Electronics and
Computer Science

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Access Structures

COMP3211 Advanced Databases

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Overview

- Index basics
  - Sequential files
  - Dense indexes
  - Sparse indexes
  - Multi-level indexes
  - Secondary indexes
  - Indirection
- B+trees
- Hash tables

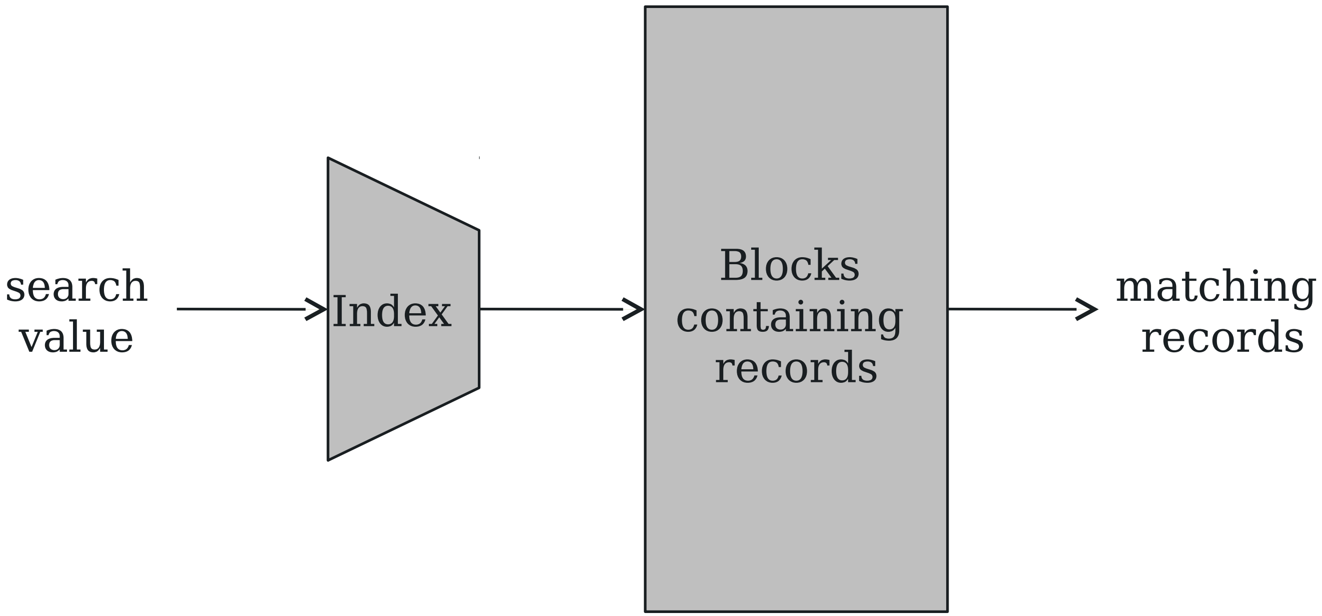
Index Basics

Index basics

- Relations are stored in files
- Files are stored as collections of blocks
- Blocks contain records that correspond to tuples in the relation

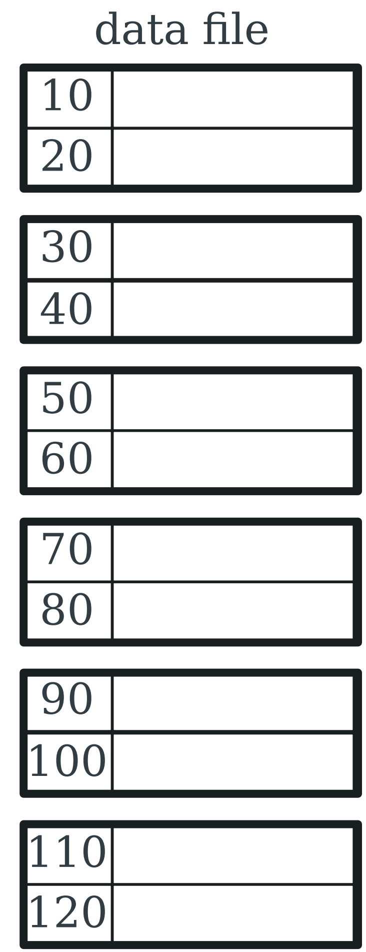
- How do we find the tuples that match some criteria?

Indexes


search value
Index
Blocks containing records
matching records

Sequential Files

- Tuples of a relation are sorted by their primary key
- Tuples are then distributed among blocks in that order
- Common to leave free space in each block to allow for later insertions

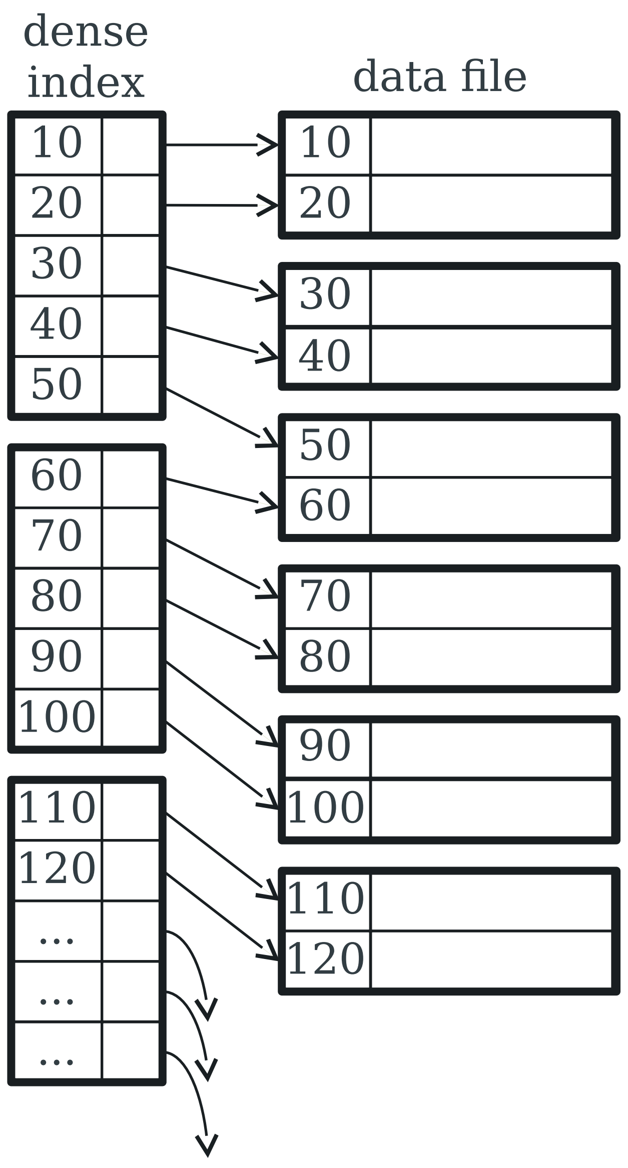


To Index or Not To Index?

- Maintaining an index costs processor time
  - When entries are added, index must be updated
  - Index must be maintained to make good use of resources
- There is a trade off between:
  - Rapid access when retrieving data
  - Speed of updating the database

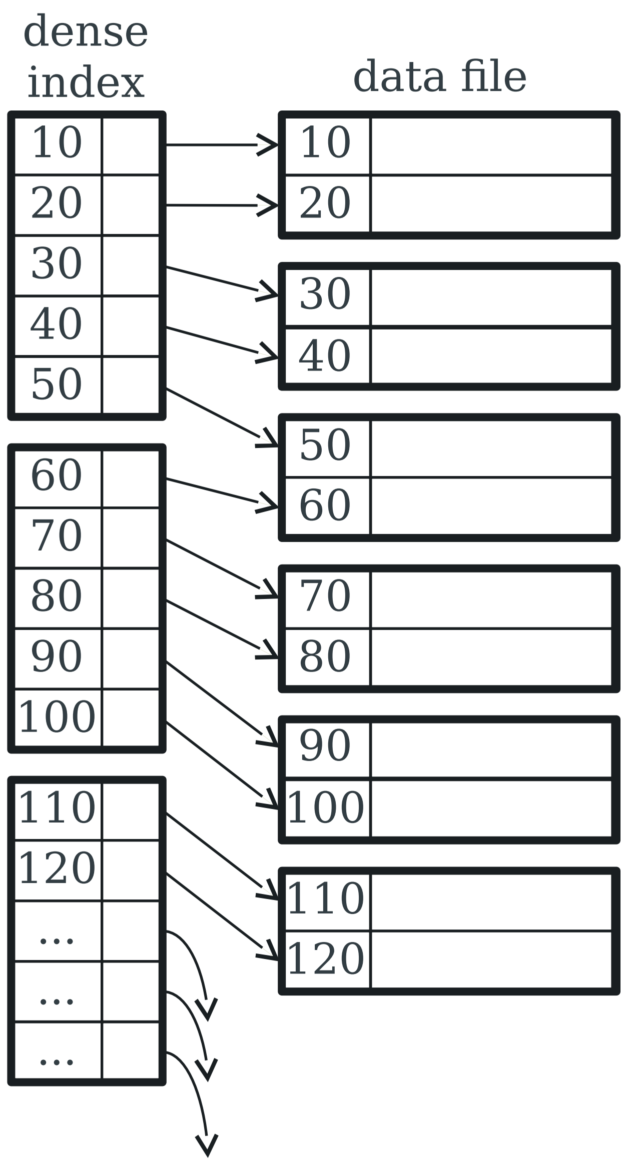
Dense Index

- Sequence of blocks holding only keys and pointers to records
- Entry for every record in data file
- Blocks of index are in same order as those of the data file
- Key-pointer pair much smaller than record



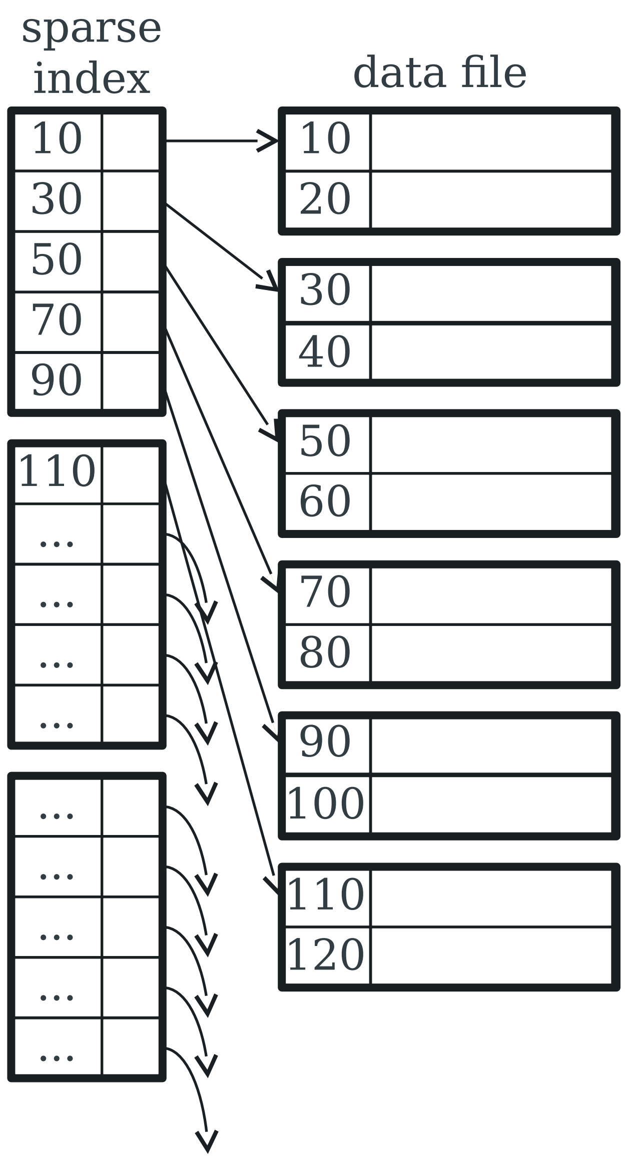
Dense Index

- Fewer blocks than data file, fewer disk accesses
- Keys are sorted, so can use binary search
- Can keep in main memory if small enough (no disk accesses)



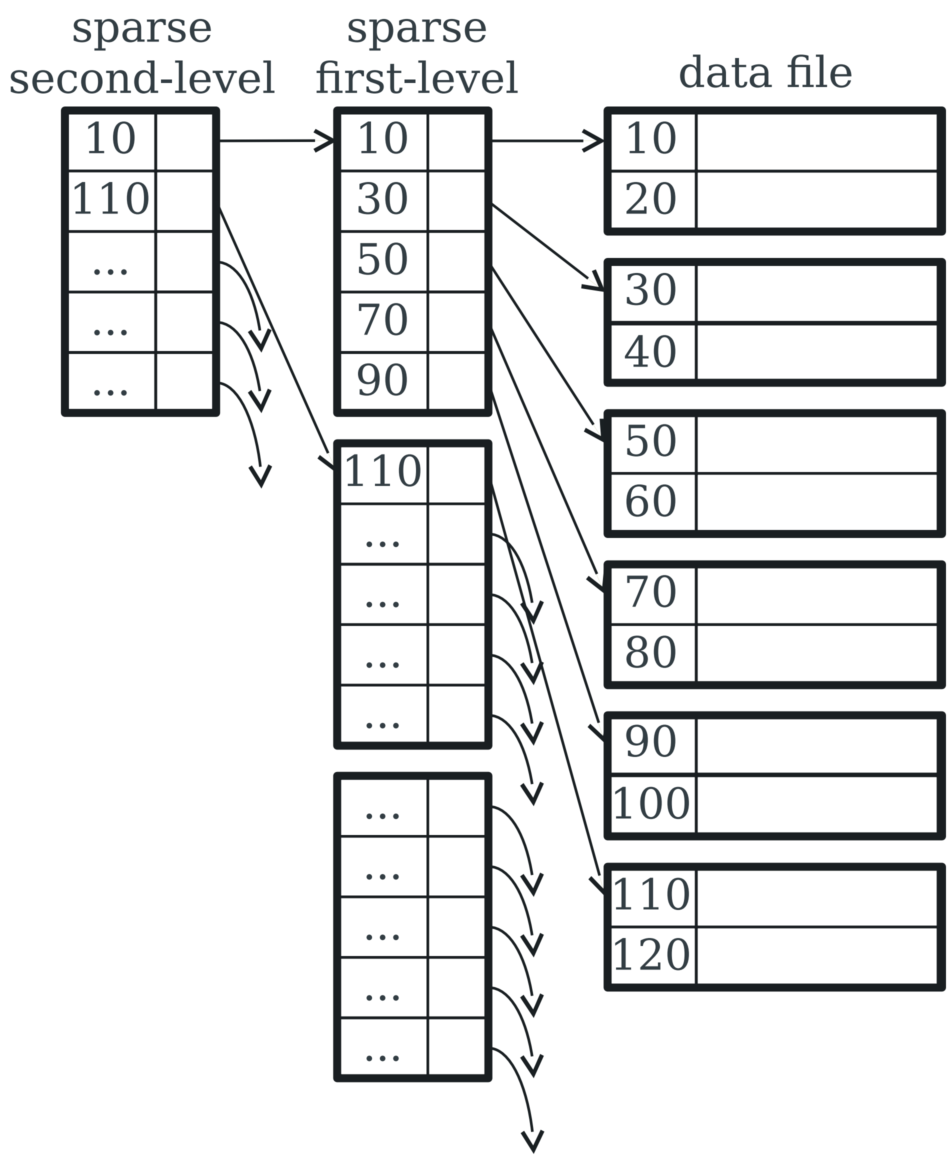
Sparse Index

- One key-pointer pair per block of data file
- Can only be used if data file is sorted by search key
- Uses less space than dense index
- Takes longer to find key than dense index



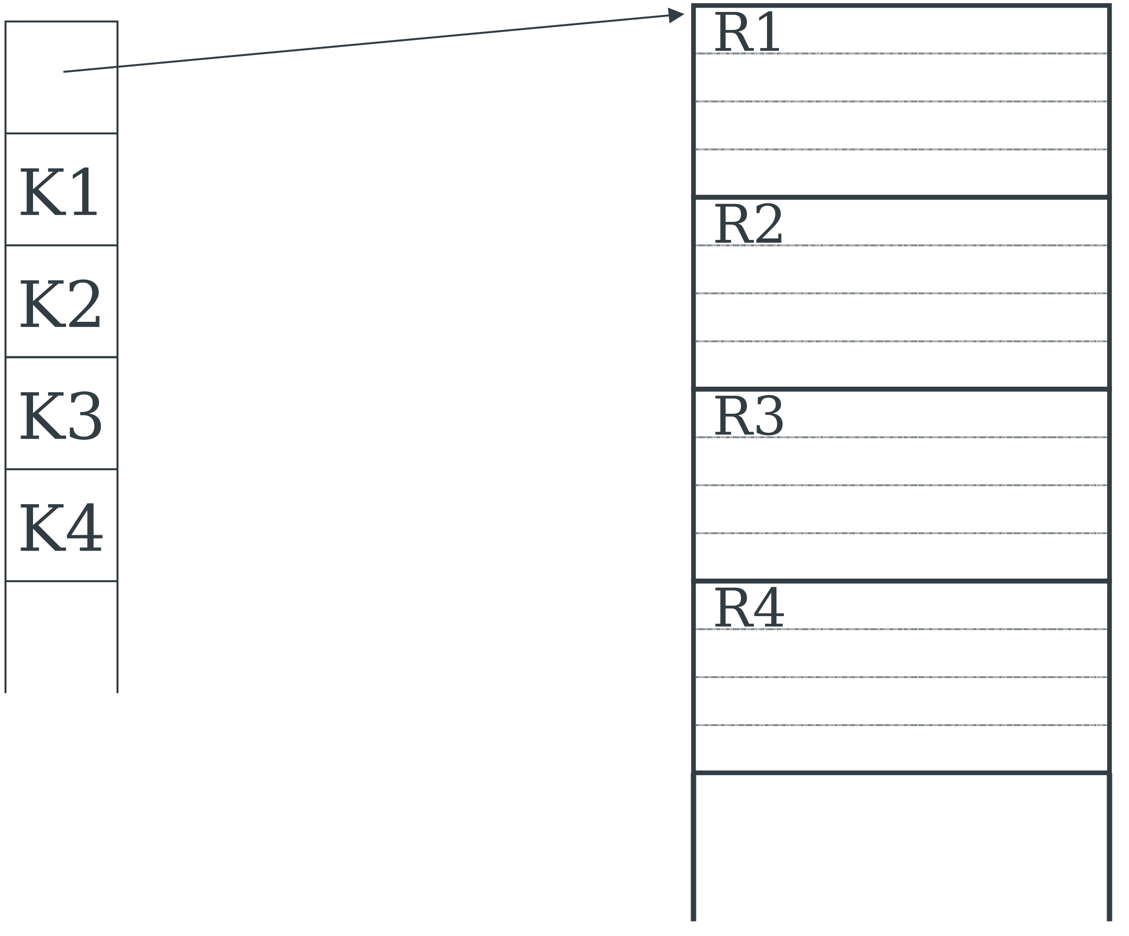
Multi-level Index

- Index file may cover many blocks
- May still need many disk accesses
- Use sparse index over the first index
  - Can't be a dense index (would use the same number of blocks as the index being indexed)
- Can create a third level index, but in general prefer B-trees

