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Q1 : zh

(a)

Q2 : let $z[n] = x[n] * y[n]$

DTFT : $Z(w) = \sum_{n=-\infty}^{\infty} z[n] \cdot e^{-jwn}$

$z[n] = x[n] * y[n]$, $Z(w) = \sum_{n=-\infty}^{\infty} (x[n] * y[n]) \cdot e^{-jwn}$

$x[n] * y[n] = \sum_{m=-\infty}^{\infty} x[m] y[n-m]$ $\quad \therefore Z(w) = \sum_{n=-\infty}^{\infty} \left(\sum_{m=-\infty}^{\infty} x[m] y[n-m] \right) e^{-jwn}$

From DTFT Properties. $x[n-n_0] \xleftrightarrow{DTFT} X(w) e^{-jwn_0}$

$\therefore Z(w) = \sum_{m=-\infty}^{\infty} x[m] \cdot Y(w) e^{-jwm}$

$= \sum_{m=-\infty}^{\infty} x[m] \cdot e^{-jwm} \cdot Y(w)$

$= X(w) \cdot Y(w)$

(b) $Z[w] = \sum_{n=-\infty}^{\infty} \left(x[-n] * y[n] \right) e^{-jwn}$

$= \sum_{n=-\infty}^{\infty} \left(\sum_{m=-\infty}^{\infty} x[m] y[n+m] \right) e^{-jwn}$

$= \sum_{m=-\infty}^{\infty} x[m] e^{jwm} Y(w)$

$= \sum_{m=-\infty}^{\infty} x[-m] e^{-jwm} Y(w)$

$= X(-w) Y(w)$

(c) if $x[n]$ is real

$X(w) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$, $X(-w) = \sum_{n=-\infty}^{\infty} x[n] e^{jwn}$

$Re\{X(w)\} = \sum_{n=-\infty}^{\infty} x[n] \cos(-wn)$

$Re\{X(-w)\} = \sum_{n=-\infty}^{\infty} x[n] \cos(wn)$

for the property of $\cos x$, $\cos(-wn) = \cos(wn)$

$\therefore$ the Real part of $X(w)$ is even.

(d) $Im\{X(w)\} = \sum_{n=-\infty}^{\infty} x[n] \cdot \sin(-wn) = -\sum_{n=-\infty}^{\infty} x[n] \sin(wn)$

$Im\{X(-w)\} = \sum_{n=-\infty}^{\infty} x[n] \cdot \sin(wn) = \sum_{n=-\infty}^{\infty} x[n] \sin(wn)$

$\therefore Im\{X(w)\} = -Im\{X(-w)\}$

$\therefore$ the imaginery part of $X(w)$ is odd.

(e) For $X^*(w)$, $Re\{X(w)\} = Re\{X(-w)\}$

$Im\{X(w)\} = -Im\{X(-w)\}$

$\therefore x[-n] \xrightarrow{DTFT} X^*(w)$

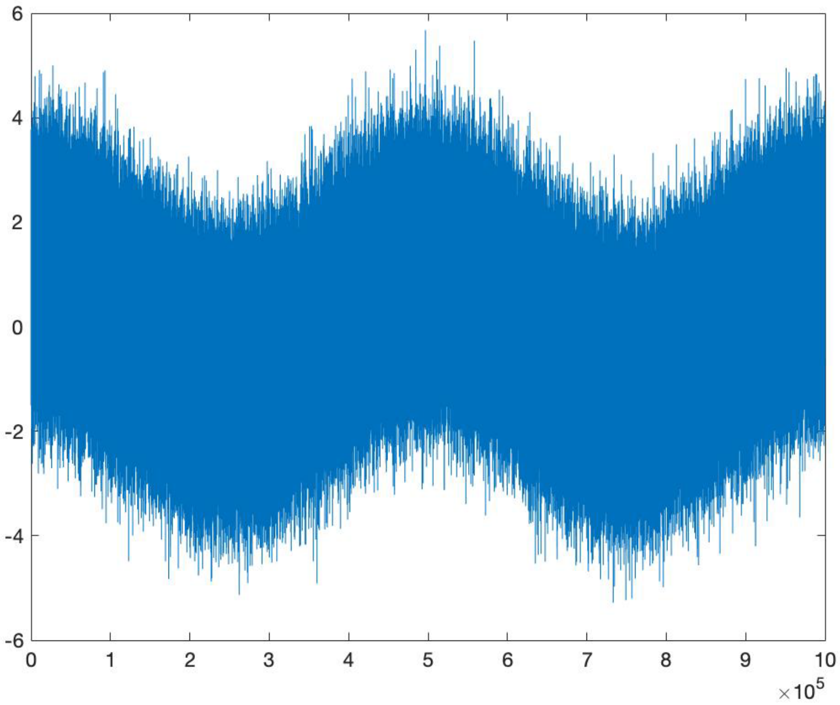
(f) $c[n] = x[-n] * y[n]$ $\qquad x[-n] \xrightarrow{DTFT} X^*(w)$

