Robust Production Function Estimation when there is Market Power*

Jordi Jaumandreu
Boston University† and CEPR

October 2024
This version: June 2025

Abstract

The production function is an engineering relationship, but recent estimators make auxiliary use of firm's optimal choices that depend on market power. The estimator called "dynamic panel," that does not use any FOC, is robust to market power, but often produces unsatisfactory results. The estimators known as OP/LP typically improve, but are frequently inconsistent in the presence of market power. We propose a test for the presence of market power based on a consistent version of OP/LP that proxies for MC. If market power cannot be rejected, it offers a robust way to estimate. Then we propose a test for the specification, based in the smaller set of assumptions used by the dynamic panel estimator. The coincidence of dynamic panel and OP/LP except by sampling error is a necessary condition for consistency.

*I am idebted to Ulrich Doraszelski for many discussions, the errors that may persist are mine. I am grateful to Manuel Arellano, Mark Roberts and the seminar audience at Penn State University, Sabien Dobbelaere, Jacques Mairesse, Markus Trunschke, and Ziyao Wang for useful comments.

†Department of Economics. Email: jordij@bu.edu.

1 Introduction

What is the relationship between production function estimation and market power? The production function is an engineering relationship, that describes how technology relates inputs and output. So there is no direct relationship. But many recent proposals for the estimation of production functions include the auxiliary use of equations based on the optimization of some objective that involves the production function. The equations use to be the derivatives of profit with respect to the variable inputs (FOCs). And these FOCs are different when the firm has some market power.

These estimators, born with the work by Olley and Pakes (1996), yearned for validity under any competitive situation. However, most of them have been in practice only justified for situations of perfect competition or almost (Levinsohn and Petrin, 2003; Gandhi, Navarro and Rivers, 2020, or the way to implement Olley and Pakes, 1996, and Levinsohn and Petrin, 2003, proposed by Ackerberg, Caves and Frazer, 2015, henceforth ACF). The urgency to have readily available methods to estimate production functions has often implied the neglect of the conditions for applicability of these estimators under imperfect competition.

Take the frequently encountered case of product differentiation. A market with product differentiation shows some market power that emerges from the ability and incentives of the firms to produce products with different characteristics. This situation generates by itself an uncomfortable context for the user of the production function, because products and possibilities of production are distinct across firms.[1] Suppose nevertheless that the empirical researcher is happy accounting for the heterogeneity of characteristics and possibilities by means of an additive random deviation in the equation to be estimated. The estimators that use the FOCs still should not be applied ignoring the consequences of market power in the FOCs.

In the current literature there are two approaches to the estimation of the production function. They diverge in how they solve the problem of controlling for unobservable productivity.[2]

[1] The production function, as concept of economic theory, was developed as describing a situation in which the characteristics of the unique product are given and the available techniques define the set of production possibilities.

[2] How to treat unobserved productivity has been the dominant worry of researchers since Marschak and Andrews (1944) pointed at the statistical problems created by the endogeneity of the inputs.

We will call the two approaches "dynamic panel," henceforth DP, that was developed in Arellano and Bond (1991) and Blundell and Bond (2000), and OP/LP approach, called in this way because it was originated in the articles of Olley and Pakes (1996) and Levinsohn and Petrin (2003). In the first approach, unobservable productivity is differentiated out after assuming that follows a first order linear Markovian or $AR(1)$ process. The second approach also assumes that the unobservable follows a Markovian first order process, and models it alternatively as a nonlinear or linear process. The unobservable is replaced by the inverse of a function representing some optimal observable choice (usually the demand for a variable input) that contains it. A big advantage of DP is hence that doesn't need to use any auxiliary relationship implying behavior. OP/LP assumes instead that some behavioral equation holds, what is often attributed the benefit of freeing the linearity of the Markov process.[3]

A well known empirical paradox is that DP often produces disappointing results while OP/LP, with more assumptions to be met, often produces more reasonable estimates (e.g. in the elasticity of scale and in the elasticity of capital). In this paper we first explore the theoretical difference between the two estimators and derive what can be learned from their divergence. More generally, we explore the effects of incorporating the FOCs to the estimation of the production function.[4]

The conclusion is that both the DP and OP/LP estimators are consistent when there is perfect competition, but OP/LP is not robust when there is market power. The consistency of a non-robust estimator depends on the detailed assumptions about how the game that firms play in the market is, and their market share consequences. A lot of symmetry and unwanted

[3] With GMM estimation this is the case, although the relaxing of the linearity is made possible by the assumption of exact holding (no errors) of the used FOCs. However, today they are available other estimation methods that can deal with the nonlinearity of a latent variable without using any FOC. Arellano, Blundell and Bonhomme (2017), for example, show the nonparametric identification of a nonlinear latent process for consumer earnings, and estimate it with quantile techniques. Aguirre, Tapia and Villacorta (2024) is a first application to production functions.

[4] Modern production function estimation is not the first time that the FOCs have been given a role. Flexible specification of the production function and its dual cost function, started by Diewert (1971), and continued by Christensen, Jorgenson and Lau (1973), and Caves, Christensen and Tretheway (1981), raised a role for system estimation of the main equation and share equations. See, for example, Berndt and Wood (1975) and Mc Elroy (1987).

restrictions should be set beforehand to ensure the consistency of the OP/LP estimator under market power.

However, we highlight an OP/LP estimator that is consistent under any form of market power. To make OP/LP robust it is enough to proxy marginal revenue MR or marginal cost MC by means of AVC and the short-run elasticity to scale, and specify the process of productivity as linear. Under this specification, it happens that both DP and OP/LP are consistent and should deliver estimates that only differ due to sampling error.

The properties of this estimator suggest that the use of the FOCs, instead of a nuisance, can be taken as an advantage to test the specification. We propose two tests. The first is a test for the presence of market power. The second, since OP/LP apparently improves but the divergence between estimators suggests that something is wrong, it is a general specification test when the researcher is estimating under market power.

The first test detects the presence of market power. OP/LP can be specified using the price of the firms or using the proxied marginal cost. Under the null of no market power both estimators should only differ due to sampling error, under the alternative of market power only the OP/LP based on proxying MC is consistent.

The second test compares the estimates obtained with DP and the OP/LP that proxies MC. The estimates should be equal under the null of consistency of both estimators. However, they will differ under the alternative if either both estimators, or only OP/LP, are inconsistent. The first variant of the alternative happens if the production function is wrongly specified, the second because what is wrong is the specification of the FOC on which OP/LP bases the demand for an input.

Estimation needs to control for econometrician-unobserved productivity. Current approaches in the estimation of the production function have treated intensively how to deal with Hicks-neutral productivity, that affects all inputs in the same way. However, empirical research has recently stressed that productivity is likely to be biased. And there have been contributions on how to apply DP and OP/LP when productivity is non-neutral and affects in particular an input. For example, Doraszelski and Jaumandreu (2018) show how to replace biased productivity from a ratio of FOCs and Demirer (2025) generalizes the technique. The dominant interest in this field is labor-augmenting productivity, henceforth LAP, presumably very related to the dominant form of current technological progress. In what follows, we systematically take into account the possibility of LAP.

Input market power can be as relevant as product market power, and it affects the first order conditions in a similar way as product market power. We also show summarily but systematically how to treat input market power when it is present, and we discuss the way to detect this mispecification if it is binding.

To illustrate the relevance of the problems, the realism of the circumstances, and the working of the procedures that we employ, we estimate the production function for the sample of US manufacturing Compustat firms used in Jaumandreu and Mullens (2024). It is a sample with more that 5,000 firms and 60,000 observations that are likely to exhibit the more diverse degrees of market power. On the one hand, the test for market power gives the unequivocal answer that market power is present. On the other, the estimation by DP and OP/LP diverges when naively applied to a Cobb-Douglas specification, and passes the specification test when applied to its enlargement into a translog with LAP that shows elasticity of substitution less than one and falling labor shares. Applied to this new specification, neither DP nor the feasible OP/LP are better. The second test hence detects that there is something wrong in the specification of the production function, and its flexibilization including LAP solves the problem. We think that this constitutes a reasonable place where to start the exploration for refinements.

The rest of the paper is organized as follows. Section 2 comments on the relation of the paper to the literature. Sections 3 and 4 explain the consistency of DP and OP/LP under perfect competition and put them in a common framework. Section 5 deals with OP/LP implementation and interprets ACF. Section 6 studies the consequences of facing market power. Section 7 develops the test for market power. Section 8 discusses estimation under market power, and section 9 the specification test. Section 10 develops the example with the sample of Compustat firms, and section 11 concludes. Four appendices deal with identification, conduct specification, statistical specification tests, and run some additional regressions, respectively.

2 Relation to the literature

The literature on the new estimators for the production function, sometimes called structural, has always been very conscious of the need to deal with market power. Olley and Pakes (1996) consider that the firms in the market are playing a dynamic oligopoly game and justify the simplification of the vector of state variables by means of symmetry that includes common input prices. Griliches and Mairesse (1998), writing contemporaneously on the "interesting new approach" of OP, worry if this treatment of the state variables may be ignoring some relevant dimensions as the expectations on the cost of investment. Levinsohn and Petrin (2003) define their setting as a competitive environment, where firms take as given output and input prices, and warn that the model can be generalized to imperfect competition but then it will depend on the specifics of competition.

Ackerberg, Caves and Frazer (2015), discussing when revenues can replace physical quantities (common output prices), introduce an explicit discussion about the difficulties to invert the demand for an input when the demand for output and/or the supply for an input are downward and upward curves respectively (i.e. there is market power). They warn that, in this situation, even assuming identical curves may be not enough. Gandhi, Navarro and Rivers (2020) make clear that their model for nonparametric estimation of the production function is developed assuming perfect competition in the output and intermediate markets. Only online Appendix O6 shows how the model can be applied specifying a parametric CES demand for output, together with the assumption of monopolistic competition, the version that in fact many researchers prefer given their needs.

More recently, a few discussions have dealt in one way or another with the ability of the OP/LP framework to address the situations with market power. Bond, Hashemi, Kaplan and Traina (2021) stress how the absence of reliable information on firm-level output prices makes difficult the estimation of structural elasticities and hence market power, and point at the robustness of the DP approach. Doraszelski and Jaumandreu (2021) develop the biases that affect an OP/LP procedure given the likely presence of correlated unobservable demand heterogeneity. Ackerberg and De Loecker (2024) is a discussion on how to expand the OP/LP estimators to include "sufficient statistics" to account for imperfect competition under behavioral and symmetry assumptions.

This paper makes, in the first place, a contribution to these discussions. It deals with how to construct estimators that are robust to market power, in the sense that they do not depend on the specification of the details of the game the firms play. This possibility builds on proxying MC as the variable that accounts for the result of the firm-level strategic interactions and heterogeneity of demand, and makes the need for other variables redundant. We show that OP/LP is nonrobust to market power, but also that there is a feasible (linear) OP/LP that can avoid this difficulty. This provides the possibility to use the feasible OP/LP as a test for the presence of market power. Also, this estimator uses more information than DP and the divergence with DP can be used as a test of specification. The result of our discussion is then rather a way to conduct the specification more than a particular estimator that fits all sizes and shapes.

