General correlator Probe — with pulse sequence

From above discussion we know

$$\langle A(t)\rangle = \sum_{n=0}^{\infty} \langle A(t)\rangle\big|_n \equiv \sum_{n=0}^{\infty} (-i)^n \int_0^t d_> t_{[n]}\ \mathrm{tr}\left\{ \rho_0 \left[A_I(t), \left[V_I(t_1), \cdots, \left[V_I(t_{n-1}), V_I(t_n)\right]\right]\right]\right\}$$

where $\langle A(t)\rangle\big|_0 \equiv \mathrm{tr}\left\{\rho_0 A_I(t)\right\}$

def $\Delta\langle A(t)\rangle \equiv \langle A(t)\rangle - \langle A(t)\rangle\big|_0$ we get responded dynamics as correlator

$$= \sum_{n=1}^{\infty} (-i)^n \int_0^t d_> t_{[n]} \left\langle \left[A_I(t), \left[V_I(t_1), \cdots, \left[V_I(t_{n-1}), V_I(t_n)\right]\right]\cdots\right]\right\rangle$$

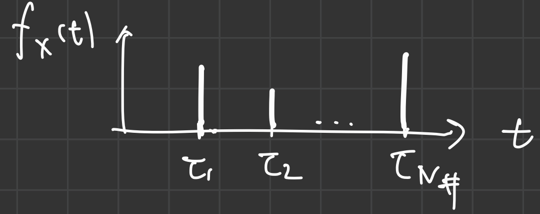
Say the driving term is $V(t) = \sum_\mu f_\mu(t)\, \hat{B}^\mu(t)$

where μ is some "mode" index, say $\mu = \{x, y, z\}$ $f_\mu(t)$ is amplitude function and $\hat{B}^\mu(t)$ is time-dependent driving operator

For example, apply continuous mag field along z-direction on spin system

is $V(t) = \cos(t) \cdot \omega \sum_i \sigma_z^{(i)}$

What I focus on, and promote, is pulse sequence at discrete times say along "x" direction gives
$$V(t) = \sum_{j=1}^{N_\#} f_x(t-\tau_j) \sum_i O_z^{(i)}$$



In such a case $\hat{B}^x(t) \equiv \sum_i O_z^{(i)}$ a collective total z-magnetization, But it does not have to be so. for example pulses at different discrete time can be magnetization along different direction; or some times the operator is Entangling (ZZ type) while others are local as $\sum_i O_z^{(i)}$

We plug the general expr of driving into the expansion-series, seperating the "modulation" (f) part and "operator" part $(B(t))$,

This gives rise to the following expr.

$$\Delta\langle A(t)\rangle = \sum_{n=1}^{\infty}(-i)^n \sum_{\mu_1\cdots\mu_n}\int_0^t d_> t_{[n]} \underbrace{\prod_{j=1}^{n} f_{\mu_j}(t_j)}_{\text{filter}} \Big\langle \underbrace{[A_I(t),[B_I^{\mu_1}(t_1),\cdots[B_I^{\mu_{n-1}}(t_{n-1}),B_I^{\mu_n}(t_n)]]]}_{\text{Green's function}} \Big\rangle$$

$$= \sum_{n=1}^{\infty}(-i)^n \sum_{\mu}\int_0^t d\, t_{[n]} \prod_{j=1}^{n} \underbrace{f^{(>)}_{\mu_j}(t_j)}_{\text{time-order}} \underbrace{\chi^{(\mu_1\cdots\mu_n)}_{(n)}(t;\, t_1>\cdots>t_n)}$$

& the order

Notice $t>t_1>t_2>\cdots t_n$ due to the casuality.

If $|\mu|\equiv 1$ for whole pulse sequene (for exemple, $\mu\equiv z$ as in one-direction magnetization case), then μ can be dropped. We will focus on this case for simplility

Question: How to probe Green's function $\chi_{(n)}(t;\, t_1>\cdots>t_n)$ for a given $(t;\, t_1>\cdots>t_n)$? And to seperate it from lower order function $\chi_{(n'<n)}(t;\, t_1>\cdots>t_{n'})$?

Answer: Set the pulse timing to be resonant with $(t_1 \cdots t_n)$

That is

$$\chi_{(n)}(t_n < t_{n-1} < t_1 \,;\, t)$$

So to probe $\chi_{(n)}(t\,;\,\vec{t}_{[n]})$ it's necessary & sufficient to set pulse-sequence at interested time $\vec{t}_{[n]}$.

Under such sequence $\chi_{(n)}(t;\,\vec{t}_{[n]})$ contribute to $\Delta\langle A(t)\rangle$ other "times" of $\chi_{(n)}$ And any $\chi_{(n'>n)}$ will not ∵ Their corresponding filter $\equiv 0$.

But some lower order $\chi_{(n'<n)}$ will contribute

What are they ? For target $\chi_{(n)}(t;\ t_1 > \cdots > t_n)$

One need to seperate $\chi_{(n'<n)}(t;\ \tilde{t}_1 > \cdots > \tilde{t}_{n'})$ where

$\{\tilde{t}_1, \ldots \tilde{t}_{n'}\} \subsetneq \{t_1, t_2, \ldots t_n\}$

Thus the General pulsed spectroscopy is

use few pulses (start from one) to estimate $\chi_{(1)}$ first.

And adding more pulses to probe higher terms.

To probe "K"-th order, One need K-pulse sequence.

One should vary $\{t_1 \ldots t_n\}$ for fixed "n" to get different

temporal points @ $\chi_{(n)}$ to get more fine information.

Ner to think: $f(t-\tau) \Longrightarrow f(t-\tau-\underbrace{\delta_t}_{\text{timing error}}) \cdot (1+\underbrace{\delta_f}_{\text{Amplitude error}})$

control-error mitigation