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Let I denote the image and M denote the mean filter. Applying the mean is equivalent to a convolution between image I and filter M.

$$f_1 = I * M \tag{1}$$

Now, we apply the filter again, leading to another convolution

$$\begin{align} f_2 &= (I * M) * M \tag{2} \\ &= I * (M * M) \tag{3} \\ \tag{4} \end{align}$$

Hence f_k is given by

$$f_k = I * (M * M * ...k - times) \tag{5}$$

If k is large enough, we can apply the Central Limit Theorem which states that if you have a bunch of distributions f_i and you convolve them all together into a distribution $F := f_1 * f_{@} * f_3 * ..f_k$, then the larger k is, the more F will resemble a Gaussian distribution. Hence the above equation is equivalent to convolving the the initial distribution with a Gaussian.

$$f_k = I * \mathcal{G} \tag{6}$$

Also, it can be noted that if the above process is repeated multiple times, it would be equivalent to taking a convolution of the image with a Gaussian multiple times. Which is equivalent to taking a convolution with a Gaussian of a larger σ. Hence the resultant image will ultimately lead to having the same average intensity at each pixel.