

Forecasting the yield curve: the role of conditional heteroscedasticity and macro-economic factors

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Agenda

1. Introduction
2. Model
3. Data set
4. Statistical and Economic Evaluation
5. Final Remarks

- Market expectations are widely used in the literature and by market agents to estimate economic cycles and forecast macro variables;
- The results for forecasting the yield curve, including macro variables, are well-known in the literature;
- We propose to follow Fernandes and Vieira (2019) and Vieira et al. (2017), however with the inclusion of three backward- and forward-looking macro variables and **conditional heteroscedasticity** as per Koopman et al. (2010);
- The results suggest that time-varying volatility does not deliver results in the economic evaluation; however, it is crucial in the economic evaluation.

Model: Equations

- Dynamic Nelson-Siegel model (DNS):

$$y_{i,t}(\tau_i) = \beta_{1,t} + \beta_{2,t} \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_{3,t} \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i} \right)$$

- State-space representation

$$y_t = \Lambda(\lambda_t) \beta_t + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon)$$

$$\beta_{t+1} = \mu + \Phi(\beta_t - \mu) + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \Sigma_\eta)$$

- Feature: GARCH by Koopman *et al.* (2010)

$$\varepsilon_t = \Gamma_\varepsilon \varepsilon_t^* + \varepsilon_t^+, \quad \varepsilon_t^* \sim \text{NID}(0, h_t), \quad \varepsilon_t^+ \sim \text{NID}(0, \Sigma_\varepsilon^+)$$

$$h_{t+1} = \gamma_0 + \gamma_1 \varepsilon_t^{*2} + \gamma_2 h_t,$$

$$\Sigma_\varepsilon(h_t) = h_t \Gamma_\varepsilon \Gamma_\varepsilon' + \Sigma_\varepsilon^+,$$

$$\Lambda(\lambda) \Sigma_\beta \Lambda(\lambda)' + \Sigma_\varepsilon(h_t)$$

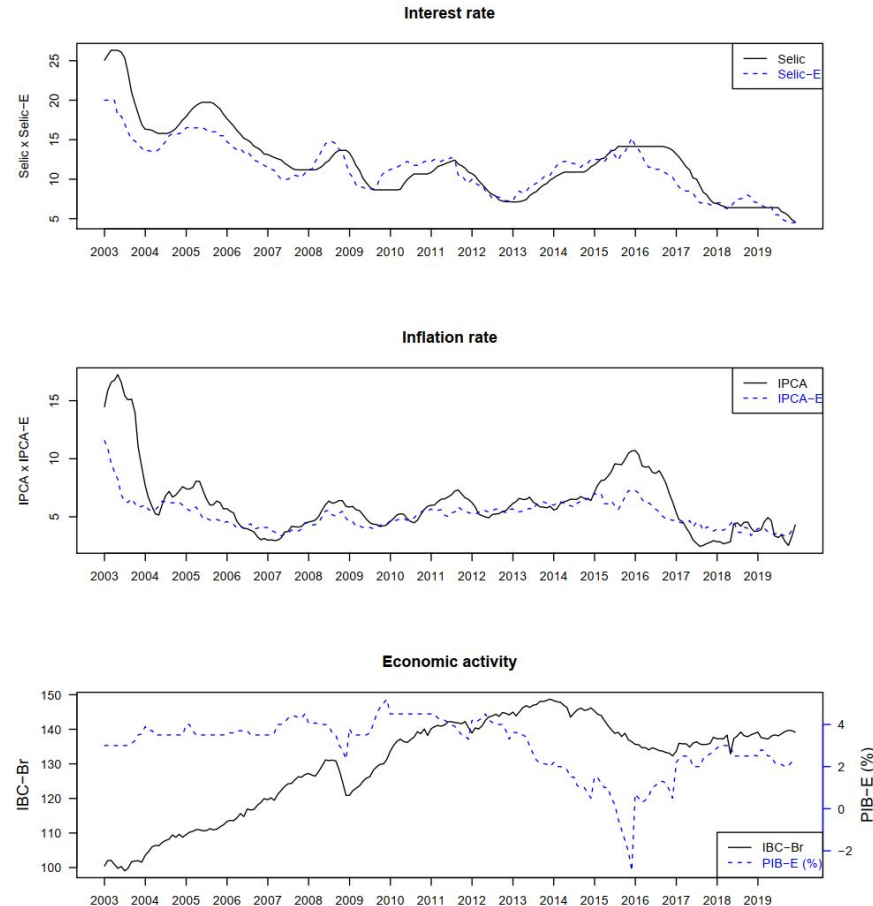
Model: Competitors

Table 1: Models and Features

#	Models	Volatility	Macro
1	DNS	constant	-
2	DNS-Macro	constant	Backward
3	DNS-MacroE1	constant	Forward
4	DNS-GARCH	time-varying	-
5	DNS-GARCH-Macro	time-varying	Backward
6	DNS-GARCH-MacroE1	time-varying	Forward