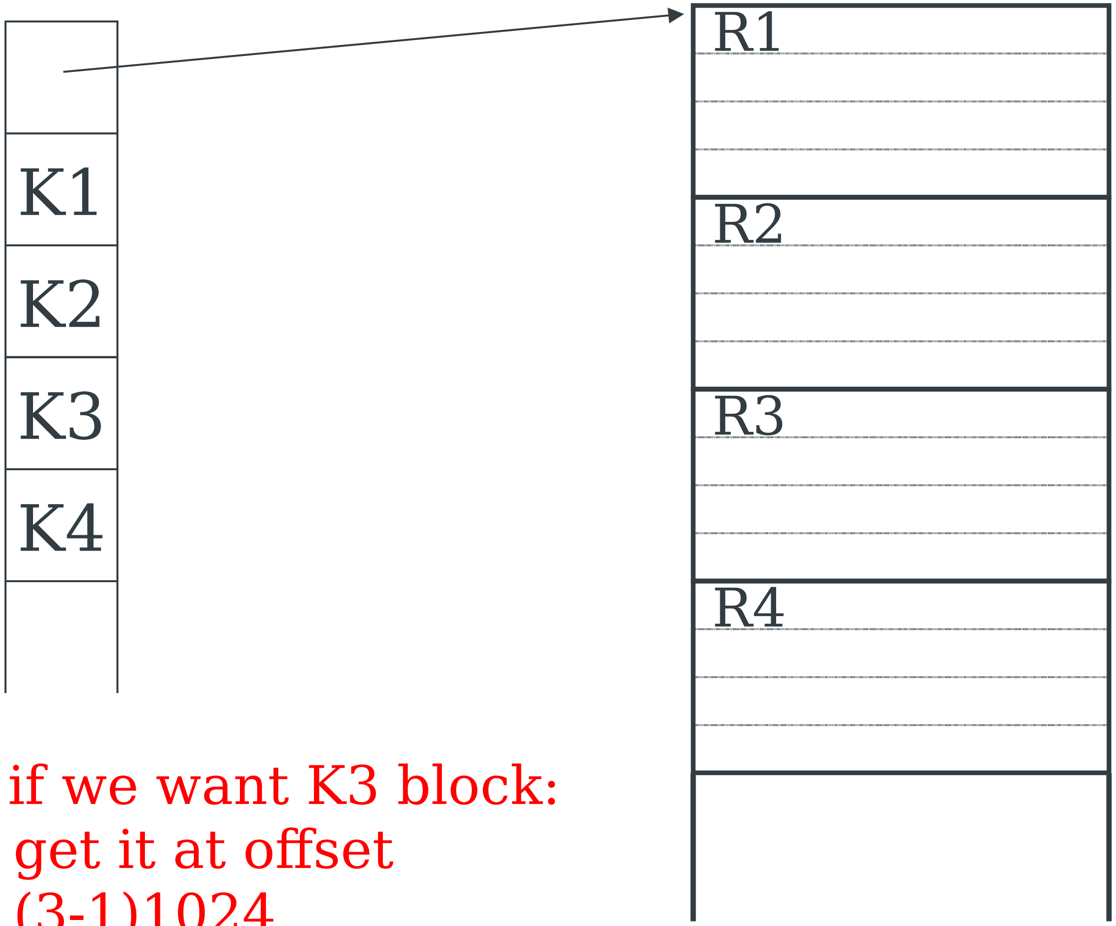


Notes on pointers:

- Block pointers (as used in sparse indexes) can be smaller than record pointers (used in dense indexes)
  - Physical record pointers consist of a block pointer and an offset
- If file is contiguous, then we can omit pointers
  - Compute offset from block size and key position



K1
K2
K3
K4
R1
R2
R3
R4



say:
1024 B
per block

- if we want K3 block:
  get it at offset
  (3-1)1024
  = 2048 bytes

Sparse vs. Dense Tradeoff

Sparse:

- Less index space per record can keep more of index in memory
- Better for insertions

Dense:

- Can tell if a record exists without accessing file
- Needed for secondary indexes

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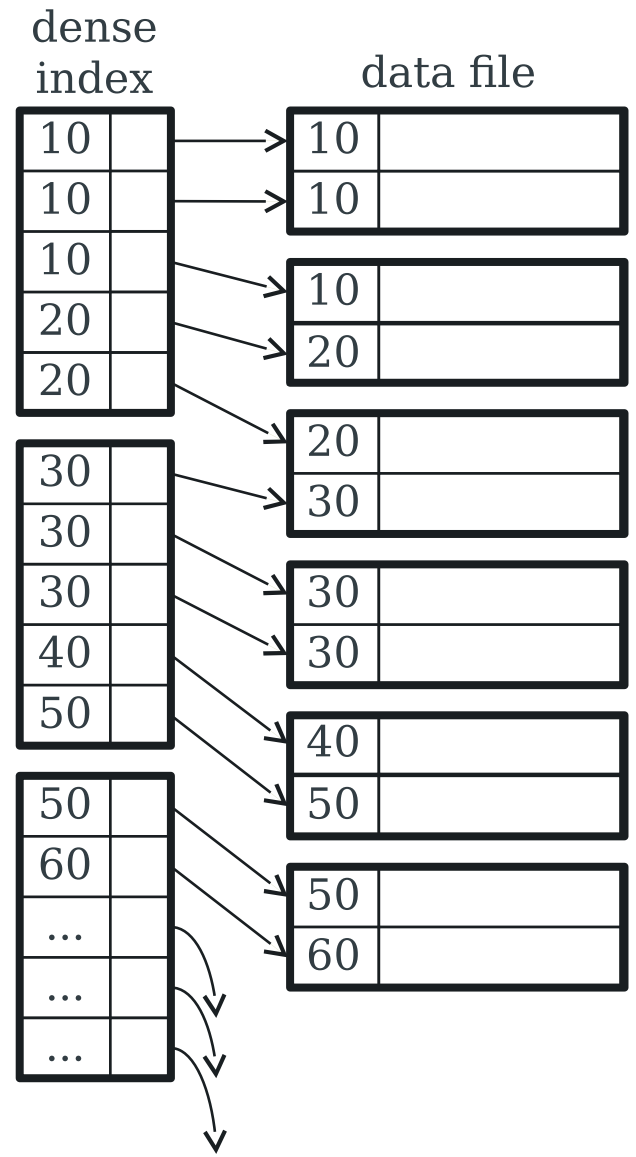
Duplicate Keys

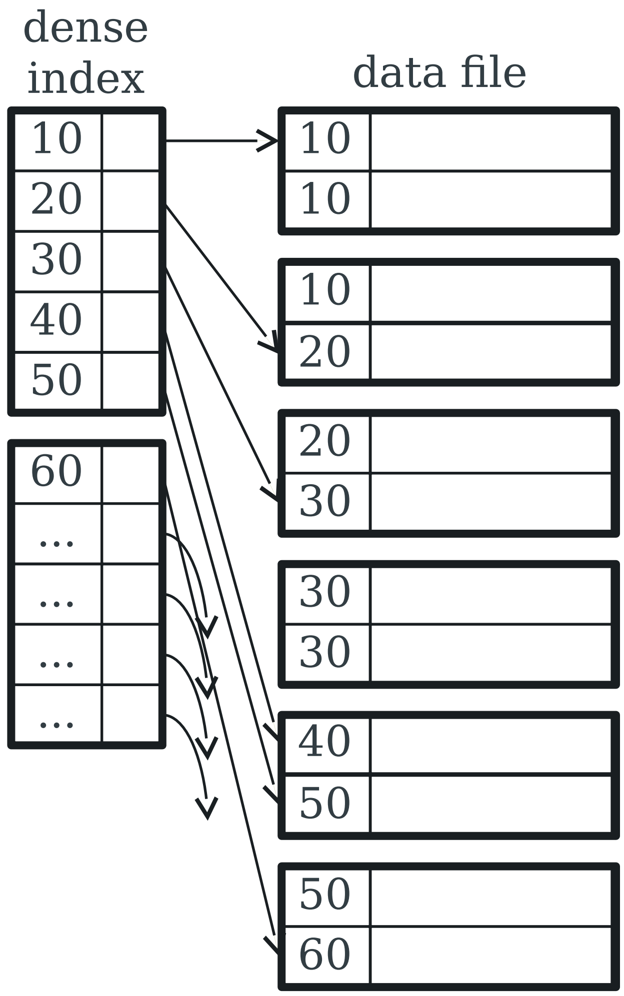
Dense index approach #1



Duplicate Keys

Dense index approach #2

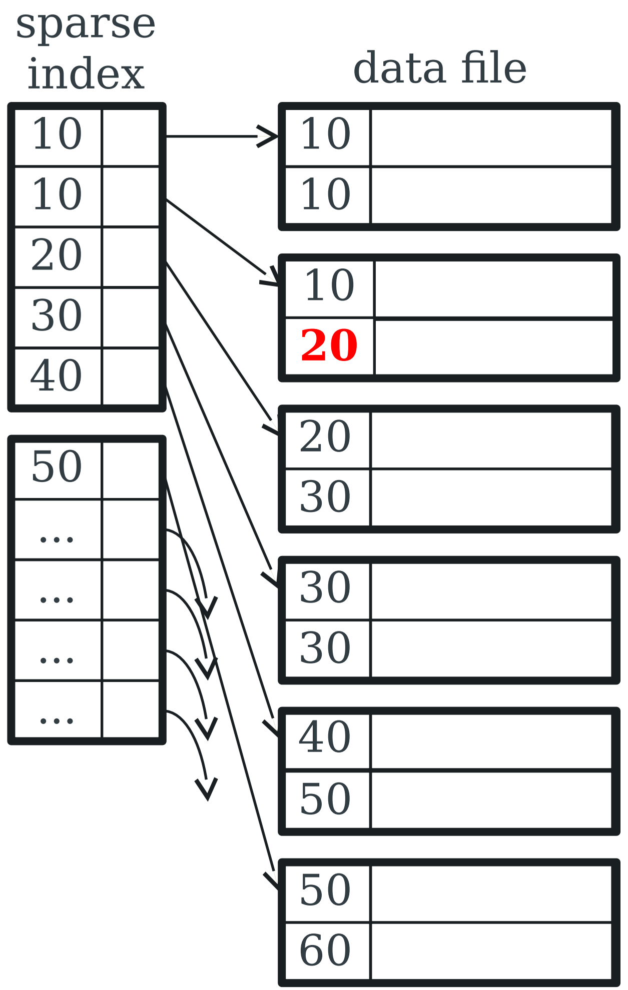
- better approach? (smaller index)



Duplicate Keys

Sparse index approach #1

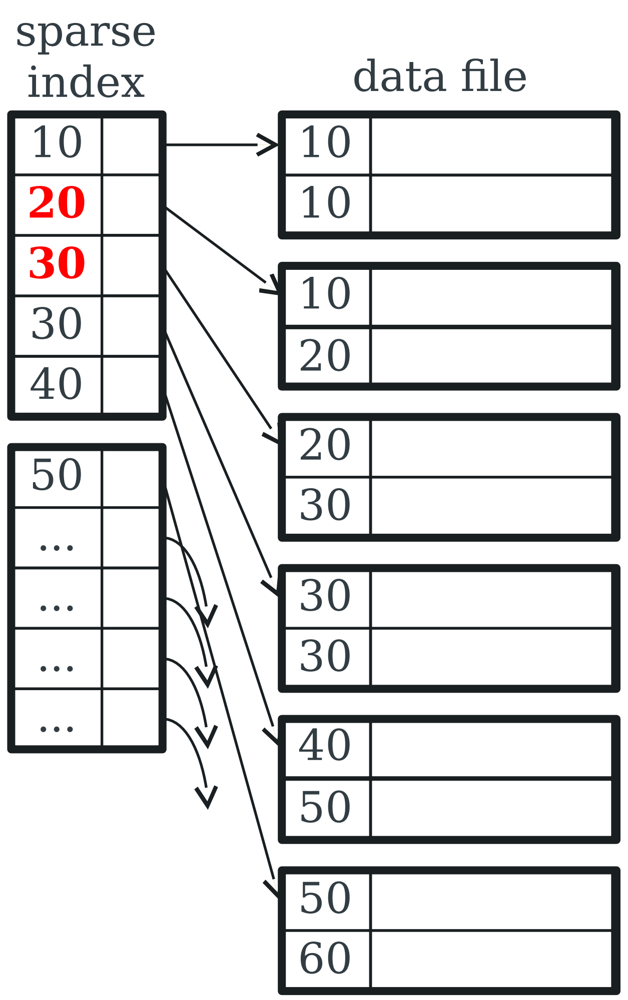
- Searching for (e.g.) 20 will give unexpected results



Duplicate Keys

Sparse index approach #2

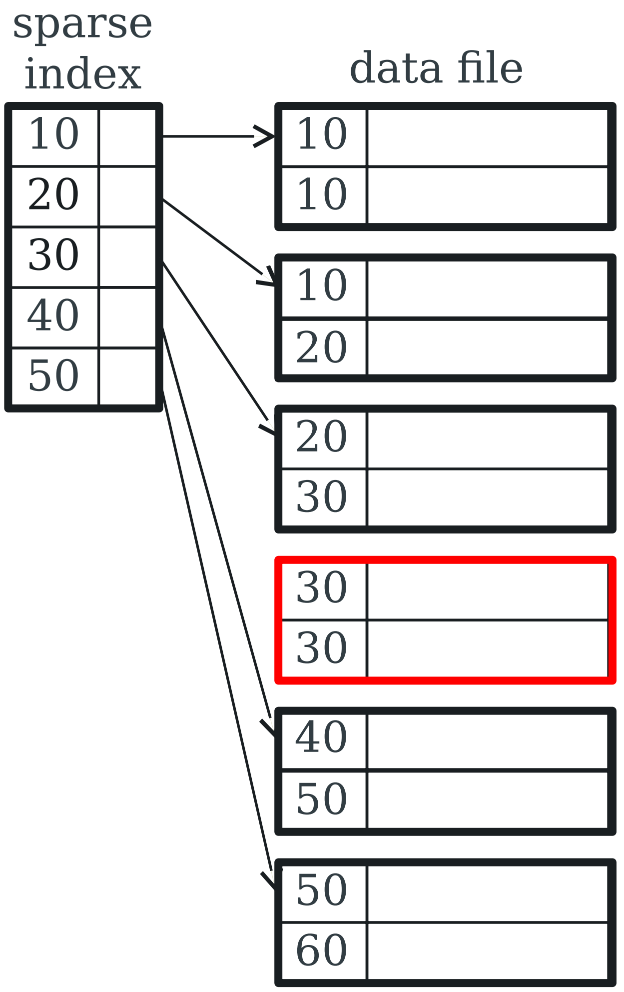
- Index contains first *new* key from each block



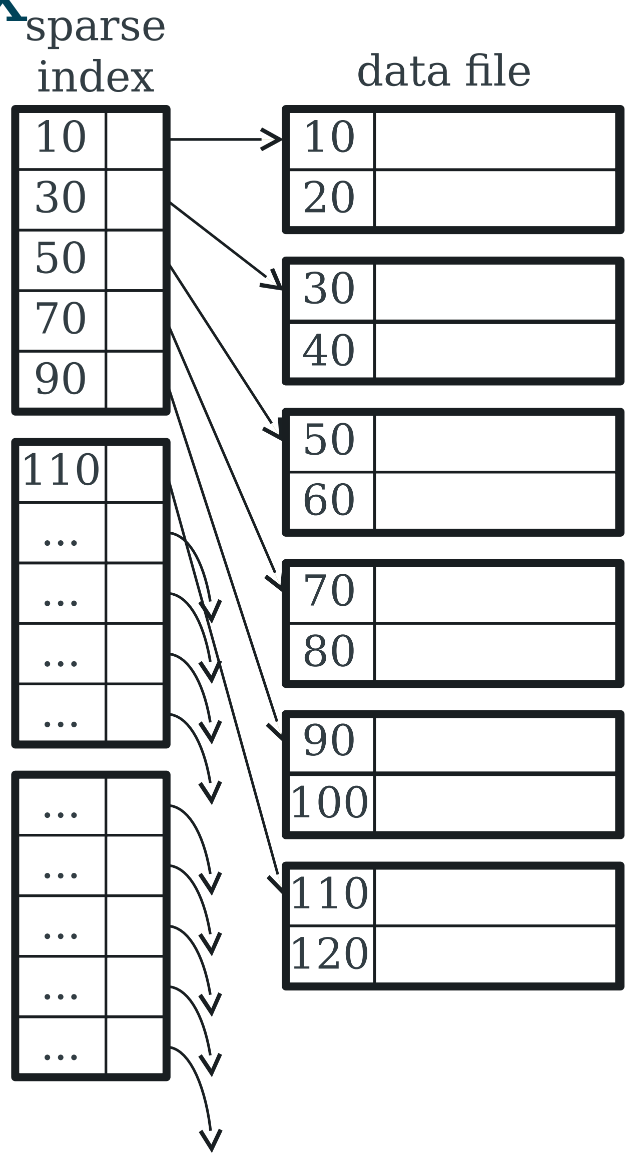
Duplicate Keys

Sparse index approach #2

- Can we exclude sequences of blocks with repeated keys?
- Point only to *first* instance of each value

