Miguel Class

Author: Wenxiao Yang

Institute: Haas School of Business, University of California Berkeley

Date: 2024

All models are wrong, but some are useful.

Contents

Chapter 1 Pricing

1.1 Monopoly

1.1.1 Base Case

The firm decides its price p to maximize $\Pi(p) = p \cdot D(p) - C(D(p))$, where $D(\cdot)$ is the demand function and $C(\cdot)$ is the cost function.

The monopoly problem is maximizing the profit

$$\max_p \Pi(p) = p \cdot D(p) - C(D(p))$$

The F.O.C. (first-order condition) is

$$\frac{\partial \Pi(p)}{\partial p} = D(p) + pD'(p) - C'(D(p))D'(p) = 0$$

and the S.O.C. (second-order condition) is

$$\frac{\partial \Pi^2(p)}{\partial p^2} < 0$$

The F.O.C. gives that

$$\begin{aligned}
(p - C')D' &= -D \\
p - C' &= -\frac{D}{D'} \\
\underbrace{\frac{p - C'}{p}}_{\text{Lerner Index}} &= -\frac{D}{D'p} \\
&= -\frac{1}{\frac{dD}{dp}\frac{p}{D}} = -\frac{1}{\frac{\frac{dD}{D}}{\frac{dp}{p}}} := \frac{1}{E}
\end{aligned}$$

where $\frac{\frac{dD}{D}}{\frac{dp}{p}} < 0$ is the elasticity of demand with respect to price. The absolute value of the elasticity is denoted by E.

E is supposed to be greater than 1, otherwise, the optimal price is negative.

In the demand function $D(p) = kp^{-E}$, where the elasticity is constant. Its elasticity is $-E$.

The monopolist gives the production that is lower than social-optimal to maximize the profit (dead weight loss).

Rent dissipation can give larger dead weight loss.

1.1.2 Multiple Products

$$\max_p \sum_{i=1}^{N} p_i D_i(p) - C(D_1(p), ..., D_N(p))$$

Related Demand and Separable Costs: $C(D_1(p), ..., D_N(p)) = C_1(D_1(p)) + ... + C_N(D_N(p))$. The optimal pricing in this case satisfies

$$\frac{p_i - C_i'}{p_i} = \frac{1}{E_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j E_{ij}}{R_i E_{ii}}$$

where $E_{ij} = \frac{\partial D_i}{\partial p_j} \frac{p_j}{D_i}$ and R_i is the revenue.

Intuition: In the case of substitutes/complements, we want to increase/decrease the price of products compared to the one product case. (Positive/negative externality by increasing price of substitutes).

Similar Intuition: Consider a two-period model that the demand at second period depends on the price at first period (assuming $\frac{\partial D_2}{\partial p_1} < 0$).

1. $q_1 = D_1(p_1)$; $C_1(q_1)$
2. $q_2 = D_2(p_2, p_1)$; $C_2(q_2)$

Then, $\frac{p_1 - C_1'}{p_1} < \frac{1}{E_1}$ (the negative externality of prices).

Independent Demands and Related Costs:

Example 1.1

Different intensity of demand across periods.

1. Period 1: Low demand. $q_1 = D_1(p_1)$.
2. Period 2: High demand. $q_2 = D_2(p_2)$, where $D_1(p) = \lambda D_2(p)$ for some $\lambda < 1$.
3. Marginal cost of Production is c and the Marginal cost of capacity is γ.

Intuition: if $\lambda \to 0$, the marginal cost at period $\to c + \gamma$ and the marginal cost at period $1 = c$. Then, we have

$$\frac{p_2 - (c + \gamma)}{p_2} = \frac{1}{E_2}, \ \frac{p_1 - c}{p_1} = \frac{1}{E_1}$$