- **Yield Curves:**
January 2003 through December 2019
- **Backward-looking Macroeconomic Variables:**
IPCA, SELIC, and IBC-Br
- **Forward-looking Macroeconomic Variables:**
IPCA (1 year), SELIC (between 0.5 - 1.5 year), and GDP (between 0.5 - 1.5 year).
- **Statistical Evaluation:**
RMSE, GW, MCS, CSE
- **Economic Evaluation:**
Using forecast mean and conditional variance of returns to assess an investor's utility

Data set: Backward- and Forward-Looking Variables



Data set: Yield Curves

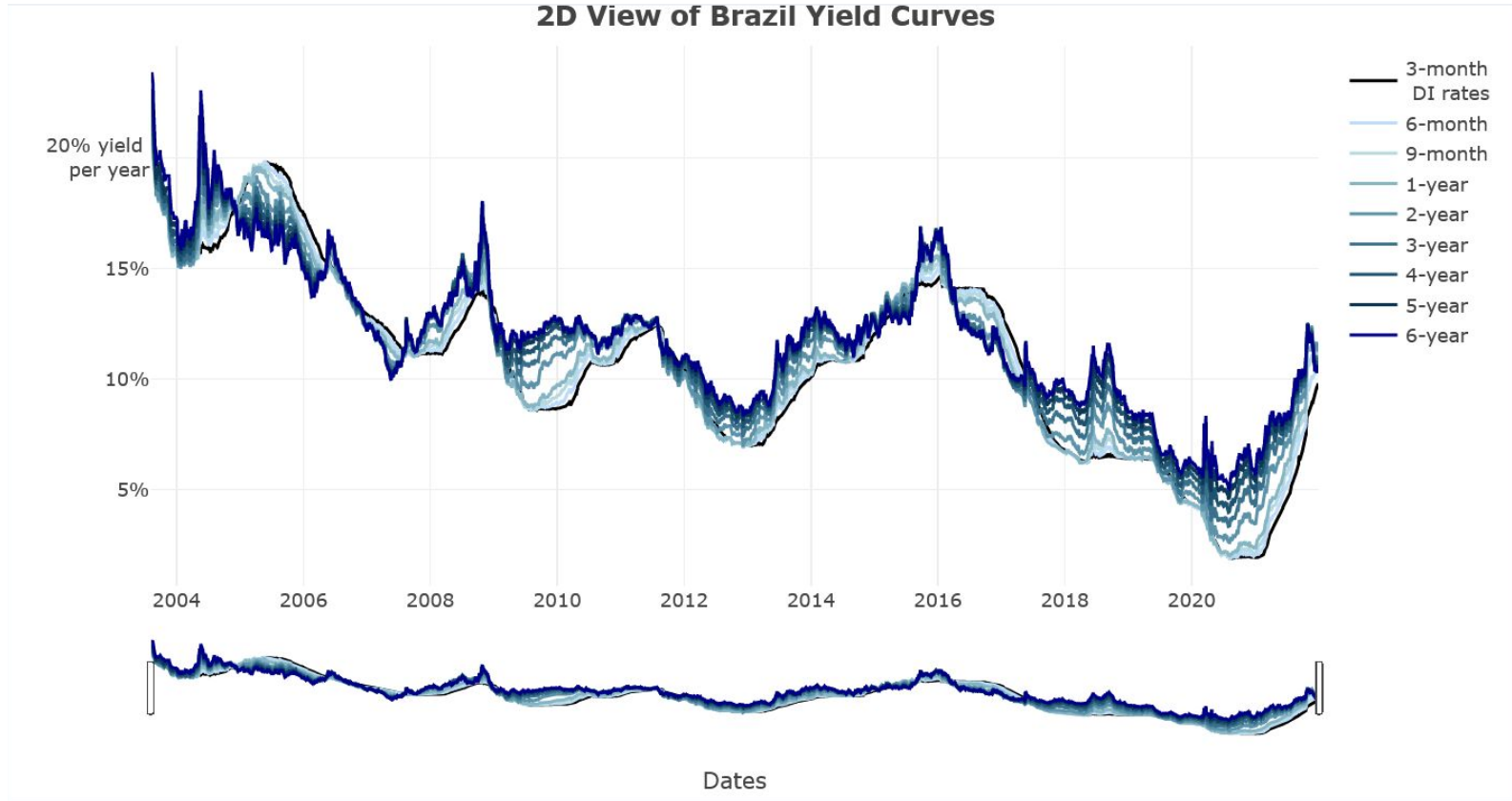


Table 2: RMSFE Against to RW (1- and 3-month ahead)

Model	Maturity													
	M3	M6	M9	M12	M15	M18	M21	M24	M27	M30	M36	M42	M48	M60
Panel A: 1-month ahead forecasts														
RW	41.26	39.69	42.97	47.79	51.04	54.47	56.47	57.74	58.80	60.69	61.63	63.07	64.53	66.56
DNS	0.933	0.992	0.999	0.995	0.982	0.972	0.971	0.963	0.960	0.959	0.967	0.966	0.972	0.971
DNS – Macro	0.940	1.010	0.989	0.976	0.970	0.971	0.977	0.976	0.978	0.978	0.992	0.991	0.996	0.995
DNS – MacroE1	0.881	0.892*	0.978	1.009	1.010	1.009	1.018	1.016	1.020	1.019	1.033	1.034	1.049	1.063
