creators join the market and $p^* = 0.46349573$

$$\begin{aligned}
&\mathbb{E}\Pi_{\text{platform}} = 0.0063293\\
&\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.0316467\\
&\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.464749067,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4650624\\
&\mathbb{E}S = 0.0425376
\end{aligned}$$

By increasing royalty rate, high-quality creators can also join the market and forms an equilibrium that has higher social welfare than separate equilibrium.

4. **Pooling Equilibrium** with $\beta = 0.1$: all low-quality creators and all high-quality creators join the market and $p^* = 0.4526483$

$$\begin{aligned}
&\mathbb{E}\Pi_{\text{platform}} = 0.005723\\
&\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.0286167\\
&\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.46398167,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.466815\\
&\mathbb{E}S = 0.038235
\end{aligned}$$

Because the outside option of low-quality creators are also relatively high in this example, if we keep increasing royalty rate, the inefficiency effect will decrease low-quality creators' profits from joining the market and then eliminate the equilibrium that only low-quality creators join the market.

5. **Pooling Equilibrium** with $\beta = 0.3$: $\Theta(\rho_l) = 0.694$ low-quality creators and all high-quality creators join the market and $p^* = 0.434$

$$\begin{aligned}
&\mathbb{E}\Pi_{\text{platform}} = 0.003224\\
&\mathbb{E}\Pi_{\text{buyer1}} = 0,\ \mathbb{E}\Pi_{\text{buyer2}} = 0.01612\\
&\mathbb{E}\Pi_{\text{creator}}(\rho_l) = 0.46,\ \mathbb{E}\Pi_{\text{creator}}(\rho_h) = 0.4665\\
&\mathbb{E}S = 0.019404
\end{aligned}$$

If keep increasing royalty rate, some low-quality creators will be kicked out the market because of the inefficiency effect, while high-quality creators can stay at the market because of the strong transfer effect for them.

CHAPTER 6

EXTENSION: BUYERS HAVE MORE INFORMATION THAN CREATORS

In this section, we examine a scenario where buyers possess more information than creators. In this context, creators have a prior probability of their NFT becoming "popular" at stage $t = 2$, represented as ρ_0. The information held by buyers is typically characterized by a probability distribution $H \in \Delta[0, 1]$ with a mean value of ρ_0 and a probability mass function (p.m.f.) $h : [0, 1] \to [0, 1]$. Specifically, we consider the binary example that $\text{supp}(h) = \{\rho_l, \rho_h\}$ where $\rho_l < \rho_h$, $h_l = h(\rho_l), h_h = h(\rho_h)$, and $h_l\rho_l + h_h\rho_h = \rho_0$.

The profit functions of agents at $t = 2$ remain the same as the main model. Assuming the first buyer acquires the NFT at stage $t = 1$, the profit function of the first buyer at $t = 2$ concerning γ is denoted by $V_{1,t=2}(\gamma)$, which is a decreasing function of γ. The creator's profit function derived from royalty payments at $t = 2$ in relation to γ is expressed as $\beta\pi_{t=2}(\gamma)$, while the platform's profits are represented by $\alpha\pi_{t=2}(\gamma)$.

To restrict our attention on the royalty payment regulation, we also don't consider the creator's outside option here.

When the first buyer's belief about NFT quality is ρ, his willingness to pay is $\rho V_{1,t=2}(\gamma)$. Then, a creator's trade-off of setting price is actually between setting a low price $\rho_l V_{1,t=2}(\gamma)$ to attract buyers even if the NFT's quality is low or setting a high price $\rho_h V_{1,t=2}(\gamma)$ to only sell high-quality NFT.

The creator's profits with $p_1 = \rho_l V_{1,t=2}(\gamma)$ are

$$\Pi^l_{\text{creator}} = M_{\rho_l}(\beta, \frac{\rho_0}{\rho_l}) = \rho_l V_{1,t=2}(\gamma) + \rho_0\beta\pi_{t=2}(\gamma)$$

The creator's profits with $p_1 = \rho_h V_{1,t=2}(\gamma)$ are

$$\Pi^h_{\text{creator}} = h_h \rho_h \overline{M}(\beta) = h_h(\rho_h V_{1,t=2}(\gamma) + \rho_h \beta \pi_{t=2}(\gamma))$$

So, the creator sets a high price if

$$\Pi^h_{\text{creator}} - \Pi^l_{\text{creator}} = (h_h\rho_h - \rho_l)V_{1,t=2}(\gamma) - h_l\rho_l\beta\pi_{t=2}(\gamma) > 0$$

6.1 Creator's optimal strategy

6.1.1 Given $h_h\rho_h \leq \rho_l$

In the situation $h_h\rho_h \leq \rho_l$, $\Pi^h_{\text{creator}} - \Pi^l_{\text{creator}}$ is always negative, so the creator always set low price $p_1 = \rho_l V_{1,t=2}(\gamma)$. If the creator can commit royalty rate at $t = 1$, he will commit $\beta = b^*_{\frac{\rho_0}{\rho_l}}$ and set price $p_1 = \rho_l V_{1,t=2}(b^*_{\frac{\rho_0}{\rho_l}})$.

6.1.2 Given $h_h\rho_h > \rho_l$

In the situation $h_h\rho_h > \rho_l$, we can find the $\Pi^h_{\text{creator}} - \Pi^l_{\text{creator}}$ is decreasing in $\beta \in [0, b^*_2]$. We consider the creator's optimal strategy under different regulations:

- "No Royalty Payment", $\beta = 0$: When there is no royalty payment, $\Pi^h_{\text{creator}} - \Pi^l_{\text{creator}} = (h_h\rho_h - \rho_l)V_{1,t=2}(\alpha) > 0$. The creator set price $p_1 = \rho_h V_{1,t=2}(\alpha)$.
- "Flexible Royalty Rate", $\beta = b^*_2$: The creator set price

$$p_1 = \begin{cases} \rho_l V_{1,t=2}(b^*_2), & \text{if } (h_h\rho_h - \rho_l)V_{1,t=2}(b^*_2) - h_l\rho_l\beta\pi_{t=2}(b^*_2) \leq 0 \\ \rho_h V_{1,t=2}(b^*_2), & \text{if } (h_h\rho_h - \rho_l)V_{1,t=2}(b^*_2) - h_l\rho_l\beta\pi_{t=2}(b^*_2) > 0 \end{cases}$$

- "Fixed Royalty Rate": The creator's optimal strategy $s = (p, \beta)$ follows

$$(p,\beta)=\begin{cases}(\rho_h V_{1,t=2}(\alpha),0), & \text{if } h_h\rho_h V_{1,t=2}(\alpha)\geq \rho_l V_{1,t=2}(b^*_{\frac{\rho_0}{\rho_l}})+\rho_0 b^*_{\frac{\rho_0}{\rho_l}}\pi_{t=2}(b^*_{\frac{\rho_0}{\rho_l}}+\alpha)\\ (\rho_l V_{1,t=2}(b^*_{\frac{\rho_0}{\rho_l}}),b^*_{\frac{\rho_0}{\rho_l}}), & \text{if } h_h\rho_h V_{1,t=2}(\alpha)< \rho_l V_{1,t=2}(b^*_{\frac{\rho_0}{\rho_l}})+\rho_0 b^*_{\frac{\rho_0}{\rho_l}}\pi_{t=2}(b^*_{\frac{\rho_0}{\rho_l}}+\alpha)\end{cases}$$

In this extension, we examine the scenario where buyers may possess more information than the creator, and explore how royalty payments influence the creator's pricing strategy. Specifically, we demonstrate that royalty payments enable the creator to set a lower initial price, thereby increasing the likelihood of a successful transaction. This is due to the fact that royalty payments facilitate the transfer of profits from the first-stage price to the royalty fees collected at the second stage.

By adopting a strategy that involves setting a lower initial price and a relatively higher royalty rate, the creator is able to appeal to a broader range of buyers and consequently enhance the chances of selling their NFT. In this manner, the creator captures a more extensive market share, while the royalties collected at the second stage compensate for the profit reduction resulting from the lower initial price. This strategy allows the creator to optimize their revenue without relying solely on the sale of high-quality NFTs at the first stage.