A long list of papers have recently stressed that the presence of Hicks neutral productivity should be complemented with the presence of biased productivity, particularly in the form of LAP. See Doraszelski and Jaumandreu (2018, 2019), Raval (2019, 2023), Zhang (2019), Demirer (2025), Jaumandreu and Mullens (2024), Kusaka, Okazaki, Onishi and Wakamori (2024), and Zhao, Malikov and Kumbhakar (2024).[5]

We add to this literature by uncovering that the mispecification revealed by our estimators, when applied to sample of Compustat firms, is redressed when we consider an specification that allows shares in cost and elasticities to change from firm to firm and over time. We could not have formally assessed that this was the specification problem without estimators robust to market power, and we had never got the coincidence of the estimators without the change in the specification of productivity in the production function.

Our addition is otherwise a pioneering modeling of firm-level different dimensions of productivity in US manufacturing, that confirms the biased technological change that Raval (2019) found with Census of Manufacturing data on plants. It provides a rich chacterization on the firm dynamics of labor-augmenting productivity (see Jaumandreu and Mullens, 2024), with a flexible production function and subject to the rigor of the specification tests.

A recent literature has stressed that market power in the input markets can be as relevant as market power in the product market. See, for example, the papers by Dobbelaere and Mairesse (2013, 2018), Yeh, Macaluso, Hershbein (2022), Rubens (2023), and Azzam, Jaumandreu and Lopez (2025). The estimators OP/LP are not robust to the presence of unspecified input market power. Given the initial character of this paper, we only focus marginally on this particular topic. However, we show how the tools that we have developed can be applied to the detection of input market power affecting the estimation of the production function and, summarily, how they can be used

[5] A recent literature is exploring the nonparametric estimation of a unique productivity term, freely interacted with the inputs. See Ackerberg, Hahn and Pan (2023) and Pan (2024).

for consistent estimation under this presence.

3 DP and OP/LP under perfect competition

Let us first clarify the properties and relationship between the two estimators under perfect competition. The assumption of perfect competition implies that the price of the output is common for all firms and equals marginal cost. Firms differ in size because differ in their marginal cost curves, though. The usual time and information assumptions are as follows. Firms choose the variable inputs labor l and materials m at time t, when productivity becomes their knowledge, but capital k needs time to build and is given as chosen one period before.

Assume a population of firms (we drop firm and time subscripts). Write the production function in logs as

$$q = f(\mathbf{x}) + \omega + \varepsilon, \tag{1}$$

where $f(\mathbf{x}) = \ln F(\mathbf{x})$, $\mathbf{x} = \{k, l, m\}$ is the logs of capital, labor and materials, ω is Hicks-neutral productivity, and ε is an error of observation, not autocorrelated and uncorrelated with all variables known at t. Sometimes we will use the notation $q^* = f(\mathbf{x}) + \omega$ for the output without error.

Everything that we are going to say is compatible with the presence of labor-augmenting productivity (henceforth LAP). To see this it is enough to suppose that the labor input is $l^* = l + \omega_L$ and LAP ω_L has been controlled for observables.[6]

A first order Markov process establishes

$$\omega = g(\omega_{-1}) + \xi, \tag{2}$$

where $g(\cdot)$ is an unknown function and ξ a mean-independent error.

DP

DP assumes that productivity follows the linear Markov process $\omega = \rho\omega_{-1} + \xi$. The implication is that we can "pseudo-differentiate" equation (1)

[6] For example, Doraszelski and Jaumandreu (2018) show that $m - l = cons - \sigma(p_M - w) + (1-\sigma)\omega_L$ is a linear approximation that can be used to solve for ω_L when the production function is separable in capital. Demirer (2025) generalizes this equation. Zhao, Malikov and Kumbhakar (2024) show that an equation of this type is possible without separability for the translog specification.

(subtract the lagged equation multiplied by ρ) and unobservable productivity drops

$$q = \rho q_{-1} + f(\mathbf{x}) - \rho f(\mathbf{x}_{-1}) + \xi + \varepsilon - \rho\varepsilon_{-1}. \tag{3}$$

From the point of view of estimation, the inputs of the $\mathbf{x}$ vector that are set at t, when the shock of the Markov process is known, are correlated with ξ and should be instrumented. Researchers usually consider variable the inputs l and m. If the production function $f(\cdot)$ only requires the estimation of three parameters (additional to the constant), we need four instruments because we have to estimate the extra parameter ρ (that introduces nonlinearity in the model). The model is exactly identified using k, k_{-1}, l_{-1} and m_{-1} as instruments (however, see the discussion on identification of Appendix A).

It can be assumed that lagged input and output prices are non-correlated with ξ. Then, using them as instruments gets overidentifying restrictions. Cost and firm-demand shifters can be used as additional instruments.

OP/LP

OP/LP is based on the first order conditions for the variable inputs. These have the form.

$$P\frac{\partial F(\mathbf{x})}{\partial X}\exp(\omega) = W_X, \tag{4}$$

where P is the price of the output, $X = L, M$ and $W_X = W, P_M$. Unobserved productivity ω can be obtained by inverting one of these FOC or using the combination of both. A combination of the first order conditions drops one variable input including both input prices in addition to the price of the output (this is the unconditional demand for the input that remains). With perfect competition we expect P to be common across firms, so the only variation of P is over time and can be subsumed in a system of time dummies. But the price (s) of the input (s) is (are) not necessarily the same and must be explicitly included except when equality across firms is assumed.

The model can be extended to the case of input market power by assuming that the relevant input price is $W_X^* = W_X(1+\tau)$, where the markdown τ is either an additional parameter to estimate or is controlled for observables.[7]

Let us use, without loss of generality, only one FOC (sometimes this has

[7] See, for example, the treatment of τ in Azzam, Jaumandreu and Lopez (2025). Rubens (2022) uses the production function augmented with a labor supply.

been called to use the demand for a variable input conditional on the other)[8]

$$\omega = w_X - p - \ln \frac{\partial F(\mathbf{x})}{\partial X}.$$

The assumption that ω follows a first order Markov process allows us to write the production function replacing ω_{-1} by its expression according to the inverse of the conditional input demand

$$q = f(\mathbf{x}) + g(w_{X,-1} - p_{-1} - \ln \frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}) + \xi + \varepsilon. \tag{5}$$

The unknown function $g(\cdot)$ is typically specified by means of polynomials and the model easily estimated in one step by nonlinear GMM.[9] Note that the derivatives of $F(\mathbf{x})$ will include at most the same parameters as $F(\mathbf{x})$, so $f(\mathbf{x})$ and $\frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}$ are linked by equality restrictions, even if we are dealing with a flexible specification.[10] See Appendix A for a discussion on identification. We face exactly the same problem of endogeneity as before: the variable inputs l and m are correlated with ξ. If we have to estimate four parameters, variables k, k_{-1}, l_{-1}, and m_{-1} are enough for identification. Prices and shifters can be used as before as additional instruments. As $g(\cdot)$ is usually made of polynomials, it seems natural to enlarge the instrument set with powers of the instruments.

4 A common framework

DP and OP/LP estimators are presented differently (pseudodifferentiation, replacement of the unobservable by the inverse of an input demand) for pedagogical reasons, but they can be seen under a more common perspective. It happens that both estimators assume a first order Markov process for productivity, and then propose to replace past productivity by an expression in terms of observables.

[8] Later we will use the demand for the input conditional on output, that can be obtained using the ratio of FOCs to replace one variable input in the production function by the relationship with the other.

[9] A recent paper that shows in practice the advantages of one step GMM estimation is Trunschke and Judd (2024).

[10] Not recognizing this may produce unproductive discussions on identification. It is customary to apply nonparametric estimation with a polynomial specification.

We are going to see that this common perspective clarifies many properties and relationships. In discussing the estimators we will assume for the moment that the specification of the production function $f(\mathbf{x})$ is correct. We relax this assumption in section 9.

We can say that both estimators start by assuming that the production function can be written as

$$q = f(\mathbf{x}) + g(\omega_{-1}) + \xi + \varepsilon, \tag{6}$$

because of the process of productivity. Then DP proposes to replace ω_{-1} by $q_{-1} - f(\mathbf{x}_{-1}) - \varepsilon_{-1}$, and OP/LP by $w_{X,-1} - p_{-1} - \ln \frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}$. DP uses the lagged production function, OP/LP the lagged FOC. Accordingly, in what comes now we will use the following definitions

DEFINITION 1 The dynamic panel estimator is the application of IV to the equation

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) - \varepsilon_{-1}) + \xi + \varepsilon, \tag{7}$$

with $g(\cdot)$ specified as linear.

DEFINITION 2 The OP/LP estimator is the application of IV to the equation

$$q = f(\mathbf{x}) + g(w_{X,-1} - p_{-1} - \ln \frac{\partial F(\mathbf{x}_{-1})}{\partial X_{-1}}) + \xi + \varepsilon, \tag{8}$$

with $g(\cdot)$ specified by means of polynomials.[11] We will call linear OP/LP the estimator that only uses a first degree polynomial for $g(\cdot)$.

Revenue-share OP/LP

The expression used by OP/LP can be written in different ways. For example, we will find useful to use the revenue-share form, based on the share of the expenses on input X in observed revenue $S_X^R = \frac{W_X X}{PQ}$.

PROPOSITION 1 The OP/LP estimator can be written in the revenue-share form

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{S_{X,-1}^R}{\beta_{X,-1}}) + \xi + \varepsilon, \tag{9}$$

[11] We will speak of parametric OP/LP estimation in opposition to the ACF estimation (see below).

where $\beta_X = \frac{X}{F(\mathbf{x})} \frac{\partial F(\mathbf{x})}{\partial X}$, the output elasticity of input X.

Proof

Add and subtract x, and subtract and add q and $f(\mathbf{x}) = \ln F(\mathbf{x})$, to the expression for ω, then do some reordering. That is, $\omega = w_X - p - \ln \frac{\partial F(\mathbf{x})}{\partial X} = w_X + x - p - q + q - f(\mathbf{x}) - (x - \ln F(\mathbf{x}) + \ln \frac{\partial F(\mathbf{x})}{\partial X}) = q - f(\mathbf{x}) + \ln \frac{S_X^R}{\beta_X}$. ♦

It is important to note that we write β_X for notational simplicity, but it should be clear that in general it is a function $\beta_X(\cdot)$ of the inputs (and labor-augmenting productivity) and so is the short-run elasticity of scale $\nu(\cdot) = \beta_L(\cdot) + \beta_M(\cdot)$.[12]

The revenue-share form of the OP/LP estimator makes clear that the OP/LP estimator is using more information than the dynamic panel estimator, and this additional information is encompassed in the term $\ln \frac{S_X^R}{\beta_X}$. Multiplying the first order condition (4) by $\frac{X}{PQ^*}$, using $\frac{Q}{Q^*} = \exp(\varepsilon)$, and taking logs, the first order condition can be also written as $\ln \beta_X = \ln S_X^R + \varepsilon$, and hence the addtional term is controlling for ε in terms of the differences between the share of X in revenue and the specification of the production elasticity of the input.[13] However, when applied to data, if the first order condition is not met, this term will contain more than ε and the estimates will usually become inconsistent because of a problem of omitted variable (the discussion of this important case continues below).

