3

(c) Here are the graphs of values of the NCC, JE, and QMI as a function of θ:

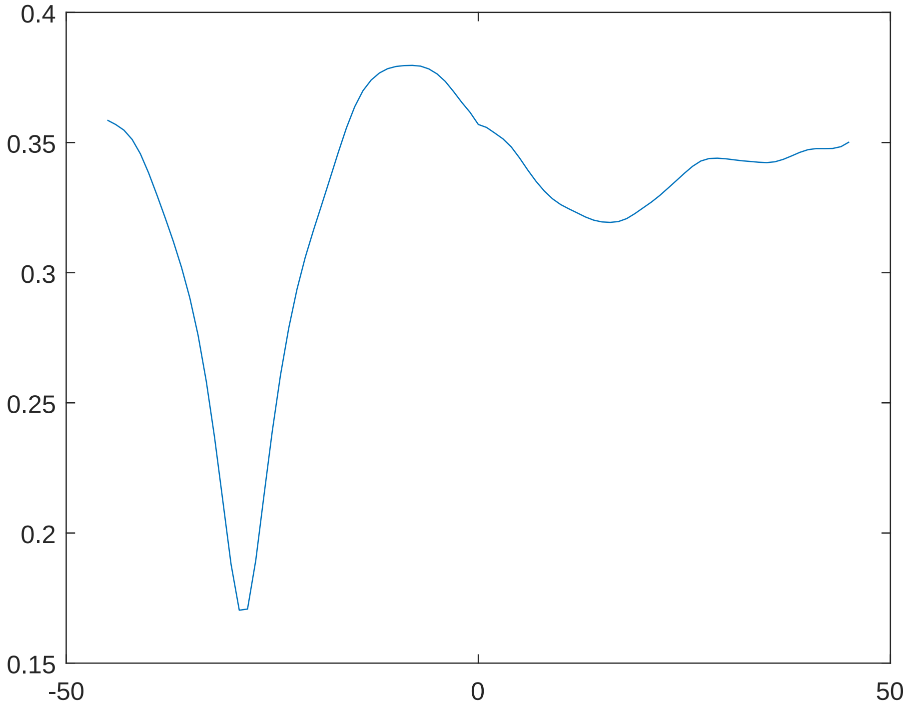

Figure 1: The graph of the NCC of $J1$ and $J4$ as a function of θ.

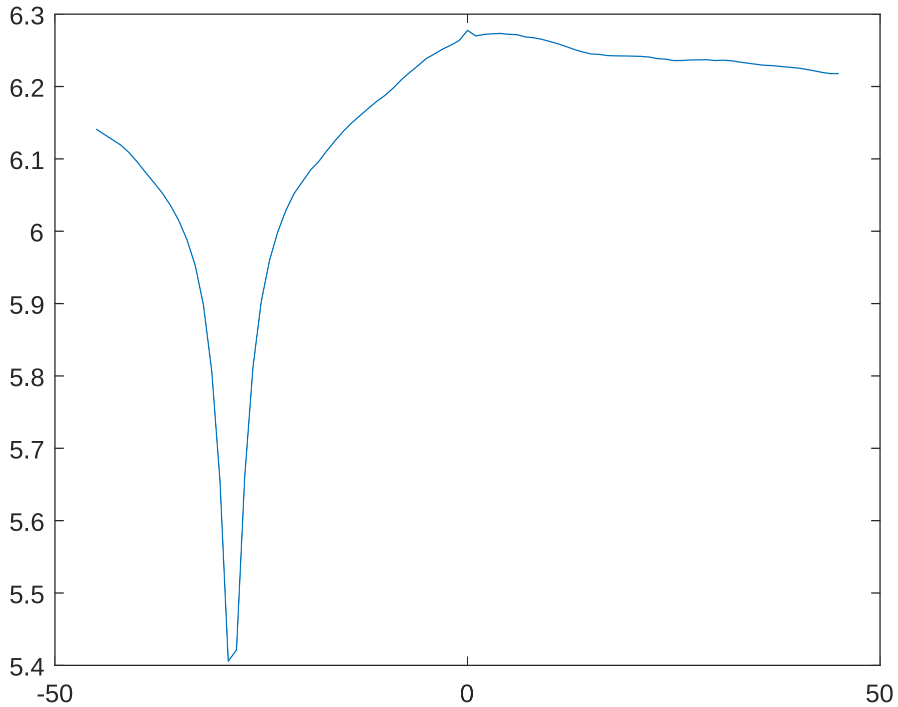

Figure 2: The graph of the JE of $J1$ and $J4$ as a function of θ.