From DTFT properties, $x[n] * y[n] \xrightarrow{DTFT} X(w) Y(w)$

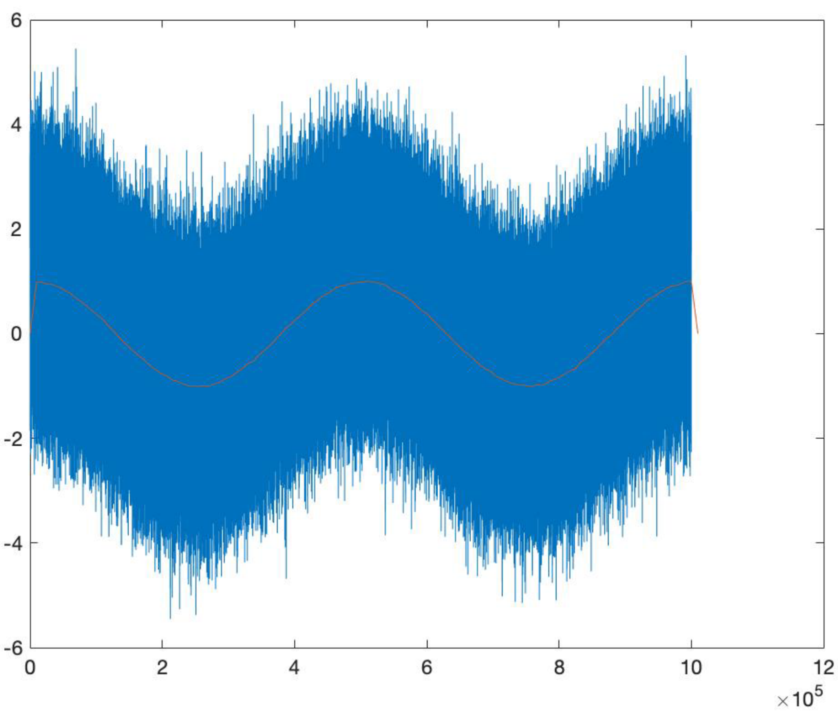
so the discrete-time Fourier transform of $c[n]$ is $X^*(w) Y(w)$

Q3

(a)

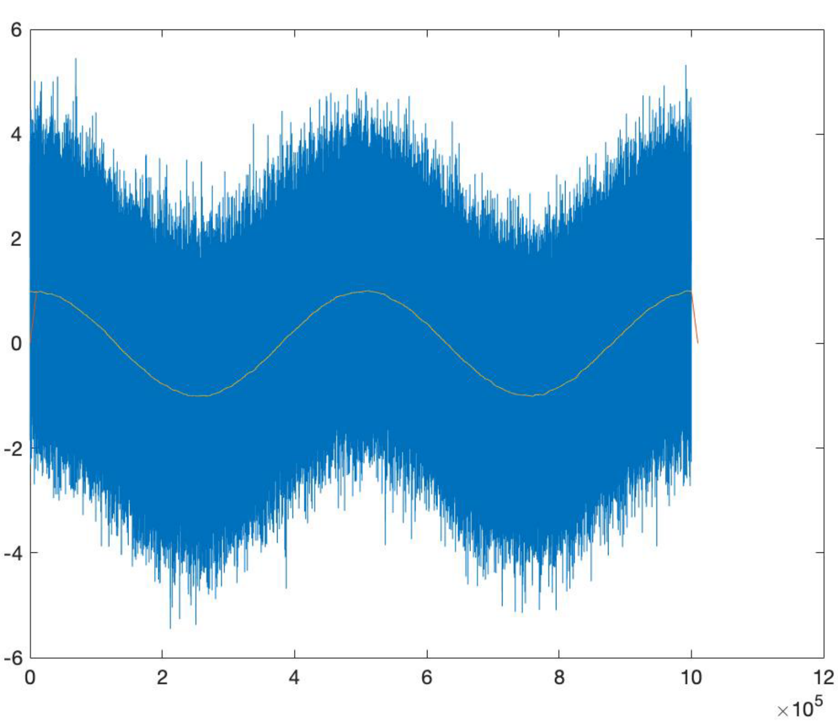


(b)



(c) when $M=1$, $X=Y1$

(d)



1e)

	Q3(b)	Q3(d)
①	0.867270	0.167519
②	0.594508	0.106424
③	0.532514	0.122293
④	0.558729	0.107008
⑤	0.531113	0.112281
⑥	0.587500	0.113346
⑦	0.566132	0.106018
⑧	0.567361	0.117288
⑨	0.522163	0.116854
⑩	0.689767	0.105543

Average:

	0.6017057 (s)	0.1174574 (s)

Q4.

UFID: 64118211

The name of the matching file: rudenko_23.mp4

Time: 1 minutes, 24.8361 seconds