

We have to estimate the set of parameters  $\theta = (\beta', \gamma')'$ , where  $\beta$  is  $k_1 \times 1$  and  $\gamma$  is  $k_2 \times 1$ . The code exploits the fact that the model can be written as

$$\nu_{jt} = y_{jt}(\beta) - w_{jt}(\beta)\gamma,$$

where  $y_{jt}$  and  $w_{jt}$  (an scalar and a  $1 \times k_2$  vector respectively) are “composite” variables which depend on the set of parameters  $\beta$ . The vector of parameters  $\gamma$  enters the equation linearly and “concentrating out” amounts to replacing them by its optimal expression given  $\beta$ .

To find the optimal expression notice that GMM minimizes

$$(\sum_j z'_j \nu_j(\theta))' A \sum_j z'_j \nu_j(\theta)$$

where  $z_j$  is the  $L \times T_j$  matrix of instruments for firm  $j$ ,  $\nu_j$  is  $T_j \times 1$ , and  $A$  is the current weighting matrix. FOCs of the problem can be written as

$$\left[ \sum_j \frac{\partial(z'_j \nu_j(\theta))}{\partial \beta}, \sum_j \frac{\partial(z'_j \nu_j(\theta))}{\partial \gamma} \right]' A \sum_j z'_j \nu_j(\theta) = 0$$

where the transpose matrix of derivatives is  $(k_1 + k_2) \times L$ . This is hence a system of  $(k_1 + k_2)$  equations. Using  $\nu_j(\theta) = y_j(\beta) - w_j(\beta)\gamma$  and noticing that  $\sum_j \frac{\partial(z'_j \nu_j(\theta))}{\partial \gamma} = -\sum_j z'_j w_j(\beta)$  the second block of  $k_2$  equations gives

$$\gamma = \left[ (\sum_j z'_j w_j(\beta))' A \sum_j z'_j w_j(\beta) \right]^{-1} (\sum_j z'_j w_j(\beta))' A \sum_j z'_j y_j(\beta).$$

The estimator of the asymptotic variance of the whole model can be written  $Avar(\hat{\theta}) = (\hat{G}' A \hat{G})^{-1} \hat{G}' A (\sum_j z'_j \hat{\nu}_j \hat{\nu}'_j z_j) A \hat{G} (\hat{G}' A \hat{G})^{-1}$ , which becomes  $Avar(\hat{\theta}) = (\hat{G}' (\sum_j z'_j \hat{\nu}_j \hat{\nu}'_j z_j)^{-1} \hat{G})^{-1}$  when  $A$  is chosen optimally. Both variances depend of the estimator  $\hat{G} = \sum_j \frac{\partial(z'_j \nu_j(\hat{\theta}))}{\partial \theta}$ . Using the fact that  $\frac{d(z'_j \nu_j)}{d\beta} = \frac{\partial(z'_j \nu_j)}{\partial \beta} - z'_j w_j \frac{\partial \gamma}{\partial \beta}$ , we compute this estimator as

$$\hat{G} = \left[ \sum_j \frac{d(z'_j \nu_j)}{d\beta} + (\sum_j z'_j w_j) \frac{\partial \gamma}{\partial \beta}, -\sum_j z'_j w_j \right].$$