Notice that, under perfect competition, the fulfillment of the first order condition implies that the dynamic panel and the linear OP/LP estimator should only diverge by sampling error. Their only difference is that, while the dynamic panel estimator leaves the ε error to become part of the error of the equation, the linear OP/LP estimator controls for ε by means of the difference between the revenue share and the elasticity.

If $g(\cdot)$ is nonlinear, OP/LP produces a different estimate that comes exclusively from adding nonlinear terms to approximate $g(\cdot)$. DP can be seen as a first order approximation to the productivity process dealt with by OP/LP. As productivity is in practice quite persistent, it should be unexpected that this creates a dramatic divergence.

[12] However we know that, under homotheticity, $\nu(\cdot)$ becomes a function of Q^* alone.

[13] The FOC under perfect competition, in the reordered form $\ln S_X^R = \ln \beta_X - \varepsilon$ is used by Gandhi, Navarro and Rivers (2020) as first step of their estimator. Notice that (9) suggests a unique-step form for the estimator.

Failure of the FOCs

First order conditions may not hold as (4) by multiple reasons. The most commonly discussed by researchers are: adjustment costs (see e.g. Bond and Van Reenen, 2007), market power in the input market (see e.g. Manning, 2011), firm optimization errors (see e.g. Marschak and Andrews, 1944), misallocation of inputs (see e.g. Hsieh and Klenow, 2009)...

We can add the case of biased technological change, as for example LAP. But it is important to take into account that this motive also changes the structure of the production function, something that we discuss in section 7.

All these circumstances may be represented in the FOC by the presence of an unobservable. Assume, without loss of generality, that the first order conditions affecting the variable inputs are

$$P\frac{\partial F(\mathbf{x})}{\partial X}\exp(\omega) = (1+u_X)W_X, \tag{10}$$

where u_X is an input-specific FOC unobservable.[14]

In order to give a more precise content to the fulfillment of the first order conditions we develop now a lemma. We will use it to define another form of the OP/LP estimator.

LEMMA The relationship between marginal cost and average variable cost is

$$MC = \frac{\theta AVC}{\nu}, \tag{11}$$

where $AVC = \frac{VC}{Q^*}$ is average variable cost, $\nu = \beta_L + \beta_M$ (or $\nu(\cdot) = \beta_L(\cdot) + \beta_M(\cdot)$) is the short-run elasticity of scale, and $\theta = 1 + \sum_X S_X u_X$, a weighted sum of the unobservables with weights equal to the cost-shares $S_X = \frac{W_X X}{\sum_X W_X X}$.

Proof

Since $P = MC$, adding the FOCs of the variable inputs multiplied by $\frac{X}{Q^*}$ we have

$$MC\sum_X \frac{X}{Q^*}\frac{\partial F}{\partial X}\exp(\omega) = \frac{\sum_X (1+u_X)W_X X}{Q^*} = (1+\sum_X S_X u_X)\frac{\sum_X W_X X}{Q^*},$$

or

$$MC\nu = \theta AVC,$$

[14] The FOC corresponding to LAP is usually written $P\frac{\partial F(\mathbf{x}^*)}{\partial L}\exp(\omega_L+\omega) = W_X$, that can also be written as $P\frac{\partial F(\mathbf{x}^*)}{\partial L}\exp(\omega) \simeq (1-\omega_L)W_X$, where $\mathbf{x}^* = \{k, l^*, m\}$.

so marginal cost can be writtent in terms of average variable cost AVC, the short-run elasticity of scale ν, and θ. ♦

Notice that the revenue-share estimation becomes

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{(1+u_{X,-1})S^R_{X,-1}}{\beta_{X,-1}}) + \xi + \varepsilon,$$

and cannot be claimed unaffected by the unobservable.

Cost-share OP/LP

Using the lemma, we can develop another form of the OP/LP estimator that, instead of using the price, proxies for marginal cost.

PROPOSITION *2* The OP/LP estimator can be written in the cost-share form

$$q = f(\mathbf{x}) + g(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{\nu_{-1}\theta_{X,-1}S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}) + \xi + \varepsilon, \qquad (12)$$

where $\theta_X = (1+u_X)/\theta$. If all FOCs for variable inputs hold with no unobservables, $\theta_X = 1$. The cost-share estimator is only consistent under linearity of $g(\cdot)$.

Proof

Since $p = mc$, by (10) we have $\omega = \ln(1+u_X) + w_X - mc - \ln \frac{\partial F(\mathbf{x})}{\partial X} = \ln(1+u_X) + w_X + x - (\ln\theta + vc - (q-\varepsilon) - \ln\nu) - f(\mathbf{x}) - (x - \ln F(\mathbf{x}) + \ln \frac{\partial F(\mathbf{x})}{\partial X}) - \varepsilon = q - f(\mathbf{x}) + \ln \frac{\nu\theta_X S_X}{\beta_X} - \varepsilon$.

In the second equality we use the lemma, that allows to proxy marginal cost by average variable cost. The price that we have to pay for this approximation is the introduction of the error ε. This error determines that $g(\cdot)$ must be linear for consistency. ♦

The cost-share form of OP/LP encompasses the information additional to the dynamic panel estimator in the term $\ln \frac{\nu_{-1}\theta_{X,-1}S_{X,-1}}{\beta_{X,-1}}$.[15] The researcher can model total or partially θ_X if knows of the presence of some unobservable (e.g. monopsony power).

[15] Notice that we can also pass from one to the other estimator with the chain $\frac{(1+u_{X,})S^R_{X,}}{\beta_X} = \frac{(1+u_{X,})\frac{VC}{PQ}\frac{W_X X}{VC}}{\beta_X} = \frac{(1+u_{X,})\frac{AVC}{MC}\exp(-\varepsilon)S_X}{\beta_X} = \frac{\nu\theta_X S_X}{\beta_X}\exp(-\varepsilon)$.

What happens when the equation is specified using the term $\ln \frac{\nu S_X}{\beta_X}$; that is, ignoring the unobservable? Multiplying the FOC (10) by $\frac{X}{MCQ^*}$ we can write $\frac{X}{Q^*}\frac{\partial F(\mathbf{x})}{\partial X}\exp(\omega) = (1+u_X)\frac{AVC}{MC}\frac{W_X X}{AVCQ^*}$. It follows that $\beta_X = \nu\theta_X S_x$. This implies that the expression $\ln \frac{\nu S_X}{\beta_X}$ (without θ_X) is going to take the value $-\ln\theta_X$ if $\theta_X \neq 1$, and zero if there are no unobservables.

We can establish an important proposition:

PROPOSITION 3 Under perfect competition, if the first order conditions for variable inputs hold without unobservables, the dynamic panel estimator and the linear cost-share OP/LP estimator should only diverge by sampling error.

Proof

The comparison of (7) and (12) shows that the two estimators only diverge by a term that, provided that $\theta_X = 1$, should be zero according to the FOCs. ♦

Summary

In summary, DP estimation under competition, and OP/LP under competition and with no unobservables in the FOCs, are consistent and they should differ only in the effect of the OP/LP nonlinear modeling of the productivity process. If the OP/LP estimator is specified using a linear Markov process, they should only diverge due to sampling error. This is regardless of how the OP/LP estimator is computed, in the revenue-share or cost-share form. However, an unobservable in the FOCs makes any OP/LP inconsistent while DP is not affected.

5 Implementing OP/LP

Expressions (9) or (12) are more or less simple to implement according to the specification chosen for the production function. For example, with a Cobb-Douglas production function they amount to estimate the production function by pseudo-differences, as in dynamic panel, including the terms $\rho\ln S^R_{X,-1}$ and $\rho\ln S_{X,-1}$ respectively (see the example of section 7).

However, this is not how OP/LP has been implemented by researchers. Under competition, LP proposed to replace ω by inverting the unconditional implicit demand for a variable input that we get by solving the system of FOCs of the firm, $X = X(K, P, W, P_M, \omega)$. That is, use $\omega =$

$h(K, P, W, P_M, X)$ that, assuming a common output price P and common input prices, it has often been implemented as the (time varying) $\omega = h(K, X)$ relationship. Implementing OP/LP substituting a flexible lagged $h(\cdot)$ for ω_{-1} in the Markov process is an expeditious way. See, for example, Wooldridge (2009). In comparison to the shares form, this form keeps more flexibility than what is implied by the production function specification. On the negative side, it wrongly omits the output and input prices when relevant.

Ackerberg, Caves and Frazer (2015), henceforth ACF, proposed a form to implement OP/LP that has become prevalent. Let us use the insights gained with proposition 2 to give an interpretation of the ACF OP/LP estimator.

An interpretation of the ACF procedure

The idea of ACF is to regress first nonparametrically output on all inputs and variables relevant to explain the demand for the input used to substitute for unobserved productivity. The goal is to identify separately ε. Let $\mathbf{z}$ be the set of variables on which the output is going to be projected (that we discuss later specifically) with $\mathbf{x} \subset \mathbf{z}$. The first stage of ACF computes

$$\widehat{\phi} = E(q|z) = E(f(\mathbf{x})|\mathbf{z}) + E(\omega|\mathbf{z}) = f(\mathbf{x}) + E(\omega|\mathbf{z}).$$

This means that, in the second stage, what ACF carries inside the $g(\cdot)$ can be written as

$$\widehat{\phi}_{-1} - f(\mathbf{x}_{-1}) = q_{-1} - f(\mathbf{x}_{-1}) + [E(\omega_{-1}|\mathbf{z}_{-1}) - \omega_{-1}] - \varepsilon_{-1}.$$

Again, as in (12), the difference with respect to the dynamic panel estimator depends from a potential bias. In this case the potential bias can be thought of as the difference of the projection of lagged productivity on the vector $\mathbf{z}_{-1}$ from the true value of productivity.

Since $E(\omega_{-1}|\mathbf{z}_{-1}) - \omega_{-1} = E(q_{-1} - f(\mathbf{x}_{-1}) + \ln \frac{\nu_{-1}\theta_{X,-1}S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1}|\mathbf{z}_{-1}) - \omega_{-1} = E(\ln \nu_{-1} + \ln S_{X,-1} - \ln \beta_{X,-1} + \ln \theta_{X,-1}|\mathbf{z}_{-1})$, everything depends on how is this expectation. With input quantities and input prices we can predict reasonably well the cost share and the elasticities, but we do not have the unobservable. Consequently, the bias is going to be zero if $\theta_X = 1$ and at least $-E(\ln \theta_X|\mathbf{z})$ otherwise. Recall that in the parametric case the presence of the unobservable imposed a bias of $-\ln \theta_X$. ACF is likely to smooth this bias by projecting it on z, since $Var(-\ln \theta_X|z) \leq Var(-\ln \theta_X)$.

From this point of view, ACF estimation using quantities and prices of the variable inputs is a semiparametric approximation to the inclusion of the term in the cost-share. With a linear Markov process, it should also only diverge from DP due to sampling error. If there is bias induced by unobservables in the FOCs, ACF estimation is likely to mitigate this bias by reducing its variance.

