Problem 2

For the ferry boat problem, find an optimal angle β with which the boat travels on the shortest path between the two banks. We assume a=8miles, the flow speed v_0=20mph and the boat's speed v_B=30mph.

Python 3.9 was used for this program. I additionally use several Python libraries. This program can be run by running Test 3.py. I used Visual Studio Code to code and run.

Algorithm Description

I first generated a temporary beta which is the E[X] +- a random number between [-0.1, 0.1].

Then, solve the differential equation using forward euler:
y[i+1] = y[i] + h * ((2/3) * (1 - ((x[i]**2) / (8**2))) / np.cos(bTemp) + np.sin(bTemp) / np.cos(bTemp))

$$\begin{cases} \frac{dy}{dx} = \frac{v_0}{v_B}\left(1 - \frac{x^2}{a^2}\right)\frac{1}{\cos\beta} + \frac{\sin\beta}{\cos\beta} \\ y(x = -a) = 0 \end{cases}$$

I calculate the distance of this path: distTemp = distTemp + np.sqrt(h**2 + (y[i+1] - y[i])**2)

I use an initial T value of 0.04, and then I do fast annealing: T = 0.04 / for loop idx

Then, I use the metropolis method to see if I will accept this new step.

$$P(Y)=\min\{1,\exp[-E(Y)-E(X)/T]\}$$

If the new state is better than the original, than I accept this state.

If not, I take it with the probability in the formula. I generate a random number from [0,1]. If this generated number is less than the probability, than I accept.

Results

Beta is around -.43 radians, with the optimal distance being around 16.38.

Performance

There are no issues with performance.