DNS – GARCH	0.964	0.978	1.040	1.056	1.055	1.044	1.050	1.048	1.048	1.042	1.045	1.044	1.047	1.045
DNS – GARCH – Macro	0.940	1.010	0.989	0.976	0.970	0.971	0.976	0.976	0.978	0.978	0.992	0.991	0.996	0.995
DNS – GARCH – MacroE1	0.881	0.892*	0.979	1.009	1.010	1.009	1.018	1.016	1.020	1.019	1.034	1.034	1.049	1.063
Panel B: 3-months ahead forecasts														
RW	90.21	90.41	94.52	99.69	103.88	108.63	111.44	112.61	114.64	117.00	120.34	121.97	124.01	125.32
DNS	0.824	0.892	0.934	0.959	0.953	0.941	0.932	0.925	0.914	0.905	0.887*	0.880*	0.878*	0.878*
DNS – Macro	0.795	0.877	0.913	0.938	0.937	0.935	0.936	0.936	0.932	0.928	0.920*	0.920*	0.922*	0.929*
DNS – MacroE1	0.668	0.800	0.877	0.93	0.942	0.945	0.953	0.955	0.951	0.947	0.934	0.935	0.939	0.952
DNS – GARCH	0.818	0.929	0.977	0.997	0.999	0.993	0.992	0.996	0.992	0.985	0.972	0.974	0.974	0.981
DNS – GARCH – Macro	0.795	0.877	0.913	0.938	0.937	0.935	0.936	0.936	0.932	0.928	0.920*	0.920*	0.922*	0.929*
DNS – GARCH – MacroE1	0.668	0.800*	0.877	0.930	0.942	0.945	0.953	0.956	0.951	0.947	0.934	0.935	0.939	0.952

Table 3: RMSFE Against to RW (1- and 3-month ahead)