However, it there is market power, it is not correct to include the output price in the first stage of ACF. This corresponds to the demand for the input under perfect competition, and is likely to bias the estimation. In our exercise of section 10 we propose a form of carrying ACF that is akin to perfect competition and another to imperfect competition.

6 The consequences of market power

When there is market power, under the assumption of short-run profit maximization, the relevant variable in the FOCs is marginal revenue MR instead of P

$$MR\frac{\partial F(x)}{\partial X}\exp(\omega) = W_X.$$

The problem is that MR is, in general, unobservable. At first sight it seems like we have no more alternative than choosing DP estimation, that does not need this relationship. Let us take a closer look at what the new variable implies.

Let us now use firm subindices for clarity. There are N firms in a market. If the firms have market power, the solution of the system of the two variable input first order conditions is going to produce for each firm the condition of equilibrium $X_j = X(K_j, MR_j, W_j, P_{Mj}, \omega_j)$.

What is in MR_j? Let us assume for economy of notation that competition, both in quantities and prices, happens with product differentiation.[16] Under product differentiation firm j demand is $Q_j^* = D_j(P, \delta)$, where P is the vector of N prices and δ is a vector of unobserved correlated heterogeneity of demand (observable heterogeneity z can always be easily included). Assume that the demand system can be inverted, $P_j = D_j^{-1}(Q^*, \delta)$, where Q^* is the vector of N quantities. Revenue is $P_j Q_j^*$. If the firm competes in quantities, $MR_j = P_j + Q_j^* \frac{\partial D_j^{-1}}{\partial Q_j^*} + T(P_j, Q_j^*, conduct)$ and, if the firm competes in prices,

[16] The discussion can be easily extended to the case of homogeneous product. We take as reference the market power models in Vives (1999).

we can write the implicit $MR_j = P_j + (Q_j^* + T(P_j, Q_j^*, conduct)) \left(\frac{\partial D_j}{\partial P_j}\right)^{-1}$. In both cases $T(\cdot)$ represents a function.

There are two problems implied by these expressions. The first is that they depend in an unspecified way on the market behavior of firms. In the absence of a specific conduct, we do not have a function of observed variables but only a correspondence. Different $MR(\cdot)$ values can be associated to exactly the same P_j or Q_j^*, depending on behavior in the market. See Appendix B for a simple example.

The second problem is the presence of Q_j^* and δ in the expressions, and the derivatives $\frac{\partial D_j^{-1}}{\partial Q_j^*}$ and $\frac{\partial D_j}{\partial P_j}$. The variables Q_j^* and δ are non observable (we observe the actual output Q_j). Maybe we can use $Q_j^* = D_j(P, \delta)$ and have $MR(P,\ \delta,\ conduct)$, but notice that in general P and δ are vectors, the second the vector of N unobserved variables. We can try also using $Q_j^* = R_j(Q_{-j}^*)$ or $P_j = R_j(P_{-j})$, where $R_j(\cdot)$ are the corresponding best response functions. But these reaction functions also include unobservables and are behavior specific.

Of course it is possible to obtain expressions in terms of observables by simplifying the cases of behavior and with specific assumptions on symmetry of the firms and their behavior. For example, it is very popular to assume that competition is monopolistic and the elasticity of demand constant and equal for all firms. Under these assumptions $MR_j = P_j(1 - \frac{1}{\eta})$, where η represents the (absolute value) of the elasticity of demand.

A discussion of possible behavior restrictions and assumptions of symmetry across oligopoly models is carried out in Ackerberg and De Loecker (2024). They are able to reduce significantly the information requirements but, for example, they confirm that there cannot be unobserved characteristics if products are differentiated, as Doraszelski and Jaumandreu (2021) pointed out in relation to the correlated unobserved demand heterogeneity.

A compact way to think of the models that are possible is the following. Start with the output-conditional demand for the variable input $X_j = \widetilde{X}(K_j, W, P_M, Q_j^*, \omega_j)$, that with common input prices can be written $X_j = \widetilde{X}(K_j, Q_j^*, \omega_j)$. Use the (sales) market share of the firm S_j divided by P_j to express the production of the firm as a function of the market aggregate sales A, $Q_j^* = \frac{S_j}{P_j} A$. If S_j is only a function of (K_j, ω_j), the demand for the input can be expressed as a time varying function $X_j = \overline{X}(K_j, P_j, \omega_j)$.

This is close to what the standard application of OP/LP assume to be the arguments of the input demand to be inverted, except for the output price. But note that we have reached the expression assuming that firms cannot be unequal because input price differences, suppressing unobserved correlated demand heterogeneity, and/or asymmetric behavior. That is, discarding factors of efficiency other than ω before starting the investigation. In general we want to avoid this.

The central question is whether it is possible to estimate the production function without taking a stance on how is competition. Estimate without having to assume things like whether competition is in prices or quantities, firms either take the rivals actions as given or collude, collusion is either with all or with part of the rivals, some firms have a particular type of advantages or not, ...and so on. The answer is yes, it is possible.

To see why notice that, in equilibrium, a short-run profit maximizing firm equates marginal revenue and marginal cost, so

$$MR(P, \delta, conduct) = MC(K_j, W_j, P_{Mj}, Q_j^*, \omega_j).$$

On the left hand side, the expression depends on the particular specification of conduct. The right hand side, on the contrary, picks up a specific single value under quite general conditions.[17] It singles out a unique marginal cost $MC(\cdot)$ for each set of values of the arguments (we have specified possibly varying input prices for the sake of generality). We can even accommodate labor market power and LAP by considering the price $W^* = (1+\tau)W/\exp(\omega_L)$ with the unobservables τ and ω_L replaced. If we have MC, we have what has been called a "sufficient statistic," a variable that contains all the relevant information of the conduct and demand conditions.

7 A test for market power

Since applying procedures of estimation that are robust to market power limits the choices, it is worthy to have a readily available method that allows for testing the presence of market power. It turns out that we have developed in section 4 the estimators that can serve to construct a test for martket power.

The linear cost-share (parametric) OP/LP estimator is based on replacing P by MC, and approximating MC by AVC. It is hence consistent when

[17] These conditions are basically convexity assumptions on the technology of the firm.

the firm has market power because it is based on the formulas for cost minimization. It is an estimator fully robust to market power in the product market, that can be robustified to market power in input markets (and LAP, of course) by means of the model specification.

On the other hand, a revenue-share (parametric) OP/LP is not consistent in the presence of market power, even in a linear version, because it is based on the price of the firms, that under market power diverges from marginal cost.

We can test the null of no market power, in which both estimators are consistent, against the alternative of market power, under which the revenue-share estimator is not consistent, while the linear cost-share estimator remains consistent.

In practice, both estimators only differ in the alternative use of the cost or revenue shares of the input, and the need to specify the short-run elasticity of scale in the first. Furthermore, if ν is a constant, the difference collapses to only the use of a different regressor, making very easy the appplication of the test.

EXAMPLE Assuming that the production function is Cobb-Douglas, the two regressions to run are

$$\begin{aligned} q &= \beta_0 + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) + \rho \ln S_{X,-1} + e, \\ q &= \beta_0' + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) + \rho \ln S_{X,-1}^R + e', \end{aligned}$$

where $e = \xi + \varepsilon - \rho\varepsilon_{-1}$, $e' = \xi + \varepsilon$, and we want to compare the estimates for β_K, β_L and β_M, say. ♦

A Hausman (1978) specification test, or a Durbin-Wu-Hausman test, can be seen as a test of the equality between the parameter estimates under two methods of estimation that are consistent under the null. Following Wooldridge (2010), we set a quadratic form of the differences in the parameters $(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})$ using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})] = V_{CS} + V_{RS} - (C + C')$. See Appendix C on the computation of these matrices. Under the null, we have

$$(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})' Avar[\sqrt{N}(\widehat{\beta}_{CS} - \widehat{\beta}_{RS})]^{-1}(\widehat{\beta}_{CS} - \widehat{\beta}_{RS}) \sim \chi^2(p),$$

where the p degrees of freedom are the number of parameters being tested.

A drawback of the test is that we have assumed the the production function is well specified. If this is not the case, the two estimators are inconsistent under the null and under the alternative, although for reasons that are different from market power. While the test can be still informative under these circumstances, it seems convenient to repeat it again after addressing the specification of the production function.

8 Estimating under market power

An estimator that is robust with respect to market power is an estimator that is consistent whatever are the details of the game that firms play in the market. DP is robust to market power but OP/LP is not. DP is robust to market power because does not use any FOC which changes when market power replaces competition. OP/LP is not robust because the usual specification, based on the first order conditions under market power, needs to model marginal revenue MR. For modeling MR in a tractable way some particular games and strong symmetry conditions must be assumed.

However, we have shown that there is a feasible OP/LP that always is possible. It consists of proxying MC by AVC and the short-run elasticity of scale, taking into account that this replacement leaves in the expression the error of the production function and the Markov process must be assumed to be linear. This linear estimator, that we have called cost-share (parametric) OP/LP estimator, is robust to market power.

To summarize the properties of the DP and linear cost-share OP/LP estimators under market power, we can formulate the following

THEOREM Under market power, and no unobservables in the FOCs additional to MC, the dynamic panel estimator and the linear cost-share OP/LP estimators are both consistent, and their estimates must only differ by sampling error. With any unobservable in the FOCs, the linear cost-share OP/LP estimator loses its consistency while the the dynamic panel estimator retains it.

Proof

That DP is always consistent under market power follows directly from the fact that the estimator doesn't need to assume anything about the variable input FOCs. That the linear cost-share OP/LP estimator is consistent follows from the FOCs under market power with short-run profit maximization. Since under profit maximization $MR = MC$, we have

$$MC\frac{\partial F(x)}{\partial X}\exp(\omega) = W_X,$$

that are also the conditions for cost minimization of variable cost. Hence we can use the inversion

$$\omega = w_X - mc - \ln\frac{\partial F(x)}{\partial X} = w_X - vc + q + \ln\nu - \ln\frac{\partial F(x)}{\partial X} - \varepsilon,$$

where in the second equality we use the property $mc = vc - q^* - \ln\nu$, shown in the lemma of section 4 for $\theta = 1$ (no FOC unobservables other than MC). Similarly to proposition 2 we can rewrite this expression lagged as

$$\omega_{-1} = q_{-1} - f(x_{-1}) + \ln\frac{\nu_{-1}S_{X,-1}}{\beta_{X,-1}} - \varepsilon_{-1},$$

where $\ln\frac{\nu_{-1}S_{X,-1}}{\beta_{X,-1}} = 0$. It follows that the dynamic panel estimator and the linear cost-share estimator should only differ by sampling error.

To see that the linear cost-share estimator loses its consistency in the presence of any FOC unobservable we can reintroduce the unobservables $(1 + u_X)$ in the FOCs. This implies that $mc = \ln\theta_X + vc - q^* - \ln\nu$ and $\ln\frac{\nu_{-1}S_{X,-1}}{\beta_{X,-1}} = -\ln\theta_X$, an unobservable that the form of estimation (parametric or ACF) cannot eliminate. ♦

Let us summarize how to estimate under market power. In perfect competition it was the price that measured marginal cost. Now under market power, if we want to estimate using the OP/LP method, we have to directly deal with marginal cost. We need to replace marginal cost by the observables variable cost and output (that makes average variable cost up to an error), and specify the short-run elasticity of scale. This collapses to an expression that simply adds the term $\ln\frac{\nu_{-1}S_{X,-1}}{\beta_{X,-1}}$ to the DP specification.