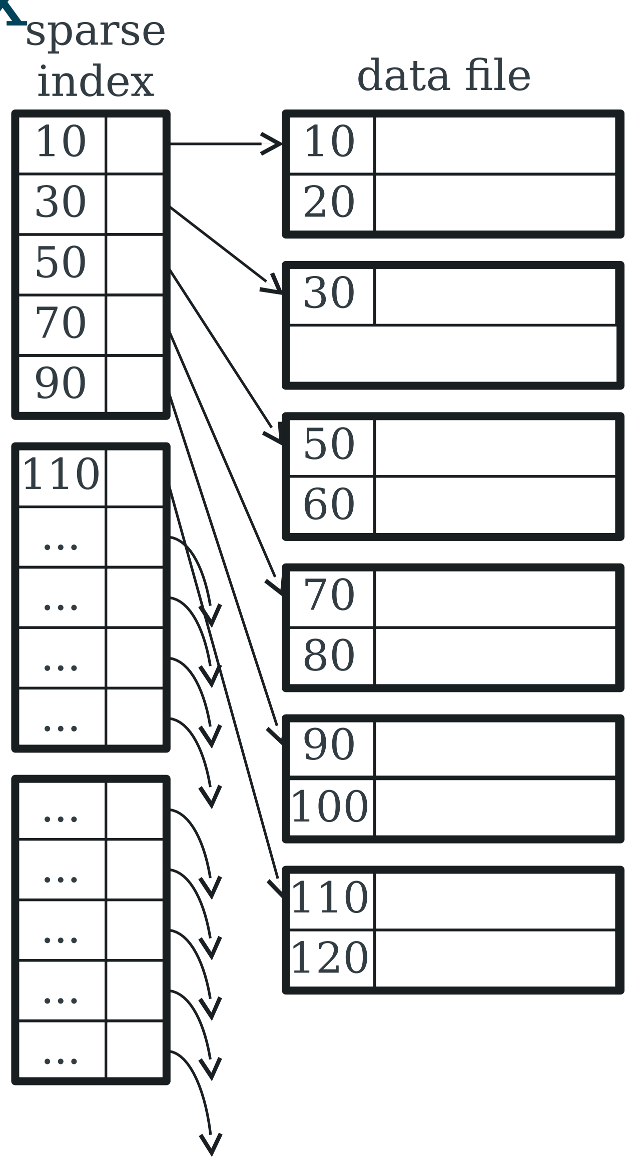


Deletion from Sparse Index



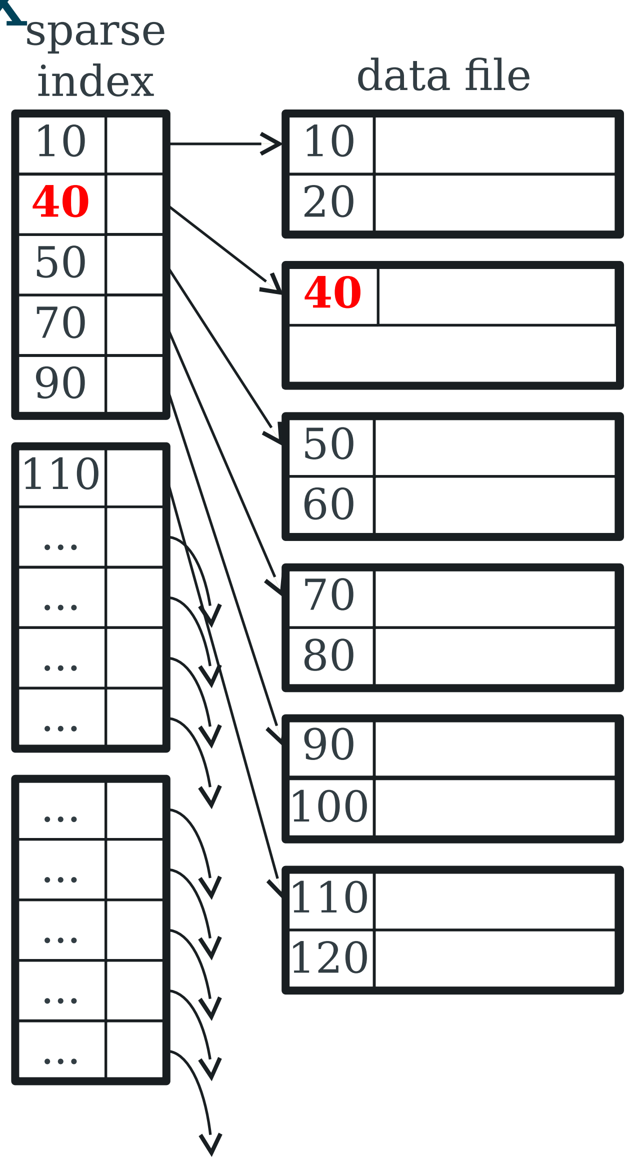
Deletion from Sparse Index

- Delete record 40



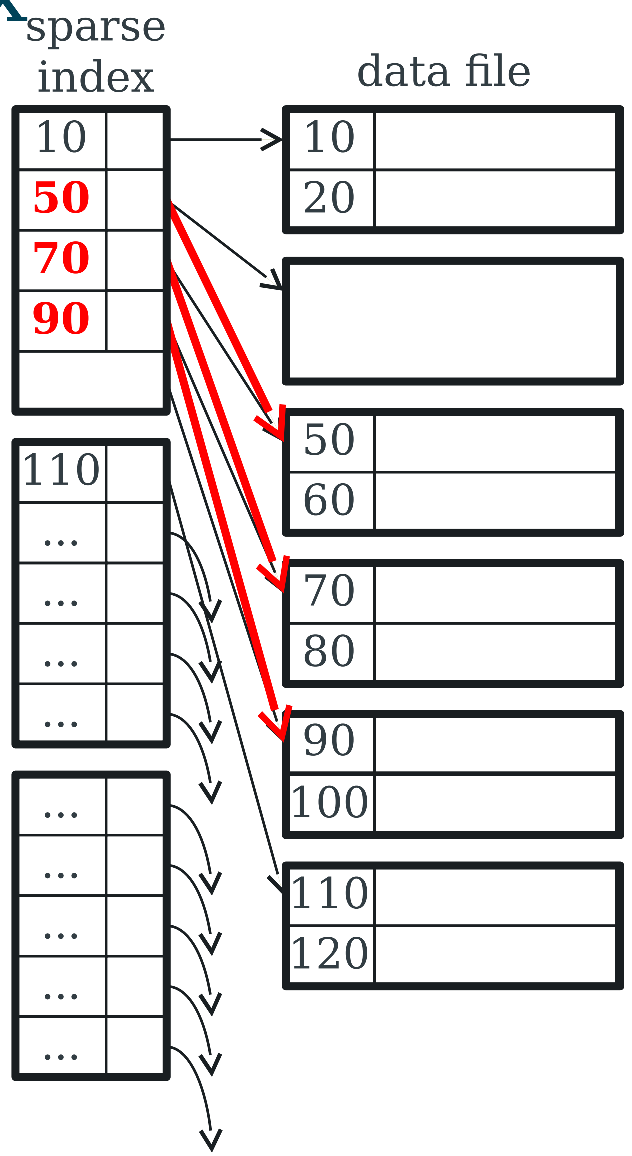
Deletion from Sparse Index

- Delete record 30
  - Delete record 30 from data file and reorder block
  - Update entry in index



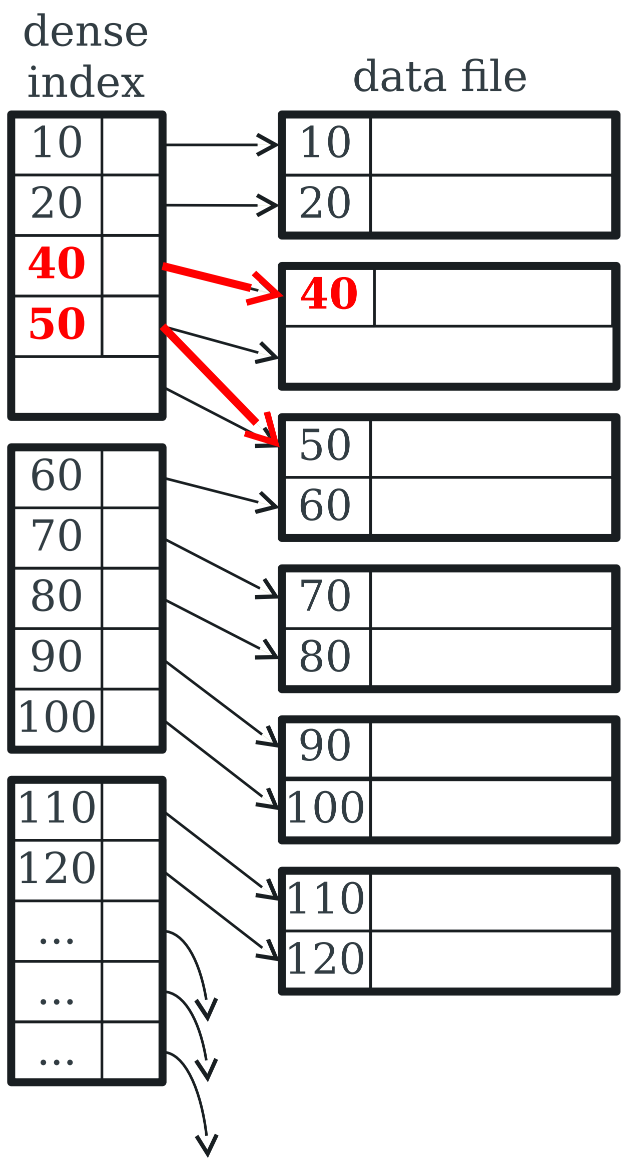
Deletion from Sparse Index

- Delete records 30 and 40
  - Delete records from data file
  - Update index

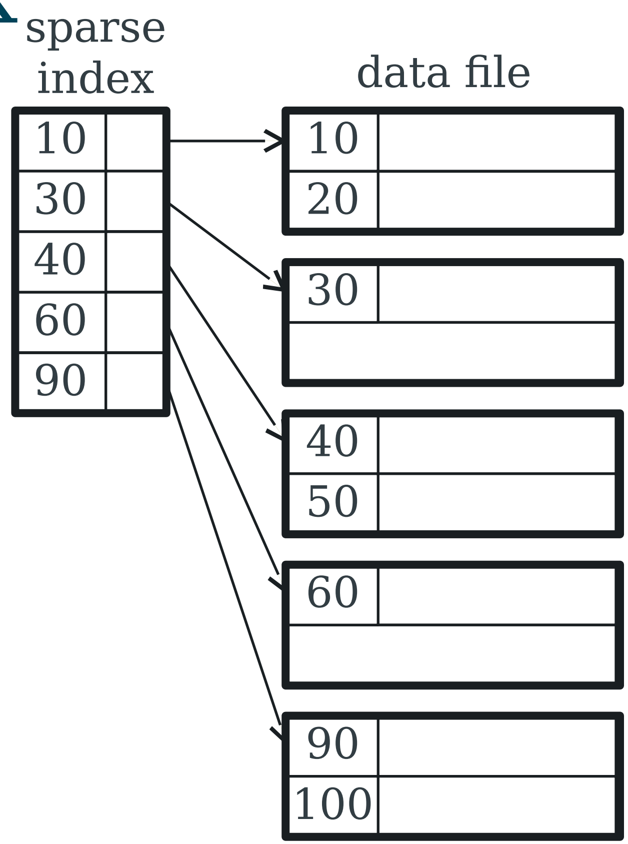


Deletion from Dense Index

- Delete record 30
  - Delete record from data file
  - Remove entry from index and update index

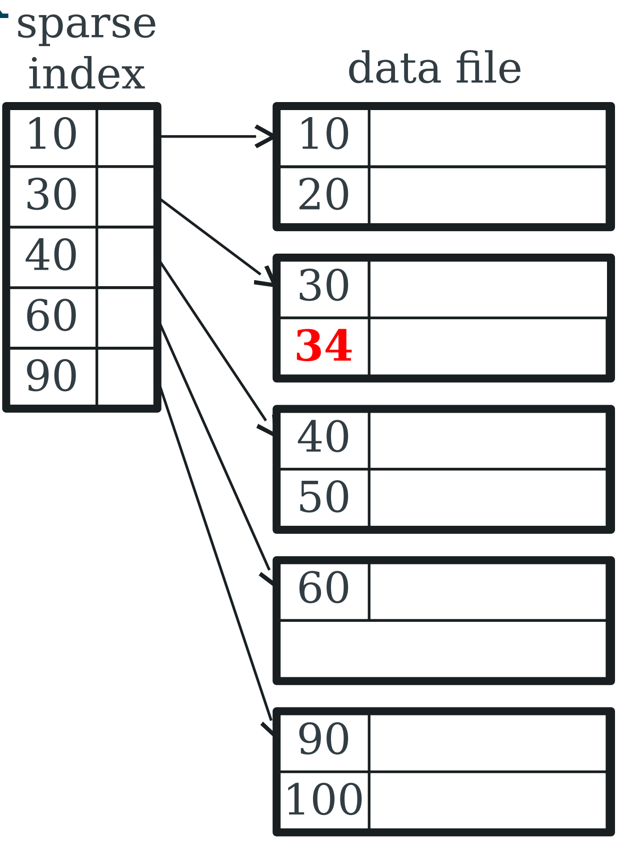


Insertion into Sparse Index



Insertion into Sparse Index

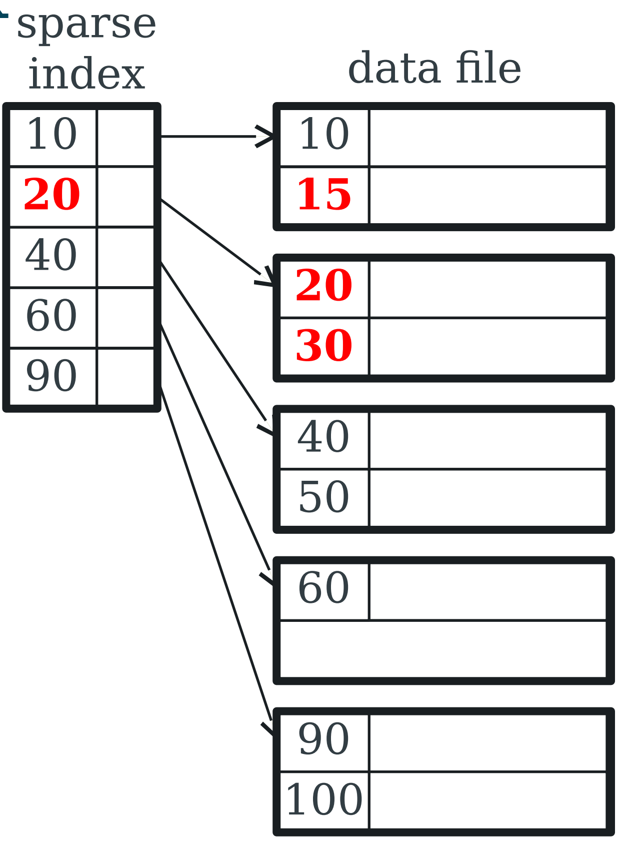
- Insert record 34
  - Easy! We have free space in the right block of the data file



Insertion into Sparse Index

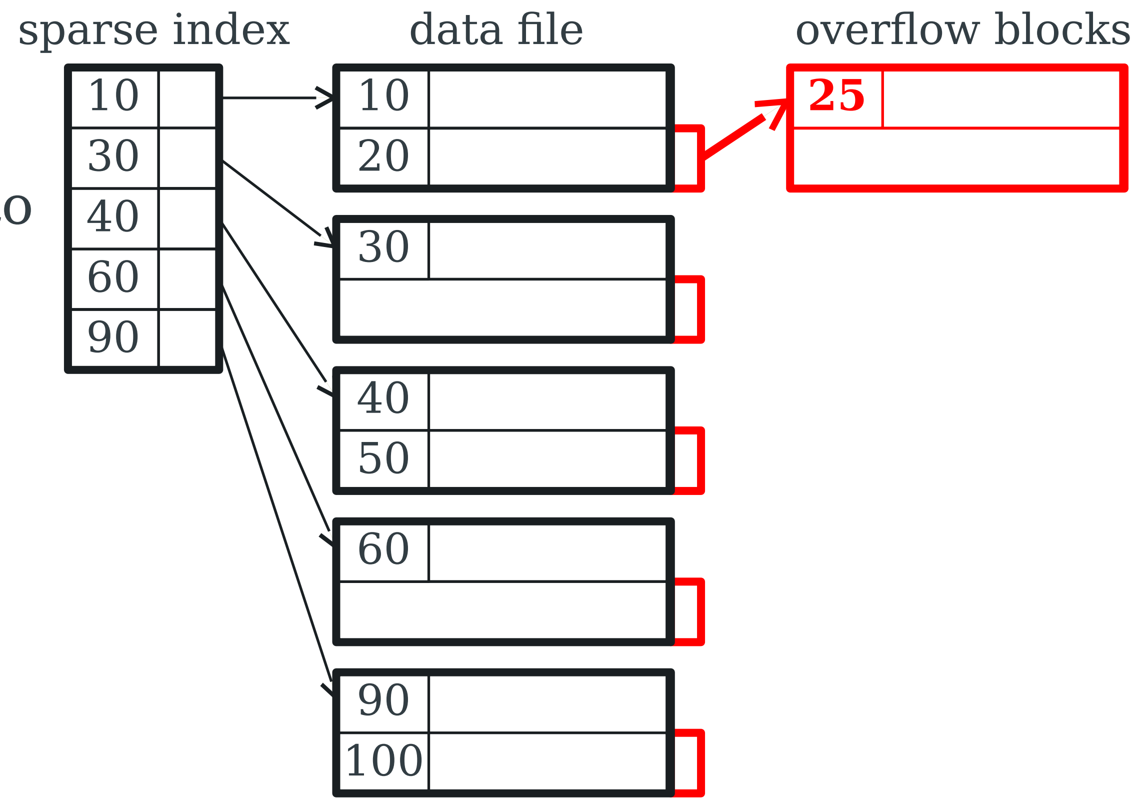
- Insert record 15
  - Add to data file and immediately reorganise
  - Update index

- Alternatively:
  - Insert new block (chained file)
  - Update index



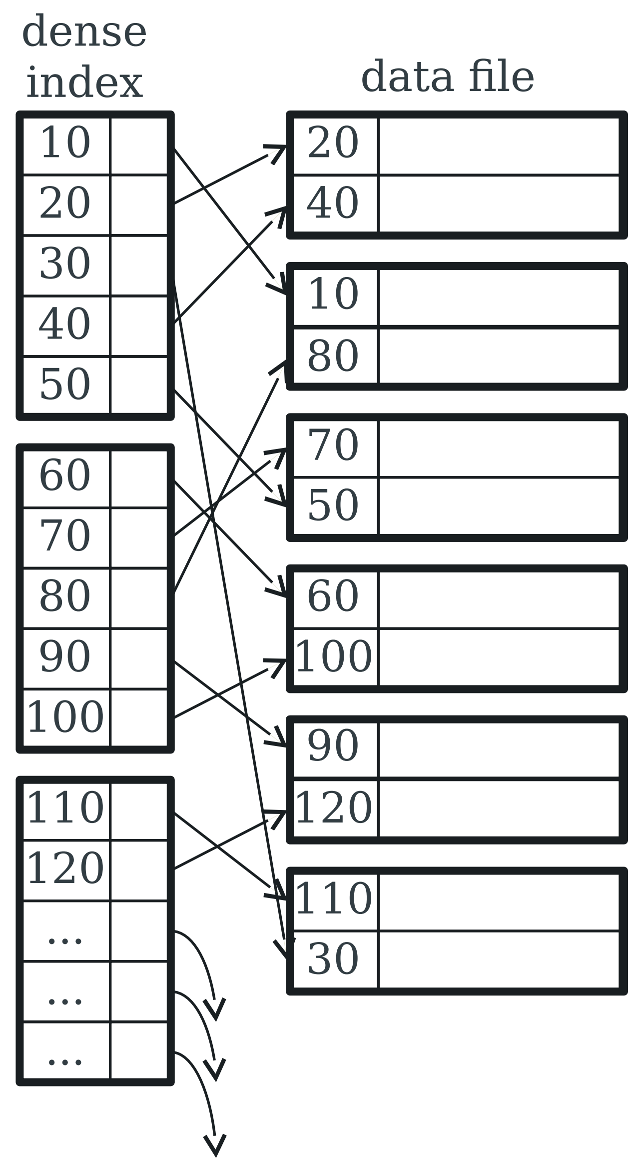
Insertion into Sparse Index

- Insert record 25
  - Block is full, so add to overflow block
  - Reorganise later...