Now, let's consider a not too small λ. The problem is given as

$$\max_{p_1, p_2, k} (p_1 - c)D_1(p_1) + (p_2 - c)D_2(p_2) - \gamma k$$

$$s.t. \ D_1(p_1) \leq k$$

$$D_2(p_2) \leq k$$

The Lagrangian is given by

$$\mathcal{L} = (p_1 - c)D_1(p_1) + (p_2 - c)D_2(p_2) - \gamma k + \lambda_1(k - D_1(p_1)) + \lambda_2(k - D_2(p_2))$$

$$\frac{\partial \mathcal{L}}{\partial k} = -\gamma + \lambda_1 + \lambda_2 = 0 \Leftrightarrow \gamma = \lambda_1 + \lambda_2$$

Skip the process: $\frac{p_1-(c+\lambda_1)}{p_1} = \frac{1}{E_1}, \frac{p_2-(c+\lambda_2)}{p_2} = \frac{1}{E_2}$. Example: If $\lambda_1 = 0, k > D_1(p_1)$, the second period pays all the capacity cost.

Example 1.2 (Learning by Doing)

Suppose there are two periods $t = 1, 2$. The demand is $q_t = D_t(p_t)$. The cost in period one is $c_1(q_1)$ and $c_2(q_2, q_1)$ ($\frac{\partial c_2}{\partial q_1} < 0$, the more you produce in period one, the lower the cost you are facing in period two).

In continuous form, the cost form is

$$C(w(t))$$

where $\dot{w}(t) = \frac{dw}{dt} = q(t)$. We want to maximize

$$\max_{q(t),w(t)} \int_0^\infty e^{-\pi t}[q(t)p(q(t)) - C(w(t))q(t)]dt$$

$$\text{s.t. } \dot{w}(t) = q(t)$$

By Hamiltonian (skip), average of future marginal costs is

$$A(t) = \int_t^\infty C(w(s))\pi e^{-\pi(s-t)}ds$$

$$\frac{P(t) - A(t)}{P(t)} = \frac{1}{E(t)}$$

1.1.3 Durable Goods

The demand in one period is substitute to demand in other periods.

Example 1.3

Two periods $t = 1, 2$. Three consumers: $v_1 = 1$ per period, $v_2 = 2$ per period, and $v_3 = 3$ per period. The cost of production is zero. The seller chooses p_1, p_2.

(Consumer may forward-looking).

Moorthy (1988), Levinthal, D. A., & Purohit, D. (1989).

$t = 1, 2$ and zero production cost. The values of consumers $v \sim U[0, 1]$ and the discount factor is $\delta < 1$. The selling price p_1, p_2.

Suppose the consumers bought in first period have $v \geq v_1^*$, which must satisfies

$$\delta(v_1^* - p_2) = v_1^* - p_1 + \delta v_1^*$$

$$v_1^* = p_1 - \delta p_2$$

The price in second period should be $p_2 = \frac{v_1^*}{2}$. Then, $v_1^* = p_1 - \delta\frac{v_1^*}{2} \Rightarrow v_1^* = \frac{2p_1}{2+\delta}$. The price in the first period is given by

$$\max_{p_1}\ p_1(1-v_1^*)+\delta\frac{(v_1^*)^2}{4}=p_1\frac{2+\delta-2p_1}{2+\delta}+\delta\frac{p_1^2}{(2+\delta)^2}$$

Leasing (instead of selling): The leasing price is p in each period, which is given by $\operatorname{argmax}_p p(1-p)=\frac{1}{2}$. Leasing may generate more profits than selling for the seller.

Intuition:

1. Too much flexibility for seller $\Rightarrow$ losses of capital of first period buyers.
2. Intertemporal price discrimination (first period buyers pay higher price) "price skimming".

Theorem 1.1 (Coase Conjecture)

Suppose the seller can change the price faster and faster. What happens to the profits of the seller? The profit goes to zero.

Why there is selling in the world?

1. Moral hazard of leasing.
2. Leasing is not anonymous. Reveal reservation price $\Rightarrow$ Price discrimination in leasing. (Even worse than selling.) (Long-term contract + renegotiation = selling).
3. Commit to sequences of prices.
   (a). Deposit to third party
   (b). Reputation
4. Increasing cost
5. "Most-favored Nation" clause.
6. Consumers are not informed about the production costs.
7. New consumers coming into the market.

1.1.4 Learning Demand

Firms may not be able to learn the demand function perfectly.