In practice, the OP/LP estimator can be specified either by extending DP with the term $\ln\frac{\nu_{-1}S_{X,-1}}{\beta_{X,-1}}$ or, in the ACF manner, including all variables $\mathbf{z}$ relevant to the demand for the input in a first nonparametric step. When the production function is not a Cobb-Douglas this can simplify notably the estimation. In practice it projects the expression $\ln\frac{\nu_{-1}S_{X,-1}}{\beta_{X,-1}}$ on the set of variables $\mathbf{z}$ (see section 5), what can smooth the bias when there is one.

9 A specification test

The previous theorem is not only important in itself, it suggests a powerful way to test the specification used under market power.

However, to transform the theorem in an specification tool we need to consider another reason by which DP and the linear cost-share OP/LP may diverge. Until now we have assumed that $f(\mathbf{x})$ was well specified, with the consequence that the theorem concludes that, under market power and no FOC unobservables, DP and OP/LP must only diverge due to sampling error. Now we want to encompass also possible mistakes of specification. The production function in itself can be not well specified, and this affects both the production function and the FOC.

To take the most relevant example of mispecified production function, think of the case of biased technological progress, LAP say. Now the relevant labor is $l^* = l + \omega_L$, the production function is $f(k, l^*, m)$ and the labor FOC is $MC\frac{\partial F(\mathbf{x}^*)}{\partial L^*}\exp(\omega_L + \omega) = W$. The DP estimator consistency fails because of the production function specification, and the consistency of the OP/LP estimator because both the specification of the production function and the presence of the unobservable in the labor FOC. Under LAP, both DP and OP/LP diverge and both are inconsistent.

Adopting this broader perspective, the situation under market power can be summarized as follows. On the one hand, both estimators can be inconsistent because the production function is not well specified. On the other hand, if the production function is well specified and the FOC contain no unobservables additional to MC, DP and the OP/LP must only diverge due to sampling error. Hence, an specification test based on the equality of the coefficients of two estimators, this time DP and OP/LP, is again available. Under the null both estimators are consistent, under the alternative either both estimators are inconsistent or only the OP/LP estimator (because only the FOCs fail) is inconsistent.

When there is market power, starting the estimation of the production function with this test is an easy and convenient way to work on the specification. If DP and OP/LP coincide and the test is passed, the researcher has the statistical evidence that a necessary condition for consistency is met. Of course this doesn't automatically ensure consistency and hence the specification may be still worked and improved.

However, if the test is not passed (DP and OP/LP diverge) we are sure that one of two things is happening: either the production function specification is wrong or there are unobservables in the FOCs. It is crucial to know it in order to address the effort to detect the origin of the problem.

EXAMPLE Assuming that the production function is Cobb-Douglas, we can run the DP and linear cost-share OP/LP regressions

$$\begin{aligned} q &= \beta_0 + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) + e, \\ q &= \beta_0' + \rho q_{-1} + \beta_K(k - \rho k_{-1}) + \beta_L(l - \rho l_{-1}) + \beta_M(m - \rho m_{-1}) + \rho \ln S_{X,-1} + e', \end{aligned}$$

where both e and e' have the form $\xi + \varepsilon - \rho\varepsilon_{-1}$, and compare the estimates for β_K, β_L and β_M. ♦

The test can be constructed again setting a quadratic form of the differences in the parameters $(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})$ and using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})] = V_{DP} + V_{OP/LP} - (C + C')$ (see Appendix C). Under the null, we have

$$(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})' Avar[\sqrt{N}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP})]^{-1}(\widehat{\beta}_{DP} - \widehat{\beta}_{OP/LP}) \sim \chi^2(p),$$

where the p degrees of freedom are the number of parameters being tested.

10 Estimating the production function with US firms

In this section we show with an example how the DP and OP/LP estimators start diverging under standard estimation, and how this divergence reflects at the same time the presence of market power and more general problems of specification. The aplication of our two tests guides the work of re-specification of the production function, and the two estimators end by coinciding. This coincidence implies that the necessary condition for consistency is met, and we leave the task where it can be continued for refinements.

We estimate the production function for the sample of Compustat US manufacturing firms (1960-2018) used in Jaumandreu and Mullens (2024). It is a sample of firms belonging to many different markets and times, so we expect that they possess various degrees of market power. In estimating the production function of these firms, it is then important to be robust to the exercise of market power.

At the beginning, the DP estimation and a standard ACF implementation of the OP/LP estimator applied to a Cobb-Douglas specification, diverge in the estimate of the elasticity of capital and in the assessment of the returns to scale. As it is often the case, for dismay of researchers, DP produces a negative elasticity for capital, and a short-run elasticity of scale well above unity. However, the ACF implementation of OP/LP shows up a nicely estimated (small) elasticity of capital and a more moderated short-run elasticity of scale (although non smaller than one as well). As odd as it may sound, this is not a sample-specific phenomenon but a quite typical finding.

A usual interpretation for the DP behavior is that the differentiation of the data exacerbates errors in measurement of a capital otherwise quite persistent over time. However, a little experimentation shows that this explanation is not convincing. For example, OLS of the Cobb-Douglas in first differences gives positive coefficients for all inputs (see Appendix D). On the other hand, if DP is inconsistent, sections 3 and 4 have shown that OP/LP cannot be consistent. The result of the estimation raises hence a puzzle: DP cannot be right, but the ACF estimation of OP, that must be equally inconsistent and probably accumulating the inconsistency due to market power, is adding something that improves the estimation.

In what follows we start by confirming that effectively we are in the presence of market power. Applying the test we are not able to discard market power, and hence we should stick to the version of the OP/LP estimator that is robust to market power. However, the result of the estimator by itself is not satisfactory as we already expected given the performance of DP. Then, we look for reasons, related to the production function specification, that must be inducing the inconsistency of both estimators. A simple inspection of the labor shares shows that the elasticity of labor must have been falling over time and should be deeply varying across firms, while the Cobb-Douglas specification fails in picking up this characteristic. When labor-augmenting productivity is allowed into the specification, enlarging the Cobb-Douglas to a translog that admits shares and elasticities that are varying, the DP estimator and the OP/LP version robust to market power fully coincide. Our specification test sanctions that.

The conclusion is that the divergence of the estimators was detecting simultaneously the presence of market power and the mispecification of the production function, and that the redressement of mispecification allows robust estimators to provide the same answer. We have needed to address both things because both things were in the data. The test for market power has limited the estimators for which the comparisons are valid, and the comparisons carried out with robust estimators confirm that the necessary condition for consistency can be met. This can be a good starting point for the researcher try to further improve the specification keeping the equality of the two estimators (after testing again, now presumably with consistency, that market power cannot be discarded).

Exercise details

In what follows, we explain in detail how the above exercise is done.

Table 1 reports the result of the two basic estimators. Column (1) of the table reports the results of applying the DP estimator to a Cobb-Douglas specification. The estimator proceeds as follows. Under the assumption that Hicks-neutral productivity ω_H follows an $AR(1)$ of parameter ρ and innovation ξ, it can be written that

$$\begin{aligned} q_{jt} &= \beta_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1}] + \xi_{jt} + \varepsilon_{jt} - \rho\varepsilon_{jt-1}, \end{aligned} \tag{13}$$

where β_t includes one year that is taken as base (constant). We estimate this equation by nonlinear GMM using as instruments the constant and time dummies, the input variables $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}$ and the (real) input prices $w_{jt-1} - p_{jt-1}$ and $p_{Mjt-1} - p_{jt-1}$. This is a fully standard choice of instruments.[18] As we have to estimate (in addition to the constant and time dummies) the four parameters $\beta_K, \beta_L, \beta_M$ and ρ, the instruments provide two overidentifying restrictions.

The result is not nice: the elasticity of capital turns out to be negative, the elasticity of labor very large and the short-run elasticity of scale ν, the sum of $\beta_L + \beta_M$, above 1.1. Economic theory tells us that it is not realistic that when changing in the short-run the variable factors we encounter increasing returns to scale (although, unfortunately, this is a quite usual result that is reported without further comments).

Column (2) reports the results of computing the OP/LP estimator applied to the Cobb-Douglas, implemented by means of a standard ACF method. In a first stage, we regress q_{jt} non parametrically on a constant, time dummies, and using a complete polynomial of order 3 on the five variables

[18] Even if there is market power, and output price differs from marginal cost, price can be assumed naturally uncorrelated with the observational error ε of the production function and hence is a legitimate instrument. The price of output scales the input prices.

$k_{jt}, l_{jt}, m_{jt}, w_{jt} - p_{jt}$ and $p_{Mjt} - p_{jt}$.[19] From the result of this first step we compute the estimate $\widehat{\phi}(k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}, p_{Mjt-1} - p_{jt-1})$ that we use in forming the second step equation

$$\begin{aligned} q_{jt} &= \beta_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[\widehat{\phi}_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1}] + \xi_{jt} + \varepsilon_{jt}. \end{aligned} \tag{14}$$

As we try to emphasize with the writing, equations (13) and (14) are very close. They only differ, in addition to the component of the error $-\rho\varepsilon_{jt-1}$, in that the nonparametric estimate $\widehat{\phi}_{jt-1}$ has replaced q_{jt-1}.

If we have thought of the demand for materials to construct the proxy for ω, the application of the analysis of section 4 tells us that

$$\begin{aligned} \widehat{\phi}_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} &= \\ q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} & \\ + E(\ln \frac{\nu S^R_{Mj,-1}}{\beta_M} | k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}, p_{Mjt-1} - p_{jt-1}). & \end{aligned}$$

Hence, we interpret the method as if we are adding into the bracket of the DP estimator the expression corresponding to the nonparametric prediction of the share $S^R_{M,-1}$ (note that β_M and ν are here irrelevant constants). Everything is like in the second stage we were using

$$\begin{aligned} q_{jt} &= \beta'_t + \beta_K k_{jt} + \beta_L l_{jt} + \beta_M m_{jt} \\ &\quad + \rho[q_{jt-1} - \beta_K k_{jt-1} - \beta_L l_{jt-1} - \beta_M m_{jt-1} + \ln \widehat{S}^R_{jt-1}] + \xi_{jt} + \varepsilon_{jt}, \end{aligned}$$

where $\ln \widehat{S}^R_{jt-1}$ stands for the empirical expectation. In the second step of the ACF implementation we use the instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}$ and $\widehat{\phi}_{-1}$, what implies one overidentifying restriction. The use of $\widehat{\phi}_{-1}$ brings as instrument the (lagged) result of the first stage estimation.

The results reported in column (2) show the outcome. The addition (the nonparametric prediction of the share) helps to redress two things with respect to the DP results. The elasticity of capital becomes positive, and the short-run elasticity of scale falls.

It is apparent that DP and OP/LP are giving different answers to the estimation of the production function. This is attributable to two facts.

[19] Notice that this implies the use of the output price dividing the input prices, what is consistent with the input demand under perfect competition. See below.

