3

(c) Here are the graphs of values of the NCC, JE, and QMI as a function of θ:

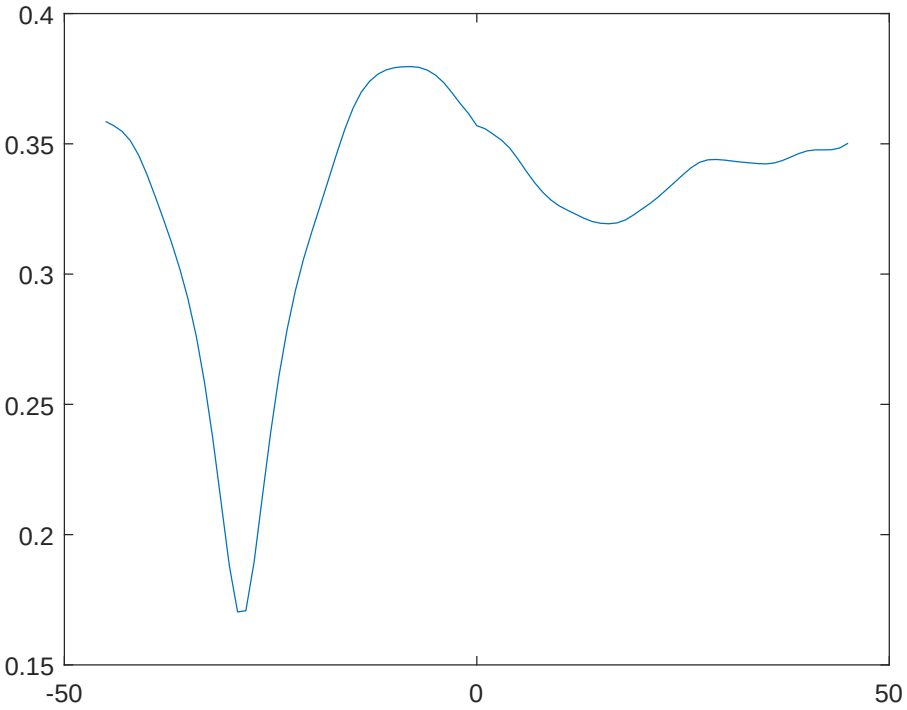

Figure 1: The graph of the NCC of $J1$ and $J4$ as a function of θ.

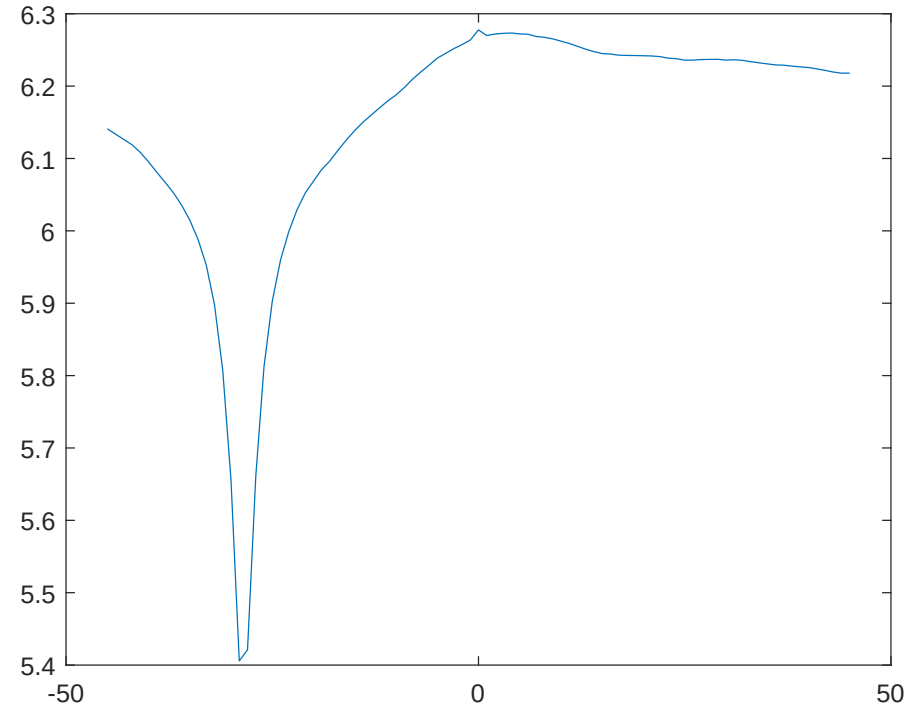

Figure 2: The graph of the JE of $J1$ and $J4$ as a function of θ.