Appendices

APPENDIX A

A.1 Proof of Lemma 1

A.1.1 Lemma 1

Suppose $f'(x) \geq -\frac{2f(x)^2}{1-F(x)}, \forall x \in \text{supp}(g)$. The optimal price of a first buyer with e_1, $p_2^*(\gamma, e_1)$, is

(1), increasing in e_1,

(2), increasing in γ,

(3), equal to 1 for all $e_1 \geq 1-\gamma$,

(4), in $(\frac{e_1}{1-\gamma}, 1)$ for all $e_1 < 1-\gamma$, and

(5), the unique solution of p_2 for equation $f(p_2)(e_1-(1-\gamma)p_2)+(1-F(p_2))(1-\gamma) = 0$

(6), $\frac{\partial v_1(\gamma,e_1,p_2)}{\partial p_2} > 0, \forall p_2 \in (\frac{e_1}{1-\gamma}, p_2^*(\gamma, e_1))$ and $\frac{\partial v_1(\gamma,e_1,p_2)}{\partial p_2} < 0, \forall p_2 \in (p_2^*(\gamma, e_1), 1)$.

A.1.2 Proof

The object function is

$$v_1(\gamma, e_1, p_2) = (1-\gamma)(1-F(p_2))p_2 + F(p_2)e_1$$

whose derivative with respect to p_2 is

$$\begin{aligned}\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2} &= (1-\gamma)\left(1 - F(p_2) - f(p_2)p_2\right) + f(p_2)e_1 \\ &= f(p_2)(e_1 - (1-\gamma)p_2) + (1 - F(p_2))\,(1-\gamma)\end{aligned}$$

For $e_1 \geq 1-\gamma$, the derivative is always positive for all $p_2 \in [0,1]$. Hence, the optimal price in this situation is $p_2^*(\gamma, e_1) = 1, \forall e_1 \in [1-\gamma, 1]$. (3) is proved.

For $e_1 < 1-\gamma$, the derivative substituting $p_2 = 1$ is $f(1)(e_1 - (1-\gamma)) < 0$ and the derivative substituting $p_2 = \frac{e_1}{1-\gamma}$ is $\left(1 - F(\frac{e_1}{1-\gamma})\right)(1-\gamma) > 0$. Since all functions in the derivative is continuous, the optimal $p_2^*(\gamma, e_1)$ must satisfy

$$f(p_2^*(\gamma, e_1))(e_1 - (1-\gamma)p_2^*(\gamma, e_1)) + (1 - F(p_2^*(\gamma, e_1)))\,(1-\gamma) = 0 \tag{1}$$

and fall in $(\frac{e_1}{1-\gamma}, 1)$. (4) is proved here.

Based on the equation (1), we can infer

$$\frac{e_1}{1-\gamma} = p_2^*(\gamma, e_1) - \frac{1 - F(p_2^*(\gamma, e_1))}{f(p_2^*(\gamma, e_1))} = K(p_2^*(\gamma, e_1))$$

where $K(p) \triangleq p - \frac{1-F(p)}{f(p)}$, which is increasing in p given assumption $f'(x) \geq -\frac{2f(x)^2}{1-F(x)}, \forall x \in \mathrm{supp}(g)$.

As e_1 and γ increases, the $p^*(e_1)$ increases. (1) and (2) are proved.

This increasing function $K(p)$ also give the uniqueness of $p_2^*(\gamma, e_1)$. (5) is proved.

Since $p_2^*(\gamma, e_1)$ is the unique zero point of $\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2}$ and $\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2}$ is continuous about p_2, we can infer $\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2} > 0, \forall p_2 \in (\frac{e_1}{1-\gamma}, p_2^*(\gamma, e_1))$ and $\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2} < 0, \forall p_2 \in (p_2^*(\gamma, e_1), 1)$. (6) is proved.

A.2 Proof of Proposition 2

A.2.1 Proposition 2

$\pi_{t=2}(\gamma)$ is decreasing in γ and $\pi_{t=2}(1) = 0$.

A.2.2 Proof

Suppose $e_1 < 1 - \gamma$. Since $p_2^*(\gamma, e_1)$ is the unique zero point of $\frac{\partial v_1(\gamma,e_1,p_2)}{\partial p_2}$ and $\frac{\partial v_1(\gamma,e_1,p_2)}{\partial p_2}$ is continuous about p_2, we can infer $\frac{\partial v_1(\gamma,e_1,p_2)}{\partial p_2} > 0, \forall p_2 \in (\frac{e_1}{1-\gamma}, p_2^*(\gamma, e_1))$ and $\frac{\partial v_1(\gamma,e_1,p_2)}{\partial p_2} < 0, \forall p_2 \in (p_2^*(\gamma, e_1), 1)$.

$$\frac{\partial v_1(\gamma, e_1, p_2)}{\partial p_2} = (1-\gamma)\frac{\partial(1-F(p_2))p_2}{\partial p_2} + \frac{\partial F(p_2)e_1}{\partial p_2}$$

Suppose $\gamma_1 < \gamma_2$. $p_2^*(\gamma_1, e_1) < p_2^*(\gamma_2, e_1)$ since $p_2^*(\gamma, e_1)$ is increasing in γ.

$$\left.\frac{\partial v_1(\gamma_1, e_1, p_2)}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} = (1-\gamma_1)\left.\frac{\partial(1-F(p_2))p_2}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} + \left.\frac{\partial F(p_2)e_1}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} = 0$$

$$\left.\frac{\partial v_1(\gamma_2, e_1, p_2)}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} = (1-\gamma_2)\left.\frac{\partial(1-F(p_2))p_2}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} + \left.\frac{\partial F(p_2)e_1}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} > 0$$

$$\left.\frac{\partial v_1(\gamma_2, e_1, p_2)}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} - \left.\frac{\partial v_1(\gamma_1, e_1, p_2)}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} = (\gamma_1-\gamma_2)\left.\frac{\partial(1-F(p_2))p_2}{\partial p_2}\right|_{p_2=p_2^*(\gamma_1,e_1)} > 0$$

Hence, we can infer $\left.\frac{\partial(1-F(p_2))p_2}{\partial p_2}\right|_{p_2=p_2^*(\gamma,e_1)} < 0, \forall\gamma$. Then,

$$\frac{\partial(1-F(p_2^*(\gamma,e_1)))p_2^*(\gamma,e_1)}{\partial\gamma} = \left.\frac{\partial(1-F(p_2))p_2}{\partial p_2}\right|_{p_2=p_2^*(\gamma,e_1)} \frac{\partial p_2^*(\gamma,e_1)}{\partial\gamma}$$

We proved in the first lemma that $\frac{\partial p_2^*(\gamma,e_1)}{\partial\gamma} > 0$, so $\frac{\partial(1-F(p_2^*(\gamma,e_1)))p_2^*(\gamma,e_1)}{\partial\gamma} < 0$.

Since $(1 - F(p_2^*(\gamma, e_1)))p_2^*(\gamma, e_1)$ is decreasing in γ given e_1, $\pi_{t=2}(\gamma) = \int_0^{1-\gamma}[1 - F(p_2^*(\gamma, e_1))]p_2^*(\gamma, e_1)\, de_1$ is decreasing in γ. $\pi_{t=2}(1) = 0$ is obviously since $p_2^*(1, e_1) =$

$1, \forall e_1$. The proposition is proved.

A.3 Proof of Lemma 6

A.3.1 A Useful Theorem

To help us prove these lemmas, we can prove a theorem firstly.

Theorem 2. *Given a continuously differentiable function $R(d)$ with domain $d \in (0, \infty)$ and a condition A:*

$$R(\epsilon) \leq 0, \text{ where } \epsilon \to 0^+ \tag{Condition A}$$

If the derivative of the function is non-negative, i.e., $R'(d) > 0, \forall d \in (0, \infty)$, the condition A is equivalent to condition A.1:

$$R(0) < 0 \tag{Condition A.1}$$

If the derivative of the function at is negative, i.e., $R'(d) \leq 0, \forall d \in (0, \infty)$, the condition A is equivalent to condition A.2:

$$R(0) \leq 0 \tag{Condition A.2}$$

Proof. For $R'(d) \geq 0, \forall d \in (0, \infty)$,

- ($R(\epsilon) \leq 0 \Rightarrow R(0) < 0$): Given $R(\epsilon) \leq 0$, we know $R(0) < R(\epsilon) \leq 0$ since $\epsilon > 0$.
- ($R(0) \leq 0 \Rightarrow R(\epsilon) < 0$): Given $R(0) < 0$, based on the continuity of $R(\cdot)$, there exists $d^* > 0$ such that $R(d) < 0, \forall d \in (0, d^*)$. Since $\epsilon \to 0^+$, we can prove $R(\epsilon) < 0$ by $\epsilon \in (0, d^*)$.