The first is that we are using an OP/LP estimator that it is inconsistent under market power. In the first stage we use the price of the product (in a specification to replace unobservable productivity that correponds to the arguments of the demand for materials), and this use is only correct under perfect competition. Market power implies that price should be replaced by marginal cost. The second is that the simultaneous obvious inconsistency of DP suggests that there is a shortcoming in the specification of the production function that should be affecting also the production function and the FOC used in OP/LP.

Testing (ex-ante) for market power

First thing to answer is if we really cannot discard the presence of market power. In order to do this we perform the test for market power, with the results reported in Table 2. In columns (1) and (2) we compute the test using the two parametric versions of OP/LP, that is, adding respectively to specification (13) the log of the cost-share of materials and the revenue-share of materials. The test sharply says that this is not equivalent, rejecting the absence of market power.

However, notice that the regressions for the implementation of the test are already pointing to the researcher that the involved problems are not only market power. Even the simple addition of the log of the cost-share of materials (column (1)) provokes a change in the estimated coefficients that can only be interpreted as revealing the presence of an unobservable strongly correlated with the variable inputs. In fact we can already guess that this unobservable is LAP by noticig that is positively correlated to materials and negatively to labor.

Columns (3) and (4) report the alternative result of applying the test with the ACF version of OP/LP. The ACF OP/LP estimator when there is perfect competition has already been presented in Table 1, and hence column (4) of Table 2 simply reproduces column (2) of Table 1 for the sake of covenience in the comparison. Column (3) estimates the version of ACF OP/LP that must be consistent in the presence of market power. A direct substitution of ω in the production function leaves the expression

$$q_{jt} = f(x_{jt}) + q_{jt}^{*} - f(x_{jt}) + \ln \frac{\nu S_{Mjt}}{\beta_M} + \varepsilon_{jt},$$

what suggests to regress q_{jt} non parametrically in the first stage on a constant, time dummies, and using a complete polynomial of order 3 on the four variables k_{jt}, l_{jt}, m_{jt},and $\ln S_{Mjt}$. In the second step of the ACF implementation we use the instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}, \widehat{\phi}_{-1}$ and $\ln S_{Mjt-1}$, what implies two overidentifying restrictions. The result of the test is again the rejection of the absence of market power. The value of the test can be only taken as a χ^2 with 2 degrees of freedom with a 1% probability.

Testing for the specification of the production function

Given these results we move to amend the specification of the production function. Using OP/LP estimators robust to market power, we compare the value of the coefficients obtained under DP and OP/LP. The results are reported in Table 3.

Now we estimate a multiproductivity production function. We use the simplest production function that admits LAP, a translog separable in capital and homogeneous of degree ν in labor and materials (we follow Doraszelski and Jaumandreu, 2019).[20] Because it is homogeneous in labor and materials can be written in terms of the log-ratios materials to labor, and these log-ratios exhibit unobserved labor-productivity,

$$q_{jt} = \alpha_0 + \alpha_K k_{jt} + \nu m_{jt} - \alpha_L (m_{jt} - l_{jt} - \omega_{Ljt}) - \frac{1}{2}\alpha (m_{jt} - l_{jt} - \omega_{Ljt})^2 + \omega_{Hjt} + \varepsilon_{jt}.$$

Using the ratio of first order conditions for labor and materials, we derive an expression to be substituted for these ratios, $m_{jt} - l_{jt} - \omega_{Ljt} = -\frac{\alpha_L}{\alpha} + \frac{\nu}{\alpha} S^*_{Ljt}$, with $S^*_{Ljt} = S_{Ljt} - \frac{\alpha}{\nu}\overline{\omega}_L$, and where $\overline{\omega}_L$ is a guess for the mean of labor-augmenting productivity.[21] Hence, the production function becomes a function of observables in which, to control for Hicks-neutral productivity, we can easily apply both the DP estimation procedure and the OP/LP method.

DP

The DP estimator is obtained by applying nonlinear GMM to the equation

$$\begin{aligned} q_{jt} &= \gamma_0 + \beta_t + \alpha_K k_{jt} + \nu m_{jt} - \frac{1}{2}\frac{\nu^2}{\alpha} S^{*2}_{Ljt} \\ &\quad + \rho[q_{jt-1} - \alpha_K k_{jt-1} - \nu m_{jt-1} + \frac{1}{2}\frac{\nu^2}{\alpha} S^{*2}_{Ljt-1}] + \xi_{jt} + \varepsilon_{jt} - \rho\varepsilon_{jt-1}, \end{aligned} \tag{15}$$

[20] See for more detail Jaumandreu and Mullens (2024). The specification can be conisdered a particular case of Demirer (2025).

[21] We use $\overline{\omega}_L = \overline{m} - \overline{l}$. This specification only estimates directly parameter ν, and the elasticities β_L and β_M are determined by the implications $\beta_L = \nu \overline{S}_L$ and $\beta_M = \nu(1 - \overline{S}_L)$.

where the expression is written in a similar format to the previous estimators for the sake of comparability. To estimate this more nonlinear equation we enlarge the instruments with the squares of the inputs, and we add the lagged share of labor cost in variable cost (that it is an important part of the translog): $k_{jt}, k_{jt}^2, l_{jt-1}, l_{jt-1}^2, m_{jt-1},\ m_{jt-1}^2, w_{jt-1} - p_{jt-1},\ p_{Mjt-1} - p_{jt-1}$,and $S_{L,-1}$. This gives nine instruments and we hence have five overidentifying restrictions.

Parameric OP/LP

We construct the parametric OP/LP version by adding the term $\ln \frac{\nu S_{Mjt}}{\beta_{Mjt}}$ inside the brackets of (15). It is easy to check that in practice it amounts to add $\ln \frac{S_{Mjt}}{S_{Mjt}+\frac{\overline{\omega}}{\nu}}$. We use the instruments $k_{jt}, k_{jt-1}, l_{jt-1}, m_{jt-1}, w_{jt-1} - p_{jt-1}$, $p_{Mjt-1} - p_{jt-1}, S_{L-1}$ and vc_{-1}, which imply four overidentifying restrictions. We replace the inputs squared by lagged capital, and add the lagged log of variable costs to help the instrumentation of S_{Mjt} (using S_{Mjt-1} would introduce perfect collinearity).

Nonparametric ACF OP/LP

To estimate the nonparametric ACF version of the OP/LP estimator we first again regress as before q_{jt} non parametrically on the variables k_{jt}, l_{jt}, m_{jt},and $\ln S_{Mjt}$. This first step gives the estimate $\widehat{\phi}(k_{jt-1}, l_{jt-1}, m_{jt-1}, \ln S_{Mjt-1})$ that we use in forming the second step equation. In the second step we use the instruments $k_{jt}, k_{jt}^2, k_{jt-1}, l_{jt-1}, l_{jt-1}^2, m_{jt-1},\ m_{jt-1}^2, \widehat{\phi}_{-1}$ and $S_{Lj,t-1}$, so that we have five overidentifying restrictions.

Test

The results for the new DP and the parametric and nonparametric OP/LP are reported in columns (1), (2) and (3) of Table 3 respectively. They clearly show that now DP and OP/LP give basically the same estimate of the production function (notice that the nonparametric estimation is however less efficient and makes the comparison less demanding).

To check statistically that the estimates can be considered the same, we apply specification tests. We construct a quadratic form of the elasticity of capital and the elasticity of scale, using as weight the inverse of a robust estimator of the asymptotic variance of the difference between the coefficients $(\beta_{DP} - \beta_{OP/LP})$, see Appendix C. The tests, reported for the parametric and nonparametric OP/LP at the bottom of columns (2) and (3) respectively, do not reject that the quadratic form is distributed as an χ^2 with 2 degrees of freedom, and hence do tell us that the differences now can be interpreted as coming from sampling error.

Notice that both the elasticity of capital and the short-run elasticity of scale are sensible. The elasticity of capital is greater than with the Cobb-Douglas specification, and the estimate of the elasticity of scale is in the range $0.70 - 0.85$, what clearly improves the unrealistic constant returns to scale for the variable inputs estimation of column (2) of Table 1. No estimator produces a clear better estimation than the other when rightly specified. From here on, the researcher can focus on improving other aspects of the estimation, as accounting explicitly for input market power, introduce the effect of adjustment cost of the variable inputs, or experiment with the way to deflate output and materials.

Testing (ex-post) for market power

We finally want to confirm that it has been relevant to use the linear cost-share OP/LP estimator, that in effect we cannot reject the presence of market power, now with estimators that meet the necessary condition for consistency. To check this we compare the OP/LP estimator that is consistent under market power with the estimator that is based on assuming perfect competition, both in the parametric version and the nonparametric ACF version, by means of the market power test. To construct the parametric OP/LP based on perfect competition it is enough to add to (15) $vc - r + \ln \frac{S_{Mjt}}{S_{Mjt} + \frac{\overline{\omega}}{\nu}}$, that implements the revenue-share. To construct the nonparametric estimator based on perfect competition we go back to the use of the price of the product in the first stage of ACF. Both constructed estimators are reported in columns (1) and (2) of Table 4 respectively. Then we compare column (1) of Table 4 with the estimator of column (1) of Table 2, and column (2) of Table 4 with column (3) of Table 2. The resulting χ^2 with 2 degrees of freedom show a strong rejection of the null of perfect competition.

11 Concluding Remarks

The production function is not affected by market power, but the estimators that employ an auxiliary FOC encompassing a derivative of the production function are sensitive to the form of the FOC under market power. DP is an estimator robust to market power because does not use any FOC, while the OP/LP approach cannot be generally robust to market power because marginal revenue, present in the FOCs, depends on the details of the game the firms play. However, marginal cost, that equals marginal revenue and summarizes all relevant effects of the strategy of the firm and its demand, can be replaced by average variable cost (corrected by the elasticity of scale) plus the uncorrelated error of the production function. Under linearity of the productivity process, the error does not affect the estimation. This gives an OP/LP estimator that is feasible under market power, that we have called the linear cost-share OP/LP.

The linear cost-share OP/LP can be written adding to the equation used by DP the (log) difference between the observed variable cost share and the normalized elasticity of the input whose demand is inverted. It can be also implemented nonparametrically as in ACF. In theory, with this addition, we should have exactly the same estimate because, if the production function is right, the theoretical value of the expression is zero. If the estimators diverge, either the production function specification is wrong or there is a problem in the specification of the FOC. This gives us a test for the specification.

The researcher who wants to estimate the production function under market power may first test if market power is relevant. If market power is relevant, it is convenient to start by estimating both the DP and the feasible OP/LP estimators, and testing the equality between them. The equality still doesn't ensure that both estimators are consistent, but they are meeting a necessary condition for consistency. Further work of specification with the two models may warrant consistent estimation under market power.

We have shown how this works with an example of estimation of the production function for a sample of US manufacturing firms. The naive Cobb-Douglas specification of the production function produces, with the DP estimator, a negative elasticity for capital and a too large short-run elasticity of scale. Recognizing LAP in addition to Hicks-neutral productivity, and allowing it into the specification, induces a matching of DP and the robust to market power linear cost-share OP/LP. The specification test is passed.