Model	Maturity													
	M3	M6	M9	M12	M15	M18	M21	M24	M27	M30	M36	M42	M48	M60
Panel A: 1-month ahead forecasts														
RW	41.26	39.69	42.97	47.79	51.04	54.47	56.47	57.74	58.80	60.69	61.63	63.07	64.53	66.56
DNS	0.933	0.992	0.999	0.995	0.982	0.972	0.971	0.963	0.960	0.959	0.967	0.966	0.972	0.971
DNS – Macro	0.940	1.010	0.989	0.976	0.970	0.971	0.977	0.976	0.978	0.978	0.992	0.991	0.996	0.995
DNS – MacroE1	0.881	0.892*	0.978	1.009	1.010	1.009	1.018	1.016	1.020	1.019	1.033	1.034	1.049	1.063
DNS – GARCH	0.964	0.978	1.040	1.056	1.055	1.044	1.050	1.048	1.048	1.042	1.045	1.044	1.047	1.045
DNS – GARCH – Macro	0.940	1.010	0.989	0.976	0.970	0.971	0.976	0.976	0.978	0.978	0.992	0.991	0.996	0.995
DNS – GARCH – MacroE1	0.881	0.892*	0.979	1.009	1.010	1.009	1.018	1.016	1.020	1.019	1.034	1.034	1.049	1.063
Panel B: 3-months ahead forecasts														
RW	90.21	90.41	94.52	99.69	103.88	108.63	111.44	112.61	114.64	117.00	120.34	121.97	124.01	125.32
DNS	0.824	0.892	0.934	0.959	0.953	0.941	0.932	0.925	0.914	0.905	0.887*	0.880*	0.878*	0.878*
DNS – Macro	0.795	0.877	0.913	0.938	0.937	0.935	0.936	0.936	0.932	0.928	0.920*	0.920*	0.922*	0.929*
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DNS – GARCH	0.818	0.929	0.977	0.997	0.999	0.993	0.992	0.996	0.992	0.985	0.972	0.974	0.974	0.981
DNS – GARCH – Macro	0.795	0.877	0.913	0.938	0.937	0.935	0.936	0.936	0.932	0.928	0.920*	0.920*	0.922*	0.929*
DNS – GARCH – MacroE1	0.668	0.800*	0.877	0.930	0.942	0.945	0.953	0.956	0.951	0.947	0.934	0.935	0.939	0.952

Tabela 4: RMSFE Against to RW (6-month ahead)

Model	Maturity													
	M3	M6	M9	M12	M15	M18	M21	M24	M27	M30	M36	M42	M48	M60
Panel C: 6-months ahead forecasts														
RW	168.06	166.53	167.34	169.92	172.07	174.62	176.65	177.09	178.23	180.37	184.08	186.97	187.46	188.79
DNS	0.760	0.835	0.892	0.922	0.926	0.919	0.907	0.896	0.884	0.868	0.834	0.815	0.806	0.790
DNS – Macro	0.722	0.819	0.886	0.925	0.936	0.939	0.935	0.930	0.924	0.914	0.889	0.877	0.873	0.864
DNS – MacroE1	0.669	0.782	0.858	0.906	0.922	0.928	0.929	0.926	0.923	0.914	0.885	0.873	0.867	0.86
DNS – GARCH	0.849	0.916	0.949	0.966	0.97	0.965	0.958	0.956	0.948	0.936	0.911	0.903	0.899	0.895
DNS – GARCH – Macro	0.722	0.819	0.886	0.925	0.936	0.939	0.935	0.930	0.924	0.914	0.889	0.877	0.873	0.864
DNS – GARCH – MacroE1	0.669	0.782	0.858	0.906	0.923	0.928	0.929	0.926	0.923	0.914	0.885	0.873	0.867	0.860

Tabela 5: RMSFE Against to RW (6 and 12-months ahead)

Model	Maturity													
	M3	M6	M9	M12	M15	M18	M21	M24	M27	M30	M36	M42	M48	M60
Panel C: 6-months ahead forecasts														
RW	168.06	166.53	167.34	169.92	172.07	174.62	176.65	177.09	178.23	180.37	184.08	186.97	187.46	188.79
DNS	0.760	0.835	0.892	0.922	0.926	0.919	0.907	0.896	0.884	0.868	0.834	0.815	0.806	0.790
DNS – Macro	0.722	0.819	0.886	0.925	0.936	0.939	0.935	0.930	0.924	0.914	0.889	0.877	0.873	0.864
DNS – MacroE1	0.669	0.782	0.858	0.906	0.922	0.928	0.929	0.926	0.923	0.914	0.885	0.873	0.867	0.86
DNS – GARCH	0.849	0.916	0.949	0.966	0.97	0.965	0.958	0.956	0.948	0.936	0.911	0.903	0.899	0.895
DNS – GARCH – Macro	0.722	0.819	0.886	0.925	0.936	0.939	0.935	0.930	0.924	0.914	0.889	0.877	0.873	0.864
DNS – GARCH – MacroE1	0.669	0.782	0.858	0.906	0.923	0.928	0.929	0.926	0.923	0.914	0.885	0.873	0.867	0.860
Panel D: 12-months ahead forecasts														
RW	296.67	294.33	290.84	286.67	281.94	277.09	272.77	269.43	267.11	265.25	264.99	263.29	262.12	260.45
DNS	0.823	0.875	0.914	0.940	0.951	0.954	0.951	0.941	0.929	0.918	0.884	0.861	0.844	0.819
DNS – Macro	0.830	0.924	0.991	1.036	1.059	1.072	1.073	1.068	1.057	1.0479	1.014	0.990	0.973	0.944
DNS – MacroE1	0.869	0.944	0.996	1.031	1.044	1.047	1.045	1.034	1.019	1.006	0.966	0.937	0.917	0.885
DNS – GARCH	0.894	0.923	0.951	0.969	0.976	0.982	0.977	0.970	0.959	0.951	0.924	0.909	0.899	0.886
DNS – GARCH – Macro	0.830	0.924	0.991	1.036	1.059	1.072	1.074	1.068	1.057	1.048	1.014	0.990	0.973	0.944
DNS – GARCH – MacroE1	0.869	0.944	0.996	1.031	1.044	1.048	1.045	1.034	1.019	1.006	0.966	0.938	0.917	0.885

