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Fuzzy Linguistic Summaries for Hidden Markov Models

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Introduction

Explainable AI strives to demystify how models make decisions. We explore linguistic summaries to clarify the reasoning behind hidden Markov models (HMMs). Illustrative example is presented for detecting early bipolar disorder symptoms using smartphone-acquired acoustic data.
Our method further extends [1] and involves translating HMMs outputs into prototype-based sentences. We validate this approach experimentally, using simulated time series data and medical datasets.

Hidden Markov Model

Hidden Markov models (HMMs) are an extension of Markov models, see e.g. [2].
We assume that there is an unobservable Markov process X_t that can be in one of N states $S = \{s_1, s_2, \ldots, s_N\}$ at time t the state at timer is denoted as (q_t): Our goal is two-fold: (1) find semi-continuous HMM $\lambda = (\mathbf{A}, \mathbf{B}, \pi)$ for the observed sample; (2) construct fuzzy linguistic summaries about this model and accuracy of the predictions.

- Probability of transitioning between hidden states is given by a transition matrix $A = \{a_{ij}\}$ where $a_{ij} = \mathbb{P}(q_t = s_j | q_{t-1} = s_i)$ for $i, j = 1, \ldots, N$.
- The choice of initial state depends on the **initial probability distribution** $\pi = \{\pi_j\}$ $\pi_i = \mathbb{P}(q_1 = s_i)$ for $i = 1, \cdots, N$.
- In each state one of M observation symbols $V = \{v_1, v_2, \ldots, v_M\}$ can be emitted from an **observed stochastic process** Y_t. Observation at time t is denoted as $\mathcal{O}_t$ and probability of emitting an observation in state s_j is given by an **emission matrix** $B = \{b_j(k)\}$ where $b_l(k) = \mathbb{P}(v_k$ at time $t | q_t = s_j)$.

We adapt the semi - continuous hidden Markov model (TMHMM) and run the Baum-Welch algorithm to estimate its parameters. It is assumed that in each of the states, we have the same normal distributions, differing only in weight coefficients:

$$b_j^*(\mathcal{O}) = \sum_{m=1}^{M} c_{jm} \mathcal{N}(\mathcal{O}, \mu_m, \sigma_m) \tag{1}$$

To compare a vector of predicted labels $\hat{\mathbf{q}}$ of length n against the vector of ground truths $\mathbf{q}$, we used **Exact Match Ratio (EMR)**. Defined as a percentage of labels that were correctly predicted:

$$\mathrm{EMR} = \frac{1}{n} \sum_{j=1}^{n} 1(\hat{q}^{(j)} == q^{(j)}). \tag{3}$$

Fuzzy Linguistic Summaries

Linguistic summaries based on the extended protoforms have the following form:

$$LS : Q\ R\ y\text{'s are } P. \tag{4}$$

where $Y = \{y_1, \cdots, y_n\}$ be a set of objects and $V = \{V_1, \cdots, V_n\}$ a set of attributes characterizing objects from Y. A summary of a data set consists of:

- a summarizer P,
- a qualifier R (optionally),
- a quantity in agreement Q,
- a measure of validity or truth of the summary T.

Degree of Truth is calculated as follows:

$$\mathbf{DoT}(LS) = Q\left(\frac{\sum_{i=1}^{n} P(x_i) * R(x_i)}{\sum_{i=1}^{n} R(x_i)}\right), \tag{5}$$

Degree of Focus:

$$\mathrm{DoF}(LS) = Q\left(\frac{1}{n}\sum_{i=1}^{n} P(x_i)\right) \tag{6}$$

Degree of Support:

$$\mathrm{DoS}(LS) = \frac{1}{n}\sum_{i=1}^{n}\{x_i : P(x_i) > 0 \wedge R(x_i) > 0\}, \tag{6}$$

where LS is a fuzzy linguistic summary, $*$ is a t-norm.