Hence, we have proved condition A is equivalent to condition A.1 when $R'(d) > 0, \forall d \in (0, \infty)$.
For $R'(d) \leq 0, \forall d \in (0, \infty)$,

- ($R(\epsilon) \leq 0 \Rightarrow R(0) \leq 0$): Assume $R(0) > 0$. Based on the continuity of $R(\cdot)$, there exists $d^* > 0$ such that $R(d) > 0, \forall d \in (0, d^*)$. Since $\epsilon \to 0^+$, we can prove $R(\epsilon) > 0$ by $\epsilon \in (0, d^*)$. However, this conclusion contradicts to our condition $R(\epsilon) \leq 0$. So, the assumption $R(0) > 0$ can't hold. $R(0) \leq 0$ is proved.

- ($R(0) \leq 0 \Rightarrow R(\epsilon) \leq 0$): Given $R(0) \leq 0$, we know $R(\epsilon) \leq R(0) \leq 0$ since $\epsilon > 0$.

Now, we have proved condition A is equivalent to condition A.2 when $R'(d) \leq 0, \forall d \in (0, \infty)$. $\square$

A.3.2 Lemma 6

The condition 2'

$$\frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho) + \rho_i \epsilon}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho) + \epsilon} V_{1,t=2}(\beta_* + \alpha) + \rho_i \beta_* \pi_{t=2}(\beta_* + \alpha) \leq u(\rho_i), \text{ for all } \rho_i \in \{y : \overline{\Theta}(y) > 0\} \tag{Condition 2'}$$

is equivalent to

$$\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta) < u(\rho_i), \text{ for all } \rho_i \in \left\{ y : \overline{\Theta}(y) > 0, y > \frac{\sum_{x \in \text{supp}(g)} x\Theta(x)}{\sum_{x \in \text{supp}(g)} \Theta(x)} \right\} \tag{Condition 2.1}$$

$$\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta) \leq u(\rho_i), \text{ for all } \rho_i \in \left\{ y : \overline{\Theta}(y) > 0, y \leq \frac{\sum_{x \in \text{supp}(g)} x\Theta(x)}{\sum_{x \in \text{supp}(g)} \Theta(x)} \right\} \tag{Condition 2.2}$$

where $\mathbb{E}\Pi^*_{\text{creator}}(\rho_i, \Theta) = \frac{\sum_{\rho \in \text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha) + \rho_i \beta_* \pi_{t=2}(\beta_* + \alpha)$

A.3.3 Proof of Lemma 6

First we set a function

$$E_\Theta(d) = \frac{\sum_{x \neq \rho} x\Theta(x) + \rho(\Theta(\rho) + d)}{\sum_{x \in \text{supp}(g)} \Theta(x) + d}$$

whose derivative is

$$E'_{\Theta}(d) = \frac{\rho \sum_{x\in \text{supp}(g)} \Theta(x) - \sum_{x\in \text{supp}(g)} x\Theta(x)}{(\sum_{x\in \text{supp}(g)} \Theta(x) + d)^2}$$

For $\rho > \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}$, $E'_{\Theta}(d) > 0, \forall d \in (0, \infty)$; For $\rho \leq \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}$, $E'_{\Theta}(d) \leq 0, \forall d \in (0, \infty)$.

Let

$$R(d) = E_{\Theta}(d)V_{1,t=2}(\gamma) + \rho\beta\pi_{t=2}(\gamma) - u(\rho)$$

which is continuously differentiable in d and $R'(d) = V_{1,t=2}(\gamma)E'_{\Theta}(d)$.

Hence, for $\rho > \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}$, $R'(d) > 0, \forall d \in (0, \infty)$; For $\rho \leq \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}$, $R'(d) \leq 0, \forall d \in (0, \infty)$.

Condition 2' can be rewritten as

$$R(\epsilon) \leq 0, \text{ for all } \rho \in \{y : \overline{\Theta}(y) > 0\}, \text{ where } \epsilon \to 0^+ \tag{Condition 2'}$$

Condition 2.1 and 2.2 can be rewritten as

$$R(0) < 0, \text{ for all } \rho \in \{y : \overline{\Theta}(y) > 0, y > \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}\} \tag{Condition 2.1}$$

$$R(0) \leq 0, \text{ for all } \rho \in \{y : \overline{\Theta}(y) > 0, y \leq \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}\} \tag{Condition 2.2}$$

- For $\rho \in \{y : \overline{\Theta}(y) > 0, y > \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}\}$, $R'(d) > 0$. Based on the theorem 2

$$R(\epsilon) \leq 0, \text{ where } \epsilon \to 0^+ \Leftrightarrow R(0) < 0$$

- For $\rho \in \{y : \overline{\Theta}(y) > 0, y \leq \frac{\sum_{x\in \text{supp}(g)} x\Theta(x)}{\sum_{x\in \text{supp}(g)} \Theta(x)}\}$, $R'(d) \leq 0$. Based on the theorem 2

$$R(\epsilon) \leq 0, \text{ where } \epsilon \to 0^+ \Leftrightarrow R(0) \leq 0$$

Hence, condition 2' is equivalent to condition 2.1 and 2.2.

A.4 Proof of Proposition 11

A.4.1 Proposition 11

Given the entry situation of creators Θ, if there is a pooling perfect Bayesian equilibrium of the signaling game, $\left(\{\theta(s_*, \rho)\}_\rho, \hat{\theta}\right)$, such that the buyer's beliefs fulfill On-Path Belief and Off-Path Belief and $\theta(s_*, \rho) = \Theta(\rho), \forall \rho \in \text{supp}(g)$, the following conditions must be fulfilled for $s_* = \{p_*, \beta_*\}$.

$$p_* = \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha) \qquad \text{(Condition 1)}$$

$$\beta_* = \min\left\{ \underset{\beta \in [0,1]}{\text{argmax}} \max_{\rho : \Theta(\rho) > 0} \mathbb{E}\Pi_{\text{creator}} \left(\rho \mid \left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta + \alpha), \beta \right\} \right) \right\} \qquad \text{(Condition 2)}$$

A.4.2 Proof

As we showed above, Condition 1 is directly given by Lemma 5.

Let $\beta_1 = \min\left\{ \text{argmax}_{\beta \in [0,1]} \mathbb{E}\Pi_{\text{creator}} \left(\bar{\rho} \mid \left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta + \alpha), \beta \right\} \right) \right\}$ and $\bar{\rho} = \max\{\rho : \Theta(\rho) > 0\}$.

Assume $\beta_* \neq \beta_1$. Based on the definition of β_1, signal $\left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_* + \alpha), \beta_* \right\}$ is always less preferable than signal $\left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1 + \alpha), \beta_1 \right\}$ for a type-$\bar{\rho}$ creator, because the former signal either generates lower profits or generates the same profits but has lower price. The only possibility for this equilibrium holds is that the signal $\left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1 + \alpha), \beta_1 \right\}$ can't induce transition, that is

$$\begin{aligned}\left\{ \frac{\sum_{\rho \in \text{supp}(g)} \rho \Theta(\rho)}{\sum_{\rho \in \text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1 + \alpha), \beta_1 \right\} &\notin H(\hat{\theta}, \Theta) \\ &= \left\{ s_i : p_i \le \frac{\sum_{\rho \in \mathcal{R}(s_i)} \rho \Theta(\rho)}{\sum_{\rho' \in \mathcal{R}(s_i)} \Theta(\rho')} V_{1,t=2}(\beta_i + \alpha) \right\}\end{aligned}$$

where $\mathcal{R}(s_i) = \{\rho \in \text{supp}(g) : \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_*) < \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_i)\}$. Hence, we can infer

$$\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} > \frac{\sum_{\rho\in\mathcal{R}\left(\left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha),\beta_1\right\}\right)} \rho\Theta(\rho)}{\sum_{\rho'\in\mathcal{R}\left(\left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha),\beta_1\right\}\right)} \Theta(\rho')}$$