We have shown that the use of the FOCs together with the production function can be taken advantage for testing the presence of market power, and for improving the specification of the production function itself. Our empirical results suggest that market power can be frequent, that is easy to detect, and its impact in estimation nonnegligible with non robust estimators. The results also suggest the need for more flexible production functions than what is regularly assumed. The good relative behavior of OP/LP seems more linked to its implicit flexibility than a generality that can only be reached in its robust version. An interesting avenue for the future is the extension of what we have done to more nonparametric (and nonseparable) contexts.

Appendix A: Identification

Assume the timing and information conditions of the text. The FOC to be used by an OP/LP procedure becomes $\omega = d - \ln \frac{\partial F(\mathbf{x})}{\partial X} = h(d, \mathbf{x}; \theta_1)$, where $d = \ln \frac{W_X}{P}$, and θ_1 represents the parameters of the derivative. The model to be estimated can be written as

$$q = f(\mathbf{x}; c_0, \theta) + g(h(d_{-1}, \mathbf{x}_{-1}; \theta_1); \theta_2) + \xi + \varepsilon,$$

where θ are the parameters of the production function additional to the constant c_0, and $\theta_1 \subset \theta$. If we are in a strictly nonparametric setting, the vectors θ, θ_1, and θ_2 are infinite-dimensional. If we want to approximate the nonparametric relationship by a flexible form, we are going to use a limited number of parameters. If the procedure for estimation is nonlinear GMM, identification basically depends on having the same or more valid moments than parameters to estimate.

For example, to start with something simple, if the production function is Cobb-Douglas with inputs k, l and m, vector θ has dimension 3, vector θ_1 the same dimension 3 (if we use the demand for m, say), and we may decide to be linear using for $g(\cdot)$ an $AR(1)$ of parameter ρ. This gives 4 parameters to estimate in addition to the constant. With instruments k, k_{-1}, l_{-1} and m_{-1} (in addition to the constant) we are exactly identified.

Notice first that this form of estimation avoids any problem of "functional dependence". Combine the FOCs and write the unconditional demands $m = m(d_M, d_L, k, \omega)$ and $l = l(d_M, d_L, k, \omega)$. Inverting these demands we have $\omega = h_M(d_M, d_L, k, m)$ and $\omega = h_L(d_M, d_L, k, l)$. Suppose that we enlarge the number of parameters of $h_M(\cdot)$ carrying out flexible ("nonparametric") estimation to control for ω as proposed by Levinsohn and Petrin (2003) using $q = f(\mathbf{x}) + h_M(d_M, d_L, k, m) + \varepsilon$. Ackerberg, Caves and Frazer (2015, p. 2422-23) argue that $l - E(l|d_M, d_L, k, m)$ is not different from zero because l is an exact function of only (d_M, d_L, k, m) and hence Robinson (1988) procedure of nonparametric control breaks down. This argument does not affect the above procedure because, if one uses $q = f(\mathbf{x}) + \rho[h_M(d_{M,-1}, d_{L,-1}, k_{-1}, m_{-1})] + \xi + \varepsilon$, there is no functional dependence and the nonparametric control works fine.

Of course this works as long as we do not have perfect collinearity between the quantities of the two variable inputs. Bond and Soderbom (2005) argued that if prices are common to all firms at a given point of time, and inputs are perfectly flexible, all variable "inputs are perfectly collinear with the productivity shocks observed by firms" (page 1). That is, it would not be identification for the elasticities of a simple Cobb-Douglas in the cross section, although adjustment costs can partially alter this. However, the first order conditions allows to identify the elasticities of the variable inputs from the "share equation," as Doraszelski and Jaumandreu (2013) show in foonote 14.

Gandhi, Navarro and Rivers (2020), looking for the limits to identification, give a theorem that can be read setting an optimistic minimum: with a unique perfectly flexible input, identification of all elasticities requires time series variation of the ratio of the price of the input to the output price, d_M say. In fact, this variation seems only needed when there are two variable inputs and there are no constant returns to scale (see Trunschke and Judd, 2024, for a Monte Carlo).

These results can be extended for flexible specifications. Assume now that we want to make an approximation to $f(\mathbf{x})$ based on a complete polynomial of order 3 (the most used approximation) and $\mathbf{x}$ has n inputs. The number of parameters in (c_0, θ) is $1+2n+\frac{n(n-1)}{2}+n^2+\frac{n(n-1)(n-2)}{6}$. If $n=3$ this gives us 20 parameters (the first is the constant). If we decide to estimate $g(\cdot)$ also by means of three powers, we have to estimate a total of 23 parameters. Using the constant and the vector (k, l_{-1}, m_{-1}) we can form 20 moments. Adding the new moments that we can form using k_{-1} and interacting it with l_{-1} and m_{-1} we have 10 more. We hence have 7 overidentifying restrictions. As long as we do not have perfect collinearity between the quantities of two variable inputs, identification is warranted.

Appendix B: Conduct specification

Let us suppose for simplicity 2 firms (the industry can have, for example, $N/2$ of each type). We drop the asterisk from the quantities Q_j^* for economy of notation. We also abstract from heterogeneity of demand. Firms 1 and 2 have demands

$$\begin{aligned} Q_1 &= P_1^{-\eta} P_2^{\gamma}, \\ Q_2 &= P_2^{-\eta} P_1^{\gamma}, \end{aligned}$$

and costs $C_1(Q_1)$ and $C_2(Q_2)$. Inverse demands are

$$\begin{aligned} P_1 &= Q_1^{-\eta^*} Q_2^{-\gamma^*}, \\ P_2 &= Q_2^{-\eta^*} Q_1^{-\gamma^*}, \end{aligned}$$

where $\eta^* = \frac{\eta}{\eta^2-\gamma^2}$ and $\gamma^* = \frac{\gamma}{\eta^2-\gamma^2}$. Firm i maximizes

$$\pi_i + \lambda \pi_j = P_i Q_i - C_i(Q_i) + \lambda(P_j Q_j - C_j(Q_j)),$$

where λ is an exogenous conduct parameter. If firms compete in prices

$$\begin{aligned}\frac{\partial(\pi_i + \lambda\pi_j)}{\partial p_i} &= Q_i - \eta P_i \frac{Q_i}{P_i} + \eta C_i' \frac{Q_i}{P_i} + \lambda[\gamma P_j \frac{Q_j}{P_i} - \gamma C_j' \frac{Q_j}{P_i}] = \\ &= P_i - \eta(p_i - C_i') + \lambda\gamma(P_j - C_j')\frac{Q_j}{Q_i} = 0,\end{aligned}$$

and if they compete in quantities

$$\frac{\partial(\pi_i + \lambda\pi_j)}{\partial Q_i} = P_i - \eta^* Q_i \frac{P_i}{Q_i} - C_i' - \lambda\gamma^* Q_j \frac{P_j}{Q_i} = 0.$$

Using symmetry, is easy to see that $MR_j = P_j(1 - \frac{1}{\eta(1-\lambda\frac{\gamma}{\eta})}) = C_j'$ under price competition and $MR_j = P_j(1 - \frac{\eta}{\eta^2-\gamma^2}(1+\lambda\frac{\gamma}{\eta})) = C_j'$ under quantity competition. If $\lambda = 1$, both marginal revenues coincide in $P_j\frac{1}{\eta-\gamma}$, the unique total collusive solution. If $\lambda = 0$, Cournot with $P_j(1 - \frac{1}{\eta(1-(\frac{\gamma}{\eta})^2)})$ is less competitive than Bertrand, which gives $P_j(1-\frac{1}{\eta})$. If $0 < \lambda < 1$, quantity competition is less competitive than price competition.

The point is how MR_j changes with conduct, in this case the type of competition and the unobservable exogenous parameter λ.

Appendix C: Specification tests

A Hausman (1978) specification test, or a Durbin-Wu-Hausman test, can be seen as a test of the equality between the parameter estimates under two methods of estimation that are consistent under the null. The alternative is in our case that one method is inconsistent (market power test) or that either one method or the two are inconsistent (general specification test). Following Wooldridge (2010), we set a quadratic form of the differences in the parameters $(\widehat{\beta}_A - \widehat{\beta}_B)$ using the inverse of a robust estimate of $Avar[\sqrt{N}(\widehat{\beta}_A - \widehat{\beta}_B)] = V_A + V_B - (C + C')$.

Let $i, l = A, B$. To estimate V_i, we use

$$\widehat{V}_i = (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1} \widehat{G}_i' A_{Ni} \widehat{\Omega}_i A_{Ni} \widehat{G}_i (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1},$$

with $\widehat{G}_i' = N^{-1}\sum_j \frac{\partial(Z_{ij}'\widehat{u}_{ij})}{\partial\beta_i}$, $A_{Ni} = N^{-1}\sum_j Z_{ij}' Z_{ij}$, and $\widehat{\Omega}_i = N^{-1}\sum_j Z_{ij}'\widehat{u}_{ij}\widehat{u}_{ij}' Z_{ij}$. To estimate $\widehat{C}$, we use

$$\widehat{C} = (\widehat{G}_i' A_{Ni} \widehat{G}_i)^{-1} \widehat{G}_i' A_{Ni} \widehat{\Omega}_{il} A_{Nl} \widehat{G}_l (\widehat{G}_l' A_{Nl} \widehat{G}_l)^{-1},$$

where $\widehat{\Omega}_{il} = N^{-1} \sum_j Z'_{ij} \widehat{u}_{ij} \widehat{u}'_{lj} Z_{lj}$. Hence $\widehat{Avar}\ (\widehat{\beta}_A - \widehat{\beta}_B) = (\widehat{V}_A + \widehat{V}_B - (\widehat{C} + \widehat{C}'))/N$.

Under the null, we have

$$(\widehat{\beta}_A - \widehat{\beta}_B)' Avar[\sqrt{N}(\widehat{\beta}_A - \widehat{\beta}_B)]^{-1} (\widehat{\beta}_A - \widehat{\beta}_B) \sim \chi^2(p),$$

where the p degrees of freedom are the number of parameters being tested.

Appendix D: Additional regressions

Table AD reports a few complementary estimates. Column (1) reports the result of carrying out an OLS estimation of the Cobb-Douglas specification in first differences. Although the coefficient on capital is small, it is positive and statistically significant.

Column (2) reports the results of the estimation of a nonlinear nonparametric ACF OP/LP. Recall that this is a theoretically inconsistent estimator under market power. The coefficients modeling the nonlinear productivity process are not separately significant, and the coefficient on capital and the short-run elasticity of scale (1.022) ressemble the values obtained with the CD specification.

References

Ackerberg, D., K. Caves and G. Frazer (2015), "Structural Identification of Production Functions," *Econometrica*, 6, 2411-2451.

Ackerberg, D., and J. De Loecker (2024), "Production Function Identification Under Imperfect Competition," mimeo, University Texas Austin.

Ackerberg, D., J. Hahn, and Q. Pan (2023), "Nonparametric Identification Using Timing and Information Set Assumptions with an Application to Non-Hicks Neutral Productivity Shocks," University of Texas-Austin.

Aguirre, A., M. Tapia, and L. Villacorta (2024), "Productivity, Investment and Wealth Dynamics under Financial Frictions," mimeo, Central Bank of Chile.