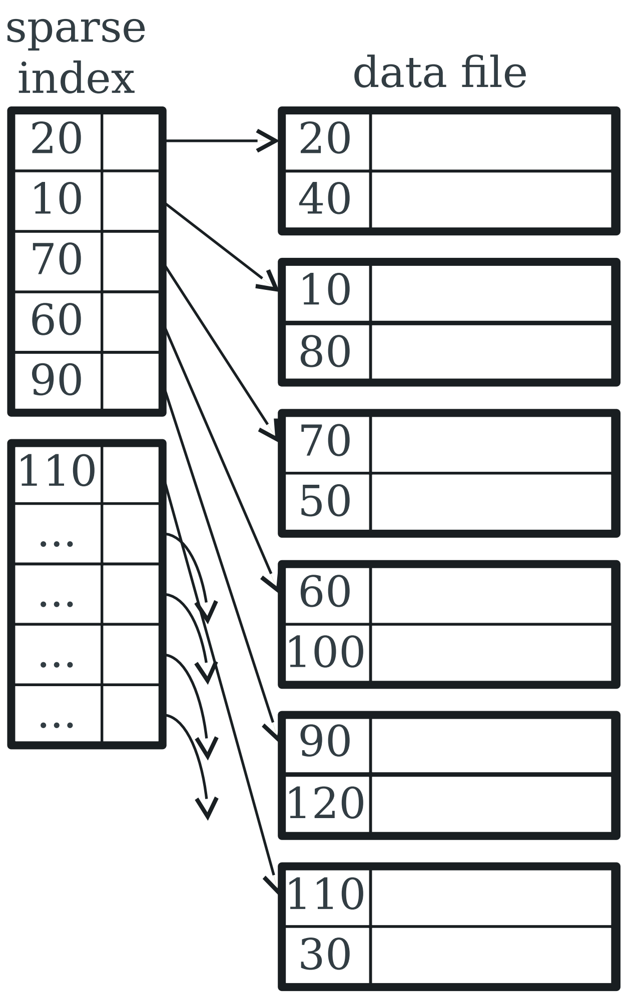
Secondary Indexes

- Unlike a primary index, does not determine placement of records in data file
- Location (order) of records may have been decided by a primary index on another field
- Secondary indexes are always dense
- Pointers are record pointers, not block pointers



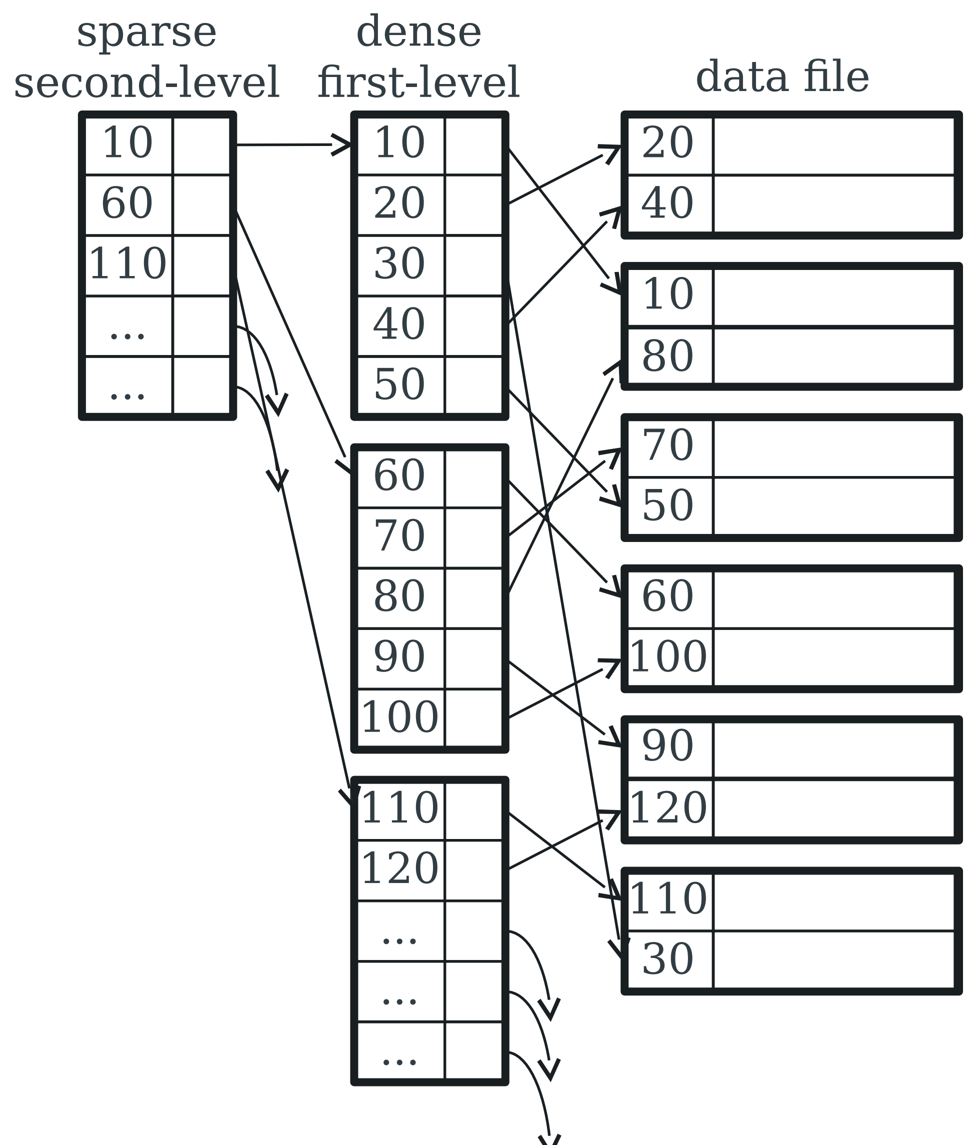
Secondary Indexes

- Sparse secondary indexes make no sense



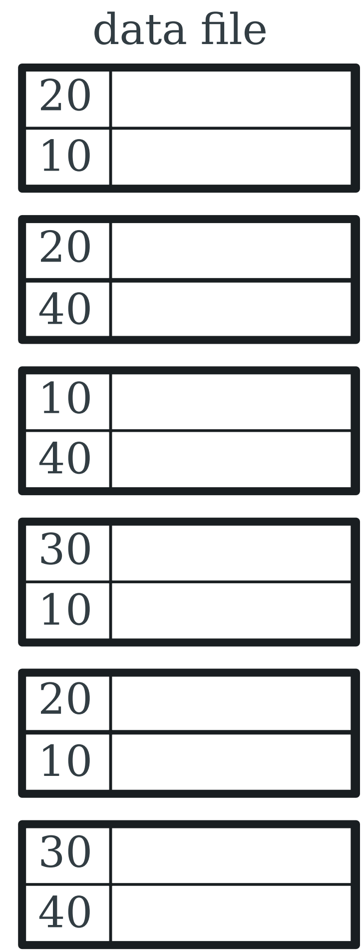
Secondary Indexes

- May have higher levels of sparse indexes above the dense index



Duplicate values

- Secondary indexes need to cope with duplicate values in the data file

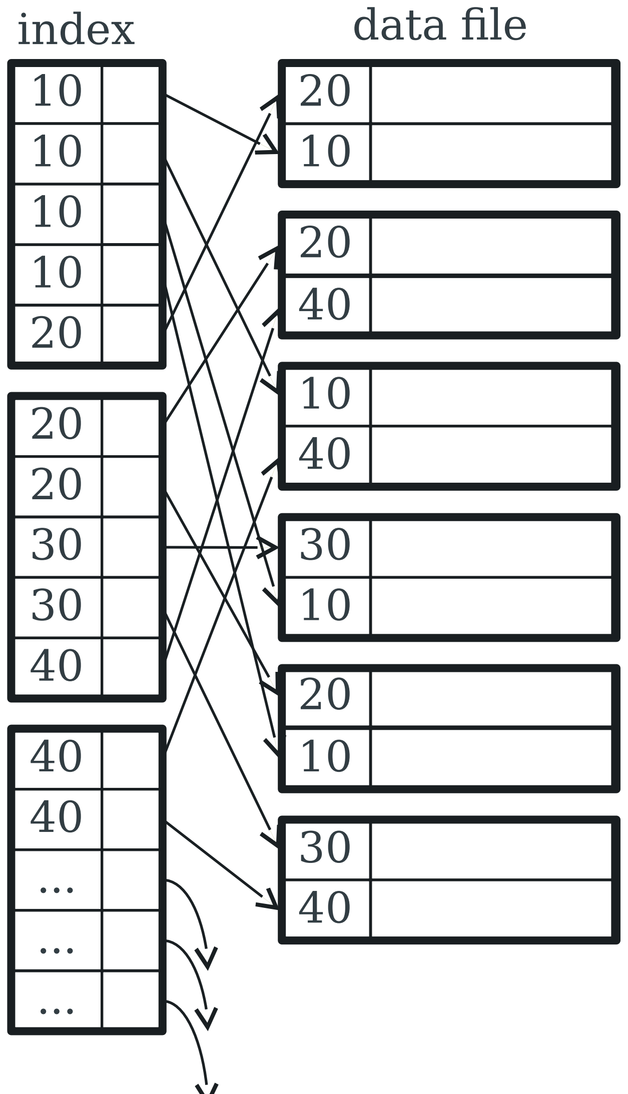


Duplicate values

Solution #1: repeated entries

Problems

- excess disk space
- excess search time

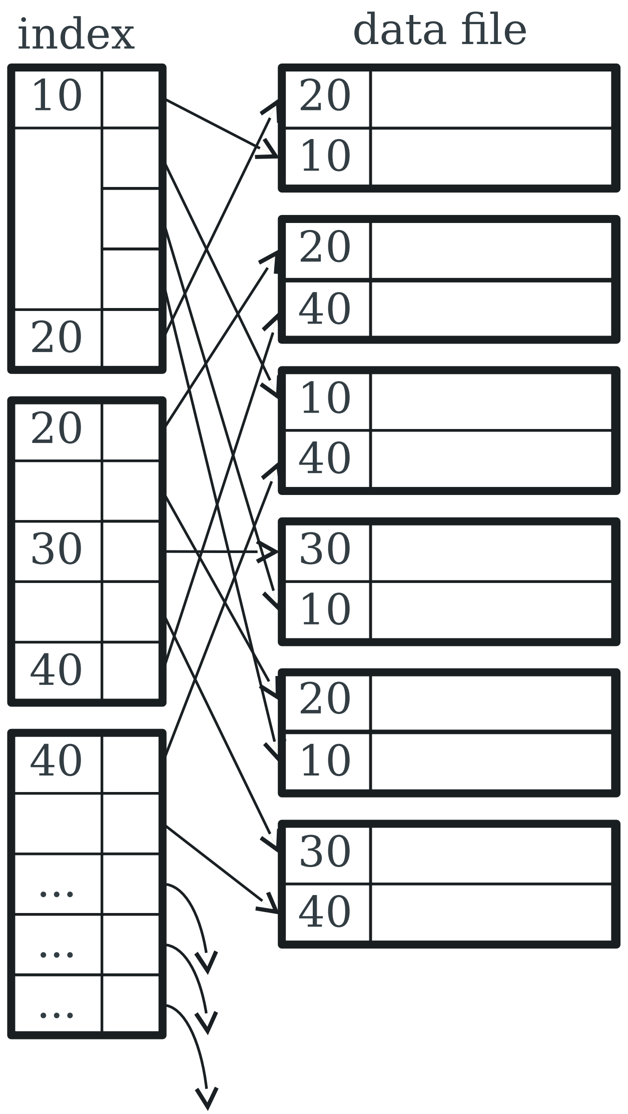


Duplicate values

Solution #2: drop repeated keys

Problems

- variable size records in index

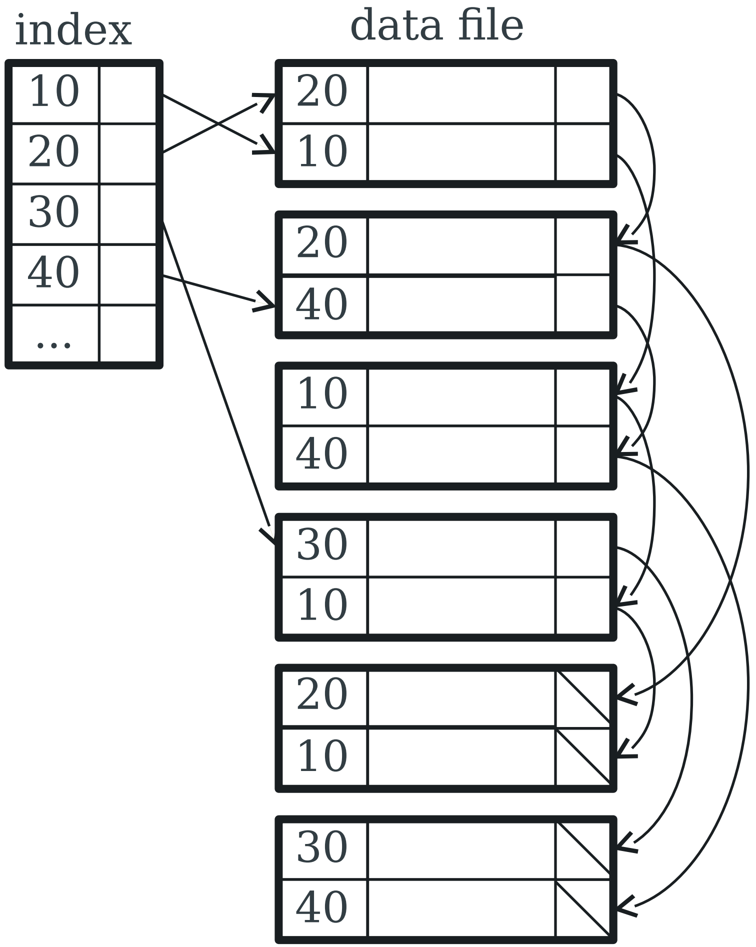


Duplicate values

Solution #3: chain records with same key

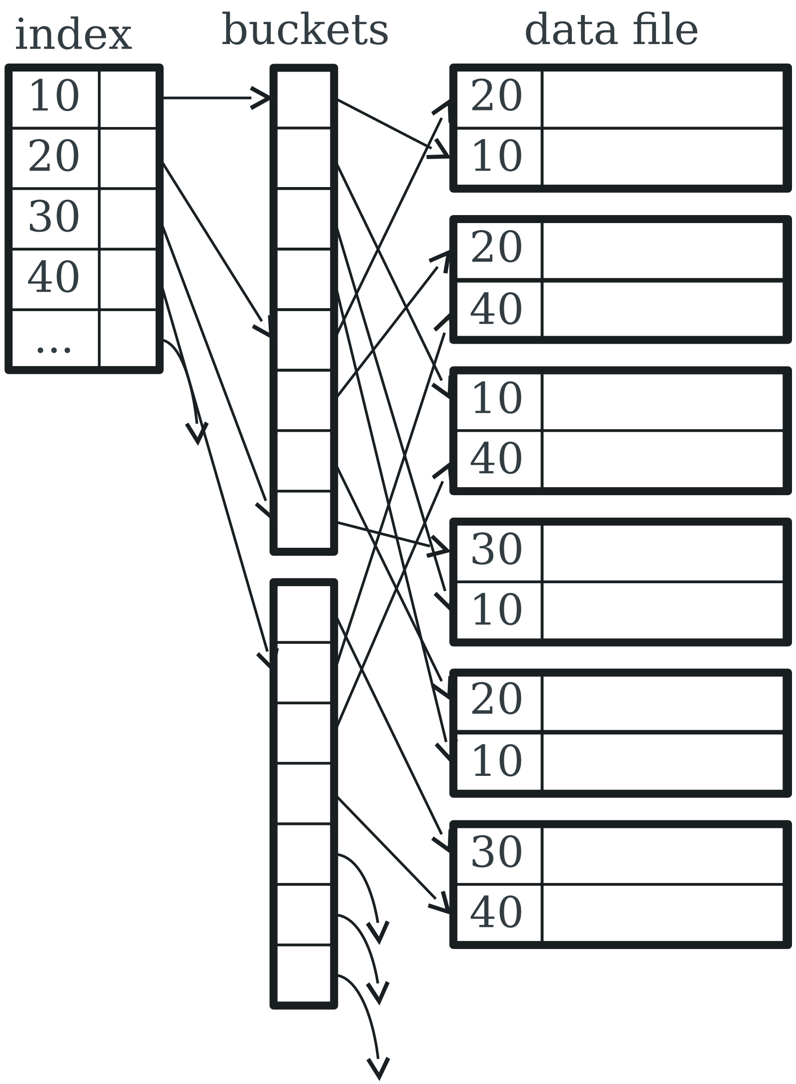
Problems

- need to add fields to records
- need to follow chain



Duplicate values

Solution #4: indirection via buckets of pointers



Advantages

- If we have multiple secondary indexes on a relation, we can calculate conjunctions by taking intersections of buckets
- Don’t need to examine data file!

Conventional indexes

Advantages:

- Simple
- Index is sequential file and good for scans

Disadvantages:

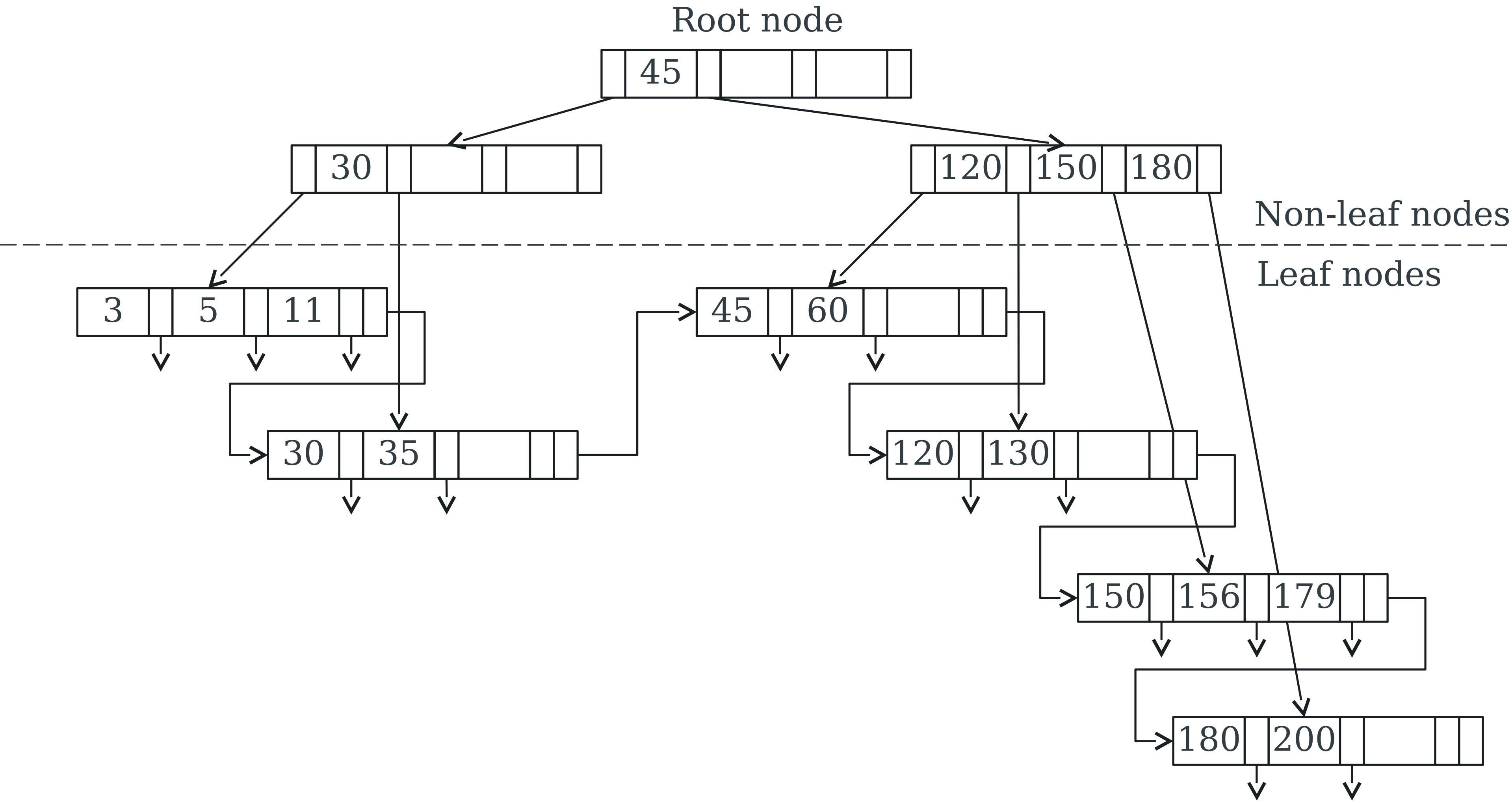
- Inserts expensive, and/or
- Lose sequentiality & balance