The necessary condition for the inequality holds is that there exists $\rho_a > \frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)}$ such that $\rho_a \notin \mathcal{R}\left(\left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha), \beta_1\right\}\right)$ (obviously $\rho_a < \bar{\rho}$) and $\rho_b < \frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)}$ such that $\rho_b \in \mathcal{R}\left(\left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha), \beta_1\right\}\right)$. Then, we have

$$\mathbb{E}\Pi_{\text{creator}}(\rho_a \mid s_*) \geq \mathbb{E}\Pi_{\text{creator}}\left(\rho_a \mid \left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha), \beta_1\right\}\right)$$

$$\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} \left[V_{1,t=2}(\beta_1+\alpha) - V_{1,t=2}(\beta_*+\alpha)\right] \leq \rho_a \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right]$$

$$\mathbb{E}\Pi_{\text{creator}}(\rho_b \mid s_*) < \mathbb{E}\Pi_{\text{creator}}\left(\rho_b \mid \left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha), \beta_1\right\}\right)$$

$$\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} \left[V_{1,t=2}(\beta_1+\alpha) - V_{1,t=2}(\beta_*+\alpha)\right] > \rho_b \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right]$$

Because $\rho_a > \rho_b$ and $\rho_a \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right] > \rho_b \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right]$, we can induce

$$\beta_*\pi_{t=2}(\beta_*+\alpha) > \beta_1\pi_{t=2}(\beta_1+\alpha)$$

Based on the definition of β_1, we also know

$$\mathbb{E}\Pi_{\text{creator}}(\bar{\rho} \mid s_*) \leq \mathbb{E}\Pi_{\text{creator}}\left(\bar{\rho} \mid \left\{\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} V_{1,t=2}(\beta_1+\alpha), \beta_1\right\}\right)$$

$$\frac{\sum_{\rho\in\text{supp}(g)} \rho\Theta(\rho)}{\sum_{\rho\in\text{supp}(g)} \Theta(\rho)} \left[V_{1,t=2}(\beta_1+\alpha) - V_{1,t=2}(\beta_*+\alpha)\right] \geq \bar{\rho} \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right]$$

Because $\rho_a \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right] \geq \bar{\rho} \left[\beta_*\pi_{t=2}(\beta_*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right]$ and $\beta_*\pi_{t=2}(\beta_*+\alpha) > \beta_1\pi_{t=2}(\beta_1+\alpha)$, we can induce $\rho_a \geq \bar{\rho}$. $\rho_a \geq \bar{\rho}$ contradicts to

$\rho_a < \bar{\rho}$. Hence, we can reject $\beta_* \neq \beta_1$. Then Condition 2 is proved.

A.5 Proof of Theorem 1

A.5.1 Theorem 1

In a separate equilibrium $\left(\{\theta(s,\rho)\}_{s,\rho}, \hat{\theta}\right)$ of the signaling game such that there are $N \geq 2$ different signals in the market. Signals are denoted as $\mathcal{S} = \{s_i, i = 1, ..., N\}$. The order of signals follows $\beta_1 \pi_{t=2}(\beta_1 + \alpha) < \beta_2 \pi_{t=2}(\beta_2 + \alpha) < ... < \beta_N \pi_{t=2}(\beta_N + \alpha)$. Then, following properties of the separate equilibrium hold

(1). $\text{argmax}\{x : \theta(s_i, x) > 0\} < \text{argmin}\{x : \theta(s_j, x) > 0\}$, $\forall i, j \in \{1, 2, ..., N\}, i < j$.

(2). $p_1 > p_2 > ... > p_N$

A.5.2 Proof

In this equilibrium, the necessary condition NC 1 should be satisfied:

$$\mathbb{E}\Pi_{\text{creator}}(\rho \mid s_i) \geq \mathbb{E}\Pi_{\text{creator}}(\rho \mid s_j), \text{ for all } i, j \in \{1, 2, ..., N\}, \rho \in \{y : \theta(s_i, y) > 0\} \tag{NC 1}$$

Hence, we know $\forall i, j \in \{1, 2, ..., N\}, i < j$,

(1). Assume $\text{argmax}\{x : \theta(s_i, x) > 0\} > \text{argmin}\{x : \theta(s_j, x) > 0\}$. We denote these two bounds by $\bar{x}_i = \text{argmax}\{x : \theta(s_i, x) > 0\}$ and $\underline{x}_j = \text{argmin}\{x : \theta(s_j, x) > 0\}$, $\bar{x}_i > \underline{x}_j$. Based on NC 1, we know

$$p_i + \bar{x}_i \beta_i \pi_{t=2}(\beta_i + \alpha) \geq p_j + \bar{x}_i \beta_j \pi_{t=2}(\beta_j + \alpha) \tag{[Ne1]}$$

$$p_i + \underline{x}_j \beta_i \pi_{t=2}(\beta_i + \alpha) \leq p_j + \underline{x}_j \beta_j \pi_{t=2}(\beta_j + \alpha) \tag{[Ne2]}$$

must be satisfied in equilibrium. We let each side of [Ne1] minus each side of [Ne2],

$$(\bar{x}_i - \underline{x}_j)\beta_i\pi_{t=2}(\beta_i + \alpha) \geq (\bar{x}_i - \underline{x}_j)\beta_j\pi_{t=2}(\beta_j + \alpha) \qquad \text{([Ne1-Ne2])}$$

Based on assumption $\bar{x}_i > \underline{x}_j$, we can simplify [Ne1-Ne2]:

$$\beta_i\pi_{t=2}(\beta_i + \alpha) \geq \beta_j\pi_{t=2}(\beta_j + \alpha) \qquad \text{([Ne1-Ne2])}$$

which contradicts to the setting $\beta_i\pi_{t=2}(\beta_i+\alpha) < \beta_j\pi_{t=2}(\beta_j+\alpha)$. We can conclude $\text{argmax}\{x : \theta(s_i, x) > 0\} \leq \text{argmin}\{x : \theta(s_j, x) > 0\}$. Based on our assumption that a creator send the signal with larger first stage price if there are two equally optimal signals. So, we can conclude $\text{argmax}\{x : \theta(s_i, x) > 0\} < \text{argmin}\{x : \theta(s_j, x) > 0\}$.

(2). The necessary condition [Ne1] is equivalent to

$$p_i - p_j \geq \bar{x}_i(\beta_j\pi_{t=2}(\beta_j + \alpha) - \beta_i\pi_{t=2}(\beta_i + \alpha))$$

since the right side $\bar{x}_i(\beta_j\pi_{t=2}(\beta_j + \alpha) - \beta_i\pi_{t=2}(\beta_i + \alpha)) > 0$, we can infer

$$p_i > p_j, \quad \forall i, j \in \{1, 2, ..., N\}, i < j$$

A.6 Proof of Proposition 12

A.6.1 Proposition 12

With assumptions A1, 3, and 4, given buyers' belief about the NFT's quality being δ and the quality of a creator's NFT being ρ_j (or directly given $\lambda = \frac{\rho_j}{\delta}$),

(1). the $M_\delta(\beta, \lambda)$ is concave in β for $\lambda > \min_{\beta \in (0, 1-\alpha)} \left\{1 + \frac{\overline{M}(0) - \overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$.

And the β that maximizes the profits of the creator $M_\delta(\beta,\lambda)$, b_λ^*, exhibits the following behavior:

(2). When $\lambda < \min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$, $b_\lambda^* = 0$.

(3). When $\lambda \geq \min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$, b_λ^* exhibits the following properties:

(3.1). b_λ^* is the unique solution of β for $\frac{\partial V_{1,t=2}(\gamma)}{\partial\beta}+\lambda\frac{\partial\beta\pi_{t=2}(\gamma)}{\partial\beta}=0$

(3.2). b_λ^* is increasing in $\lambda\in[\min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\},\infty)$.

(3.3). $b_\lambda^*=0$ when $\lambda=\min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$.

(3.4). $b_\lambda^*\to b_2^*$ as $\lambda\to\infty$.

A.6.2 Proof

Given δ and λ, a creator's expected profits are $M_\delta(\beta,\lambda)=\delta\left(V_{1,t=2}(\gamma)+\lambda\beta\pi_{t=2}(\gamma)\right)$, which has first-order and second-order derivatives in β

$$\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta}=\delta\left(\frac{\partial V_{1,t=2}(\gamma)}{\partial\beta}+\lambda\frac{\partial\beta\pi_{t=2}(\gamma)}{\partial\beta}\right)$$
$$\frac{\partial^2 M_\delta(\beta,\lambda)}{\partial\beta^2}=\delta\left(\frac{\partial^2 V_{1,t=2}(\gamma)}{\partial\beta^2}+\lambda\frac{\partial^2\beta\pi_{t=2}(\gamma)}{\partial\beta^2}\right)$$

Based on proposition 10, we can know the profits of a creator satisfy $M_\delta(\beta,\lambda)\leq M_\delta(0,\lambda)$ for any $\beta\in[0,1-\alpha)$ given $\lambda\leq\min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$. So, the maximizer of a creator's profits with $\lambda\leq\min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$ is $b_\lambda^*=0<b_2^*$.