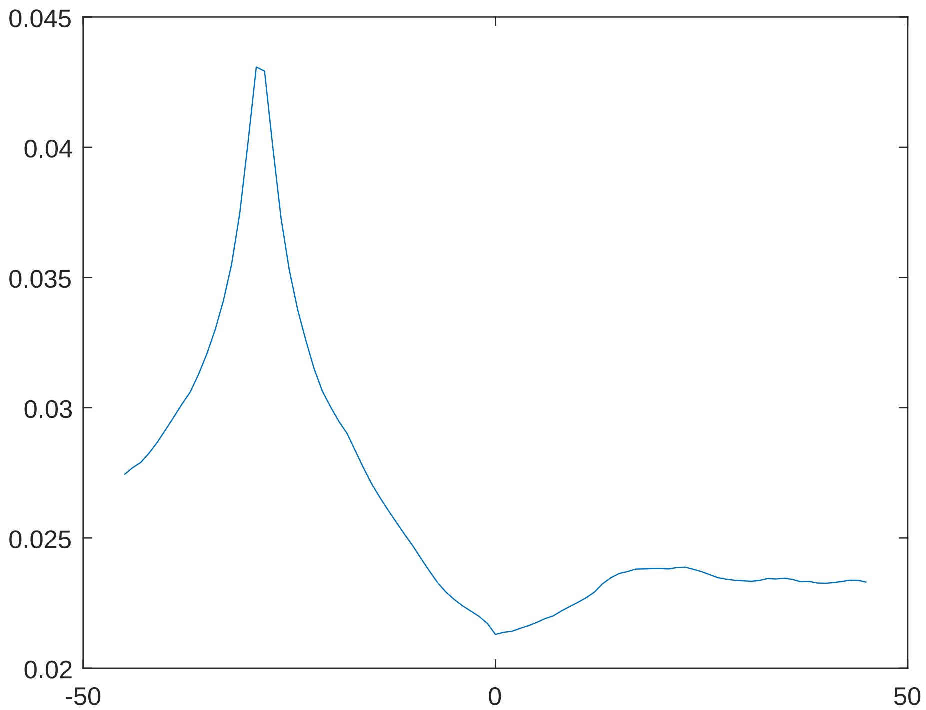

Figure 3: The graph of the QMI of $J1$ and $J4$ as a function of θ.

(d) Based on the Joint Entropy graph and the QMI graph, the optimal rotation is $\theta = -29°$.

For the NCC graph, the argmax is $-8°$, but the argmin is $-29°$. Ideally, we check the argmax for NCC. However, in this case, the argmax result is nowhere near the correct rotation, and the argmin, surprisingly, is the optimal rotation in this case. It also agrees with the answer generated by the JE and QMI algorithms.

Our theory for this odd behavior is the fact that images `T1.jpg` and `T2.jpg` have differing intensities at physically corresponding pixels, which is throwing off the NCC and for this reason, the rotation for which they are the least correlated (intensity-wise) is the optimal rotation for image alignment.

(e) Here is the joint histogram between $J1$ and $J4$:

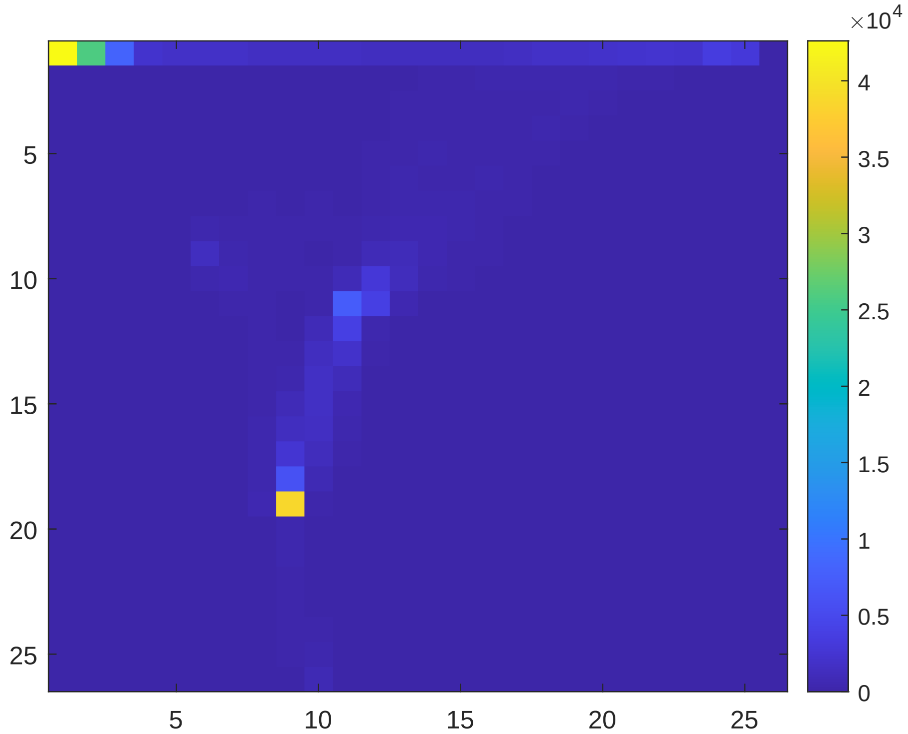

Figure 4: The joint histogram between images $J1$ and $J4$.

(f) The intuition behind Quadratic Mutual Information (QMI)is simple: we want to quantify the amount of 'mutual information' we have between the two images. That is, given the pixel intensity data in image $J1$, how much uncertainty (or dually, certainty) do we have about the corresponding pixel intensity data in image $J4$?

The reason for using quadratic terms is the same as the reason we use it for least squares: We don't want the errors to cancel out.

If we have high mutual information, that means that given some pixel intensity data in image $J1$, we can say with (relatively) high certainty what the corresponding pixel intensity data is in image $J4$.

The QMI is at its lowest when the random variables I_1 and I_2 are statistically independent. If we assume that I_1 and I_2 are statistically independent, then $p_{I_1 I_2}(i_1, i_2) = p_{I_1}(i_1) p_{I_2}(i_2)$, so we get

$$\begin{aligned}
\mathrm{QMI} &= \sum_{i_1} \sum_{i_2} \Big(p_{I_1 I_2}(i_1, i_2) - p_{I_1}(i_1) p_{I_2}(i_2) \Big)^2 \\
&= \sum_{i_1} \sum_{i_2} \Big(p_{I_1}(i_1) p_{I_2}(i_2) - p_{I_1}(i_1) p_{I_2}(i_2) \Big)^2 \\
&= \sum_{i_1} \sum_{i_2} 0 \\
&= 0
\end{aligned}$$

The intuitive reason for this is because I_1 and I_2 are independent, the pixel intensity data in the first image has no bearing on the pixel intensity data in the second image, so we have, loosely speaking, 0 ‘mutual information’.