Because we are wise, we know the m.g.f. of normal distribution is

where D is the variance.

We send it in matrix terms. Let X=(x1,x2) 。Mx(t)=exp(uTti-0.5tT\*D\*t)，D is(σ1^2,σ2^2)，A is the transform matrix for Y=AX，u is matrix of expression。

We already know x follows bi-normal distribution for fx1x2(X1,X2)=P(x1=X1,x2=X2)=P(x1=X1)\*P(x2=X2)=fx1(x)\*fx2(x)=√(A)\*⌠e^(n^2)^Bdx1\*√(A)\*⌠e^(m^2)^Bdx2=(A)\*⌠e^(m^2+n^2)^Bdx1dx2. It’s Bi-normal distribution’s pdf when ρ=0.Since x1,x2 are independent, ρ=0 obviously. So X follows Bi-normal distribution. Y=AX

In matrix term, mY(t)=E（e^YtTi）=E（e^AXtT）=E（e^(ATt)TXi）=exp(uTATti-0.5(ATt)T\*D\*ATt)=exp((Au)Tti-0.5(ATt)T\*D\*ATt)=exp((Au)Tti-0.5(t)T\*(A\*D\*AT)\*t)=exp((u’)Tti-0.5(t)T\*(D’)\*t)

Obviously Y follows the same kind of distribution with only different parameters. Y follows bi-normal distribution.

According to 3.10.3,fx1x2(x1,x2)=A\*exp(-0.5<x-ux,∑-1x(x-ux)>)=> ,fy1y2(y1,y2)=A\*exp(-0.5<y-uy,∑-1y(y-uy)>). And fy(y)=Ae^(m^2-2ρmn+n^2)^B

with:

A=1/2piσ1σ2√(1-ρ2)

B=-0.5/(1-ρ2)

m=(y2-u2)/σ2

n=(y1−µ1)/σ1