For creator with $\lambda>\min_{\beta\in(0,1-\alpha)}\left\{1+\frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$, there exists $\beta>0$ such that $\overline{M}(0)<\overline{M}(\beta)$. Let's consider the second derivatives firstly. Based on the assumption 4 that $\left.\frac{\partial^2 M_\delta(\beta,\lambda)}{\partial\beta^2}\right|_{\lambda=1}=\delta\left(\frac{\partial^2 V_{1,t=2}(\gamma)}{\partial\beta^2}+\frac{\partial^2\beta\pi_{t=2}(\gamma)}{\partial\beta^2}\right)<0$ and assumption A1 that

$\frac{\partial^2 \beta\pi_{t=2}(\gamma)}{\partial\beta^2} < 0$, we have

$$\begin{aligned}
\frac{\partial^2 M_\delta(\beta,\lambda)}{\partial\beta^2} &= \delta\left(\frac{\partial^2 V_{1,t=2}(\gamma)}{\partial\beta^2} + \lambda\frac{\partial^2 \beta\pi_{t=2}(\gamma)}{\partial\beta^2}\right) \\
&= \left.\frac{\partial^2 M_\delta(\beta,\lambda)}{\partial\beta^2}\right|_{\lambda=1} + \delta(\lambda-1)\frac{\partial^2 \beta\pi_{t=2}(\gamma)}{\partial\beta^2} < 0, \\
&\forall\lambda > \min_{\beta\in(0,1-\alpha)}\left\{1 + \frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\} \text{ and } \beta\in[0,1-\alpha)
\end{aligned}$$

That is the $M_\delta(\beta,\lambda)$ is concave in β for $\lambda > \min_{\beta\in(0,1-\alpha)}\left\{1 + \frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$.

Then, we can know the first derivate $\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta}$ is strictly decreasing in $\beta\in[0,1-\alpha)$ given $\lambda > \min_{\beta\in(0,1-\alpha)}\left\{1 + \frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\}$. Since there exists $\beta > 0$ such that $\overline{M}(0) < \overline{M}(\beta)$, we can infer $\left.\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta}\right|_{\beta=0} > 0$. Hence, a continuous function $\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta}$, with $\left.\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta}\right|_{\beta=0} > 0$ and $\left.\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta}\right|_{\beta=b_2^*} = \delta\frac{\partial^2 V_{1,t=2}(\gamma)}{\partial\beta^2} < 0$, must have a zero point in $(0,b_2^*)$ and the zero point should be the maximizer of $M_\delta(\beta,\lambda)$.

Suppose $\lambda_1,\lambda_2\in[\min_{\beta\in(0,1-\alpha)}\left\{1 + \frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\},\infty)$ with $\lambda_1<\lambda_2$ and $b_{\lambda_1}^*\in(0,b_2^*)$ with $\left.\frac{\partial M_\delta(\beta,\lambda_1)}{\partial\beta}\right|_{\beta=b_{\lambda_1}^*} = 0$. Then,

$$\begin{aligned}
\left.\frac{\partial M_\delta(\beta,\lambda_2)}{\partial\beta}\right|_{\beta=b_{\lambda_1}^*} &= \delta\left(\left.\frac{\partial V_{1,t=2}(\gamma)}{\partial\beta}\right|_{\beta=b_{\lambda_1}^*} + \lambda_2\left.\frac{\partial\beta\pi_{t=2}(\gamma)}{\partial\beta}\right|_{\beta=b_{\lambda_1}^*}\right) \\
&> \delta\left(\left.\frac{\partial V_{1,t=2}(\gamma)}{\partial\beta}\right|_{\beta=b_{\lambda_1}^*} + \lambda_1\left.\frac{\partial\beta\pi_{t=2}(\gamma)}{\partial\beta}\right|_{\beta=b_{\lambda_1}^*}\right) = 0
\end{aligned}$$

The inequality holds because $\frac{\partial\beta\pi_{t=2}(\gamma)}{\partial\beta} > 0, \forall\beta\in(0,b_2^*)$. Since $\frac{\partial M_\delta(\beta,\lambda_2)}{\partial\beta}$ is decreasing in β, so the $b_{\lambda_2}^*$ with $\left.\frac{\partial M_\delta(\beta,\lambda_2)}{\partial\beta}\right|_{\beta=b_{\lambda_2}^*} = 0$ satisfies $b_{\lambda_2}^* > b_{\lambda_1}^*$. Hence, we can conclude b_λ^* is increasing in $\lambda\in[\min_{\beta\in(0,1-\alpha)}\left\{1 + \frac{\overline{M}(0)-\overline{M}(\beta)}{\beta\pi_{t=2}(\gamma)}\right\},\infty)$

Specifically, when $\lambda = \frac{\rho_j}{\delta}\to\infty$ (i.e. $\delta\to 0$), $\lim_{\delta\to 0}\frac{\partial M_\delta(\beta,\lambda)}{\partial\beta} = \rho_j\frac{\partial\beta\pi_{t=2}(\gamma)}{\partial\beta}$, which is maximized at b_2^*.

A.7 Proof of Proposition 13

A.7.1 Proposition 13

$\Theta(\rho_l) = \theta(s_l^*, \rho_l) = g_l, \Theta(\rho_h) = \theta(s_h^*, \rho_h) = g_h$ with the buyer's belief updating rule $\delta(\cdot)$ forms a separate equilibrium if and only if the following conditions are satisfied

(1). When receiving signal s_l^*, the first buyer forms belief ρ_l and purchases, which requires high-quality creators have no incentive to deviate from s_h^* to this signal:

$$\delta(s_l^*) = \rho_l, \ s_l^* \in H(\rho_l), \text{ and } s_l^* \notin \overline{S}(s_h^*) \tag{(1)}$$

(2). When receiving signal s_h^*, the first buyer forms belief ρ_h and purchases, which requires low-quality creators have no incentive to deviate from s_l^* to this signal:

$$\delta(s_h^*) = \rho_h, \ s_h^* \in H(\rho_h), \text{ and } s_h^* \notin \overline{S}(s_l^*) \tag{(2)}$$

(3). A low-quality creator can get more profits from joining the market than from outside option:

$$p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha) \geq u_l \tag{(3)}$$

(4). A high-quality creator can get more profits from joining the market than from outside option:

$$p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \geq u_h \tag{(4)}$$

(5). Compared to s_l^*, a low-quality creator doesn't have a signal that can get higher profits for him and can also sell the NFT to the first buyer:

$$H(\rho_l) \cap \left(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\right) = H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset \tag{(5)}$$

(6). Compared to s_h^*, a high-quality creator doesn't have a signal that can get higher profits for him and can also sell the NFT to the first buyer:

$$H(\rho_h) \cap \left(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\right) = H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset \tag{(6)}$$

A.7.2 Proof

Based on definition 1 and definition 2, a separate equilibrium holds if and only if

- Con. 1: $s_l^* \in \operatorname{argmax}_{s \in H(\hat{\theta},\Theta)} \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s)$ and $s_h^* \in \operatorname{argmax}_{s \in H(\hat{\theta},\Theta)} \mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s)$.
- Con. 2: $\hat{\theta}(\rho_l \mid \Theta, s_l^*) = 1$, $\hat{\theta}(\rho_h \mid \Theta, s_h^*) = 1$.
- Con. 3: $p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha) \geq u_l$ and $p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \geq u_h$ (Exactly the (3) and (4) of the Proposition 13)
- Con. 4: No need to discuss since all creators join the market in the separate equilibrium.

