

Let I be the original image with a given pdf distribution  $P_I(i)$ . The addition of Gaussian noise can be treated as the addition of an image J to I where the pdf of J is a Gaussian. Let the resultant image be K with pdf  $P_K(k)$ . We have:

$$P_K(k) = \int_{-\infty}^{\infty} P_{I,J}(i, k-i) di \quad (1)$$

Which is nothing but the convolution of I and J

Since it is mentioned that the additive noise is applied independently, we can assume that the pdf of I and J are independent. Thus,

$$P_K(k) = \int_{-\infty}^{\infty} P_I(i) P_J(k-i) di \quad (2)$$

The form for the pdf of J is,

$$P_J(j) = \frac{e^{\frac{-j^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \quad (3)$$

Hence, the expression for the pdf of the Final image is,

$$P_K(k) = \int_{-\infty}^{\infty} P_I(i) \frac{e^{\frac{-(k-i)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} di \quad (4)$$