Q3) DFT of $f(x,y)$ of size $W_1 \times W_2$ is

$$F_d(\mu,\nu) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-j2\pi\left(\frac{\mu x}{W_1}+\frac{\nu y}{W_2}\right)} \quad \text{--- (1)}$$

a) To show: $F_d^*(\mu,\nu) = F_d(-\mu,-\nu)$

LHS

$$\Rightarrow F_d^*(\mu,\nu) = \left(\frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-j2\pi\left(\frac{\mu x}{W_1}+\frac{\nu y}{W_2}\right)}\right)^*$$

(By (1))

(As $f(x,y)$, $W_1, W_2 \in \mathbb{R}$)

$$= \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y) \left(e^{-j2\pi\left(\frac{\mu x}{W_1}+\frac{\nu y}{W_2}\right)}\right)^*$$

(As $(e^{+jx})^* = e^{-jx}$)

$$= \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-\left(-j2\pi\left(\frac{\mu x}{W_1}+\frac{\nu y}{W_2}\right)\right)}$$

$$= \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-j2\pi\left(\frac{-\mu x}{W_1}+\frac{-\nu y}{W_2}\right)} \quad \text{--- (*)}$$

$$= F_d(-\mu, -\nu) = \text{RHS}$$

LHS = RHS

Hence Proved!

b) To show: $f(x,y)$ is real & even $\Rightarrow F_d(\mu,\nu)$ is real & even

$(f(x,y) = f(-x,-y))$

we know that $F_d^*(\mu,\nu) = F_d(-\mu,-\nu)$ (using a) & ⊛)

$$\Rightarrow F_d^*(\mu,\nu) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-j2\pi\left(\frac{-\mu x}{W_1} + \frac{-\nu y}{W_2}\right)}$$

$$= \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{+j2\pi\left(\frac{-\mu(-x)}{W_1} + \frac{(-\nu)(-y)}{W_2}\right)}$$

Now substitute $x \to -x$
$y \to -y$

$$= \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(-x,-y)\, e^{j2\pi\left(\frac{-\mu x}{W_1} + \frac{-\nu y}{W_2}\right)}$$

(As $f(x,y) = f(-x,-y)$)
$$= \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-j2\pi\left(\frac{\mu x}{W_1} + \frac{\nu y}{W_2}\right)}$$

$$= F_d(\mu,\nu) \quad \forall \mu,\nu$$

As $F_d^*(\mu,\nu) = F_d(\mu,\nu) \Rightarrow$ $\underline{F_d(\mu,\nu) \text{ is real}}$

Now, $F_d(\mu,\nu) = F_d^*(\mu,\nu) = F_d(-\mu,-\nu)$ (using a))

$\Rightarrow F_d(\mu,\nu) = F_d(-\mu,-\nu) \Rightarrow$ $\underline{F_d(\mu,\nu) \text{ is even}}$

Hence Proved!