1 Approach

We are assuming the images are 2-D and carry out Fourier analysis in continuous domain. For given equations,

$$\begin{aligned} g_1(x,y) &= f_1(x,y) + h_2(x,y) * f_2(x,y) \\ g_2(x,y) &= h_1(x,y) * f_1(x,y) + f_2(x,y) \end{aligned} \tag{1}$$

Let F_i, G_i, H_i be the Fourier transformations of f_i, g_i, h_i respectively. After applying Fourier transformation on both sides to 1, we get

$$G_1(\mu,\nu) = F_1(\mu,\nu) + H_2(\mu,\nu)F_2(\mu,\nu) \tag{2a}$$
$$G_2(\mu,\nu) = H_1(\mu,\nu)F_1(\mu,\nu) + F_2(\mu,\nu) \tag{2b}$$

Now, we can solve for F_1, F_2 by multiplying 2a by H_1 and 2b by H_2 and then subtracting these equations from 1

$$\begin{aligned} H_1G_1 &= H_1F_1 + H_1H_2F_2 \\ H_2G_2 &= H_2H_1F_1 + H_2F_2 \end{aligned} \Rightarrow \begin{aligned} H_1G_1 - (G_2) &= H_1F_1 + H_1H_2F_2 - (H_1F_1 + F_2) \\ H_2G_2 - (G_1) &= H_2H_1F_1 + H_2F_2 - (F_1 + H_2F_2) \end{aligned} \Rightarrow \begin{aligned} H_1G_1 - G_2 &= H_1H_2F_2 - F_2 \\ H_2G_2 - G_1 &= H_2H_1F_1 - F_1 \end{aligned} \Rightarrow \begin{aligned} H_1G_1 - G_2 &= F_2(H_1H_2 - 1) \\ H_2G_2 - G_1 &= F_1(H_2H_1 - 1) \end{aligned} \tag{3}$$

Now, we can get f_1, f_2 by taking inverse Fourier transform of F_1, F_2 respectively.

$$\begin{aligned} f_1(x,y) &= \mathcal{F}^{-1}\left(\frac{H_1G_1 - G_2}{H_1H_2 - 1}\right)(\mu,\nu) \\ f_2(x,y) &= \mathcal{F}^{-1}\left(\frac{H_2G_2 - G_1}{H_1H_2 - 1}\right)(\mu,\nu) \end{aligned} \quad \text{where} \quad \left(\begin{aligned} F_1 &= \frac{H_1G_1 - G_2}{H_1H_2 - 1} \\ F_2 &= \frac{H_2G_2 - G_1}{H_1H_2 - 1} \end{aligned} \quad \text{and} \quad \begin{aligned} f_1(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_1(\mu,\nu)e^{j2\pi(\mu x + \nu y)}\, \mathrm{d}\mu \\ f_2(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2(\mu,\nu)e^{j2\pi(\mu x + \nu y)}\, \mathrm{d}\mu \end{aligned} \right) \tag{4}$$

2 Formula