B+trees

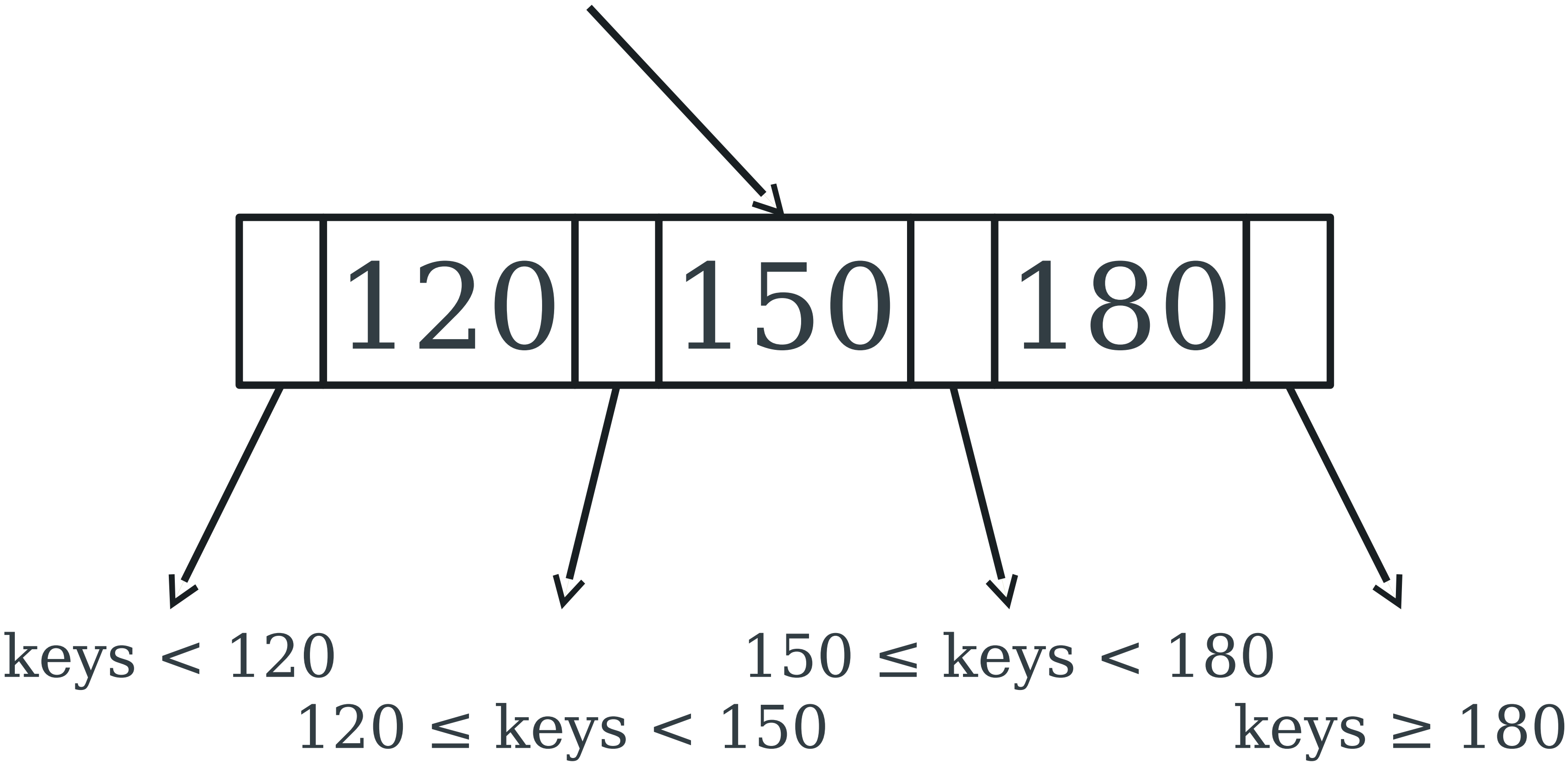
B+trees

- The most widely used tree-structured indexes
- Balanced multi-way tree
  - Yields consistent performance
  - Sacrifices sequentiality

B+tree example



Example non-leaf node



Non-leaf nodes

- Root node typically kept in memory
  - Entrance point to index – used as frequently as any other node
  - Some nodes from second level may also be kept in memory

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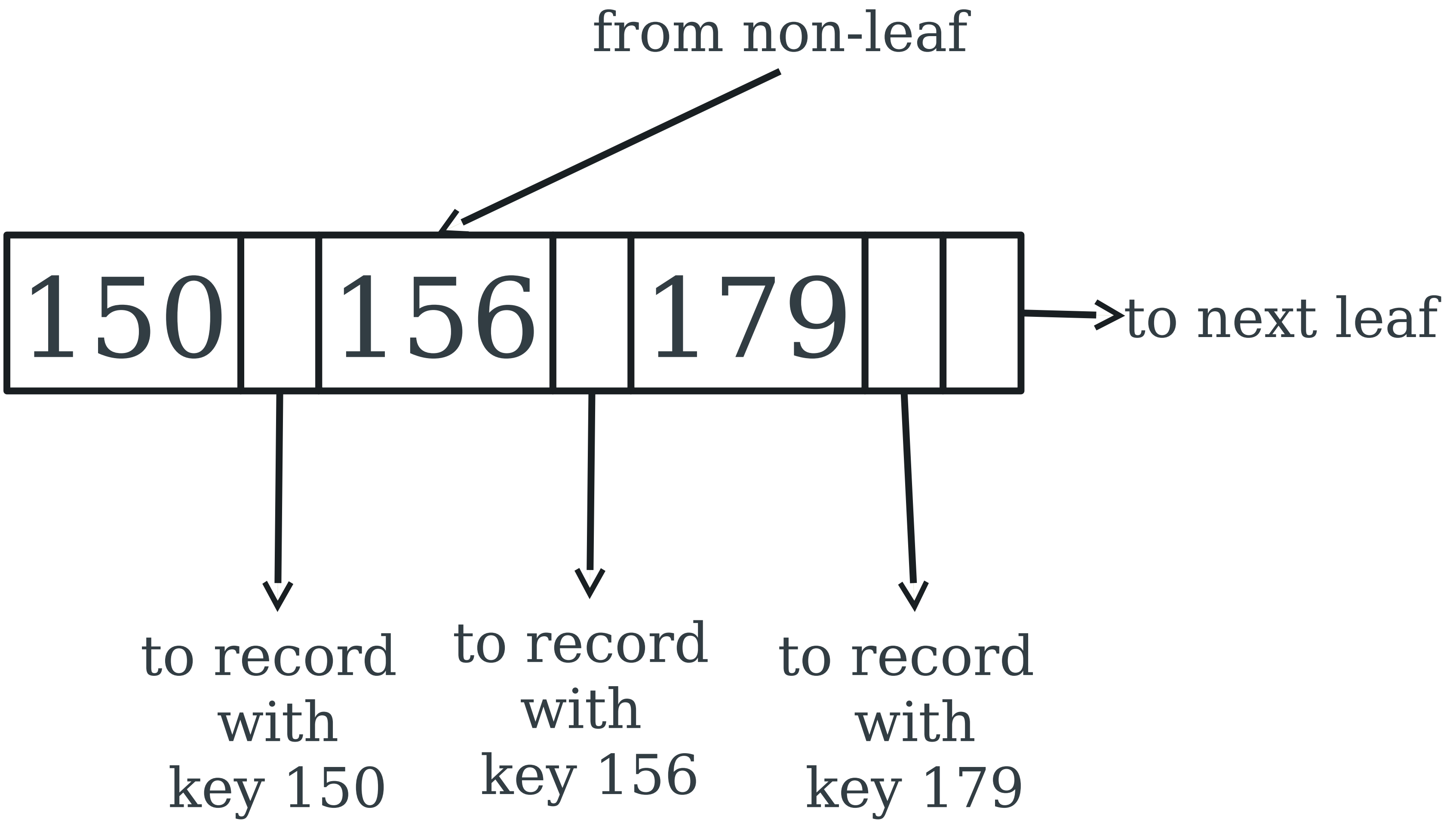
Example leaf node



Leaf nodes

If the index is a primary index

- Leaf nodes are records containing data, stored in the order of the primary key
- The index provides an alternative to a sequential scan

If the index is a secondary index

- Leaf nodes contain pointers to the data records
- Data can be accessed in the sequence of the secondary key
- A secondary index can point to any sort of data file, for example one created by hashing

Node size

Each node is of fixed size and contains

- n keys
- n+1 pointers

non-leaf

	120		150		180	

leaf

150		156		179		

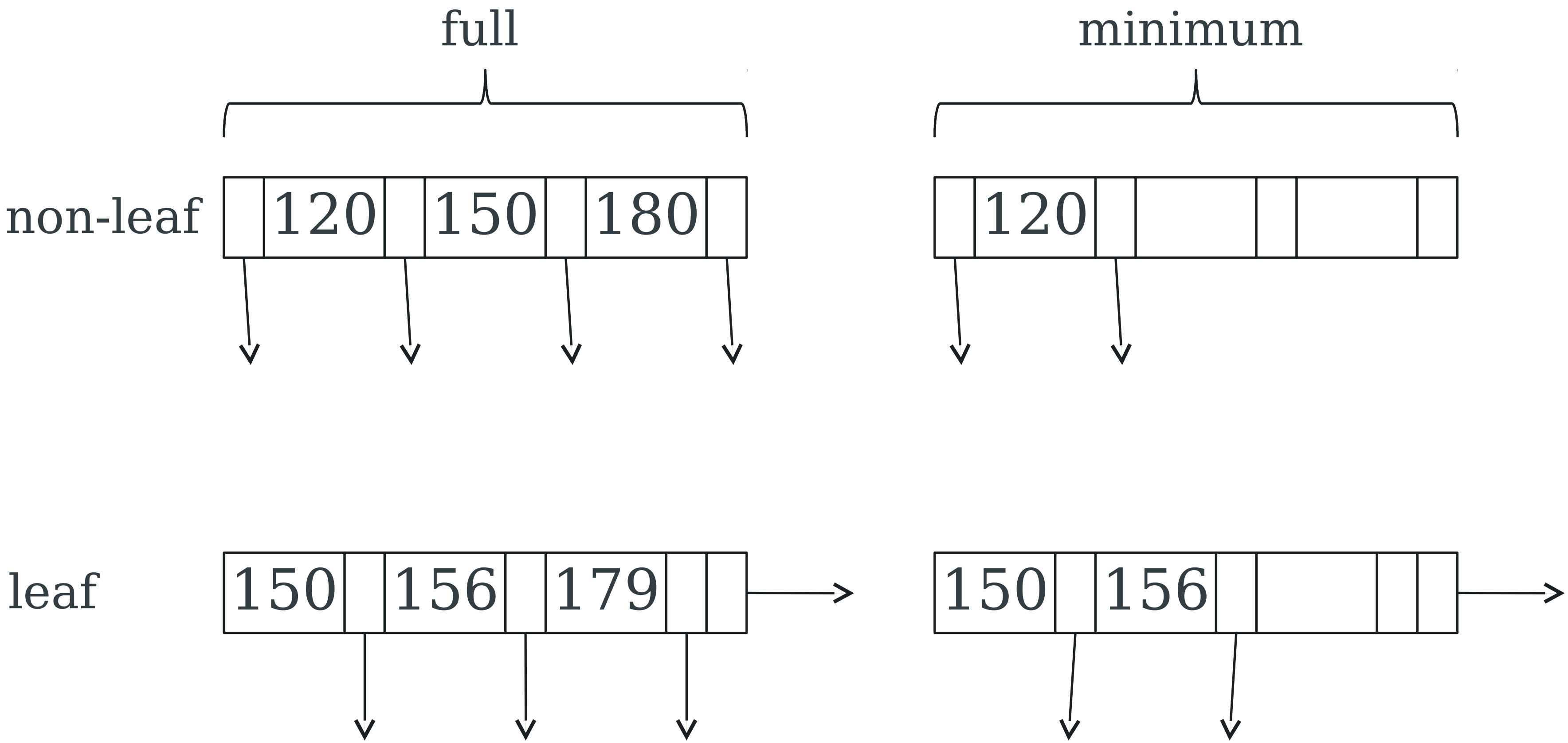
Minimum nodes

Don’t want nodes to be too empty (efficient use of space)

Non-leaf: $\lceil (n+1)/2 \rceil$ pointers

Leaf: $\lfloor (n+1)/2 \rfloor$ pointers

Minimum node examples (n=3)



B+tree rules

1. All leaves same distance from root (balanced tree)
2. Pointers in leaves point to records except for “sequence pointer”
3. Number of pointers/keys for B+tree of order n:

	max ptrs	max keys	min ptrs to data	min keys
Non-leaf	n+1	n	$\lceil (n+1)/2 \rceil$	$\lceil (n+1)/2 \rceil$ - 1
Leaf	n+1	n	$\lfloor (n+1)/2 \rfloor$	$\lfloor (n+1)/2 \rfloor$
Root	n+1	n	1	1

B+tree arithmetic example

First, some parameters:

- block size 8kb, of which:
  b = 8000 bytes available for storage of records
- key length
  k = 10 bytes
- record length
  r = 100 bytes (including the key)
- record pointer
  p = 6 bytes

B+tree arithmetic example

A leaf node in a primary index can accommodate lp records, where lp = $\lfloor b/r \rfloor$ = 80 records

A leaf node in a secondary index can accommodate ls records,
where ls = $\lfloor (b-p)/(k+p) \rfloor$ = 499 records

A non-leaf node could accommodate i entries, where i = $\lfloor (b-p)/(k+p) \rfloor$ = 499 records

To allow for expansion, assume initial node occupancy of u, where u = 0.6

B+tree primary index

For a primary index (the leaf nodes hold the records)

- A non-leaf node initially points to
  i*u = blocks
- Each leaf initially contains
  lp*u = records
- 1 level of non-leaf nodes initially points to
  (lp*u)(i*u) = records
- 2 levels of non-leaf nodes initially point to
  $(i*u)^2$ = blocks
  $(lp*u)(i*u)^2$ = records

B+tree primary index

For a primary index (the leaf nodes hold the records)

- A non-leaf node initially points to

  i*u = 299 blocks

- Each leaf initially contains

  lp*u = 48 records

- 1 level of non-leaf nodes initially points to

  (lp*u)(i*u) = 14,352 records

- 2 levels of non-leaf nodes initially point to

  $(i*u)^2$ = 89,401 blocks

  $(lp*u)(i*u)^2$ = 4,291,248 records

B+tree secondary index

For a secondary index (the leaf nodes hold record pointers)

- A non-leaf node initially points to
  $i*u =$ blocks
- A leaf node initially points at
  $ls*u =$ records
- 1 level of non-leaf nodes initially points to
  $(ls*u)(i*u) =$ records
- 2 levels of non-leaf nodes initially point to
  $(ls*u)(i*u)^2 =$ records

B+tree secondary index

For a secondary index (the leaf nodes hold record pointers)

- A non-leaf node initially points to
  $i*u = 299$ blocks
- A leaf node initially points at
  $ls*u = 299$ records
- 1 level of non-leaf nodes initially points to
  $(ls*u)(i*u) = 89{,}401$ records
- 2 levels of non-leaf nodes initially point to
  $(ls*u)(i*u)^2 = 26{,}730{,}899$ records

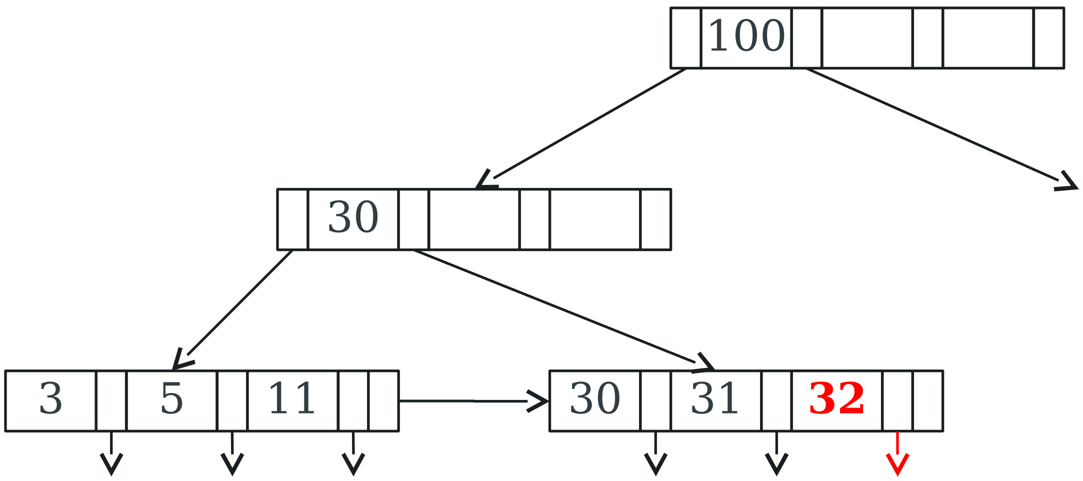
It is not normally necessary to go more than about three levels deep in the index

B+tree Insertion

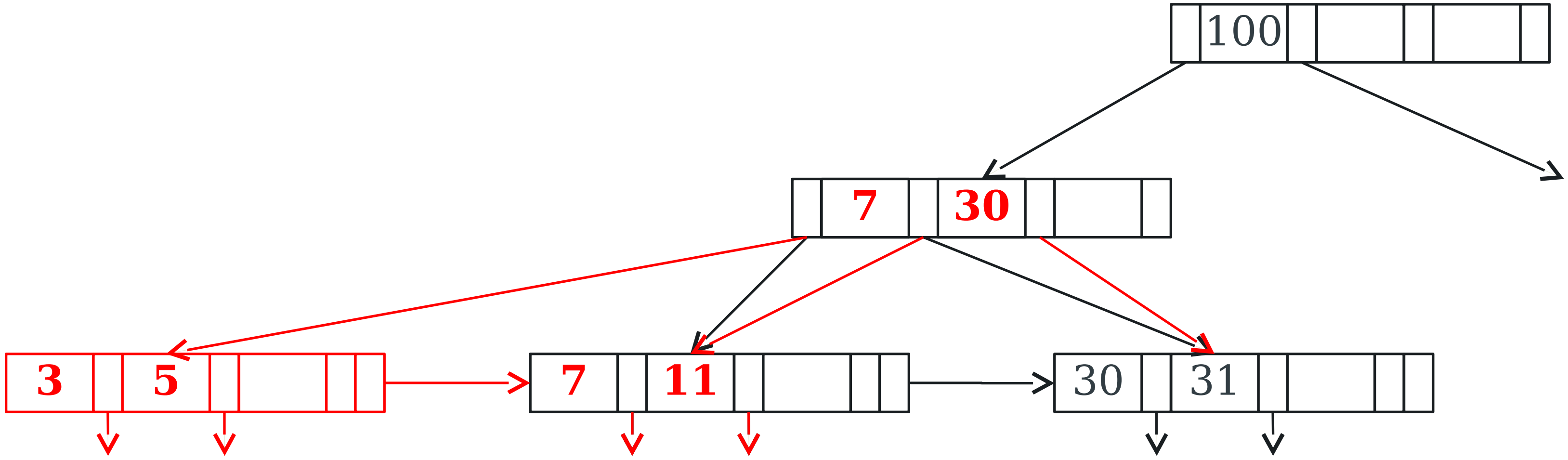
Four cases to consider:

