1. For this part, we have to derive an expression for $I(x)$ (where I is a 1D ramp image) after filtering it by a zero-mean Gaussian filter with standard deviation σ. In other words, we convolve g with I, where g is the PDF associated with $N(0, \sigma^2)$ (the zero-mean Gaussian distribution). So, we get

$$\begin{aligned} J(x) &= (g * I)(x) \\ &= \int_{-\infty}^{\infty} g(y) I(x-y) \, \mathrm{d}y \\ &= \int_{-\infty}^{\infty} g(y) \left(c(x-y) + d \right) \, \mathrm{d}y \\ &= \underbrace{-\int_{-\infty}^{\infty} cyg(y) \, \mathrm{d}y}_{\mathcal{I}_1} + \int_{-\infty}^{\infty} (cx+d) g(y) \, \mathrm{d}y \end{aligned}$$

In the above expression for $J(x)$, we observe that $\mathcal{I}_1$ is 0 because the integrand is an odd function. Therefore, upon simplifying, we get

$$\begin{aligned} J(x) &= \int_{-\infty}^{\infty} (cx+d) g(y) \, \mathrm{d}y \\ &= (cx+d) \int_{-\infty}^{\infty} g(y) \, \mathrm{d}y \qquad (1) \\ &= cx + d \qquad (2) \end{aligned}$$

where (1) follows because c and d are constants in $\mathbb{R}$, and x is constant w.r.t. y, and (2) follows because the integral of a PDF over the entire space is equal to 1.