Miguel Class

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All models are wrong, but some are useful.

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Chapter 1 Pricing

1.1 Monopoly

1.1.1 Base Case

The firm decides its price p to maximize $\Pi(p) = p \cdot D(p) - C(D(p))$, where $D(\cdot)$ is the demand function and $C(\cdot)$ is the cost function.

The monopoly problem is maximizing the profit

$$\max_p \Pi(p) = p \cdot D(p) - C(D(p))$$

The F.O.C. (first-order condition) is

$$\frac{\partial \Pi(p)}{\partial p} = D(p) + pD'(p) - C'(D(p))D'(p) = 0$$

and the S.O.C. (second-order condition) is

$$\frac{\partial \Pi^2(p)}{\partial p^2} < 0$$

The F.O.C. gives that

$$\begin{aligned}
(p - C')D' &= -D \\
p - C' &= -\frac{D}{D'} \\
\underbrace{\frac{p - C'}{p}}_{\text{Lerner Index}} &= -\frac{D}{D'p} \\
&= -\frac{1}{\frac{dD}{dp}\frac{p}{D}} = -\frac{1}{\frac{\frac{dD}{D}}{\frac{dp}{p}}} := \frac{1}{E}
\end{aligned}$$

where $\frac{\frac{dD}{D}}{\frac{dp}{p}} < 0$ is the elasticity of demand with respect to price. The absolute value of the elasticity is denoted by E.

E is supposed to be greater than 1, otherwise, the optimal price is negative.

In the demand function $D(p) = kp^{-E}$, where the elasticity is constant. Its elasticity is $-E$.

The monopolist gives the production that is lower than social-optimal to maximize the profit (dead weight loss).

Rent dissipation can give larger dead weight loss.

1.1.2 Multiple Products

$$\max_{p} \sum_{i=1}^{N} p_i D_i(p) - C(D_1(p), ..., D_N(p))$$

Related Demand and Separable Costs: $C(D_1(p), ..., D_N(p)) = C_1(D_1(p)) + ... + C_N(D_N(p))$. The optimal pricing in this case satisfies

$$\frac{p_i - C_i'}{p_i} = \frac{1}{E_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j E_{ij}}{R_i E_{ii}}$$

where $E_{ij} = \frac{\partial D_i}{\partial p_j} \frac{p_j}{D_i}$ and R_i is the revenue.

Intuition: In the case of substitutes/complements, we want to increase/decrease the price of products compared to the one product case. (Positive/negative externality by increasing price of substitutes).

Similar Intuition: Consider a two-period model that the demand at second period depends on the price at first period (assuming $\frac{\partial D_2}{\partial p_1} < 0$).

1. $q_1 = D_1(p_1)$; $C_1(q_1)$
2. $q_2 = D_2(p_2, p_1)$; $C_2(q_2)$

Then, $\frac{p_1 - C_1'}{p_1} < \frac{1}{E_1}$ (the negative externality of prices).

Independent Demands and Related Costs:

Example 1.1

Different intensity of demand across periods.

1. Period 1: Low demand. $q_1 = D_1(p_1)$.
2. Period 2: High demand. $q_2 = D_2(p_2)$, where $D_1(p) = \lambda D_2(p)$ for some $\lambda < 1$.
3. Marginal cost of Production is c and the Marginal cost of capacity is γ.

Intuition: if $\lambda \to 0$, the marginal cost at period $\to c + \gamma$ and the marginal cost at period $1 = c$. Then, we have

$$\frac{p_2 - (c + \gamma)}{p_2} = \frac{1}{E_2}, \; \frac{p_1 - c}{p_1} = \frac{1}{E_1}$$

Now, let's consider a not too small λ. The problem is given as

$$\max_{p_1, p_2, k} (p_1 - c)D_1(p_1) + (p_2 - c)D_2(p_2) - \gamma k$$

$$s.t. \; D_1(p_1) \leq k$$

$$D_2(p_2) \leq k$$

The Lagrangian is given by

$$\mathcal{L} = (p_1 - c)D_1(p_1) + (p_2 - c)D_2(p_2) - \gamma k + \lambda_1(k - D_1(p_1)) + \lambda_2(k - D_2(p_2))$$

$$\frac{\partial \mathcal{L}}{\partial k} = -\gamma + \lambda_1 + \lambda_2 = 0 \Leftrightarrow \gamma = \lambda_1 + \lambda_2$$

Skip the process: $\frac{p_1-(c+\lambda_1)}{p_1} = \frac{1}{E_1}, \frac{p_2-(c+\lambda_2)}{p_2} = \frac{1}{E_2}$. Example: If $\lambda_1 = 0, k > D_1(p_1)$, the second period pays all the capacity cost.

