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0.1 MATLAB Plot

Below, we can see the original image that was described in the question:

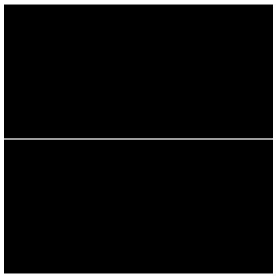

Figure 1: The original 201×201 black image with a white row in the middle.

A plot of the logarithm of the modulus of the DFT of the image can be found below:

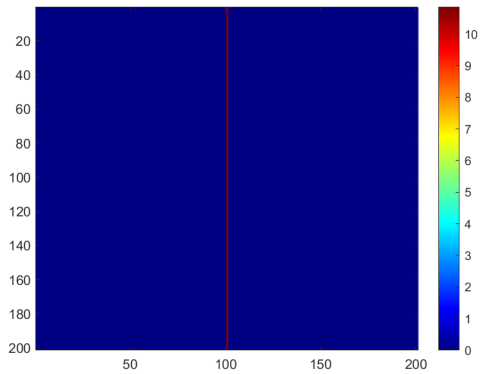

Figure 2: The log plot of the modulus of the DFT of Figure (1).

0.2 Analytical Derivation

We know that the formula for the DFT of a 2D signal is as follows:

$$F_d(u,v) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y) \exp\left(-i2\pi\left(\frac{ux}{W_1} + \frac{vy}{W_2}\right)\right)$$

Since we know that in our image, $W_1 = W_2 = 201$, we can substitute this into the formula:

$$F_d(u,v) = \frac{1}{201}\sum_{x=0}^{200}\sum_{y=0}^{200} f(x,y)\exp\left(-i2\pi\left(\frac{ux}{201}+\frac{vy}{201}\right)\right) \tag{1}$$

In our case, our 2D signal is the image shown in Figure 1, and its definition is

$$f(x,y) = \begin{cases} 0, y \in \{0,1,2,\dots,200\} \text{ and } y \neq 100, & x \in \{0,1,2,\dots,200\} \\ 1, y = 100, & x \in \{0,1,2,\dots,200\} \end{cases}$$

Observe that we normalized the image intensities, so the range of intensity values is $[0,1]$, instead of the usual $\{0,1,\dots,255\}$. Moreover, due to the indices starting from 0 instead of 1 (like in MATLAB), the index of the white pixel row is 100 instead of 101.

Since most of the image consists of 0-intensity pixels, Equation (1) simplifies to

$$\begin{aligned} F_d(u,v) &= \frac{1}{201}\sum_{x=0}^{200} f(x,100)\exp\left(\frac{-2\pi i(ux+100v)}{201}\right) \\ &= \frac{1}{201}\sum_{x=0}^{200}\exp\left(\frac{-2\pi i(ux+100v)}{201}\right) \quad \square \end{aligned}$$

(Note that x here corresponds to the x-axis and y here corresponds to the y-axis.)

0.3 Intuition

Intuitively, we can explain why we get the image we do in Figure 1. The 2D DFT is equivalent to first doing a DFT of each column, and then doing the DFT over the rows.

Each column is essentially a delta function, so its Fourier Transform is a constant matrix. Then, once we compute the row-wise Fourier Transform of that again, we get a delta function once again on each row.