It is relatively easy to learn a quasi-concave profit function. In the case that the profit function is not quasi-concave, the firm may not be able to learn the profit function. (stay at local maximum because of the loss from learning).

Learning the optimal features: By assuming the distribution of marginal increase $\frac{\partial\pi_j}{\partial x_j}$ is symmetric about 0, we can use Brownian motion to model the continuous profit function.

1.2 Short-run Competition

1.2.1 Bertrand Paradox

Consider two firms with marginal cost c:

$$\Pi^i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$$

where

$$D_i(p_i, p_j) = \begin{cases} D_i(p_i), & p_i < p_j, \\ \frac{1}{2}D_i(p_i), & p_i = p_j, \\ 0, & p_i > p_j \end{cases}$$

The NE is $p_i = p_j = c$. $\Pi^i = \Pi^j = 0$.

1.2.2 Static Solution to Bertrand Paradox

Capacity Constraints

Edgeworth: there may exist some constraints of the capacity.

1. Firms choose capacity K_i, K_j.
2. Firms choose prices p_i, p_j.

Solving by backward induction: That is, firstly solve $p_i^*(K_i, K_j)$ such that

$$\max_{p_i} \, (p_i - c)D_i(p_i, p_j)$$

$$\text{s.t. } D_i(p_i, p_j) \leq K_i$$

and then solve

$$\max_{K_i} \, (p_i^*(K_i, K_j) - c)\, D_i(p_i^*(K_i, K_j), p_j^*(K_i, K_j)) - \gamma K_i$$

where γ is the marginal cost of capacity.

Best response in prices: positive correlated, which is called “strategic complements”.

Best response in quantities (Cournot competition): negative correlated, which is called “strategic substitutes”.

(Quantity competition gives higher profits.)

Example 1.4 (Simple Example of Couront Competition)

$P(q_1, q_2) = 1 - q_1 - q_2$ and $\gamma \in (\frac{3}{4}, 1)$.

$$\max_{q_1} \, q_1 \,(1 - q_1 - q_2 - \gamma)$$

$$\Rightarrow q_1^*(q_2) = \frac{1 - q_2 - \gamma}{2}$$

Similarly, $q_2^*(q_1) = \frac{1-q_1-\gamma}{2}$. Thus, $q_1^* = q_2^* = \frac{1-\gamma}{3}$.

Similar to the Cournot competition, we can get positive profits with capacity constraints.

Differentiation

Idea: it is easier to change prices than to change products.

Basic case: Spatial Competition: There are consumers in $[0,1]$ (uniform distribution). The position chosen by both firms is $\frac{1}{2}$ (the center of the market).

With price competition: Transportation cost is tx^2, where x is the distance. Suppose the firm A locates at 0 and the firm B locates at 1. The profit of consumer x from purchasing A is $v - p_A - tx^2$ and the profit of consumer x from purchasing B is $v - p_B - t(1-x)^2$. The indifferent consumer is

$$v - p_A - tx^2 = v - p_B - t(1-x)^2$$

$$p_A - p_B = t(1-2x)$$

$$\Rightarrow x = \frac{1}{2} - \frac{p_A - p_B}{2t}$$

Therefore, the demand of A is

$$D_A(p_A, p_B) = \frac{1}{2} - \frac{p_A - p_B}{2t}$$

and the demand of B is

$$D_B(p_A, p_B) = \frac{1}{2} + \frac{p_A - p_B}{2t}$$

Note *If the transportation cost is tx, the demand function is the same as above.*

The p_A^* and p_B^* are given by

$$\left.\begin{aligned} p_A^*(p_B) &= \text{argmax}_{p_A}(p_A - c)\left(\tfrac{1}{2} - \tfrac{p_A - p_B}{2t}\right) = \tfrac{c+t+p_B}{2} \\ p_B^*(p_A) &= \text{argmax}_{p_B}(p_B - c)\left(\tfrac{1}{2} + \tfrac{p_A - p_B}{2t}\right) = \tfrac{c+t+p_A}{2} \end{aligned}\right\} \Rightarrow p_A^* = p_B^* = c + t$$

Then, the profits are $\Pi_A^* = \Pi_B^* = \frac{t}{2}$.

