Estimating π Using the Monte Carlo Method

The Monte Carlo Simulation is a computational technique that uses repeated random sampling to estimate the likelihood of various outcomes. In this method, we simulate randomly generated points within a unit square that contains a quarter circle. By calculating the ratio of points that fall inside the quarter circle to the total number of points, we can approximate the value of π.

Visualization of Monte Carlo Estimation for π

The scatter plots illustrate the Monte Carlo method for estimating the value of π using different numbers of randomly generated points: 100, 1000, 10000, and 100000.

Each subplot represents a simulation where:

- **blue dots** represent points that fall inside the quarter circle
- **red dots** represent points outside the quarter circle but within the unit square
- the **black arc** represents the boundary of the quarter circle with a radius of 1.

As the number of points increases, the estimated value of π becomes more accurate, and the distribution of points more closely reflects the actual area ratio between the circle and the square. This demonstrates that, with a sufficiently large sample size, random sampling can provide a reliable approximation of π.

Tracking Convergence of Monte Carlo Estimation

Next, we observe how the Monte Carlo estimation of π behaves across multiple repeated simulations with the same number of points. By setting n_points = 10000 and running the simulation 10 times, we can track fluctuations in the estimated values. While each run generates slightly different results due to the randomness in point generation, the estimates tend to stabilize around the actual value of π, as shown in the convergence plot.

Boxplot Analysis of π Estimation

The boxplot below shows the distribution of π estimates across multiple runs for different sample sizes: 100, 1000, 10000, and 100000 random points. Each box represents the spread of estimates from 10 independent runs of the Monte Carlo simulation for a given sample size.

As the sample size increases, the estimates of π become more concentrated around the true value (approximately 3.14159). With a larger number of points, the variability of the estimates decreases, leading to more consistent results.

This boxplot analysis demonstrates how the precision of the Monte Carlo method improves with larger sample sizes, offering insight into the variance and reliability of the π estimation technique.

Conclusions

The results from the Monte Carlo simulations show that as the number of randomly generated points increases, the estimated value of π becomes more accurate and stable. The visualizations in the 2x2 grid clearly illustrate this trend: with fewer points, the estimates fluctuate more widely (as seen in the plots for 100 and 1000 points). However, as the number of points increases to 10000 and 100000, the estimates converge toward the true value of π (≈ 3.14159), demonstrating the effectiveness and reliability of the Monte Carlo method. These findings highlight the importance of using large sample sizes to improve the accuracy of the estimation, reinforcing that Monte Carlo simulations provide a simple yet powerful approach for approximating complex mathematical constants like π.