We first prove these conditions of the Proposition 13 are necessary:

1). $\hat{\theta}(\rho_l \mid \Theta, s_l^*) = 1$, $\hat{\theta}(\rho_h \mid \Theta, s_h^*) = 1 \Rightarrow \delta(s_l^*) = \rho_l, \delta(s_h^*) = \rho_h$.
$s_l^* \in \operatorname{argmax}_{s \in H(\hat{\theta},\Theta)} \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s)$ and $s_h^* \in \operatorname{argmax}_{s \in H(\hat{\theta},\Theta)} \mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s) \Rightarrow$ $s_l^* \in H(\hat{\theta},\Theta)$, $s_h^* \in H(\hat{\theta},\Theta)$, and

$$H(\hat{\theta},\Theta) \cap \overline{S}(s_l^*) = H(\hat{\theta},\Theta) \cap \overline{S}(s_h^*) = \emptyset$$

$\delta(s_l^*) = \rho_l, s_l^* \in H(\hat{\theta},\Theta) \Rightarrow p_l^* \leq \delta(s_l^*) V_{1,t=2}(\beta_l^* + \alpha) = \rho_l V_{1,t=2}(\beta_l^* + \alpha) \Rightarrow$ $s_l^* \in H(\rho_l)$.
$\delta(s_h^*) = \rho_h, s_h^* \in H(\hat{\theta},\Theta) \Rightarrow p_h^* \leq \delta(s_h^*) V_{1,t=2}(\beta_h^* + \alpha) = \rho_h V_{1,t=2}(\beta_h^* + \alpha) \Rightarrow$ $s_h^* \in H(\rho_h)$.
$s_l^* \in H(\hat{\theta},\Theta), H(\hat{\theta},\Theta) \cap \overline{S}(s_h^*) = \emptyset \Rightarrow s_l^* \notin \overline{S}(s_h^*)$.
$s_h^* \in H(\hat{\theta},\Theta), H(\hat{\theta},\Theta) \cap \overline{S}(s_l^*) = \emptyset \Rightarrow s_h^* \notin \overline{S}(s_l^*)$.
(1) and (2) are proved.

2). (3) and (4) are directly showed by Con. 3.

3). Obviously, $H(\rho_l)\cap\big(\overline{S}(s_l^*)\backslash\overline{S}(s_h^*)\big) \subseteq \overline{S}(s_l^*)$. According to 10, $H(\rho_l)\cap\big(\overline{S}(s_l^*)\backslash\overline{S}(s_h^*)\big) \subseteq H(\hat{\theta},\Theta)$. So, $H(\rho_l)\cap\big(\overline{S}(s_l^*)\backslash\overline{S}(s_h^*)\big) \subseteq H(\hat{\theta},\Theta)\cap\overline{S}(s_l^*) = \emptyset \Rightarrow H(\rho_l)\cap\big(\overline{S}(s_l^*)\backslash\overline{S}(s_h^*)\big) = \emptyset$.
Obviously, $H(\rho_h)\cap\big(\overline{S}(s_h^*)\backslash\overline{S}(s_l^*)\big) \subseteq \overline{S}(s_h^*)$. According to 10, $H(\rho_h)\cap\big(\overline{S}(s_h^*)\backslash\overline{S}(s_l^*)\big) \subseteq H(\hat{\theta},\Theta)$. So, $H(\rho_h)\cap\big(\overline{S}(s_h^*)\backslash\overline{S}(s_l^*)\big) \subseteq H(\hat{\theta},\Theta)\cap\overline{S}(s_h^*) = \emptyset \Rightarrow H(\rho_h)\cap\big(\overline{S}(s_h^*)\backslash\overline{S}(s_l^*)\big) = \emptyset$.
Obviously, $H(\rho_0)\cap\big(\overline{S}(s_l^*)\cap\overline{S}(s_h^*)\big) \subseteq \overline{S}(s_l^*)$. According to 10, $H(\rho_0)\cap\big(\overline{S}(s_l^*)\cap\overline{S}(s_h^*)\big) \subseteq H(\hat{\theta},\Theta)$. So, $H(\rho_0)\cap\big(\overline{S}(s_l^*)\cap\overline{S}(s_h^*)\big) \subseteq H(\hat{\theta},\Theta)\cap\overline{S}(s_l^*) = \emptyset \Rightarrow H(\rho_0)\cap\big(\overline{S}(s_l^*)\cap\overline{S}(s_h^*)\big) = \emptyset$.
(5) and (6) are proved.

We then prove these conditions are sufficient.

1) $\delta(s_l^*) = \rho_l,\ s_l^* \in H(\rho_l) \Rightarrow p_l^* \le \rho_l V_{1,t=2}(\beta_l^* + \alpha) = \delta(s_l^*)V_{1,t=2}(\beta_l^* + \alpha) \Rightarrow s_l^* \in H(\hat{\theta},\Theta)$; $\delta(s_h^*) = \rho_h,\ s_h^* \in H(\rho_h) \Rightarrow p_h^* \le \rho_h V_{1,t=2}(\beta_h^* + \alpha) = \delta(s_h^*)V_{1,t=2}(\beta_h^* + \alpha) \Rightarrow s_h^* \in H(\hat{\theta},\Theta)$.
Assume there is a signal $s_i \in H(\hat{\theta},\Theta)$ such that $\mathbb{E}\Pi(\rho_l \mid s_i) > \mathbb{E}\Pi(\rho_l \mid s_l^*)$ i.e., $s_i \in \overline{S}(s_l^*)$.

   - If $s_i \in \overline{S}(s_l^*)\backslash\overline{S}(s_h^*)$, $\delta(s_i) = \rho_l$. Because $s_i \in H(\hat{\theta},\Theta)$, we have $p_i \le \delta(s_i)V_{1,t=2}(\beta_i+\alpha) = \rho_l V_{1,t=2}(\beta_i+\alpha)$, which means $s_i \in H(\rho_l)$. Hence, $s_i \in H(\rho_l)\cap\big(\overline{S}(s_l^*)\backslash\overline{S}(s_h^*)\big)$ which contradicts to (5).
   - If $s_i \in \overline{S}(s_l^*)\cup\overline{S}(s_h^*)$, $\delta(s_i) = \rho_0$. Because $s_i \in H(\hat{\theta},\Theta)$, we have $p_i \le \delta(s_i)V_{1,t=2}(\beta_i + \alpha) = \rho_0 V_{1,t=2}(\beta_i + \alpha)$, which means $s_i \in H(\rho_0)$. Hence, $s_i \in H(\rho_0)\cap\big(\overline{S}(s_l^*)\cup\overline{S}(s_h^*)\big)$ which contradicts to (5).

   We can conclude that there can't exist $s_i \in H(\hat{\theta},\Theta)$ such that $\mathbb{E}\Pi(\rho_l \mid s_i) > \mathbb{E}\Pi(\rho_l \mid s_l^*)$. Then, $s_l^* \in \operatorname{argmax}_{s\in H(\hat{\theta},\Theta)} \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s)$ is proved.
Assume there is a signal $s_i \in H(\hat{\theta},\Theta)$ such that $\mathbb{E}\Pi(\rho_h \mid s_i) > \mathbb{E}\Pi(\rho_h \mid s_h^*)$ i.e., $s_i \in \overline{S}(s_h^*)$.

- If $s_i \in \overline{S}(s_h^*) \backslash \overline{S}(s_l^*)$, $\delta(s_i) = \rho_h$. Because $s_i \in H(\hat{\theta}, \Theta)$, we have $p_i \leq \delta(s_i) V_{1,t=2}(\beta_i + \alpha) = \rho_h V_{1,t=2}(\beta_i + \alpha)$, which means $s_i \in H(\rho_h)$. Hence, $s_i \in H(\rho_h) \cap \big(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\big)$ which contradicts to (6).
- If $s_i \in \overline{S}(s_l^*) \cup \overline{S}(s_h^*)$, $\delta(s_i) = \rho_0$. Because $s_i \in H(\hat{\theta}, \Theta)$, we have $p_i \leq \delta(s_i) V_{1,t=2}(\beta_i + \alpha) = \rho_0 V_{1,t=2}(\beta_i + \alpha)$, which means $s_i \in H(\rho_0)$. Hence, $s_i \in H(\rho_0) \cap \big(\overline{S}(s_l^*) \cup \overline{S}(s_h^*)\big)$ which contradicts to (5).

We can conclude that there can't exist $s_i \in H(\hat{\theta}, \Theta)$ such that $\mathbb{E}\Pi(\rho_h \mid s_i) > \mathbb{E}\Pi(\rho_h \mid s_h^*)$. Then, $s_h^* \in \operatorname{argmax}_{s \in H(\hat{\theta}, \Theta)} \mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s)$ is proved.
Con. 1 is proved.