Endogenous Differentiation: Denote the position of firm A as a and the position of firm B as $1-b$. Then, the indifferent consumer is

$$p_A + t(x-a)^2 = p_B + t(1-b-x)^2$$

$$\Rightarrow x = \frac{p_B - p_A + t\left[(1-b)^2 - a^2\right]}{2t(1-b-a)},$$

and the demands are

$$D_A(p_A,p_B) = \frac{p_B - p_A + t\left[(1-b)^2 - a^2\right]}{2t(1-b-a)},\ D_B(p_A,p_B) = 1 - D_A(p_A,p_B)$$

Then, the equilibrium prices given a and b are

$$p_A^* = t(1-a-b)\left(1+\frac{a-b}{3}\right)$$
$$p_B^* = t(1-a-b)\left(1+\frac{b-a}{3}\right)$$

where $c := 0$. The corresponding profits are

$$\Pi_A(a,b) = p_A^* D_A^* = \left(1+\frac{a-b}{3}\right)\frac{t(1-b-a)(1-b+a)}{2}$$
$$\Pi_B(a,b) = p_B^* D_B^* = \left(1+\frac{b-a}{3}\right)\frac{t(1-b-a)(1-a+b)}{2}$$
$$\frac{\partial \Pi_A(a,b)}{\partial a} = \underbrace{(p_A^* - c)\frac{\partial D_A^*}{\partial a}}_{\text{direct effect} > 0} + \underbrace{\frac{\partial D_A^*}{\partial p_A^*}}_{=0}\frac{\partial p_A^*}{\partial a} + \underbrace{(p_A^* - c)\frac{\partial D_A^*}{\partial p_B^*}\frac{\partial p_B^*}{\partial a}}_{\text{strategic effect} < 0}$$

Which effect dominates depends on the model. In this model, the strategic effect dominates the direct effect. That is, $a = b = 0$. (If allowing negative values, $a = b = -\frac{1}{4}$.)

Note *If the transportation cost is tx, the equilibrium may not exist.*

Other models:

1. vertical differentiation (S. Moorthy);
2. defender model (John Hauser);
3. logit/limited defender model, $u_{ij} = v_j - \alpha p_j + \epsilon_{ij}$, where $\epsilon_{ij} \sim EVI$. The outside option is modeled as $u_{i0} = \epsilon_{i0} \sim EVI$. The market share is given as

$$s_j = \Pr(u_{ij} \geq u_{ij'}, \forall j') = \frac{\exp(v_j - \alpha p_j)}{1 + \sum_{j'} \exp(v_{j'} - \alpha p_{j'})}$$

The demand is given by

$$D_j = s_j \cdot \text{Market Size}$$

Estimated by taking inversion,

$$\ln s_j - \ln s_0 = v_j - \alpha p_j$$

Proliferation of Products to Deter Entry: Entry with products locating uniformly to deter entry.

Spoke Model.

1.3 Search

1.3.1 Individual Choice

1. *Simple example*, where the cost of search for each price is c and a consumer buys one or zero unit.

Definition 1.1 (Optimal Stopping Rule)

The optimal rule of searching is an **optimal stopping rule**: Stop and buy once the consumer finds a price less or equal to a reservation price, R.

The R is constructed as the critical value when the expected return from an extra search equals to the marginal cost:

$$\begin{aligned} c &= \mathbb{E}[R-p \mid p<R]\Pr(p<R) \\ &= \int_0^R (R-p)f(p)dp \end{aligned}$$

where $f(\cdot)$ is the density distribution of prices.

2. *Consuming multiple units (general case, the one unit case can be modeled by assuming the demand function)*, Remind that the consumer surplus given price p is given as

$$s(p) = \max_q \{U(q) - p \cdot q\}$$

and its derivatives are

$$s'(p) = -D(p),\ s''(p) = -D'(p) > 0 \text{ (convexity)}$$

In this case, the optimal stopping rule R that maximizing the consumer surplus is given as

$$c = \int_0^R [s(p) - s(R)]f(p)dp$$

Note *Given a greater variance of the price distribution, the R decreases.*

1.3.2 Homogeneous Markets

Can we find a fixed point of search behavior (i.e., $f(p)$)?