Example 1.2 (Learning by Doing)

Suppose there are two periods $t = 1, 2$. The demand is $q_t = D_t(p_t)$. The cost in period one is $c_1(q_1)$ and $c_2(q_2, q_1)$ ($\frac{\partial c_2}{\partial q_1} < 0$, the more you produce in period one, the lower the cost you are facing in period two).

In continuous form, the cost form is

$$C(w(t))$$

where $\dot{w}(t) = \frac{dw}{dt} = q(t)$. We want to maximize

$$\max_{q(t),w(t)} \int_0^\infty e^{-\pi t}[q(t)p(q(t)) - C(w(t))q(t)]dt$$

$$\text{s.t. } \dot{w}(t) = q(t)$$

By Hamiltonian (skip), average of future marginal costs is

$$A(t) = \int_t^\infty C(w(s))\pi e^{-\pi(s-t)}ds$$

$$\frac{P(t) - A(t)}{P(t)} = \frac{1}{E(t)}$$

1.1.3 Durable Goods

The demand in one period is substitute to demand in other periods.

Example 1.3

Two periods $t = 1, 2$. Three consumers: $v_1 = 1$ per period, $v_2 = 2$ per period, and $v_3 = 3$ per period. The cost of production is zero. The seller chooses p_1, p_2.

(Consumer may forward-looking).

Moorthy (1988), Levinthal, D. A., & Purohit, D. (1989).

$t = 1, 2$ and zero production cost. The values of consumers $v \sim U[0, 1]$ and the discount factor is $\delta < 1$. The selling price p_1, p_2.

Suppose the consumers bought in first period have $v \geq v_1^*$, which must satisfies

$$\delta(v_1^* - p_2) = v_1^* - p_1 + \delta v_1^*$$

$$v_1^* = p_1 - \delta p_2$$

The price in second period should be $p_2 = \frac{v_1^*}{2}$. Then, $v_1^* = p_1 - \delta \frac{v_1^*}{2} \Rightarrow v_1^* = \frac{2p_1}{2+\delta}$. The price in the first period is given by

$$\max_{p_1} \ p_1(1 - v_1^*) + \delta \frac{(v_1^*)^2}{4} = p_1 \frac{2 + \delta - 2p_1}{2 + \delta} + \delta \frac{p_1^2}{(2 + \delta)^2}$$

Leasing (instead of selling): The leasing price is p in each period, which is given by $\operatorname{argmax}_p p(1-p) = \frac{1}{2}$. Leasing may generate more profits than selling for the seller.

Intuition:

1. Too much flexibility for seller $\Rightarrow$ losses of capital of first period buyers.
2. Intertemporal price discrimination (first period buyers pay higher price) "price skimming".

Theorem 1.1 (Coase Conjecture)

Suppose the seller can change the price faster and faster. What happens to the profits of the seller? The profit goes to zero.

Why there is selling in the world?

1. Moral hazard of leasing.
2. Leasing is not anonymous. Reveal reservation price $\Rightarrow$ Price discrimination in leasing. (Even worse than selling.) (Long-term contract + renegotiation = selling).
3. Commit to sequences of prices.
   (a). Deposit to third party
   (b). Reputation
4. Increasing cost
5. "Most-favored Nation" clause.
6. Consumers are not informed about the production costs.
7. New consumers coming into the market.

1.1.4 Learning Demand

Firms may not be able to learn the demand function perfectly.

It is relatively easy to learn a quasi-concave profit function. In the case that the profit function is not quasi-concave, the firm may not be able to learn the profit function. (stay at local maximum because of the loss from learning).

Learning the optimal features: By assuming the distribution of marginal increase $\frac{\partial \pi_j}{\partial x_j}$ is symmetric about 0, we can use Brownian motion to model the continuous profit function.

1.2 Short-run Competition

1.2.1 Bertrand Paradox

Consider two firms with marginal cost c:

$$\Pi^i(p_i, p_j) = (p_i - c) D_i(p_i, p_j)$$

where

$$D_i(p_i, p_j) = \begin{cases} D_i(p_i), & p_i < p_j, \\ \frac{1}{2} D_i(p_i), & p_i = p_j, \\ 0, & p_i > p_j \end{cases}$$

The NE is $p_i = p_j = c$. $\Pi^i = \Pi^j = 0$.

1.2.2 Static Solution to Bertrand Paradox

Capacity Constraints

Edgeworth: there may exist some constraints of the capacity.

1. Firms choose capacity K_i, K_j.
2. Firms choose prices p_i, p_j.

Solving by backward induction: That is, firstly solve $p_i^*(K_i, K_j)$ such that

$$\max_{p_i} \, (p_i - c) D_i(p_i, p_j)$$

$$\text{s.t. } D_i(p_i, p_j) \leq K_i$$

and then solve

$$\max_{K_i} \, \left(p_i^*(K_i, K_j) - c\right) D_i(p_i^*(K_i, K_j), p_j^*(K_i, K_j)) - \gamma K_i$$

where γ is the marginal cost of capacity.