2) $\delta(s_l^*) = \rho_l = \rho_l \hat{\theta}(\rho_l \mid \Theta, s_l^*) + \rho_h \hat{\theta}(\rho_h \mid \Theta, s_l^*) = \rho_h + (\rho_l - \rho_h)\hat{\theta}(\rho_l \mid \Theta, s_l^*) \Rightarrow \hat{\theta}(\rho_l \mid \Theta, s_l^*) = 1$; $\delta(s_h^*) = \rho_h = \rho_l \hat{\theta}(\rho_l \mid \Theta, s_h^*) + \rho_h \hat{\theta}(\rho_h \mid \Theta, s_h^*) = \rho_l + (\rho_h - \rho_l)\hat{\theta}(\rho_h \mid \Theta, s_h^*) \Rightarrow \hat{\theta}(\rho_h \mid \Theta, s_h^*) = 1$.
Con. 2 is proved.

3) Con. 3 is directly showed by (3) and (4).

A.8 Proof of Lemma 11

A.8.1 Lemma 11

$\beta_l^* \in [0, \beta_2^*]$ and $\beta_h^* \in [0, \beta_2^*]$.

A.8.2 Proof

We divide the proof by two part:

1. Assume $\beta_l^* > \beta_2^*$. Based on (1) of Proposition 13 that $s_l^* \notin \overline{S}(s_h^*)$, we know

$$p_l^* + \rho_h \beta_l^* \pi_{t=2}(\beta_l^* + \alpha) \leq p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$$

As we showed in Theorem 1, two kinds of creators are separated based on quality such that $\beta_l^* \pi_{t=2}(\beta_l^* + \alpha) < \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$ and $p_l^* > p_h^*$. Then, we can find a $\beta_l' \in [0, b_2^*]$ such that $\beta_l' \leq \beta_h^*$ and $\beta_l' \pi_{t=2}(\beta_l' + \alpha) = \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) > \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)$.

Based on

$$p_l^* + \rho_h \beta_l' \pi_{t=2}(\beta_l' + \alpha) > p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$$
$$p_l^* + \rho_l \beta_l' \pi_{t=2}(\beta_l' + \alpha) > p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)$$

we can conclude

$$\{p_l^*, \beta_l'\} \in \overline{S}(s_l^*) \cap \overline{S}(s_h^*)$$

According to the (1) of Proposition 13 that $s_l^* \in H(\rho_l)$, we can infer $\{p_l^*, \beta_l'\} \in H(\rho_0)$ by

$$p_l^* \leq \rho_l V_{1,t=2}(\beta_l^* + \alpha) < \rho_l V_{1,t=2}(\beta_l' + \alpha) < \rho_0 V_{1,t=2}(\beta_l' + \alpha)$$

The second inequality holds because the $V_{1,t=2}(\cdot)$ is a decreasing function. We can conclude $\{p_l^*, \beta_l'\} \in H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right)$, which contradicts to the (5) of Proposition 13 that $H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset$. Hence, $\beta_l^* \in [0, \beta_2^*]$.

2. Assume $\beta_h^* > b_2^*$. Based on (2) of Proposition 13 that $s_h^* \notin \overline{S}(s_l^*)$, we know

$$p_h^* + \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \leq p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)$$

Consider an off-path signal

$$s' = \{p_h^* + \rho_l(\beta_h^* \pi_{t=2}(\beta_h^* + \alpha) - b_2^* \pi_{t=2}(b_2^* + \alpha)), b_2^*\}$$

Since $\mathbb{E}(\rho_l \mid s') = p_h^* + \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \leq p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)$, $s' \notin \overline{S}(s_l^*)$. The signal s' can generate more profits for high-quality creator:

$$\begin{aligned}\mathbb{E}(\rho_h \mid s') &= p_h^* + \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) + (\rho_h - \rho_l) b_2^* \pi_{t=2}(b_2^* + \alpha) \\ &> p_h^* + \rho_h \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) = \mathbb{E}(\rho_h \mid s_h^*)\end{aligned}$$

The inequality holds because b_2^* maximizes convex $\beta\pi_{t=2}(\beta+\alpha)$. From the equation, we know $s' \in \overline{S}(s_h^*)\backslash\overline{S}(s_l^*)$.
Because $s_h^* \in H(\rho_h)$, $p_h^* \le \rho_h V_{1,t=2}(\beta_h^*+\alpha)$. Then,

$$\begin{aligned} & p_h^* + \rho_l(\beta_h^*\pi_{t=2}(\beta_h^*+\alpha) - b_2^*\pi_{t=2}(b_2^*+\alpha)) \\ < \; & p_h^* \le \rho_h V_{1,t=2}(\beta_h^*+\alpha) < \rho_h V_{1,t=2}(b_2^*+\alpha) \end{aligned}$$

From the equation, we can know $s' \in H(\rho_h)$.
All in all, we can conclude $s' \in H(\rho_h) \cap \left(\overline{S}(s_h^*)\backslash\overline{S}(s_l^*)\right)$, which contradicts to the (6) of Proposition 13 that $H(\rho_h) \cap \left(\overline{S}(s_h^*)\backslash\overline{S}(s_l^*)\right) = \emptyset$. Hence, $\beta_h^* \in [0, \beta_2^*]$.

A.9 Proof of Lemma 12

A.9.1 Lemma 12

For a low-quality creator, his signal $s_l^* = (p_l^*, \beta_l^*)$ with $\beta_l^* > 0$ in separate equilibrium fulfills

$$p_l^* = \rho_l V_{1,t=2}(\beta_h^*+\alpha)$$

A.9.2 Proof

Assume $p_l^* < \rho_l V_{1,t=2}(\beta_l^*+\alpha)$. We can know

$$T(\beta) = p_l^* + \rho_h\left(\beta_l^*\pi_{t=2}(\beta_l^*+\alpha) - \beta\pi_{t=2}(\beta+\alpha)\right) - \rho_l V_{1,t=2}(\beta+\alpha)$$

is a continuous function for $\beta \in [0, b_2^*]$. We can find $T(\beta_l^*) = p_l^* - \rho_l V_{1,t=2}(\beta_l^*+\alpha) < 0$. Because of the continuity of $T(\cdot)$, there must exist $\beta_1 \in [0, \beta_l^*)$ such that $T(\beta_1) \le 0$, i.e.,

$$p_l^* + \rho_h\left(\beta_l^*\pi_{t=2}(\beta_l^*+\alpha) - \beta_1\pi_{t=2}(\beta_1+\alpha)\right) \le \rho_l V_{1,t=2}(\beta_1+\alpha) \tag{A1}$$

Let $s_1 = \left(p_l^* + \rho_h \left(\beta_l^* \pi_{t=2}(\beta_l^* + \alpha) - \beta_1 \pi_{t=2}(\beta_1 + \alpha)\right), \beta_1\right)$, we find $s_1 \in \overline{S}(s_l^*)$ as

$$p_l^* + \rho_h \left(\beta_l^* \pi_{t=2}(\beta_l^* + \alpha) - \beta_1 \pi_{t=2}(\beta_1 + \alpha)\right) + \rho_l \beta_1 \pi_{t=2}(\beta_1 + \alpha) > p_l^* + \rho_l \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)$$

Since the high-quality creator's profits under signal s_l^* and under signal s_1 are the same

$$p_l^* + \rho_h \left(\beta_l^* \pi_{t=2}(\beta_l^* + \alpha) - \beta_1 \pi_{t=2}(\beta_1 + \alpha)\right) + \rho_h \beta_1 \pi_{t=2}(\beta_1 + \alpha) = p_l^* + \rho_h \beta_l^* \pi_{t=2}(\beta_l^* + \alpha)$$

which means $s_1 \notin \overline{S}(s_h^*)$ because it provides the same profits for high-quality creator as s_l^* and $s_l^* \notin \overline{S}(s_h^*)$.

Equation A1 also shows the $s_1 \in H(\rho_l)$. Hence, we can conclude

$$s_1 \in H(\rho_l) \cap \left(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\right)$$

which contradicts to the (5) in Proposition 13 that $H(\rho_l) \cap \left(\overline{S}(s_l^*) \backslash \overline{S}(s_h^*)\right)$ should be empty.

A.10 Proof of Lemma 15

A.10.1 Lemma 15

Given $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha)$,

$$p_h^* = \rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$$

must hold in separate equilibrium.

A.10.2 Proof

Assume the $s_h^* = (p_h^*, \beta_h^*)$ in separate equilibrium satisfy $p_h^* < \rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$.

Then, $\{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\}$ is in $\overline{S}(s_h^*)$ as $\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) > p_h^*$.

Based on Lemma 14, $\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \leq \rho_h V_{1,t=2}(\beta_h^* + \alpha)$. We can conclude $\{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\} \in H(\rho_h)$.