1. Space available in leaf
2. Leaf overflow
3. Non-leaf overflow
4. New root

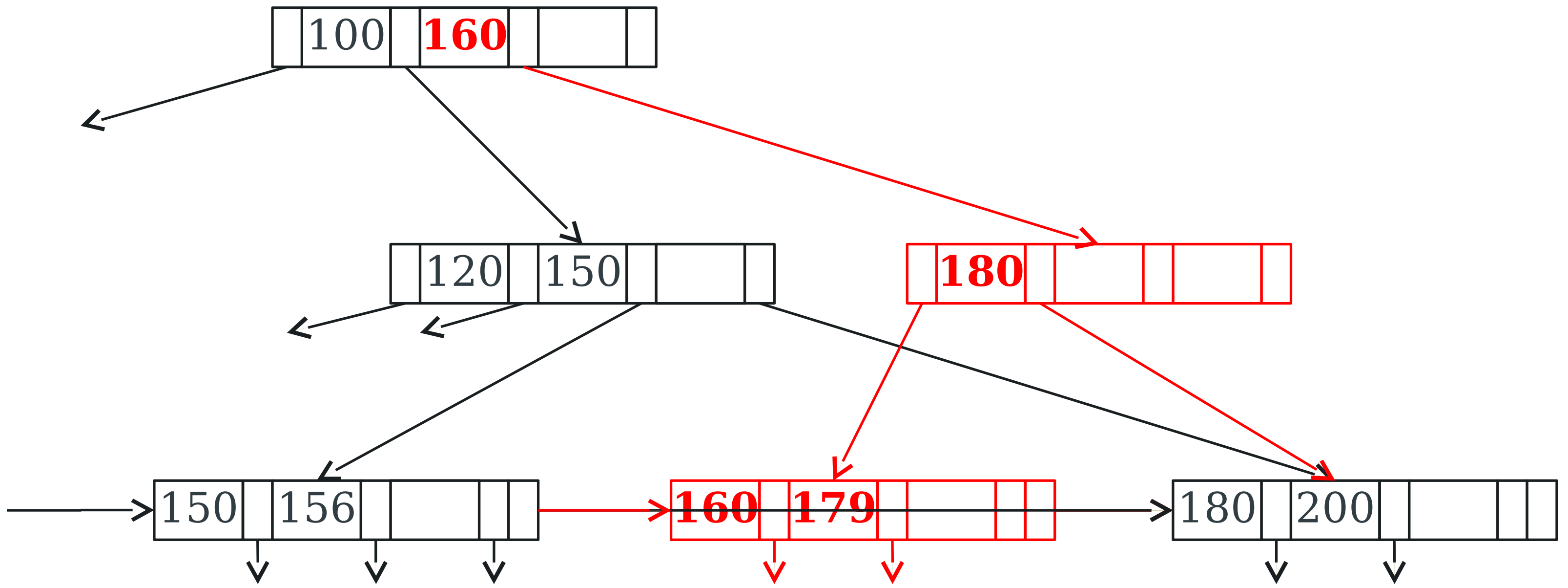
Case 1: insert key=32



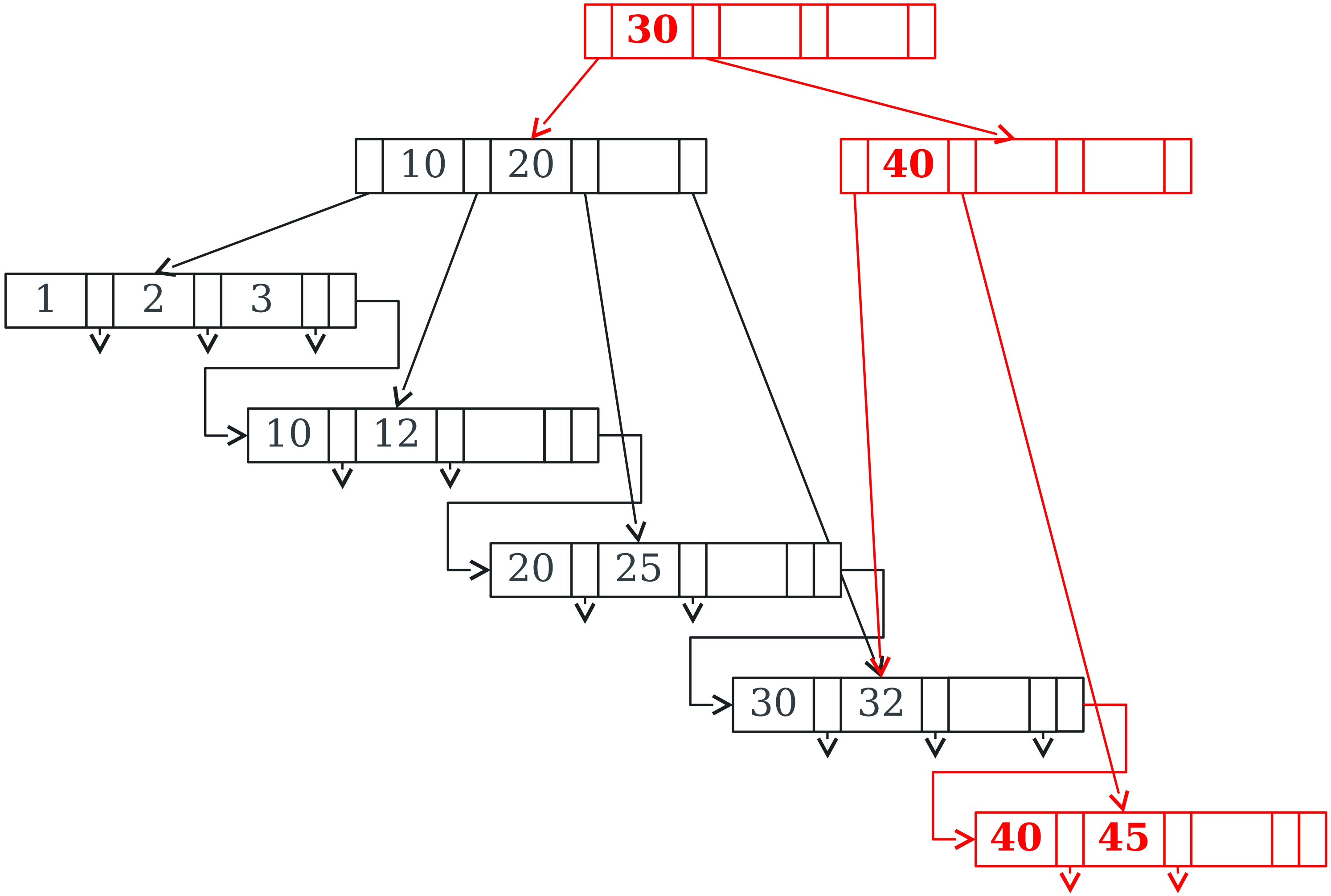
Case 2: insert key=7



Case 3: insert key=160



Case 4: insert 45

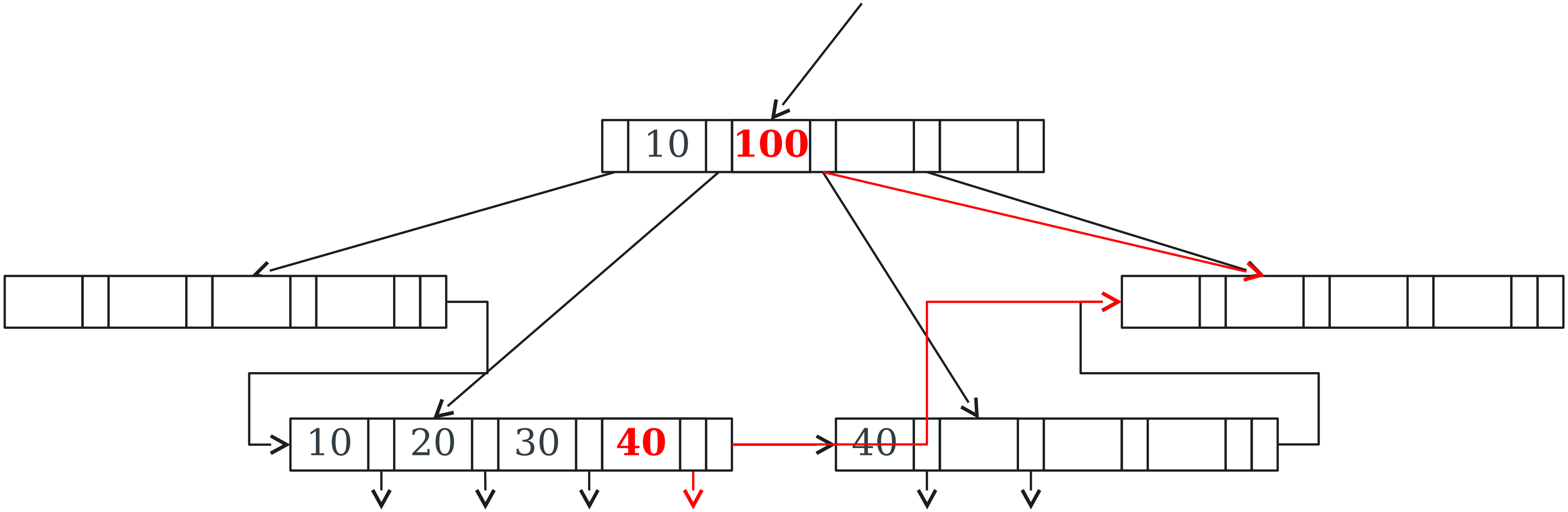


B+tree Deletion

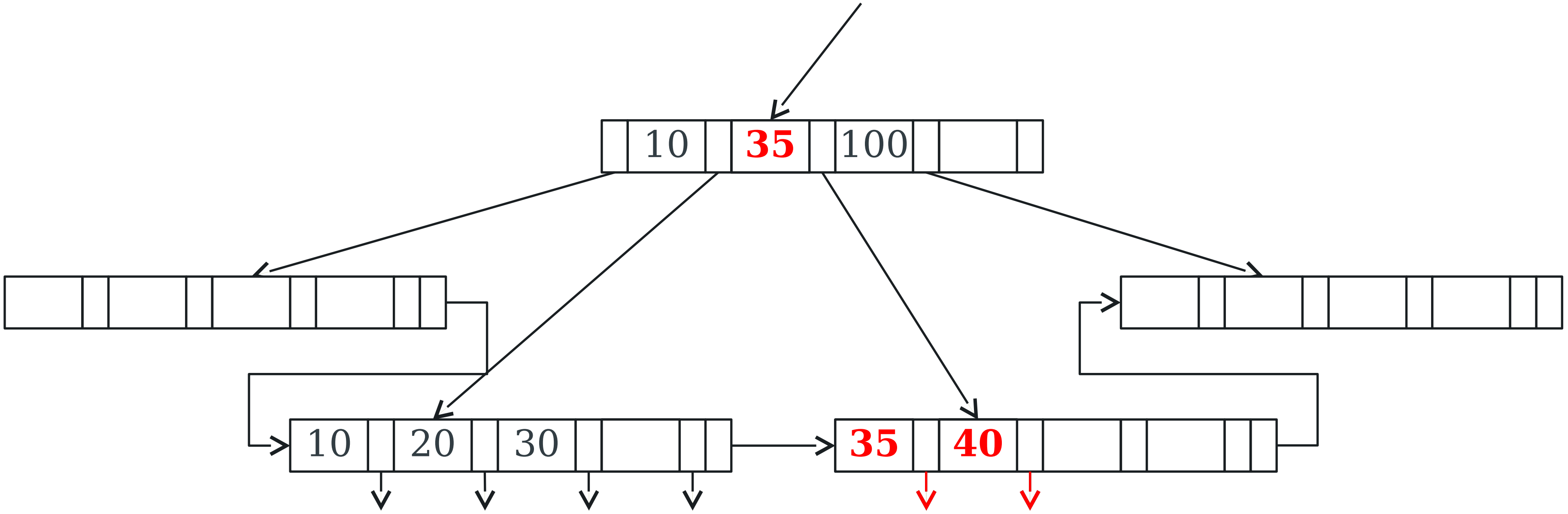
Four cases to consider:

1. Simple case
2. Coalesce with sibling
3. Re-distribute keys
4. Cases 2. or 3. at non-leaf

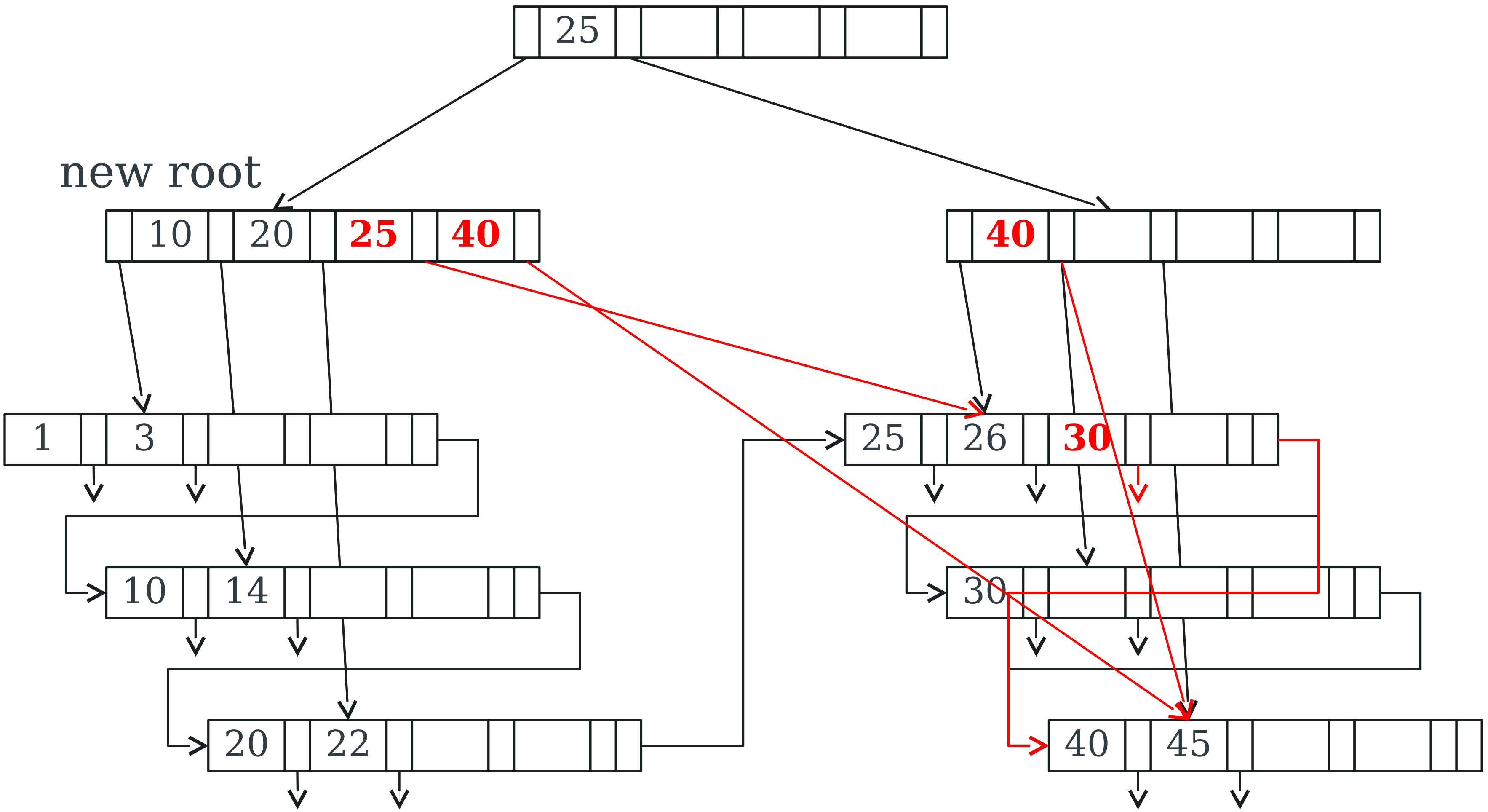
Case 2: delete key=50 (n=4)



Case 3: delete key=50 (n=4)



Case 4: delete key=37 (n=4)



B+tree deletions in practice

Often, coalescing is not implemented

- Too hard and not worth it!

B-trees versus static indexed sequential files

B-trees consume more space

- Blocks are not contiguous
- Fewer disk accesses for static indexes, even allowing for reorganisation

Concurrency control is harder in B-trees

but

DBA does not know:

- when to reorganize
- how full to load pages of new index

Hashing

Hashing

Main memory hash table

- *Hash function* h() takes a key and computes an integer value
- Value is used to select a bucket from a *bucket array*
- Bucket array contains linked lists of records

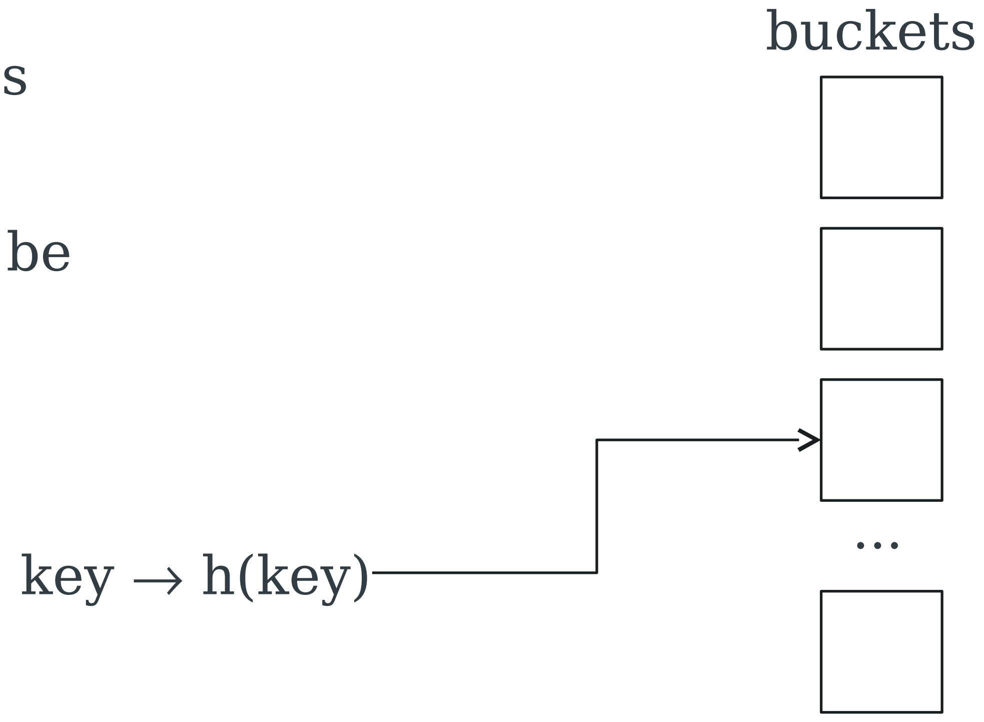
Secondary storage hash table

- Stores many more records than a main memory hash table
- Bucket array consists of disk blocks