Assumption *All firms are identical with marginal cost c_f. All consumers are identical with search costs c.*

Define the "monopoly price":

$$p^M = \underset{p}{\operatorname{argmax}} \{p \cdot D(p) - c_f D(p)\}$$

Assumption *The consumers want to visit the first store in equilibrium, $s(p^M) > c$.*

Theorem 1.2 (Diamond Theorem)

The unique equilibrium is for all firms to charge monopoly price p^M and consumers do not search.

Proof 1.1 (Sketch)

Firstly, we can prove this proposed equilibrium exists: all monopoly prices $\Rightarrow$ no search; no search $\Rightarrow$ all monopoly prices.

Secondly, we prove the uniqueness: Given $f(p)$, the corresponding reservation price of search is R. All firms charge $p = R$ and the consumers do not search.

1. Firstly, no firms should charge below R: if $p < R$, the consumer purchases once he visits the store. So, there is no firm charging $p < R$.
2. Secondly, we prove no firms charge above R: Suppose by the way of contradiction that there is a firm charging $p = R + \epsilon$. Once a consumer visits the store, the consumer's highest surplus from an extra search is $s(R) - s(R+\epsilon) - c$. There always exists an $\epsilon > 0$ such that purchases at $R + \epsilon$ is profitable for the consumer.

Therefore, the consumers do not search, and then all firms charge p^M.

1.3.3 Solutions for Diamond Paradox

1. Firms are different.
   (a). Different costs, which give different p^M.
   (b). Inflation + menu costs.
2. Consumers are different.
   (a). Different search costs.
      I. If mass zero at zero search cost, then everything is the same.
      II. If no mass zero at zero, then equal breaks down.
   (b). Different preferences [Choi, Dai, and Kim(2018)].
   (c). Lack of common knowledge [Kuksov(2006)].

1.3.4 [Choi, Dai, and Kim(2018)]

Suppose the utility of consumer from consuming product j is

$$U(v_j, z_j, p_j, N) = v_j + z_j - p_j - \underbrace{\sum_{k \in N} s_k}_{\text{search costs based on his search history}}$$

where (v_j, z_j) are the parameters of the consumer that reflects his preference, h v_j is known before search, z_j is learning during search, p_j is the price, N is set of the products searched by the consumer, s_k is the search cost of product (firm) k.

Assumption $v_j \sim F_j \in \Delta[\underline{v}_j, \bar{v}_j]$ *and* $z_j \sim G_j \in \Delta[\underline{z}_j, \bar{z}_j]$.

Definition 1.2 (Gittins index)

Weitzman index (Gittins index) z_j^* is given by solving

$$s_j = \int_{z_j^*}^{\bar{z}_j} [(1 - G_j(z_j))] dz_j$$

which is equivalent to $s_j = \int_{z_j^*}^{\bar{z}_j} (z_j - z_j^*) g(z_j) dz_j$ (integration by part).

Remark Upper Confidence Bound (UCB) Algorithm

Proposition 1.1 (Optimal Search Policy)

The optimal policy: Visit stores in descending order of $v_j + z_j^* - p_j$ and stop if

$$\max \left\{ u_0, \max_{j \in N} v_j + z_j - p_j \right\} > \max_{j \notin N} v_j + z_j^* - p_j$$

where u_0 is the utility from the outside option.

Note *Since it is descending order of $v_j + z_j^* - p_j$, stop after visiting i if $z_i \geq z_i^*$ holds.*

Define $w_i := v_i + \min\{z_i, z_i^*\}$. Then, shopping outcome is that consumer buys product i iff

1. $w_i - p_i > u_0$ and
2. $w_i - p_i > w_j - p_j$ for all $j \neq i$.

Note *Latent utility framework that maximizes $w_i - p_i$.*

Proof 1.2

Prove the sufficiency: Given $w_i - p_i > v_0$,

$$w_i - p_i > v_0 \Rightarrow v_i + z_i^* - p_i > u_0$$

By the search policy, we must have $v_i + z_i - p_i > u_0$.