Economic Evaluation

We transform the yields in returns and their volatility to perform the economic evaluation.

$$w_{i,t} = \left(\frac{1}{\gamma} \right) \left(\frac{\hat{r}_{t+h}^{(\tau_i)}}{\hat{\sigma}_{t+h}^{2,(\tau_i)}} \right)$$

$$\hat{r}_{t+h}^{(\tau_i)} = \tau_i y_t^{\tau_i} - (\tau_i - h) \hat{y}_{t+h}^{\tau_i - h}$$

$$\hat{\nu}_i = \hat{\mu}_i - 0.5\gamma\hat{\sigma}_i^2$$

RW is also the benchmark in economic evaluation.

Table 6: Average Utility Gains in Annualized Percent Return (1 and 3-months ahead)

Model	$\gamma = 0.1$					$\gamma = 0.5$					$\gamma = 1$					$\gamma = 5$				
	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$
<i>horizon = 1-month ahead</i>																				
DNS	197.70	2.403	-0.279	-0.901	-0.643	50.16	1.843	0.516	0.138	0.072	31.72	1.773	0.615	0.268	0.161	16.97	1.717	0.695	0.372	0.233
DNS – Macro	226.09	7.118	1.431	-0.296	-0.504	54.65	2.587	0.786	0.234	0.094	33.22	2.021	0.705	0.300	0.169	16.07	1.568	0.641	0.353	0.229
DNS – MacroE1	213.77	8.047	3.377	1.657	1.286	52.70	2.734	1.093	0.542	0.377	32.57	2.07	0.808	0.403	0.263	16.46	1.538	0.579	0.291	0.172
DNS – GARCH	449.44	34.41	12.92	6.247	3.711	89.91	6.896	2.600	1.267	0.760	44.97	3.457	1.31	0.644	0.391	9.019	0.706	0.278	0.146	0.095
DNS – GARCH – Macro	449.54	34.48	12.98	6.272	3.701	89.93	6.907	2.609	1.271	0.758	44.98	3.461	1.313	0.646	0.39	9.016	0.704	0.276	0.145	0.096
DNS – GARCH – MacroE1	449.41	34.40	12.92	6.258	3.726	89.91	6.895	2.601	1.268	0.762	44.97	3.457	1.310	0.645	0.391	9.020	0.706	0.278	0.146	0.095
<i>horizon = 3-month ahead</i>																				
DNS	38.21	7.202	4.000	2.762	2.271	13.71	2.135	1.054	0.673	0.512	10.65	1.501	0.685	0.412	0.292	8.204	0.995	0.391	0.203	0.116
DNS – Macro	54.28	10.606	5.521	3.529	2.668	16.25	2.672	1.294	0.794	0.574	11.50	1.680	0.765	0.452	0.313	7.697	0.887	0.343	0.179	0.103
DNS – MacroE1	47.23	10.08	5.635	3.899	3.122	15.14	2.590	1.312	0.852	0.646	11.13	1.653	0.771	0.472	0.337	7.919	0.904	0.339	0.167	0.089
DNS – GARCH	181.10	23.13	9.533	5.22	3.377	36.28	4.649	1.927	1.061	0.686	18.17	2.34	0.977	0.541	0.350	3.692	0.492	0.216	0.125	0.081
DNS – GARCH – Macro	181.30	23.25	9.65	5.309	3.423	36.31	4.668	1.946	1.075	0.694	18.18	2.346	0.983	0.546	0.353	3.685	0.488	0.212	0.122	0.080
DNS – GARCH – MacroE1	181.04	23.17	9.609	5.326	3.497	36.27	4.656	1.939	1.078	0.705	18.17	2.342	0.981	0.547	0.356	3.694	0.490	0.214	0.122	0.077