Hashing approach #1

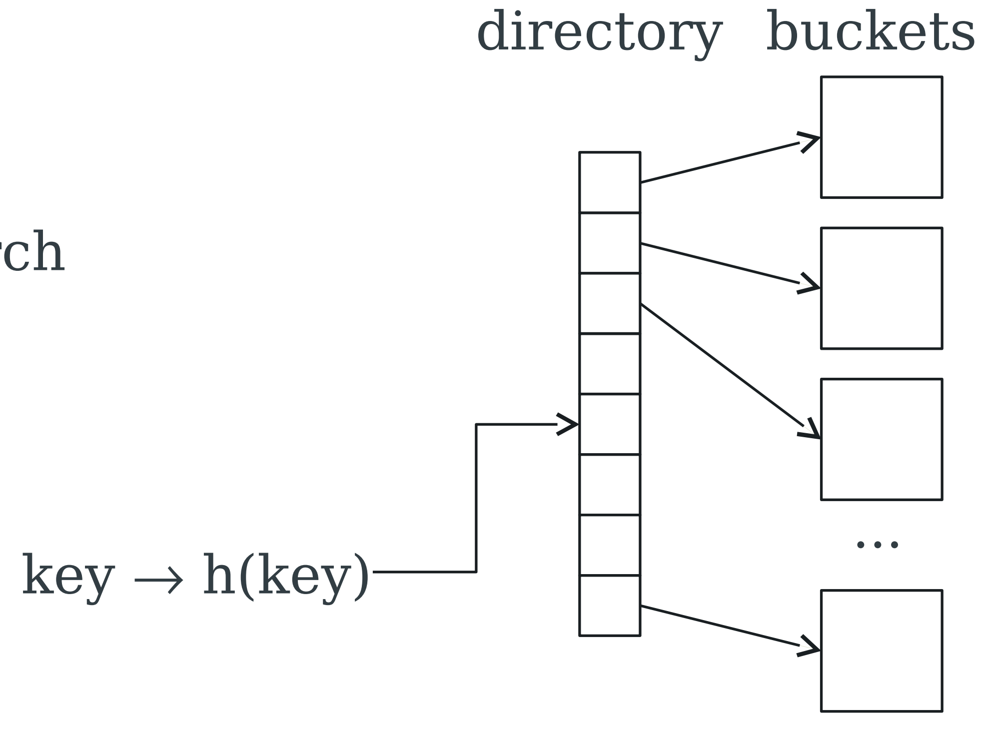
- Hash function calculates block pointer directly, or as offset from first block

- Requires bucket blocks to be in fixed, consecutive locations



Hashing approach #2

- Hash function calculates offset in array of block pointers (directory)
- Used for “secondary” search keys



Example hash function

Key = ‘x1 x2 … xn’ (n byte character string), b buckets

h: add x1 + x2 + ….. xn, compute sum modulo b

Not a particularly good function

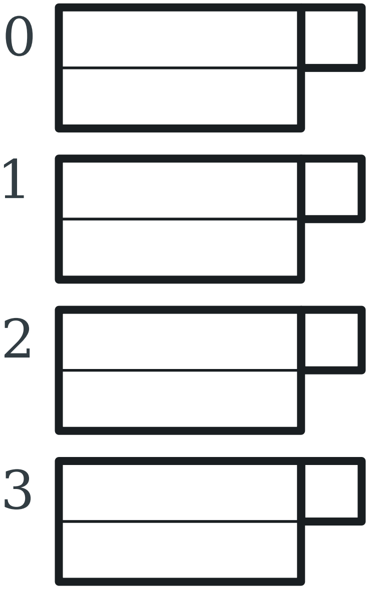
Good hash function has the same expected number of keys per bucket for each bucket

Buckets

- Do we keep keys sorted?
- Yes, if CPU time is critical and inserts/deletes are relatively infrequent

Hashing example

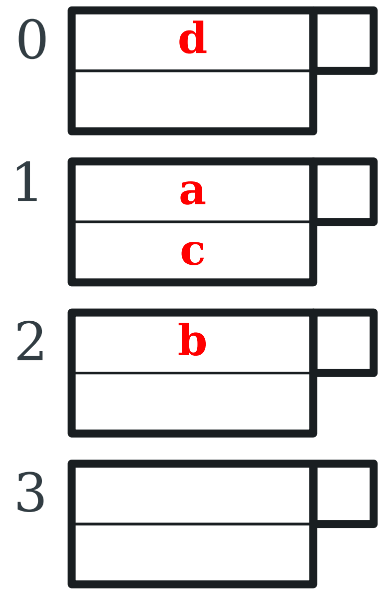
Two records per bucket



Hashing example

Insert a, b, c, d

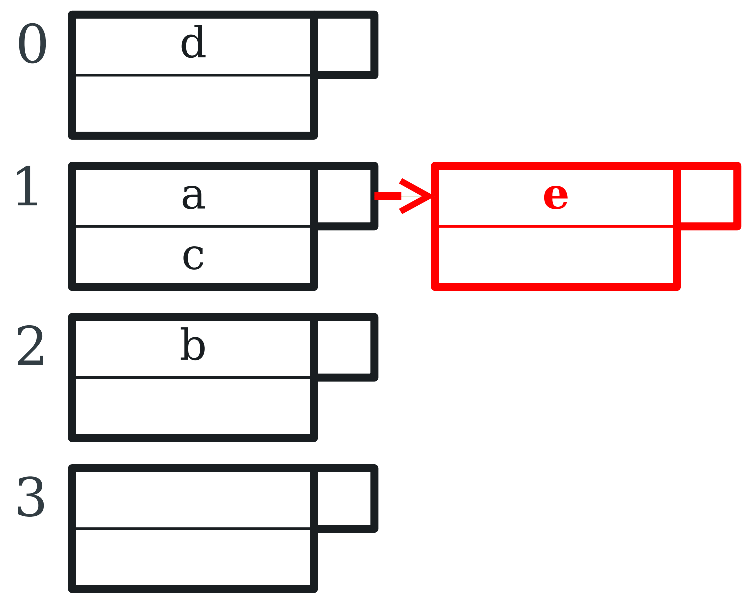
- h(a) = 1
- h(b) = 2
- h(c) = 1
- h(d) = 0



Hashing example: Overflow

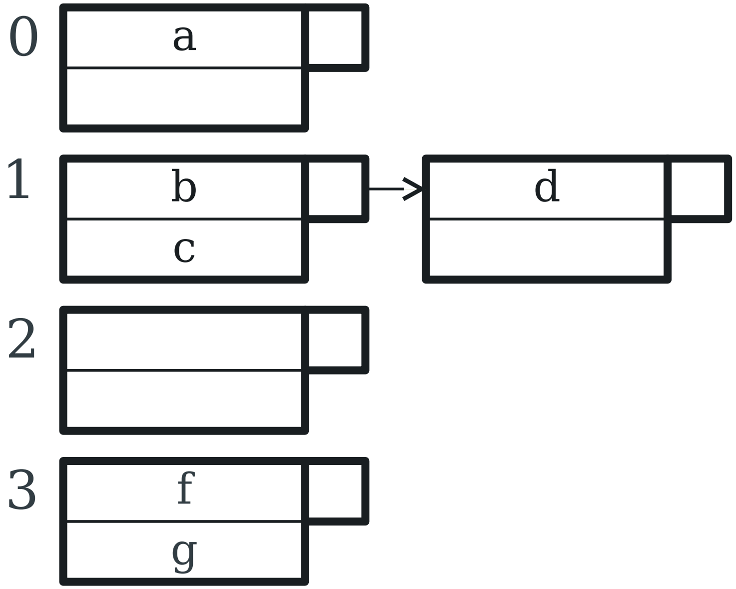
Insert e

- h(e) = 1



Hashing example: Deletion

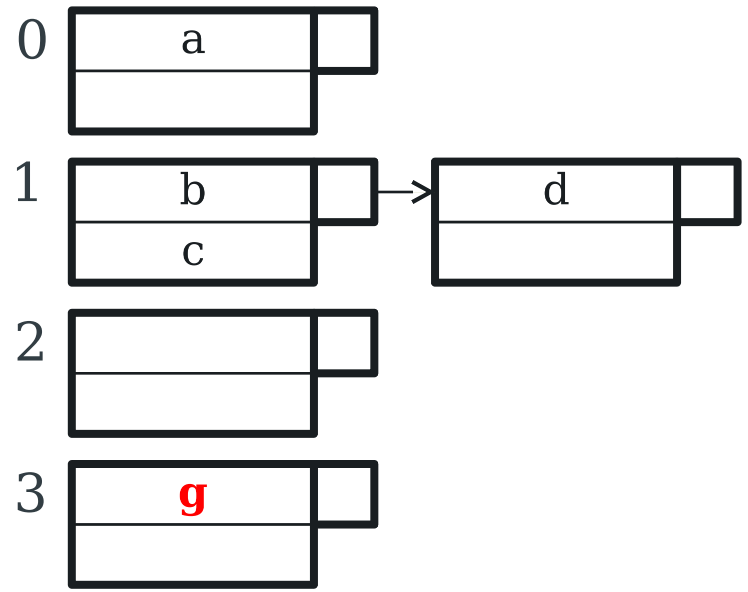
Delete e



Hashing example: Deletion

Delete f

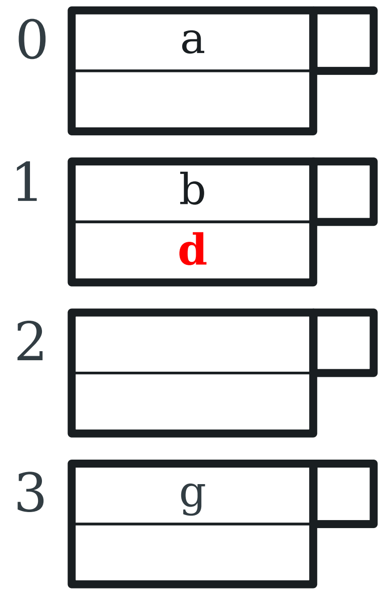
(move g up)



Hashing example: Deletion

Delete c

(move d from overflow block)



Rule of thumb:

Space utilisation should be between 50% and 80%

- Utilisation = #keys used / total #keys that fit

If < 50%, wasting space

If > 80%, overflows significant

- depends on how good hash function is and on #keys/bucket

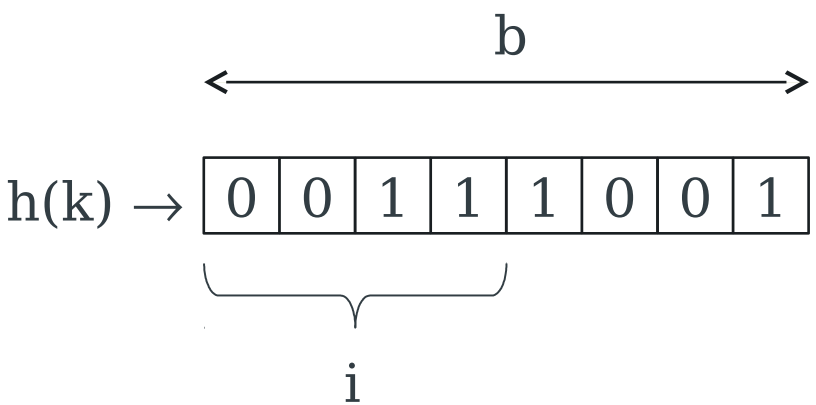
How do we cope with growth?

- Overflows and reorganizations
- Dynamic hashing
  - Extensible
  - Linear

Extensible hashing

Combines two ideas:

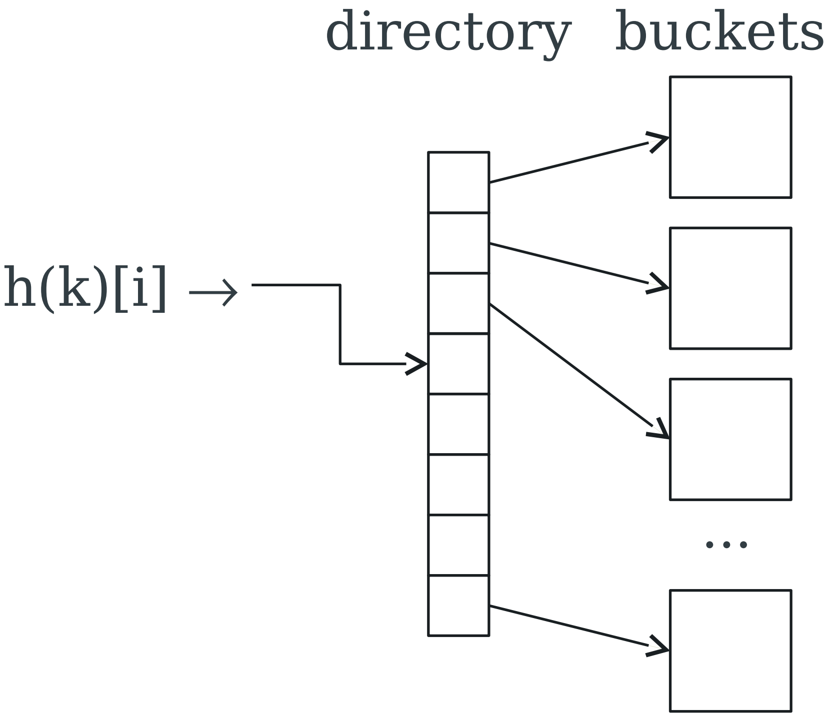
1. Use i of b bits output by hash function, where i grows over time



Extensible hashing

Combines two ideas:

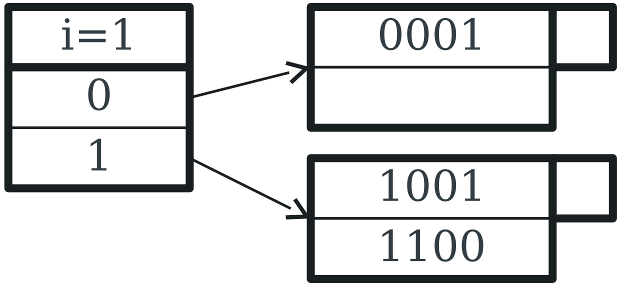
1. Use i of b bits output by hash function, where i grows over time
2. Use a directory



Example

h(k) gives 4 bits

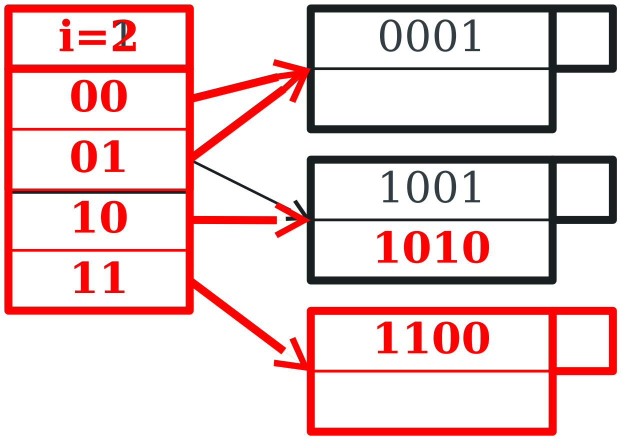
2 keys/bucket



Example

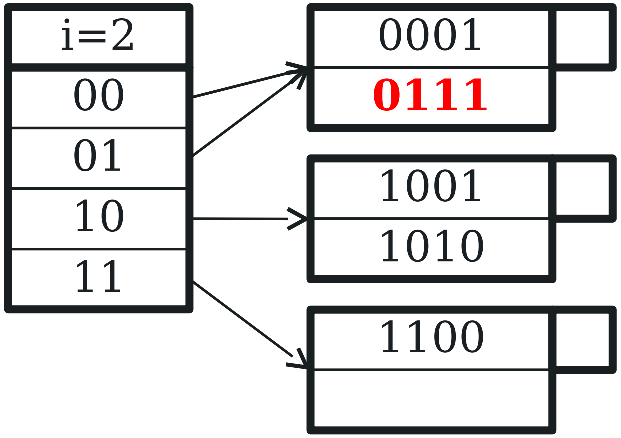
Insert 1010

- Bucket overfull
- Extend (double) directory
- Split bucket



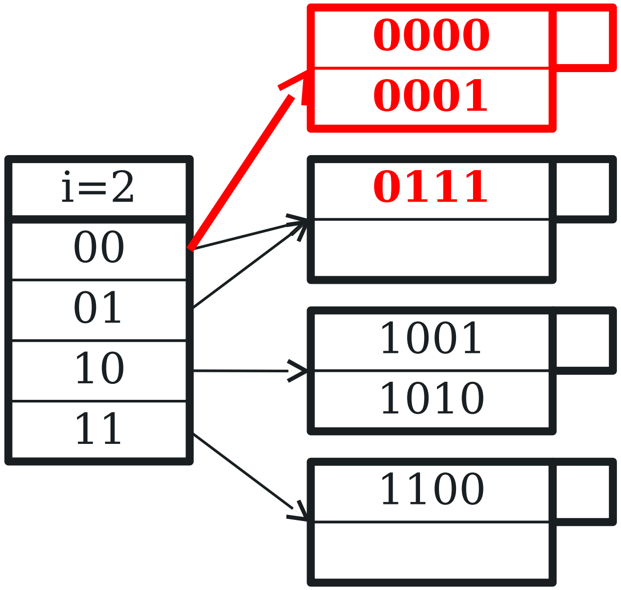
Example

Insert 0111



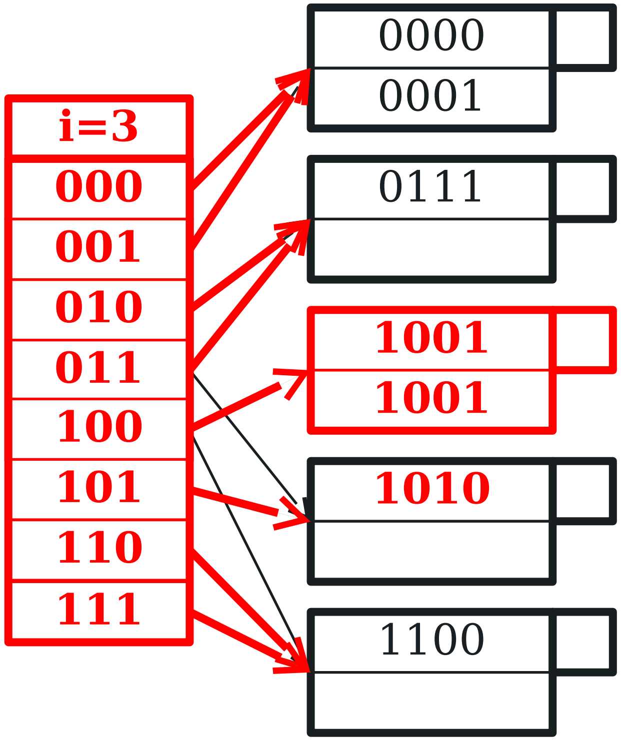
Example

Insert 0000



Example

Insert 1001



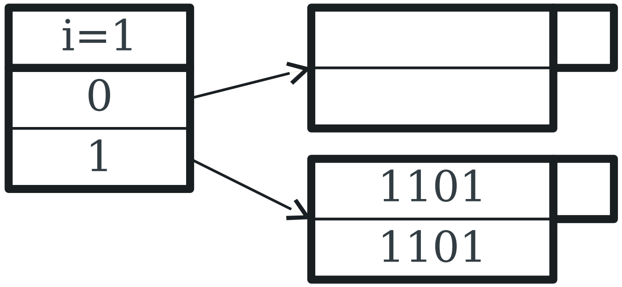
Extensible hashing: deletion

- No merging of blocks
- Merge blocks and cut directory if possible
- (Reverse insert procedure)