Best response in prices: positive correlated, which is called “strategic complements”.

Best response in quantities (Cournot competition): negative correlated, which is called “strategic substitutes”.

(Quantity competition gives higher profits.)

Example 1.4 (Simple Example of Couront Competition)

$P(q_1, q_2) = 1 - q_1 - q_2$ and $\gamma \in (\frac{3}{4}, 1)$.

$$\max_{q_1} \, q_1 \left(1 - q_1 - q_2 - \gamma\right)$$

$$\Rightarrow q_1^*(q_2) = \frac{1 - q_2 - \gamma}{2}$$

Similarly, $q_2^*(q_1) = \frac{1-q_1-\gamma}{2}$. Thus, $q_1^* = q_2^* = \frac{1-\gamma}{3}$.

Similar to the Cournot competition, we can get positive profits with capacity constraints.

Differentiation

Idea: it is easier to change prices than to change products.

Basic case: Spatial Competition: There are consumers in $[0, 1]$ (uniform distribution). The position chosen by both firms is $\frac{1}{2}$ (the center of the market).

With price competition: Transportation cost is tx^2, where x is the distance. Suppose the firm A locates at 0 and the firm B locates at 1. The profit of consumer x from purchasing A is $v - p_A - tx^2$ and the profit of consumer x from purchasing B is $v - p_B - t(1-x)^2$. The indifferent consumer is

$$v - p_A - tx^2 = v - p_B - t(1-x)^2$$

$$p_A - p_B = t(1-2x)$$

$$\Rightarrow x = \frac{1}{2} - \frac{p_A - p_B}{2t}$$

Therefore, the demand of A is

$$D_A(p_A, p_B) = \frac{1}{2} - \frac{p_A - p_B}{2t}$$

and the demand of B is

$$D_B(p_A, p_B) = \frac{1}{2} + \frac{p_A - p_B}{2t}$$

Note *If the transportation cost is tx, the demand function is the same as above.*

The p_A^* and p_B^* are given by

$$\left.\begin{aligned} p_A^*(p_B) &= \text{argmax}_{p_A}(p_A - c)\left(\tfrac{1}{2} - \tfrac{p_A - p_B}{2t}\right) = \tfrac{c+t+p_B}{2} \\ p_B^*(p_A) &= \text{argmax}_{p_B}(p_B - c)\left(\tfrac{1}{2} + \tfrac{p_A - p_B}{2t}\right) = \tfrac{c+t+p_A}{2} \end{aligned}\right\} \Rightarrow p_A^* = p_B^* = c + t$$

Then, the profits are $\Pi_A^* = \Pi_B^* = \frac{t}{2}$.

Endogenous Differentiation: Denote the position of firm A as a and the position of firm B as $1-b$. Then, the indifferent consumer is

$$p_A + t(x-a)^2 = p_B + t(1-b-x)^2$$

$$\Rightarrow x = \frac{p_B - p_A + t\left[(1-b)^2 - a^2\right]}{2t(1-b-a)},$$

and the demands are

$$D_A(p_A,p_B) = \frac{p_B - p_A + t\left[(1-b)^2 - a^2\right]}{2t(1-b-a)},\ D_B(p_A,p_B) = 1 - D_A(p_A,p_B)$$

Then, the equilibrium prices given a and b are

$$\begin{aligned} p_A^* &= t(1-a-b)\left(1+\frac{a-b}{3}\right) \\ p_B^* &= t(1-a-b)\left(1+\frac{b-a}{3}\right) \end{aligned}$$

where $c := 0$. The corresponding profits are

$$\begin{aligned} \Pi_A(a,b) = p_A^* D_A^* &= \left(1+\frac{a-b}{3}\right)\frac{t(1-b-a)(1-b+a)}{2} \\ \Pi_B(a,b) = p_B^* D_B^* &= \left(1+\frac{b-a}{3}\right)\frac{t(1-b-a)(1-a+b)}{2} \\ \frac{\partial \Pi_A(a,b)}{\partial a} &= \underbrace{(p_A^*-c)\frac{\partial D_A^*}{\partial a}}_{\text{direct effect} > 0} + \underbrace{\frac{\partial D_A^*}{\partial p_A^*}}_{=0}\frac{\partial p_A^*}{\partial a} + \underbrace{(p_A^*-c)\frac{\partial D_A^*}{\partial p_B^*}\frac{\partial p_B^*}{\partial a}}_{\text{strategic effect} < 0} \end{aligned}$$

Which effect dominates depends on the model. In this model, the strategic effect dominates the direct effect. That is, $a = b = 0$. (If allowing negative values, $a = b = -\frac{1}{4}$.)

Note *If the transportation cost is tx, the equilibrium may not exist.*

Bibliography