Arellano, M., R. Blundell, and S. Bonhomme (2017), "Earnings and Consumption Dynamics: A Nonlinear Panel Data Framework," *Econometrica*, 85, 693-734.

Arellano, M. and S. Bond (1991), "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *Review of Economic Studies,* 58, 277-297.

Azzam, Jaumandreu and Lopez (2025), "Input and Output Market Power with Non-neutral Productivity: Livestock and Labor in U.S. Meatpacking," Boston University.

Berndt, E., and D. Wood (1975), "Technology, Prices and the Derived Demand for Energy," *Review of Economics and Statistics*, 57, 3, 259-268.

Blundell, R. and S. Bond (2000), "GMM Estimation with Persistent Panel Data: An Application to Production Functions," *Econometric Reviews* 19, 321-340.

Bond, S., A. Hashemi, G. Kaplan and P. Zack (2023), "Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data," *Journal of Monetary Economics,* 121, 1-14.

Bond, S. and M. Soderbom (2005), "Adjustment Costs and the Identification of Cobb Douglas Production Functions," WP 05/04, The Institute for Fiscal Studies.

Bond, S., and J. Van Reenen (2007), "Microeconometric models of investment and employment," Handbook of Econometrics 6, 4417-4498.

Caves, D., L. Christensen, and M. Tretheway (1981), "Flexible Cost functions for Multiproduct Firms," *Review of Economics and Statistics*, 62, 3, 477-481.

Christensen, L., D. Jorgenson, and L. Lau (1973), "Trascendental Logarithmic Production Frontiers," *Review of Economics and Statistics*, 55, 1, 28-45.

Demirer, M. (2025), "Production function Estimation with Factor-Augmenting Technology: An Application to Markups," MIT.

Diewert, W. E. (1971), "An Application of the Shepard Duality Theorem: A Generalized linear Production Function," *Journal of Political Economy,* 79, 3, 482-507.

Dobbelaere, S. and J. Mairesse (2013), "Panel Data Estimation of the Production Function and Production and Labor Market Imperfections," *Journal of Applied Econometrics,* 28, 1-46.

Dobbelaere, S. and J. Mairesse (2018), "Comparing micro-evidence on rent shring from two different econometric models," *Labour Economics*, 52, 18-26.

Doraszelski, U. and J. Jaumandreu (2013), "R&D and Productivity: Estimating Endogenous Productivity," *Review of Economic Studies,* 80, 1338-1383.

Doraszelski, U. and J. Jaumandreu (2018), "Measuring the Bias of Technological Change," *Journal of Political Economy,* 126, 1027-1084.

Doraszelski, U. and J. Jaumandreu (2019), "Using Cost Minimization to Estimate Markups," CEPR Discussion Paper 14114.

Doraszelski, U. and J. Jaumandreu (2021), "Reexamining the De Loecker and Warzynski (2012) Method for Estimating Markups," CEPR Dscussion Paper 16027.

Gandhi, A., S. Navarro and D. Rivers (2020), "On the Identification of Gross Output Production Functions," *Journal of Political Economy,* 128, 2973-3016.

Griliches, Z. and J. Mairesse (1998), "Production Functions: The Search for Identification," in *Econometrics and Economic Theory in th Twentieth Century: The Ragnar Frish Centennial Symposium,* S. Strom, ed. Cambridge University Press

Hausman, J. (1978), "Specification Tests in Econometrics," *Econometrica*, 46, 1251-1271.

Hsieh, C.T. and P. Klenow (2009), "Misallocation and manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403-1448.

Jaumandreu, J. and I. Mullens (2024), "Technology and US Manufacturing Employment (Lessons from the estimation of a microeconomic multiproductivity production function)," Boston University.

Kusaka, S., T. Okazaki, K. Onishi and N. Wakamori (2024), "New Technology Diffusion: Implications for Monopsony Power, Markups, and Labor

Share," Yale University.

Levinsohn, J and A. Petrin (2003), "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies,* 70, 317-341.

Manning, A. (2011), "Imperfect Competition in the Labor market," Handbook of Labor Economics 4b, 976-1021.

Marschak, J. and W.Andrews (1944), Random simultaneous equations and the theory of production," *Econometrica,* 143-205.

McElroy, M. B. (1987), "Additive General Error Models for Production, Cost, and Derived Demand or Share Equations," *Journal of Political Economy,* 95, 4, 737-757.

Olley, S. and A. Pakes (1996), "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64, 1263-1298.

Pan (2024), "Identification of Gross Output functions with a Nonseparable Productivity Shock," Shandong University.

Raval, D. (2019), "The Micro Elasticity of Substitution and Non-neutral Technology," *Rand Journal of Economics*, 50, 147-167.

Raval, D. (2023), "Testing the Production Approach to Markup Estimation," *Review of Economic Studies,* 90, 2592-2611.

Rubens, M. (2022), "Market Structure, Oligopsony Power, and Productivity," *American Economic Review,* 113, 2382-2410.

Trunschke, M. and K. Judd (2024), "Estimating Gross Output Production Functions," mimeo, Stanford University.

Vives, X. (1999), *Oligopoly Pricing: Old Ideas and New Tools*, Harvard University Press.

Wooldridge, J. (2009), "On Estimating Firm-Level Production Functions Using Proxy Variables to Control for Unobservables," *Economic Letters*, 104, 112-114.

Wooldridge, J. (2010), *Econometric Analysis of Cross Section and Panel Data,* MIT Press.

Yeh, C., C. Macaluso and B. Hershbein (2022), "Monopsony in the US Labor Market," *American Economic Review*, 112, 7, 2099-2138.

Zhang, H. (2019), "Non-neutral Technology, Firm Heterogeneity, and Labor Demand," *Journal od Development Economics*, 140, 145-168.

Zhao, S. , E. Malikov and S. Kumbhakar (2024), "On the estimation of Nonneutral Production Technologies without Separability," Binghamton University.

Table 1: Standard DP and OP/LP estimation

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas with neutral productivity: DP[a]	(ACF) OP/LP[b]
	(1)	(2)
ρ	1.022	1.018
	(0.002)	(0.004)
β_K	-0.023	0.061
	(0.010)	(0.020)
β_L	0.486	0.486
	(0.020)	(0.050)
β_M	0.622	0.532
	(0.014)	(0.024)
Overidentifying restrictions	2	1
Function value	8.043	0.701
No. of firms	5621	5621
No. of observations	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}$.

[b] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w - p, p_M - p)$. Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}$.

Table 2: (Ex-ante) testing for market power

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas with neutral productivity			
	Parametric OP/LP		ACF OP/LP	
	Cost-share[a]	Revenue-share[b]	Cost-share[c]	Revenue-share[d]
	(1)	(2)	(3)	(4)
ρ	1.054	0.964	0.977	1.018
	(0.008)	(0.002)	(0.030)	(0.056)
β_K	0.180	-0.679	0.097	0.061
	(0.025)	(0.137)	(0.101)	(0.020)
β_L	-1.176	-0.424	0.521	0.486
	(0.091)	(0.334)	(0.283)	(0.050)
β_M	2.057	4.337	0.323	0.532
	(0.079)	(0.173)	(0.075)	(0.024)
Market power test				
$\chi^2(df)$		363.581(3)		11.387(3)
$p-value$		0.000		0.010
Overidentifying restrictions	3	3	2	1
Function value	223.199	776.678	12.392	0.701
No. of firms	5621	5621	5621	5621
No. of observations	65006	65006	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, \ln S_{M,-1}$.

[b] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, vc_{-1} - r_{-1} + \ln S_{M,-1}$.

[c] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, \ln S_M)$.
Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}, \ln S_{M,-1}$.

[d] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w - p, p_M - p)$.
Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}$.

Table 3: Testing the specification

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Translog multiproductivity		
	DP[a]	Cost-share OP/LP[b]	Cost-share ACF OP/LP[c]
	(1)	(2)	(3)
ρ	1.012	1.012	0.966
	(0.002)	(0.004)	(0.040)
β_K	0.166	0157	0.103
	(0.041)	(0.023)	(0.112)
$\beta_L + \beta_M$	0.700	0.681	0.845
	(0.071)	(0.062)	(0.319)
α	0.089	0.038	0.260
	(0.016)	(0.006)	(0.193)
σ	0.578	0.753	0.367
Specification test			
$\chi^2(df)$		2.031(2)	0.558(2)
$p - value$		0.362	0.757
Overidentifying restrictions	5	4	5
Function value	44.663	113.647	5.783
No. of firms	5621	5621	5621
No. of observations	65006	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, l_{-1}, m_{-1}, k^2, l_{-1}^2, m_{-1}^2$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, S_{L,-1}$.
[b] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, S_{L,-1}, vc_{-1}$.
[c] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, \ln S_M)$.
Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $k^2, l_{-1}^2, m_{-1}^2, \widehat{\phi}_{-1}, S_{L,-1}$.

Table 4: (Ex-post) testing for market power

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Translog multiproductivity: Parametric OP/LP Revenue-share[a]	Translog multiproductivity: ACF OP/LP Revenue-share[b]
	(1)	(2)
ρ	0.969	0.981
	(0.002)	(0.001)
β_K	-1.114	-0.876
	(0.107)	(0.090)
$\beta_L + \beta_M$	4.634	3.626
	(0.293)	(0.241)
α	0.862	10.349
	(0.093)	(7.194)
σ	0.486	0.061
Specification test		
$\chi^2(df)$	234.659(2)	57.851(2)
$p - value$	0.000	0.000
Overidentifying restrictions	4	6
Function value	1892.609	963.820
No. of firms	5621	5621
No. of observations	65006	65006

[a] Instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}$, $w_{-1} - p_{-1}, p_{M,-1} - p_{-1}, S_{L,-1}, r_{-1} - vc_{-1}$.

[c] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w - p, p_M - p)$. Second stage instruments are (in addition of constant and time dummies): $k, k_{-1}, l_{-1}, m_{-1}, k^2, l^2_{-1}, m^2_{-1}$, $\widehat{\phi}_{-1}, S_{L,-1}, r_{-1} - vc_{-1}$.

Table AD: Complementary estimates

US manufacturing production function estimation,
firm-level sample, Compustat, 1960-2018.

Parameter	Cobb-Douglas OLS in differences	Translog multiproductivity Nonlinear nonparametric OP/LP[a]
	(1)	(5)
ρ		0.779
		(0.238)
ρ_2		0.070
		(0.075)
ρ_3		-0.006
		(0.007)
β_K	0.027	0.057
	(0.009)	(0.029)
β_L	0.339	0.506
	(0.003)	(0.079)
β_M	0.555	0.516
	(0.002)	(0.036)
Overidentifying restrictions		1
Function value		0.629
No. of firms	5621	5621
No. of observations	65006	65006

[a] First stage: Constant, time dummies, and nonparametric $\phi(k, l, m, w-p, p_M-p)$.
Second stage instruments are (in addition of constant and time dummies):
$k, k_{-1}, l_{-1}, m_{-1}, \widehat{\phi}_{-1}, \widehat{\phi}^2_{-1}, \widehat{\phi}^3_{-1}$.