1 Approach

We are assuming the images are 2-D and carry out Fourier analysis in continuous domain. For given equations,

$$\begin{aligned} g_1(x,y) &= f_1(x,y) + h_2(x,y) * f_2(x,y) \\ g_2(x,y) &= h_1(x,y) * f_1(x,y) + f_2(x,y) \end{aligned} \tag{1}$$

Let F_i, G_i, H_i be the Fourier transformations of f_i, g_i, h_i respectively. After applying Fourier transformation on both sides to 1, we get

$$G_1(\mu,\nu) = F_1(\mu,\nu) + H_2(\mu,\nu)F_2(\mu,\nu) \tag{2a}$$
$$G_2(\mu,\nu) = H_1(\mu,\nu)F_1(\mu,\nu) + F_2(\mu,\nu) \tag{2b}$$

Now, we can solve for F_1, F_2 by multiplying 2a by H_1 and 2b by H_2 then subtracting these equations from 1 and rearranging the terms

$$\begin{aligned} H_1G_1 &= H_1F_1 + H_1H_2F_2 \\ H_2G_2 &= H_2H_1F_1 + H_2F_2 \end{aligned} \Rightarrow \begin{aligned} H_1G_1 - (G_2) &= H_1F_1 + H_1H_2F_2 - (H_1F_1 + F_2) \\ H_2G_2 - (G_1) &= H_2H_1F_1 + H_2F_2 - (F_1 + H_2F_2) \end{aligned} \Rightarrow \begin{aligned} H_1G_1 - G_2 &= H_1H_2F_2 - F_2 \\ H_2G_2 - G_1 &= H_2H_1F_1 - F_1 \end{aligned} \Rightarrow \begin{aligned} H_1G_1 - G_2 &= F_2(H_1H_2 - 1) \\ H_2G_2 - G_1 &= F_1(H_2H_1 - 1) \end{aligned} \tag{3}$$

Now, we can get f_1, f_2 by taking inverse Fourier transform of F_1, F_2 respectively.

$$\begin{aligned} f_1(x,y) &= \mathcal{F}^{-1}\left(\frac{H_1G_1 - G_2}{H_1H_2 - 1}\right)(\mu,\nu) \\ f_2(x,y) &= \mathcal{F}^{-1}\left(\frac{H_2G_2 - G_1}{H_1H_2 - 1}\right)(\mu,\nu) \end{aligned} \quad \text{where} \quad \left(\begin{aligned} F_1(\mu,\nu) &= \frac{H_1(\mu,\nu)G_1(\mu,\nu) - G_2(\mu,\nu)}{H_1(\mu,\nu)H_2(\mu,\nu) - 1} \\ F_2(\mu,\nu) &= \frac{H_2(\mu,\nu)G_2(\mu,\nu) - G_1(\mu,\nu)}{H_1(\mu,\nu)H_2(\mu,\nu) - 1} \end{aligned} \quad \text{and} \quad \begin{aligned} f_1(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_1(\mu,\nu)e^{j2\pi(\mu x + \nu y)}\,\mathrm{d}\mu\,\mathrm{d}\nu \\ f_2(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_2(\mu,\nu)e^{j2\pi(\mu x + \nu y)}\,\mathrm{d}\mu\,\mathrm{d}\nu \end{aligned} \right) \tag{4}$$

2 Problem

An issue can arise when the denominator in 4 is zero, i.e., $H_1(\mu,\nu)H_2(\mu,\nu) = 1$ for some (μ,ν) . In this section we show that this is indeed possible. Since the blur kernels preserves the brightness of the image, we can assume they are normalized, i.e.,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1 \quad \text{and} \quad |H(\mu,\nu)| \leq 1\ \forall(\mu,\nu) \tag{5}$$

Under this assumption, we can see that $H_1(0,0) = H_2(0,0) = 1$ as

$$
\begin{aligned}
H(\mu,\nu) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)e^{-j2\pi(\mu x+\nu y)}\,\mathrm{d}x\,\mathrm{d}y \quad \Rightarrow \quad H(0,0) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)e^{-j2\pi(0x+0y)}\,\mathrm{d}x\,\mathrm{d}y \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)e^{0}\,\mathrm{d}x\,\mathrm{d}y \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,\mathrm{d}x\,\mathrm{d}y \\
H(0,0) &= 1
\end{aligned}
\tag{6}
$$

So at $(\mu,\nu) = (0,0)$, both F_1, F_2 are undefined due to division by zero.
Hence, the formula given in 1 isn't foolproof.