However, $\{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\}$ is not in $\overline{S}(s_l^*)$ because

$$
\begin{aligned}
&\mathbb{E}\Pi_{\text{creator}}(\rho_l \mid \{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\}) \\
=&\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) + \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha) \\
=&\rho_l V_{1,t=2}(\alpha) = \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s_l^*)
\end{aligned}
$$

Hence,

$$
\{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\} \in H(\rho_h) \cap \left(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\right)
$$

which contradicts to the (6) in proposition 15.

A.11 Proof of Proposition 16

A.11.1 Proposition 16

Given $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha)$, high-quality creators release signal

$$
s_h^* = \{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}
$$

in separate equilibrium.

A.11.2 Proof

As we proved in lemma 15, a high-quality creator's signal in separate equilibrium can be represented by

$$s_h^* = \{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\}$$

So, we want to prove the $\beta_h^* = b_2^*$ must hold in separate equilibrium.
A high-quality creator's profits from signal $s_h^* = \{\rho_l V_{1,t=2}(\alpha) - \rho_l \beta_h^* \pi_{t=2}(\beta_h^* + \alpha), \beta_h^*\}$ is

$$\rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l)\beta_h^* \pi_{t=2}(\beta_h^* + \alpha)$$

which is increasing in $\beta_h^* \in [0, b_2^*]$.
Assume $\beta_h^* < b_2^*$ in the separate equilibrium, we can find $\{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}$ induce higher profits for high-quality creators than s_h^*, i.e.,

$$\{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\} \in \overline{S}(s_h^*)$$

According to the assumption $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^*+\alpha) + \rho_l b_2^* \pi_{t=2}(b_2^*+\alpha)$, $\{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}$ is in $H(\rho_h)$ as

$$\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha)$$

And $\{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}$ is not in $\overline{S}(s_l^*)$, because

$$\begin{aligned} &\mathbb{E}\Pi_{\text{creator}}(\rho_l \mid \{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}) \\ =&\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha) \\ =&\rho_l V_{1,t=2}(\alpha) = \mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s_l^*) \end{aligned}$$

Hence, we can conclude

$$\{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\} \in \overline{S}(s_h^*) \backslash \overline{S}(s_l^*) \cap H(\rho_h)$$

which contradicts to the (6) in proposition 15.

A.12 Proof of Proposition 17

A.12.1 Proposition 17

Given

- $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$ and $s_h^* = \{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}$,
- $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha)$,
- $\mathbb{E}\Pi_{\text{creator}}(\rho_l \mid s_l^*) = \rho_l V_{1,t=2}(\alpha) \geq u_l$,
- $\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) = \rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l) b_2^* \pi_{t=2}(b_2^* + \alpha) \geq u_h$,
- and the buyer's belief updating $\delta(\cdot)$ that fulfills $\delta(s_l^*) = \rho_l$, $\delta(s_h^*) = \rho_h$, and Off-Path Updating Rule,

there exists unique separate equilibrium that $\{\Theta(\rho_l) = \theta(s_l^*, \rho_l) = g_l, \Theta(\rho_h) = \theta(s_h^*, \rho_h) = g_h, \hat{\theta}\}$ if and only if

$$\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) \geq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0})$$

A.12.2 Proof

As we proved in proposition 14 and 16, given $\rho_l V_{1,t=2}(\alpha) \leq \rho_h V_{1,t=2}(b_2^* + \alpha) + \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha)$, the form of separate equilibrium must fulfill $\{\Theta(\rho_l) = \theta(s_l^*, \rho_l) = g_l, \Theta(\rho_h) = \theta(s_h^*, \rho_h) = g_h, \hat{\theta}\}$ where $s_l^* = \{\rho_l V_{1,t=2}(\alpha), 0\}$, $s_h^* = \{\rho_l V_{1,t=2}(\alpha) - \rho_l b_2^* \pi_{t=2}(b_2^* + \alpha), b_2^*\}$, and the buyer's belief updating rule $\delta(\cdot)$ fulfills $\delta(s_l^*) = \rho_l$, $\delta(s_h^*) = \rho_h$, and Off-Path Updating Rule. So, once we can show the separate equilibrium exits, it must also be unique.
Considering all conditions in proposition 15, (1), (2), (3), and (4) are easily showed by our assumptions and the structure of signal s_l^*, s_h^*. So, we only need to focus on

(5): $H(\rho_0) \cap \left(\overline{S}(s_l^*) \cap \overline{S}(s_h^*)\right) = \emptyset$ and (6): $H(\rho_h) \cap \left(\overline{S}(s_h^*) \backslash \overline{S}(s_l^*)\right) = \emptyset$.

$$\begin{aligned}
\overline{S}(s_l^*) &= \{(p_i, \beta_i) \mid p_i + \rho_l \beta_i \pi_{t=2}(\beta_i + \alpha) > \rho_l V_{1,t=2}(\alpha)\} \\
\overline{S}(s_h^*) &= \{(p_i, \beta_i) \mid p_i + \rho_h \beta_i \pi_{t=2}(\beta_i + \alpha) > \rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l) b_2^* \pi_{t=2}(b_2^* + \alpha)\} \\
&= \{(p_i, \beta_i) \mid p_i + \rho_l \beta_i \pi_{t=2}(\beta_i + \alpha) > \\
&\qquad \rho_l V_{1,t=2}(\alpha) + (\rho_h - \rho_l)[b_2^* \pi_{t=2}(b_2^* + \alpha) - \beta_i \pi_{t=2}(\beta_i + \alpha)]\}
\end{aligned}$$

Since $b_2^* \pi_{t=2}(b_2^* + \alpha) - \beta_i \pi_{t=2}(\beta_i + \alpha) \geq 0, \forall \beta_i \in [0, b_2^*]$, we can infer $\overline{S}(s_h^*) \subseteq \overline{S}(s_l^*)$. So,

$$\overline{S}(s_h^*) \backslash \overline{S}(s_l^*) = \emptyset, \quad \overline{S}(s_l^*) \cap \overline{S}(s_h^*) = \overline{S}(s_h^*)$$

Then, (6) in proposition 15 must be fulfilled since $\overline{S}(s_h^*) \backslash \overline{S}(s_l^*) = \emptyset$.
Hence, the proposition proof is simplified to prove

"$\overline{S}(s_h^*) \cap H(\rho_0) = \emptyset$ if and only if $\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) \geq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0})$"

(Proof of $\Leftarrow$): Any $(p_i, \beta_i) \in H(\rho_0)$ satisfies

$$\begin{aligned}
\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid (p_i, \beta_i)) &= p_i + \rho_h \beta_i \pi_{t=2}(\beta_i + \alpha) \\
&\leq \rho_0 V_{1,t=2}(\beta_i + \alpha) + \rho_h \beta_i \pi_{t=2}(\beta_i + \alpha) \\
&= M_{\rho_0}(\beta_i, \frac{\rho_h}{\rho_0}) \leq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0})
\end{aligned}$$

As $\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) \geq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0}) \geq \mathbb{E}\Pi_{\text{creator}}(\rho_h \mid (p_i, \beta_i))$, we know $(p_i, \beta_i) \notin \overline{S}(s_h^*), \forall (p_i, \beta_i) \in H(\rho_0)$. Hence, $\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) \geq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0}) \Rightarrow \overline{S}(s_h^*) \cap H(\rho_0) = \emptyset$ is proved.

(Proof of $\Rightarrow$): Assume there is a $\beta' \in [0, b_2^*]$ such that $\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) < M_{\rho_0}(\beta', \frac{\rho_h}{\rho_0})$, that is

$$\begin{aligned}
\mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) &< \rho_0 V_{1,t=2}(\beta' + \alpha) + \rho_h \beta' \pi_{t=2}(\beta' + \alpha) \\
&= \mathbb{E}\Pi_{\text{creator}}(\rho_h \mid (\rho_0 V_{1,t=2}(\beta' + \alpha), \beta'))
\end{aligned}$$

We can find $(\rho_0 V_{1,t=2}(\beta' + \alpha), \beta') \in \overline{S}(s_h^*) \cap H(\rho_0)$, which contradicts to $\overline{S}(s_h^*) \cap H(\rho_0) = \emptyset$. Hence, $\overline{S}(s_h^*) \cap H(\rho_0) \Rightarrow \mathbb{E}\Pi_{\text{creator}}(\rho_h \mid s_h^*) \geq \max_{\beta \in [0, b_2^*]} M_{\rho_0}(\beta, \frac{\rho_h}{\rho_0})$ is proved.

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