Contents

Chapter 5

Eleastische Eigenschaften

$\vec{r}$ nach Deformation $\rightarrow \vec{r} + \vec{U}(\vec{r})$ mit $\vec{U}$-Verformung. Die Freie Energie:

$$\mathcal{F} = \int_V d\vec{r} \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \underbrace{E_{\alpha\beta\gamma\delta}}_{\text{Tensor}} \frac{\partial U_\alpha(\vec{r})}{dr_\beta} \cdot \frac{\partial U_\gamma(\vec{r})}{dr_\gamma}$$

$E_{\alpha\beta\gamma\delta} \rightarrow 81 = 3^4$ Komponenten: Symmetrie mit 45 unabhängigen Komponenten $\alpha\beta \leftrightarrow \gamma\delta$ ändert sich nicht unter Drehung von <u>nicht</u> deformierte Festk.
$E_{\alpha\beta\gamma\delta} - E_{\beta\alpha\gamma\delta} + E_{\alpha\beta\delta\gamma} + E_{\beta\alpha\delta\gamma} = 0$;nur 21 Komponenten.
Dehnungstensor (strain tensor):

$$e_{\alpha\beta} = e^{\leftrightarrow} = [e] = \frac{1}{2}[\frac{\partial U_\alpha(\vec{r})}{dr_\beta} \cdot \frac{\partial U_\gamma(\vec{r})}{dr_\gamma}]$$

Spannungstensor:

$$\sigma_{\alpha\beta} = \sigma^{\leftrightarrow} = [\sigma] = \sum_{\alpha\beta} \underbrace{C_{\alpha\beta\gamma\delta}}_{elastizitaetstensor} e_{\gamma\delta}$$

$$\mathcal{F} = \sum_{\alpha\beta} \int \vec{r} \frac{1}{2} e_{\gamma\delta} \sigma_{\gamma\delta}$$

$$C_{\alpha\beta\gamma\delta} = \frac{1}{4}(E_{\alpha\beta\gamma\delta} + E_{\beta\alpha\gamma\delta} + E_{\alpha\beta\delta\gamma} + E_{\beta\alpha\delta\gamma})$$

Arbeit = Länge * Kraft

5.1 Elastische Konstanten für kubische Kristalle

$$C_{\alpha\alpha\alpha\alpha} \stackrel{\text{def}}{=} C_{11}$$
$$C_{\alpha\alpha\beta\beta} \stackrel{\text{def}}{=} C_{12}$$
$$C_{\alpha\beta\alpha\beta} \stackrel{\text{def}}{=} C_{44}$$

für isotropen Materialien $C_{11} - C_{12} = 2C_{44}$ nur 2 unabhänige Komponenten
Lamé-Konstanten $\lambda \stackrel{\text{def}}{=} C_{12}$; $\mu \stackrel{\text{def}}{=} C_{44}$

$$\mathcal{F} = \frac{1}{2} C_{11}(e_{xx}^2 + e_{yy}^2 + e_{zz}^2) + \frac{1}{2} C_{44}(e_{yz}^2 + e_{zx}^2 + e_{xy}^2) + C_{12}(e_{yy}^2 e_{zz}^2 + e_{zz}^2 e_{xx}^2 + e_{xx}^2 e_{yy}^2)$$