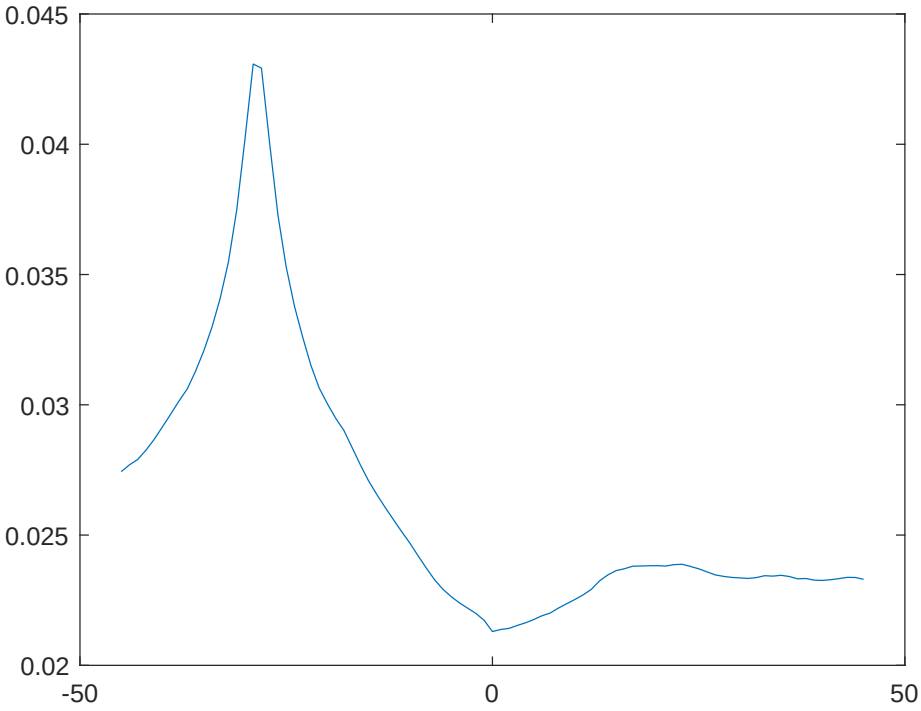

Figure 3: The graph of the QMI of $J1$ and $J4$ as a function of θ.

(d) Based on the NCC graph, the optimal rotation is $\theta = -8°$. Based on the Joint Entropy graph and the QMI graph, the optimal rotation is $\theta = -29°$.

(e) Here is the joint histogram between $J1$ and $J4$:

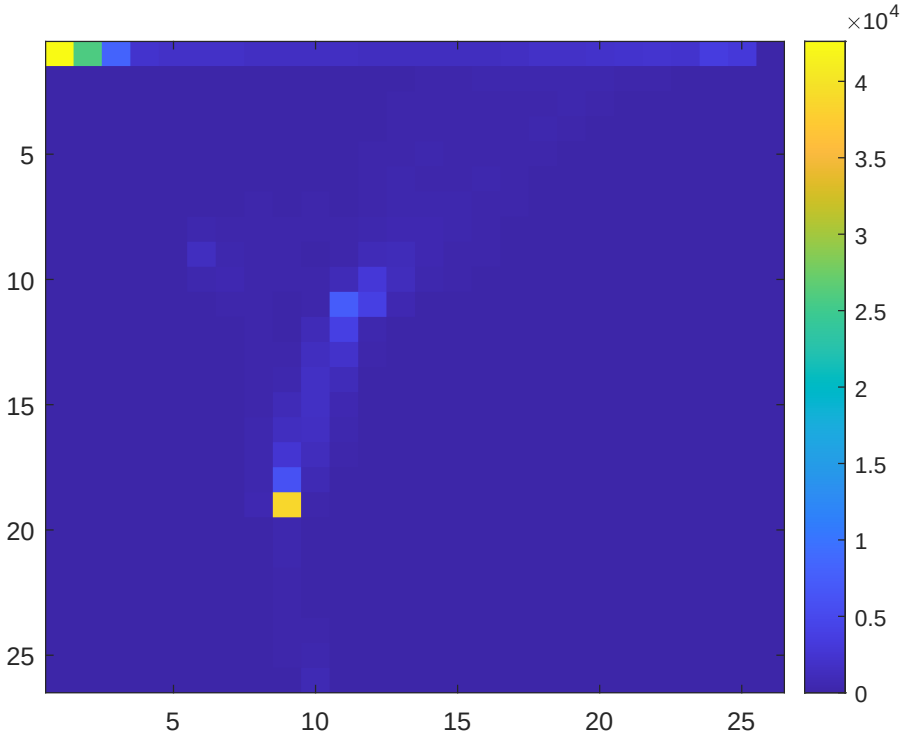

Figure 4: The joint histogram between images $J1$ and $J4$.

(f) The intuition behind Quadratic Mutual Information (QMI)is simple: we want to quantify the amount of 'mutual information' we have between the two images. That is, given the pixel intensity data in image $J1$, how much uncertainty (or dually, certainty) do we have about the corresponding pixel intensity data in image $J4$?

The reason for using quadratic terms is the same as the reason we use it for least squares: We don't want the errors to cancel out.

If we have high mutual information, that means that given some pixel intensity data in image $J1$, we can say with (relatively) high certainty what the corresponding pixel intensity data is in image $J4$.

The QMI is at its lowest when the random variables I_1 and I_2 are statistically independent. If we assume that I_1 and I_2 are statistically independent,

then $p_{I_1 I_2}(i_1, i_2) = p_{I_1}(i_1) p_{I_2}(i_2)$, so we get

$$\begin{aligned}
\text{QMI} &= \sum_{i_1} \sum_{i_2} \Big(p_{I_1 I_2}(i_1, i_2) - p_{I_1}(i_1) p_{I_2}(i_2) \Big)^2 \\
&= \sum_{i_1} \sum_{i_2} \Big(p_{I_1}(i_1) p_{I_2}(i_2) - p_{I_1}(i_1) p_{I_2}(i_2) \Big)^2 \\
&= \sum_{i_1} \sum_{i_2} 0 \\
&= 0
\end{aligned}$$

The intuitive reason for this is because I_1 and I_2 are independent, the pixel intensity data in the first image has no bearing on the pixel intensity data in the second image, so we have, loosely speaking, 0 'mutual information'.