3

Let us assume that we have 2 discrete signals $f(x, y)$ and $g(x, y)$. The size of both is $W_1 \times W_2$. If the sizes of both signals are not the same, we can employ zero-padding to make them of the same size.
The 2D discrete convolution is defined as:

$$f * g(u, v) = \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x, y) g(u - x, v - y)$$

The 2D Fourier Transform of a discrete signal is given by:

$$\mathscr{F}(f)(u, v) = \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x, y) e^{-2\pi j(\frac{ux}{W_1} + \frac{vy}{W_2})}$$

Hence,

$$\mathscr{F}(f * g)(u, v) = \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} \sum_{x'=0}^{W_1-1} \sum_{y'=0}^{W_2-1} f(x', y') g(x - x', y - y') e^{-2\pi j(\frac{ux}{W_1} + \frac{vy}{W_2})}$$

We can rearrange the summations to obtain the following:

$$\mathscr{F}(f * g)(u, v) = \sum_{x'=0}^{W_1-1} \sum_{y'=0}^{W_2-1} f(x', y') \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} g(x - x', y - y') e^{-2\pi j(\frac{ux}{W_1} + \frac{vy}{W_2})}$$

We note that,

$$\sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} g(x - x', y - y') e^{-2\pi j(\frac{ux}{W_1} + \frac{vy}{W_2})} = \mathscr{F}(g(m - x', n - y'))(u, v)$$

Also, using the translation property of Fourier transform, we can write:

$$\mathscr{F}(g(m - x', n - y'))(u, v) = \mathscr{F}(g)(u, v) e^{-2\pi j(\frac{ux'}{W_1} + \frac{vy'}{W_2})}$$

Hence we can rewrite the original equation as:

$$\mathscr{F}(f * g)(u, v) = \sum_{x'=0}^{W_1-1} \sum_{y'=0}^{W_2-1} f(x', y') \mathscr{F}(g)(u, v) e^{-2\pi j(\frac{ux'}{W_1} + \frac{vy'}{W_2})}$$

Rearranging:

$$\mathscr{F}(f * g)(u, v) = \mathscr{F}(g)(u, v) \sum_{x'=0}^{W_1-1} \sum_{y'=0}^{W_2-1} f(x', y') e^{-2\pi j(\frac{ux'}{W_1} + \frac{vy'}{W_2})}$$

Also,

$$\sum_{x'=0}^{W_1-1}\sum_{y'=0}^{W_2-1} f(x',y')e^{-2\pi j(\frac{ux'}{W_1}+\frac{vy'}{W_2})} = \mathscr{F}(f)(u,v)$$

Hence,

$$\mathscr{F}(f*g)(u,v) = \mathscr{F}(g)(u,v)\mathscr{F}(f)(u,v)$$

which is the convolution theorem for 2D Discrete Fourier transforms.