Given $w_i - p_i > w_j - p_j$.

1. If $z_j^* \leq z_j$, we have $w_j = v_j + z_j^* - p_j$. As $v_i + z_i^* - p_i \geq w_i - p_i > w_j - p_j = v_j + z_j^* - p_j$ and the order is descending in $v_j + z_j^* - p_j$, i must be visited before j. The consumer does not want to visit j after visit i because $v_i + z_i - p_i \geq w_i - p_i > v_j + z_j^* - p_j$.
2. If $z_j^* > z_j$, ...

Prove the necessary:

Solving the Equilibrium Let $H_i(\cdot)$ be the CDF of $w_i := v_i + \min\{z_i, z_i^*\}$.

$$H_i(w_i) := \int_{\underline{z}_i}^{z_i^*} F_i(w_i - z_i)dG(z_i) + \int_{z_i^*}^{\bar{z}_i} F_i(w_i - z_i^*)dG(z_i)$$

The best alternative is defined as $x_i = \max\{u_0, \max_{j\neq i} w_j - p_j\}$. Its CDF is given as

$$\tilde{H}(x) = \text{Prob}[x_i \leq x]$$

Note that $\tilde{H}(x)$ depends on p_{-i} but not p_i.

Therefore, the demand for the product i given the price vector p can be given as

$$D_i(p) = \int \underbrace{[1 - H_i(x_i + p_i)]}_{\text{Prob}[w_i - p_i \geq x_i]} d\tilde{H}(x_i)$$

Thus, the optimization problem of pricing is given by

$$\max_{p_i} (p_i - MC_i)D_i(p_i, p_{-i})$$

Main Results

1. Observable prices: as the search costs increase $\Rightarrow$ the benefit from search decreases $\Rightarrow$ attract consumers to search first is more important $\Rightarrow$ Lower prices.
2. Unobservable prices: as the search costs increase $\Rightarrow$ the benefit from search increases $\Rightarrow$ try to exploit consumers $\Rightarrow$ Higher prices.

Given more information pre-search, there are two effects:

1. Less benefit from search $\Rightarrow$ attract consumers to search first is more important $\Rightarrow$ Lower prices.
2. More dispersed consumer preferences $\Rightarrow$ Larger differentiation among products $\Rightarrow$ Higher prices.

1.3.5 [Kuksov(2006)]: Lack of Common Knowledge

Suppose there are identical buyers have valuation v for products (1 unit). Sellers are uncertain about the valuation v and choose their own prices based on private signals.

Suppose a seller j gets a signal

$$x_j = v + \eta_j = \begin{cases} v + 0, & \text{with prob } \frac{1}{2} \\ v - \delta(v), & \text{with prob } \frac{1}{2} \end{cases}$$

The cost of first search is zero and the cost of sequential search is s.

If the signal v I received is the high signal, then others who receive the low signal gets $f(v) := v - \delta(v)$

If the signal v I received is the low signal, then others who receive the high signal gets $g(v) := f^{-1}(v)$

Thus, if a seller receives signal x_j, he believes the possible other sellers' signals are $\{f(x_j), x_j, g(x_j)\}$.

Equilibrium Consumers are facing two kinds of prices set by firms $P(v)$ and $P(f(v))$.

1. If consumers see $P(f(v))$, they stop.
2. If consumers see $P(v)$, they buy the product if

$$\frac{1}{2}(P(v) - P(f(v))) \leq s$$

Therefore, the equilibrium prices should satisfy

$$P(v) = P(f(v)) + 2s$$

Results

1. Prices are below the monopoly price.
2. As s goes to 0, the price $P(v)$ goes to MC.
3. As $\delta(v)$ goes to zero, the price $P(v)$ goes to the monopoly price.

1.3.6 [Kuksov and Villas-Boas(2010)]: Alternative Overload

Consumers can either search with some costs or buy at random.