Table 7: Average Utility Gains in Annualized Percent Return (6 and 12-months ahead)

Model	$\gamma = 0.1$					$\gamma = 0.5$					$\gamma = 1$					$\gamma = 5$				
	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 5$
<i>horizon = 6-month ahead</i>																				
DNS	12.64	14.251	8.698	6.667	5.362	5.676	2.809	1.637	1.218	0.953	4.806	1.379	0.754	0.537	0.402	4.110	0.235	0.048	-0.008	-0.039
DNS – Macro	20.07	15.762	9.355	6.940	5.422	6.850	3.048	1.741	1.261	0.963	5.197	1.459	0.789	0.551	0.405	3.875	0.187	0.028	-0.017	-0.041
DNS – MacroE1	17.61	15.845	9.206	6.741	5.257	6.461	3.061	1.717	1.23	0.937	5.067	1.463	0.781	0.541	0.397	3.952	0.185	0.032	-0.010	-0.036
DNS – GARCH	85.76	13.442	6.843	4.953	4.096	17.22	2.682	1.344	0.947	0.753	8.654	1.337	0.657	0.447	0.335	1.800	0.261	0.107	0.046	0.001
DNS – GARCH – Macro	85.96	13.50	6.872	4.954	4.031	17.25	2.691	1.349	0.947	0.743	8.665	1.34	0.658	0.447	0.332	1.794	0.259	0.106	0.046	0.003
DNS – GARCH – MacroE1	85.69	13.576	7.027	5.190	4.380	17.21	2.703	1.373	0.985	0.798	8.650	1.344	0.666	0.459	0.350	1.803	0.256	0.101	0.039	-0.008
<i>horizon = 12-month ahead</i>																				
DNS	120.24	55.49	21.874	13.88	10.21	23.99	9.131	3.294	2.034	1.480	11.969	3.337	0.972	0.553	0.388	2.344	-1.298	-0.886	-0.632	-0.485
DNS – Macro	120.84	53.57	19.94	12.146	8.716	24.09	8.829	2.989	1.76	1.244	12.00	3.237	0.870	0.462	0.310	2.326	-1.238	-0.825	-0.577	-0.437
DNS – MacroE1	120.72	54.56	20.53	12.469	8.892	24.07	8.985	3.081	1.811	1.272	11.99	3.288	0.901	0.479	0.319	2.329	-1.269	-0.844	-0.587	-0.443
DNS – GARCH	118.88	14.42	3.330	4.670	6.527	23.78	2.648	0.366	0.579	0.898	11.89	1.176	-0.004	0.068	0.195	2.387	-0.001	-0.301	-0.341	-0.368
DNS – GARCH – Macro	119.27	14.72	3.402	4.635	6.398	23.85	2.695	0.377	0.574	0.878	11.92	1.192	-0.001	0.066	0.188	2.375	-0.011	-0.303	-0.340	-0.364
DNS – GARCH – MacroE1	118.99	15.36	4.483	6.039	8.130	23.80	2.796	0.548	0.796	1.151	11.90	1.225	0.056	0.140	0.279	2.384	-0.031	-0.337	-0.384	-0.419

Final Remarks

1. The model with **backward macroeconomic** variables outperforms in **long maturities**;
2. The model with **forward macroeconomic** variables outperforms in **short maturities**;
3. Conditional heteroscedasticity doesn't play a role in point yields forecast;
4. However, the **density forecast of returns** outperforms the benchmark model;
 - a. Mainly in short maturities and horizons.

Forecasting the yield curve: the role of conditional heteroscedasticity and macro-economic factors

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Comments:

“What events matter for exchange rate volatility?”

Igor Martins, Hedibert Freitas Lopes

Werley Cordeiro

Comments: “What events matter for exchange rate volatility?”

1. Quantifies the effect of macroeconomics events of on exchange rate volatility using high frequency currency returns;
2. Accounting for persistent stochastic volatility effects and **seasonal components** capturing time of the day patterns;
3. **Results:**
 - a. **Identify the macroeconomic events** that drives currency volatility;
 - b. **Find a connection between intraday seasonality**, trading volume and opening hours of majors markets across the globe;