3

Let I be the original image with a given pdf distribution $P_I(i)$. The addition of Gaussian noise can be treated as the addition of an image J to I where the pdf of J is a Gaussian. Let the resultant image be K, then for each pixel (x, y)

$$K(x, y) = I(x, y) + J(x, y) \tag{1}$$

Now, let the PDF of K be $P_K(k)$. We have:

$$P_K(k) = \int_{-\infty}^{\infty} P_{I,J}(i, k - i)di \tag{2}$$

Which is nothing but the convolution of I and J.
Since it is mentioned that the additive noise is applied independently, we can assume that the pdf of I and J are independent. Thus,

$$P_K(k) = \int_{-\infty}^{\infty} P_I(i)P_J(k - i)di \tag{3}$$

The form for the pdf of J is,

$$P_J(j) = \frac{e^{\frac{-j^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \tag{4}$$

Hence, the expression for the pdf of the final image K is,

$$P_K(k) = \int_{-\infty}^{\infty} P_I(i)\frac{e^{\frac{-(k-i)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}di \tag{5}$$