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We are assuming that I and J are of same dimensions. Also, we assume that in $I + J$ corresponding pixel values of each image gets added directly. There is no after-scaling to compensate when the intensities go out of range.

Let I, J, K be the random variables for these three images respectively. Now, for each location

$$\begin{aligned}
\mathsf{Prob}_K(k) &= \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \mathbb{1}_{i+j=k} \cdot \mathsf{Prob}_{IJ}(i,j) \\
\mathsf{Prob}_K(k) &= \sum_{i=-\infty}^{\infty} \mathsf{Prob}_{IJ}(i,k-i) && \text{(since } \mathbb{1}_{i+j=k} = 0 \text{ for} j \neq k-i\text{)}
\end{aligned} \tag{1}$$

Hence the PMF of image $I + J$ for each location is given by $\sum_{i=-\infty}^{\infty} \mathsf{Prob}_{IJ}(i, k-i)$.

Now, we can see that there is a presence of i and $k - i$ akin to a convolution. In fact, we can make this idea more robust if both the distributions $p_I(i)$ and $p_J(j)$ were independent. Then,

$$\begin{aligned}
\mathsf{Prob}_K(k) &= \sum_{i=-\infty}^{\infty} \mathsf{Prob}_I(i) \cdot \mathsf{Prob}_J(k-i) \\
\mathsf{Prob}_K(k) &= p_I(i) * p_J(j) && \text{(definition of convolution)}
\end{aligned} \tag{2}$$

Hence the PMF of image $I+J$ for each location is given by $p_I(i)*p_J(j)$ when the distributions are independent. This is exaclty the convolution operation as studied in class.