VE401 Assignment

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Exercise 1. Elementary Probability

Solution. We use Cardano's principle to get the probability. The number of ways to pick 120 people from 2000 individuals is

$$n_1 = \frac{2000!}{120! \times (2000 - 120)!}$$

The number of ways that me and my friend are both chosen is equal to the number of ways to choose 118 people from 1998 individuals, which is

$$n_2 = \frac{1998!}{118! \times (1998 - 118)!}$$

Therefore the probability that me and my friend will both be chosen is

$$\frac{n_2}{n_1} = \frac{\frac{1998!}{118! \times (1998-118)!}}{\frac{2000!}{120! \times (2000-120)!}} = 0.357\%$$

□

Exercise 2. Some Routine Calculations

i) **Proof.** Since $A \subset B$, $B = A + B \backslash A$. Note that $A \cap B \backslash A = \emptyset$. Thus $P[B] = P[A] + P[B \backslash A] \geq P[A]$. Therefore $P[A] \leq P[B]$. □

ii) **Proof.** Since A and B are independent, we have $P[A \cap B] = P[A]P[B]$. We know that $P[A]P[B] > 0$ so $P[A \cap B] > 0$. Thus $P[A \cap B] = \frac{|A \cap B|}{|S|} > 0$, which means that $|A \cap B| > 0$. Therefore, $A \cap B \neq \emptyset$ and hence they are not mutually exclusive. □

iii) **Proof.** First we have two trivial equations:

$$P[A] = P[A \backslash (A \cap B)] + P[A \cap B]$$

$$P[B] = P[B \backslash (A \cap B)] + P[A \cap B]$$

We also have

$$P[A \cup B] = P[A \backslash (A \cap B)] + P[B \backslash (A \cap B)] + P[A \cap B]$$

. Therefore,

$$P[A \cup B] = P[A] - P[A \cap B] + P[B] - P[A \cap B] + P[A \cap B] = P[A] + P[B] - P[A \cap B]$$

□

Exercise 3. D'Alembert's Coins

i) **Solution.** No, it is not possible. If the coin is fair, we know that

$$P[two\ heads] = P[no\ heads] < P[onehead]$$

. Now if the coin is biased, for that coin $P[head] \neq P[nohead]$. And hence if it is tossed twice,

$$P[two\ heads] \neq P[no\ heads]$$

. Therefore, even though the coin can be biased, the three outcomes cannot have the same probability. □

ii) **Solution.** No, it is not possible as well. Denote one coin as A, with $P[A, head] = a$. We know that $P[A, head] + P[A, tail] = 1$ so $P[A, tail] = 1 - a$. Similarly, we also have coin B with b and $1 - b$. Now, if the three outcomes have same probability, then

$$a * b = (1 - a) * (1 - b) = 1/3$$

. From here we get that $a + b = 1$. Now we calculate

$$a * b = a * (1 - a) = \frac{1}{3}$$

which has no real solution. Therefore, it is impossible to make the coins so that D'Alembert's claim is true. $\square$

Exercise 4. Independence

i) **Solution.** Denotion of events:
P1: a participant from the first group is chosen;
P2: a participant from the second group is chosen;
A: the participant replies "yes" to the second question;
Now we list the known probabilities: $P[P1] = 50\%$, $P[P2] = 50\%$, $P[A|P1] = 17\%$, $P[A|P2] = 3\%$. We want to know the total probability of $P[A]$.

$$P[A] = P[A|P1] * P[P1] + P[A|P2] * P[P2] = 17\% * 50\% + 3\% * 50\% = 10\% \tag{1}$$

Therefore, this probability is 10%. $\square$

ii) **Solution.** No. We have $P[A|P1] > P[A]$. Therefore it is not independent. $\square$