Partialwellenzerlegung (Drehimpulserhaltung) $\psi(r) = \phi_{\text{ein}} + \phi_{\text{gestr}}$

$$\psi(r) = e^{ikr\cos\theta} + f(\theta)\frac{e^{ikr}}{r} = \sum_{l} i^{l}(2l+1)j_{l}(kr)P_{l}(\cos\theta) + f(\theta)\frac{e^{ikr}}{r}$$

$$\underbrace{j_l(x) \stackrel{r \to \infty}{=} \frac{1}{x} \sin\left(x - l\frac{\pi}{2}\right)}_{p_l(x)} \underbrace{n_l(x) \stackrel{r \to \infty}{=} -\frac{1}{x} \cos\left(x - l\frac{\pi}{2}\right)}_{p_l(x)}$$

Bessel-Funktion

Neumann-Funktion

$$\psi(r) \stackrel{m=0}{=} \sum_{l} a_{l} R_{kl}(r) \cdot Y_{l0}(\theta) = \sum_{l} a_{l} R_{lk}(r) \cdot P_{l}(\cos \theta)$$

$$\left[\frac{d^2}{dr^2} + k^2\right] (rR_{kl}) = 0 \to R_{kl} = A_l j_l + B_l n_l = \frac{C_l}{kr} \sin\left(kr - l\frac{\pi}{2} + \delta_l\right)$$

$$f(\theta) = \sum_{l} f_l(\theta) = \sum_{l} \frac{1}{k} (2l+1)e^{i\delta_l} \sin \delta_l P_l(\cos \theta)$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int |f(\theta)| d\Omega = \sum_{l} \sigma_{l} = \sum_{l=0} \frac{4\pi}{k^{2}} (2l+1) \sin^{2} \delta_{l}$$