The value of consumers is defined as

$$V(v,x,I,N) = \max\Bigg\{ \underbrace{0}_{\text{not choose an alternative}} , \underbrace{\max_{i\in I}(v - t|x - z_i|)}_{\text{choose the best checked product}} , \underbrace{\sum_{i\notin I} \frac{1}{N-M}(v - t|x - z_i|)}_{\text{randomly choose from unchecked products}} , \underbrace{-c + \sum_{i\notin I} \frac{1}{N-M} V\left(v, x, I \cup \{i\}, N\right)}_{\text{(randomly) search one more}} \Bigg\}$$

where v is the basic value of choosing an alternative, x is the location of the consumer, I is the set of products checked, M is the number of products checked, N is the number of total products, and z_i is the location of the alternative i.

Infinite Alternatives: In this case the expected values from different strategies are given as

1. Random purchase: x gets expected disutility $d = \int_0^1 t|x - z|dz$.
2. Search: the search strategy can be given as
   (a). Stop search if $z \in (x - \delta, x + \delta)$. This gives disutility $d = \int_{x-\delta}^{x+\delta} t|x - z|dz$, which is minimized for consumers located at $x = \frac{1}{2}$ who gets disutility $d = \frac{t}{4}$.
   (b). Search otherwise. This requires a search cost of c.

In equilibrium, the search costs equal to the disutility from stopping:

$$c = \int_{x-\delta}^{x+\delta} t|x-z|dz \Rightarrow \delta = \sqrt{\frac{c}{t}}$$

The corresponding expected disutility is

$$d(\infty) = t\delta = \sqrt{ct}$$

Therefore, all consumers choose to search instead of randomly purchase if and only if $\frac{c}{t} \leq \frac{1}{16}$.

Results There can be too many products. To show this, we compare $\hat{N}$ products with infinity. ($\hat{N}$ products uniformly locate on $[0,1]$ in equilibrium).

Let's consider a strategy (not optimal): <u>Search until find the closest alternative</u>: without search costs, it gives the expected disutility $\frac{t}{4\hat{N}}$ for a consumer on average.

1. Without search costs, it gives the expected disutility $\frac{t}{4\hat{N}}$ for a consumer on average.
2. The expected search cost to find this product is

$$c\frac{1}{\hat{N}} + 2c\frac{\hat{N}-1}{\hat{N}}\frac{1}{\hat{N}-1} + \cdots \left[(N-1)c\frac{\hat{N}-1}{\hat{N}}\frac{\hat{N}-2}{N\hat{-}1}\cdots\frac{1}{2}\right] = (\hat{N}-1)\left(\frac{1}{2}+\frac{1}{\hat{N}}\right)c$$

The expected disutility for a consumer on average is

$$\frac{t}{4\hat{N}} + (\hat{N}-1)\left(\frac{1}{2}+\frac{1}{\hat{N}}\right)c$$

its derivate of $\hat{N}$ is $-\frac{t}{4\hat{N}^2} + \left(\frac{1}{2}+\frac{1}{\hat{N}}\right)c - \frac{(\hat{N}-1)}{\hat{N}^2}c \propto -\frac{t}{4} + \left(\frac{\hat{N}^2}{2}+1\right)c$.

For some $\hat{N} \approx \frac{1}{2\sqrt{c/t}}$ such that

$$\frac{t}{4\hat{N}} + (\hat{N}-1)\left(\frac{1}{2}+\frac{1}{\hat{N}}\right)c < \frac{3}{4}\sqrt{ct} + c \leq \sqrt{ct} < d(\infty)$$

Therefore, there is a finite number of products $\hat{N}$ that benefits consumers compared to infinity.

Bibliography

[Choi, Dai, and Kim(2018)] Choi, M., Dai, A. Y., and Kim, K. (2018). Consumer search and price competition. *Econometrica*, 86(4):1257–1281.

[Kuksov(2006)] Kuksov, D. (2006). Search, common knowledge, and competition. *Journal of Economic Theory*, 130(1):95–108.

[Kuksov and Villas-Boas(2010)] Kuksov, D. and Villas-Boas, J. M. (2010). When more alternatives lead to less choice. *Marketing Science*, 29(3):507–524.