Overflow chains

Example: many records with duplicate keys

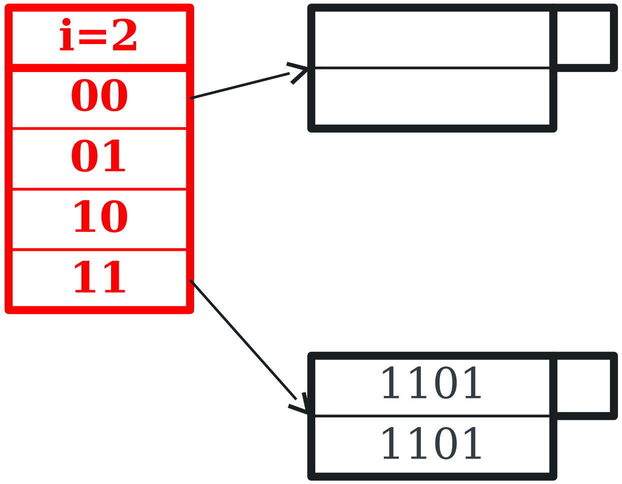
- Insert 1100



Overflow chains

Example: many records with duplicate keys

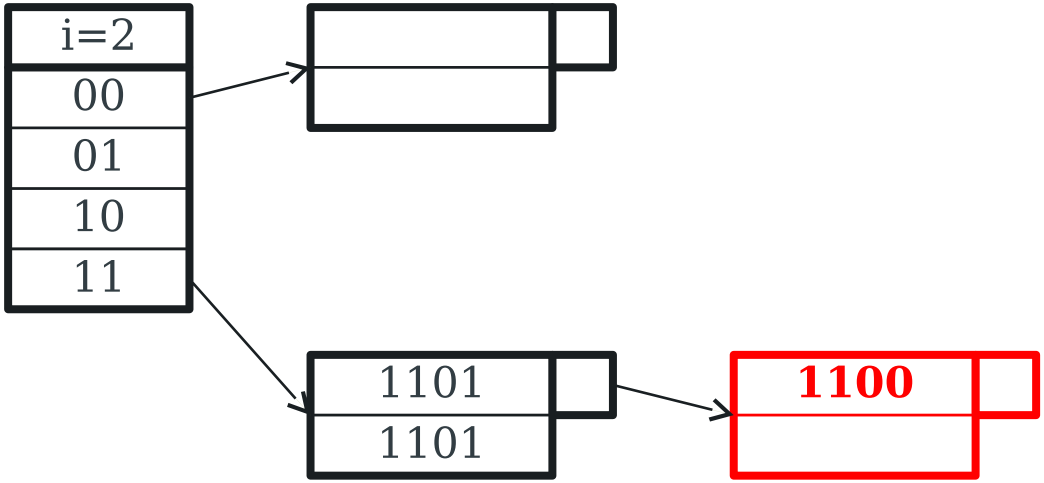
- Insert 1100



Overflow chains

Example: many records with duplicate keys

- Insert 1100
- Add overflow block



Summary

Pro

- Can handle growing files
  - with less wasted space
  - with no full reorganizations

Con

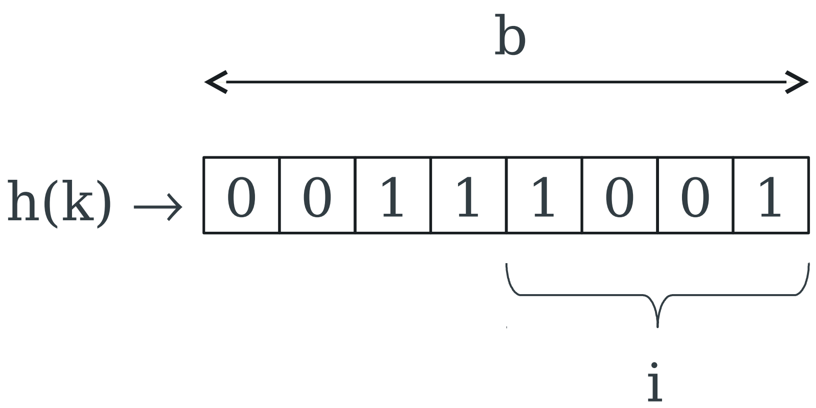
- Indirection
  - not bad if directory in memory
- Directory doubles in size
  - now it fits in memory, now it doesn't
  - suddenly increase in disk accesses!

Linear hashing

- Another dynamic hashing scheme

- Combines two ideas

1. Use i least significant bits of hash, where i grows over time



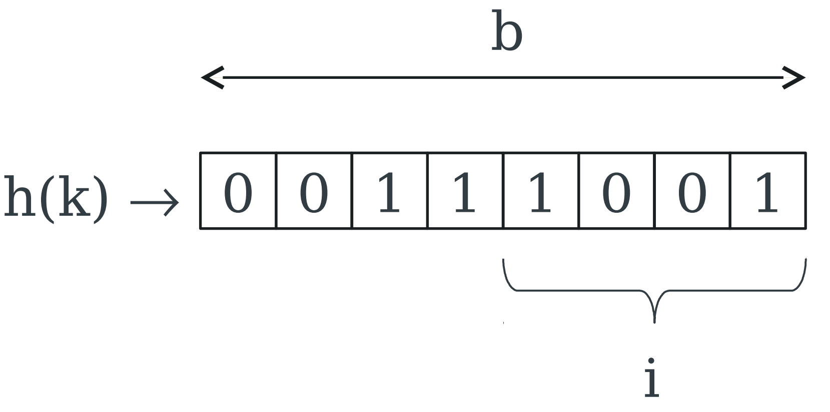
Linear hashing

- Another dynamic hashing scheme

- Combines two ideas

1. Use i least significant bits of hash, where i grows over time

2. Hash file grows incrementally and linearly (unlike extensible hash file, which periodically doubles)

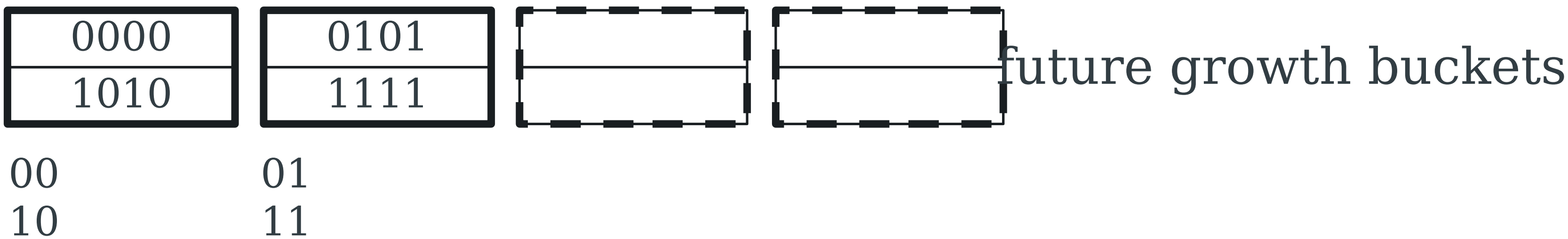


Example: b=4 bits, i=1, 2 keys/bucket

0	1
0000	0101
1010	1111

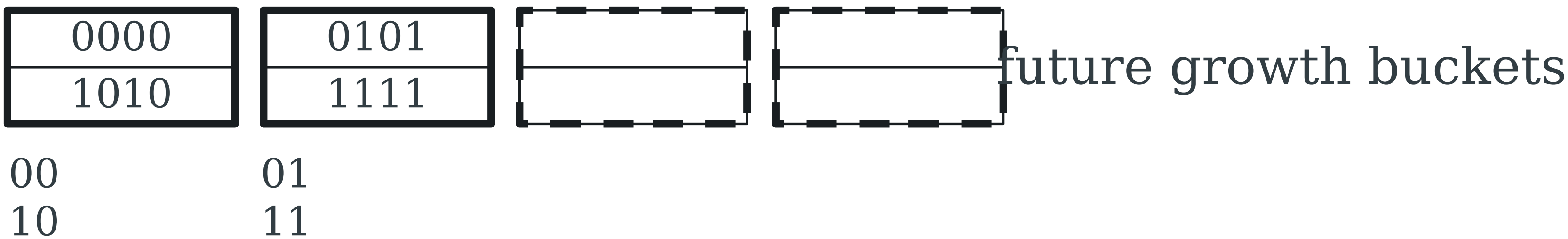
m = max used bucket = 1

Example: b=4 bits, i=2, 2 keys/bucket



m = max used bucket = 01

Example: b=4 bits, i=2, 2 keys/bucket



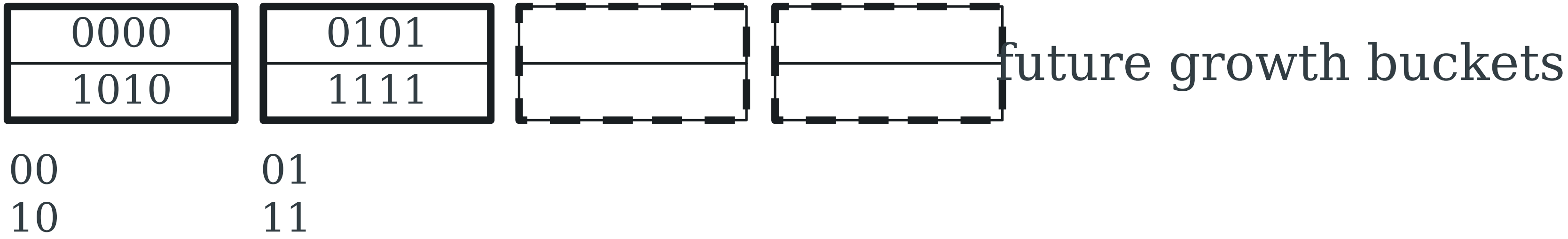
m = max used bucket = 01

Rule: If $h(k)[i] \le m$, then

look at bucket $h(k)[i]$

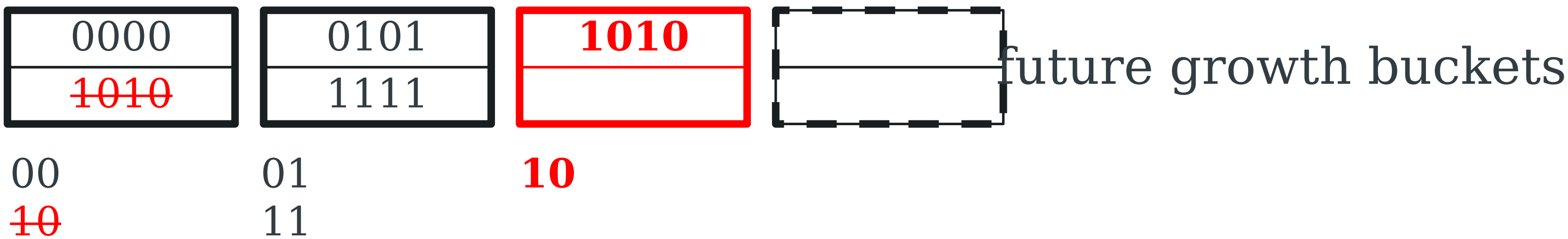
else, look at bucket $h(k)[i] - 2^{i-1}$

Example: b=4 bits, i=2, 2 keys/bucket



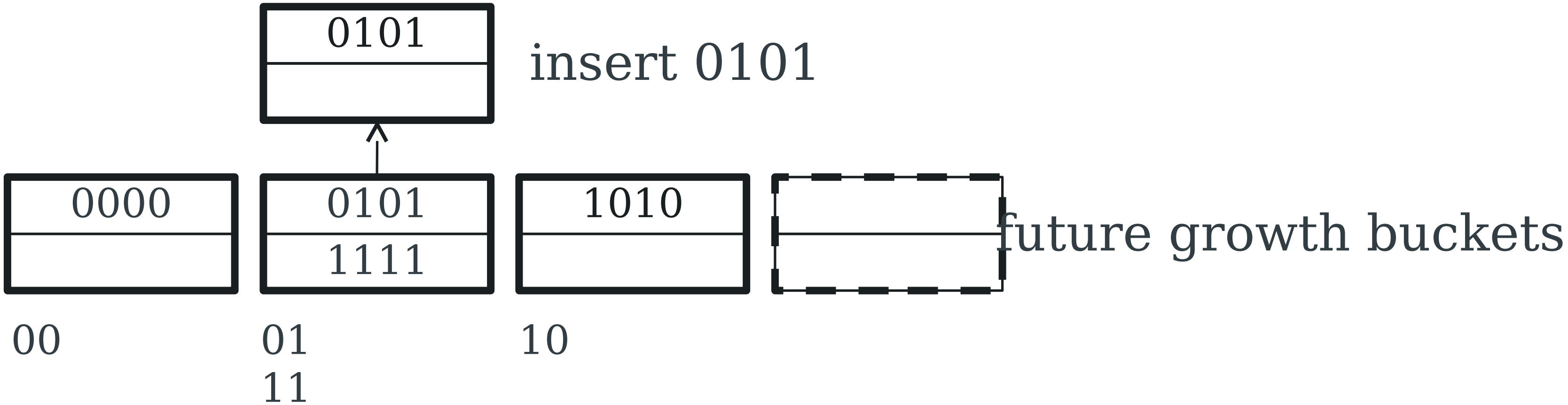
m = max used bucket = 01

Example: b=4 bits, i=2, 2 keys/bucket



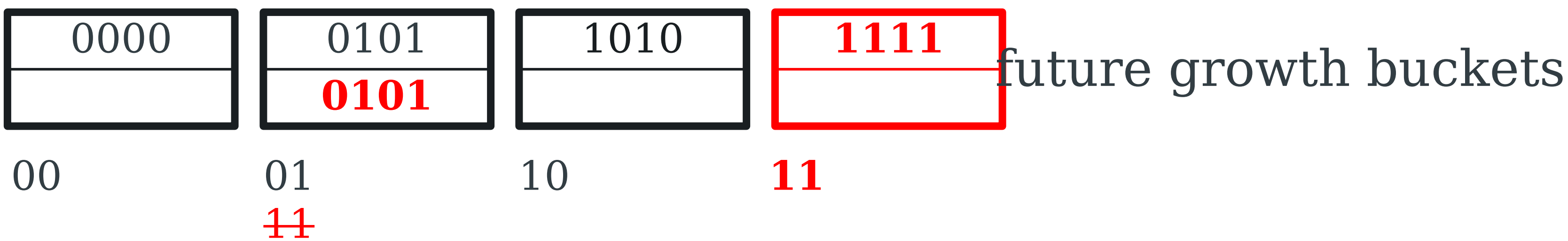
m = max used bucket = **10**

Example: b=4 bits, i=2, 2 keys/bucket



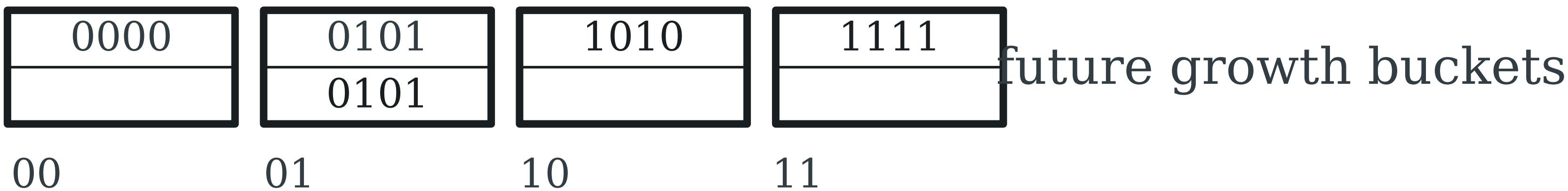
m = max used bucket = 10

Example: b=4 bits, i=2, 2 keys/bucket



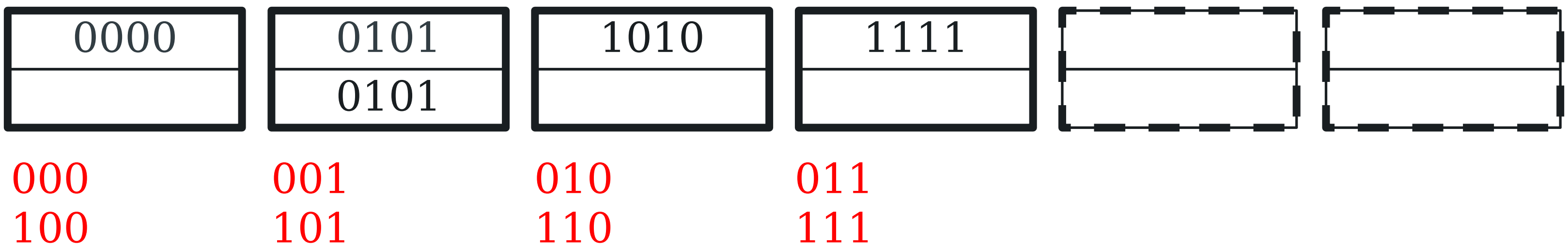
m = max used bucket = **11**

Example: further growth



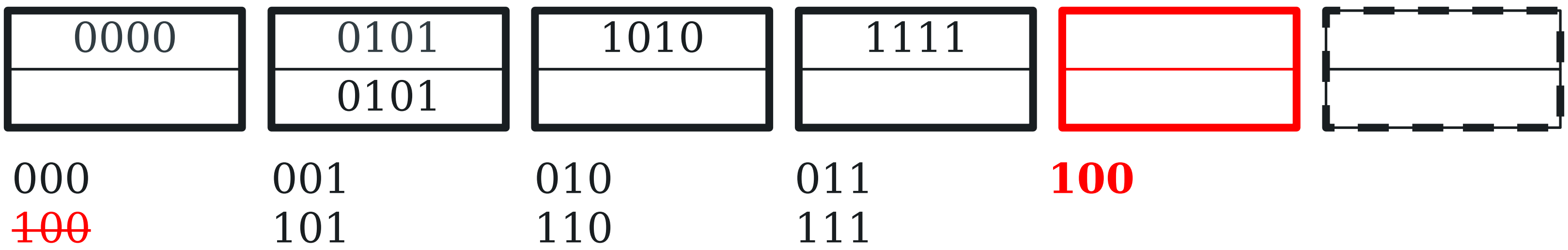
m = max used bucket = 11

Example: i=3



m = max used bucket = 11

Example: i=3



m = max used bucket = **100**

Example: i=3

0000	~~0101~~	1010	1111		**0101**
	~~0101~~				**0101**
000	001 ~~101~~	010 110	011 111	100	**101**

m = max used bucket = **101**

Example: i=3

0000		1010	1111		0101
					0101
000	001	010 110	011 111	100	101

m = max used bucket = 101

When do we expand file?

Keep track of utilisation

- U = #used slots / total #slots

If U > threshold, then increase m (and maybe i)

Linear Hashing

Pro

- Can handle growing files
  - with less wasted space
  - with no full reorganizations
- No indirection like extensible hashing

Con

- Can still have overflow chains

Indexing versus Hashing

Indexing vs Hashing

Hashing good for probes given a key:

SELECT ...
FROM R
WHERE R.A = 5

Indexing vs Hashing

Indexing (Including B-trees) good for *range searches*:

SELECT
FROM R
WHERE R.A > 5

Further Reading

Further Reading

- Chapter 14 of Garcia-Molina et al
  - Sections 14.1-14.3

- Next week: Multi-key Indexing
  - Sections 14.4-14.7