Experimental Results

The goal of experiments is to estimate semi - continuous HMM $\lambda = (A, B, \pi)$ for the observed time series in two states and to construct fuzzy linguistic summaries about it and the accuracy of the predictions.

Simulated Datasets:

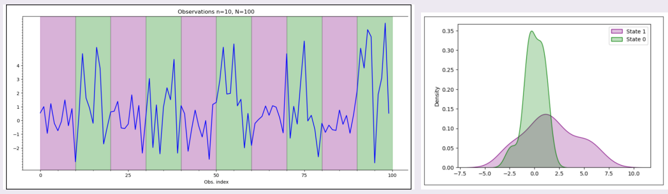

Figure: (left) Simulated samples: $\{x_i\} \sim \mathcal{N}(1.5, 3)$ for state 0 and $\{x_i\} \sim \mathcal{N}(0, 1)$, for state 1, next split into $k = 10$ segments; (right) estimated density functions in two states

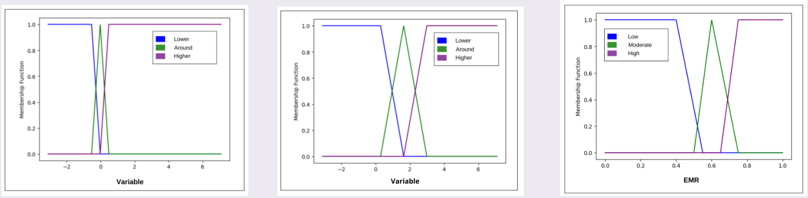

Figure: Membership functions for the linguistic terms, e.g., *high exact match rate*

LS description	DoT	DoS	DoF
For majority of observations in state 0 with variable lower than 1.64 we have EMR high	1	0.98	0.81
For majority of observations in state 1 with variable around 1.64 we have EMR high	1	0.42	0.21
For majority of observations (all states) with variable lower than 1.64 we have EMR high	1	0.77	0.62
For majority of observations (all states) with variable higher than -0.03 we have EMR high	1	0.6	0.56

Real-life data: Acoustic features collected from smartphones within prospective study for patients in euthymia and depression.

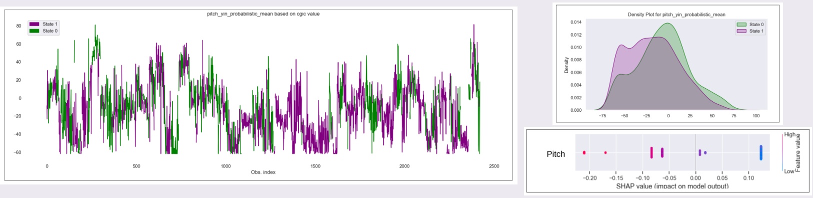

Figure: (left) time series of the pitch variable in euthymia vs depression (two states); (right top) estimated density functions in both states; (right bottom) plot with Shapley additive explanations

LS decription	DoT	DoS	DoF
For majority of observations in state 0 with pitch higher than -17.28 we have EMR low	1	0.66	0.56
For majority of observations in state 0 with pitch around -6.59 we have EMR low	1	0.4	0.21
For majority of observations (all states) with pitch around -6.59 we have EMR moderate	1	0.36	0.19
For majority of observations (all states) with pitch around -17.28 we have EMR moderate	1	0.34	0.17

Conclusions and Future Work

This method enables the construction of linguistically quantified sentences linking both, the observed time series and the estimated hidden Markov model. Further developments of the methodology is planned aiming to explain all parameters of the hidden Markov model, in particular of the transition matrix and the conditional probability density functions.

References

[1] Kaczmarek-Majer K. et al, *PLENARY: Explaining black-box models in natural language through fuzzy linguistic summaries*, Information Sciences, 2022.
[2] Rabiner, L. R. and Juang, B. H. *An introduction to hidden markov models. IEEE ASSP Magazine, 3:4–16, 1986.*