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Let I denote the image and M denote the mean filter. Applying the mean is equivalent to a convolution between image I and filter M.

$$f_1 = M * I \tag{1}$$

Now, we apply the filter again, leading to another convolution

$$\begin{aligned} f_2 &= M * (M * I) && (2) \\ &= (M * M) * I && \text{(associativity of convolution)} \quad (3) \\ & && (4) \end{aligned}$$

Hence, f_k is given by

$$\begin{aligned} f_k &= M * f_{k-1} && (5) \\ &= M * (\underbrace{M * M \cdots M * M}_{k-1\ M\text{'s}} * I) && (6) \\ f_k &= (\underbrace{M * M \cdots M * M}_{k\ M\text{'s}}) * I && (7) \end{aligned}$$

$$\text{Now, let } N = \underbrace{M * M \cdots M * M}_{k\ M\text{'s}} \Rightarrow f_k = N * I \tag{8}$$

Thus, N is a filter that gives the same result when applied to I as a mean filter when applied to I, k times.
Note that if k is large enough, we can apply the Central Limit Theorem which states that if you have a bunch of distributions f_i and you convolve them all together into a distribution $F := f_1 * f_2 * f_3 * \cdots f_k$, then the larger k is, the more F will resemble a Gaussian distribution. Hence in the limit, the above equation is equivalent to convolving the the initial distribution with a Gaussian.

$$\lim_{k \to \infty} f_k = I * \mathcal{G} \tag{9}$$