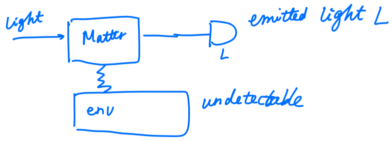
Quatm sensing for estimating parameters of Quantum matter   Datta 2025



- $\rho_L = \mathrm{Tr}_{ML}\left[\rho_{LME}\right]$ is the state we can measure and manipulate (M and E are inaccessible)

  The Hamiltonians:

  $$H^L = \int_0^\infty d\omega \; a^\dagger(\omega) a(\omega)\, \omega$$

  $$H^{ML}_{(t)} = H^M(t) + i\left(a^\dagger(t) J - a(t) J^\dagger\right) \quad , \quad a(t) = \frac{1}{\sqrt{2\pi}} \int d\omega \; a(\omega) e^{-i(\omega-\omega_0)t}$$

  - In Markovian noise, regardless of microscopic origin of Lindblad on M, purify dynamics by introducing $b_j(t)$, for each $L_j \in \mathcal{H}^E$

    $$H^{MLE}(t) = H^M(t) + i\left(J a^\dagger(t) + \sum_{j=1}^{p} L_j b_j^\dagger(t) - h.c\right) \quad , \; L_j \propto J$$

- parameter estimation:

  e.g. Two examples:

  ① $H_M[\theta] = \Omega(\sigma_+ + \sigma_-)$.
  $J = \sqrt{\eta\gamma}\,\sigma_-$; $\hat{L} = \sqrt{(1-\eta)\gamma}\,\sigma_-$

  ② $H_M(\theta) = F\alpha(t)(\sigma_+ - \sigma_-)$
  $J_\Gamma = \sqrt{\Gamma}\sigma_-$, $L = \sqrt{\Gamma_L}\sigma_-$ ($\because L \propto J_\Gamma$)

- Methods: Datta 2025

  QFI: the limits of parameter estimation precision

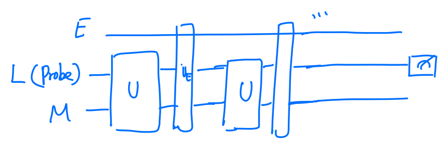
  $$\mathrm{QFI}[\rho_\theta] = \sup_X \; 2\,\mathrm{Tr}[\dot{\rho}_\theta X] - \mathrm{Tr}[\rho_\theta X^2] \quad . \quad \rho_\theta \equiv \rho_\theta^L(t_{final})$$

- Upper bound of QFI: Two-sided master equation $\rho_\theta^{LE} = \mathrm{Tr}_M[\,|\psi_\theta^{MLE}\rangle\langle\psi_\theta^{MLE}|\,]$

- Limitation: Only few-body parameters can be estimated ?

- <u>What we can do ?</u>

  infer parameters in Matter with circuit below



$\boxed{U}$: Matter and probe dynamics

$\boxed{\psi_E}$: another coupling with env.
can represent noise process / something undetectable

1. get estimation precision by measuring QFI . Measured by $\mathrm{QFI} \sim \chi'' \sim \int_0^\infty \langle S(t) S(0)\rangle e^{i\omega t} dt$
2. Use Maximum likelihood method to infer the parameters $H_M(\theta)$

A few subtlety :

1. Dynamics time t(final) ? Is it necessary to tune it ? How does it affect ?

2. If we just do tomography on the final L state ; Can we estimate M's info ?

   What is the size of "L", minimal size ?

   Should it be a bosonic field, like a coherent state in Dalla's set up

3. Is the microscopic details of the interactions like H_{ML}, H_{MLE} important (relevant) ?

   I hope NOT !

   Do we need to really know H_{MLE}, or the state ρ_{MLE} as a prerequiste for Hamiltonian learning?