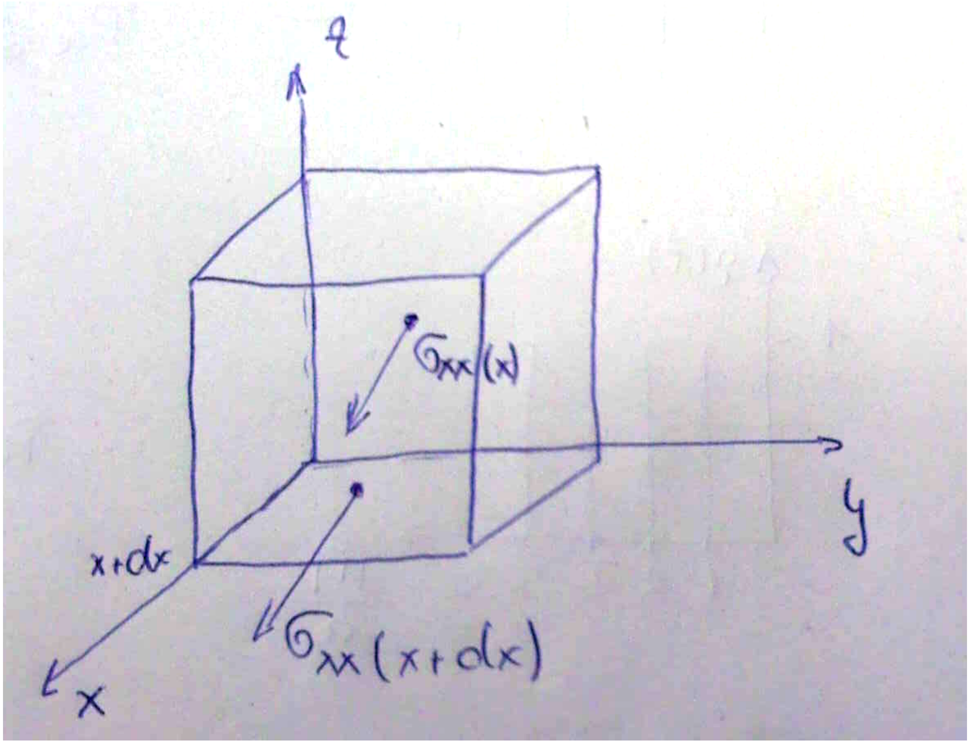
Volumen-Kompression: $e_{xx}^2 = e_{yy}^2 = e_{zz}^2 = \frac{1}{3}\delta$; $\mathcal{F} = \frac{1}{6}(C_{11} + C_{12})\delta^2$;
Kompressionsmodul: $B \frac{C_{11}+2C_{12}}{3} = \frac{3\lambda+2\mu}{3}$, $\mathcal{F} = \frac{1}{2} B\delta^2$
Kompressibilität: $\kappa = \frac{1}{B} = -\frac{1}{V} \cdot \frac{dV}{dp}$
Elastizitätsmodul (Youngscher Modul) $E = \frac{\text{Spannung}}{\text{Dehnung}} \approx \frac{d\sigma}{de} = \frac{\mu(3\lambda+2\mu}{\lambda+\mu}$

5.2 Konstanten für kubische Kristalle

Elastizitätskraft: $f_\alpha(\vec{r}) = \rho \ddot{u}_\alpha(\vec{r}) = \sum_\beta \frac{\partial \sigma_{\alpha\beta}(\vec{r})}{dr_\beta}$

5.3 Schallwellen

- longitudinale $\vec{q}$ Wellenvektor. $\vec{q}||\vec{u}$
- transversale $\vec{q}\perp\vec{u}$



$$df_x = \sigma_{xx}(x+dx) - \sigma_{xx}(x)dxdz = \frac{\partial \sigma_{xx}}{\partial x} dxdydz$$

$$\rho \frac{\partial^2 U_x}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x}$$

$$\sigma_{xx} = O_{11} e_{xx} = C_{11} \frac{\partial U_x}{\partial x}$$

$$\Rightarrow \rho \frac{\partial^2 U_x}{\partial t^2} = C_{11} \frac{\partial^2 U_x}{\partial x^2}$$

allgemeine Form

$$\rho \frac{\partial^2 U_x}{\partial t^2} = \sum_{\beta\gamma\delta} C_{\alpha\beta\gamma\delta} \frac{\partial U_\delta}{\partial r_\beta \partial_\gamma}$$

Lösung:

$$U_\alpha = U_{o\alpha} exp(-i\omega t + i\vec{q}\cdot\vec{r})$$

f kubische Kristalle

$$\rho \frac{\partial^2 U_x}{\partial t^2} = C_{11} \frac{\partial^2 U_x}{\partial x^2} + C_{44} \left(\frac{\partial^2 U_x}{\partial y^2} + \frac{\partial^2 U_x}{\partial z^2} \right) + (C_{12} + C_{44}) \left(\frac{\partial^2 U_y}{\partial x \partial y} + \frac{\partial^2 U_z}{\partial x \partial z} \right)$$

Nach Symmetrie: zyklische Vertauschung x,y,z $\rightarrow$

$$\rho \frac{\partial^2 U_y}{\partial t^2} = ...$$

$$\rho \frac{\partial^2 U_z}{\partial t^2} = ...$$

Dispersionsrelation: $\omega_\alpha = v_\alpha \cdot q$, 3 Modedn $\forall$ Richtung 1 $||$(Longitudinale) + 2 $\perp$(Transversale).
für isotropes Medium
$\vec{u}_1 || \vec{q}$; $v_{||} = \frac{\omega_{||}}{q} \sqrt{\frac{C_{11}}{\rho}}$ Elastizitätsmodul
$v_\perp = \frac{\omega_\perp}{q} \sqrt{\frac{C_{44}}{\rho}} \begin{cases} \vec{u}_2 \perp \vec{q} \\ \vec{u}_3 \perp \vec{q} \end{cases}$ Schubmodul
Messungen [100] $\rightarrow C_{11}, C_{44}$; [110]$C_{12}$
Richtungsabh. von Schallgeschw.