Heat-transfer Homework 2

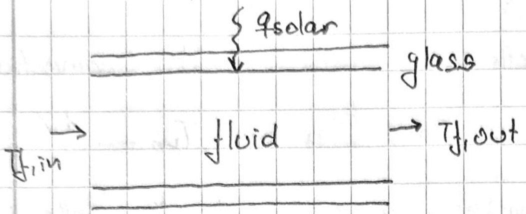
Émile Wertz

Q1. Using the 7-step methodology:

1. Known: $\dot{m}_f = 1000\,kg/h$, $q''_{solar} = 750\,W/m^2$, $L = 30\,m$, $W = 2{,}5\,m$, $T_{f,in} = 150°C$.

2. Find: q_{fluid}, $T_{f,out}$

3. Schematic:



4. Assumptions:

Steady-state conditions, negligible radiation, (not solar) negligible conduction, variable properties.

5. Properties: Cylindrical geometry, C_p (fluid) at $T_m = \frac{(T_{f,in} + T_{f,out})}{2}$

6. Analysis: By performing an energy balance in the tube, we can obtain the following expression: $q_{conv} = \dot{m}_f\, C_p\,(T_{f,out} - T_{f,in})$

where $q_{conv} = q''_{solar} \cdot L \cdot W = q_{solar}$. (Where L·W is the sun-irradiated surface)

Calculating this will require iterating over $T_{f,out}$, but to obtain an iterable form, we need to express C_p as a function of $T_{f,out}$. To achieve this, we can perform a linear regression for C_p at an arbitrary $T_m = 483\,K$

$$\text{slope} = \frac{2{,}067 \cdot 10^3 - 2{,}040 \cdot 10^3}{483 - 473} = 2{,}7 \Rightarrow C_p(483) = 2{,}067 \cdot 10^3 - 2{,}7(483) = p$$

$\Rightarrow C_p(T_m) = 2{,}7(T_m) + p$ in J/kgK. This will hold for values of $T_m \simeq 480K$.

The expression upon which iterations will be performed is then:

$$q''_{solar} \cdot L \cdot W - \dot{m}_f\, C_p(T_m)(T_{f,out} - T_{f,in}) = 0$$

With an initial guess of $T_{f,out} = 500K$, this yields $\underline{\underline{T_{f,out} = 522{,}441\,K}}$ over 5 iterations. Note that $\dot{m}_f$ is expressed in kg/s and not in kg/h, so the conversion $\dot{m}_f = \frac{1000}{3600}$ was applied.

As for q_{fluid}, this will equal q_{solar}, since there are no losses: $q_{fluid} = 750 \cdot 30 \cdot 2{,}5 = \underline{\underline{56250\,W}}$

7. Comments: This result assumes that, since the tube of the material is glass, radiation is directly absorbed in the fluid, and that conduction through glass and convection to outside air is null, so not really accurate.

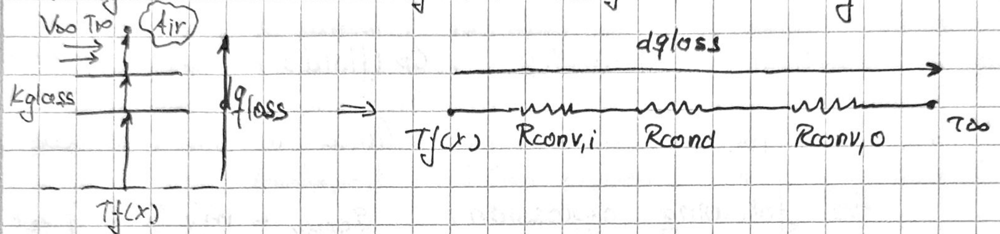
Q2. Now that all the aspects of heat-transfer are taken into account; we can perform a new energy balance over an infinitesimal portion of the tube :

$$dq_{solar} - dq_{loss} = \dot{m}_f \, c_p \left(\cancel{T_f(x)} - \cancel{T_f(x)} + dT_f \right)$$

$$\Leftrightarrow dq_{solar} - dq_{loss} = \dot{m}_f \, c_p \, (dT_f) \qquad (1)$$

Calculating dq_{loss} will involve taking into account all of the methods of heat transfer (convection & conduction)

By definition: $dq_{loss} = U_i \, dA_i \, (T_f(x) - T_\infty)$ where $U_i = 1/R_{tot}$, this is just the balance of heat-transfer according to the following circuit:



We can further develop the thermal resistances, without going in detail, since a numerical result is expected in Q3.

$$R_{conv,i} = \frac{1}{h_i} \qquad R_{conv,o} = \frac{D_i}{D_i + 2e} \cdot \frac{1}{h_o} \qquad R_{cond} = \frac{D_i}{2\pi k_{glass}} \cdot \ln\left(\frac{D_i + 2e}{D_i}\right)$$

where D_i is the internal diameter of the cylinder, and e its thickness.

Rewriting (1) yields: $\dot{m}_f \, c_p \, (dT_f) = dq_{solar} - U_i \, dA_i \, (T_f(x) - T_\infty)$

Dividing by $\dot{m}_f c_p$: $dT_f = \dfrac{dq_{solar}}{\dot{m}_f c_p} - \dfrac{U_i \, dA_i \, (T_f(x) - T_\infty)}{\dot{m}_f c_p}$

$$\Rightarrow dT_f = \frac{q''_{solar} \cdot W \cdot dx}{\dot{m}_f c_p} - \frac{\pi D_i dx \, (T_f(x) - T_\infty)}{\left(\frac{1}{h_i} + \frac{D_i}{D_i + 2e}\frac{1}{h_o} + \frac{D_i}{2\pi k_{glass}} \ln\left(\frac{D_i + 2e}{D_i}\right) \right) \dot{m}_f c_p}$$

This expression allows an evaluation of dT_f and will be used in Q3. Note that the th. resistance expression for conduction in a cylinder was derived from the formula (3.33) in the book.

Q3. Using the previous equation will allow us to calculate a new outlet temperature of the fluid. To achieve this, we will need numerical values for h_i (internal convection heat-transfer coefficient), an h_o (external)

For h_i, we have: $\dot{m}_f = \rho_{fluid}\, u_m A_i \Rightarrow u_m = \dfrac{\dot{m}}{\rho_{fluid}\, \pi \frac{D_i^2}{4}}$.

Since by definition Re_i (Reynolds' number) $= \dfrac{u_m D_i}{\nu}$

$$\Rightarrow Re_i = \frac{u_m \rho D_i}{\mu} = \frac{\dot{m}_f \rho D_i}{\rho \pi \frac{D_i^2}{4} \mu} = \frac{4 \dot{m}_f}{\pi D_i} = 12\,259 \geq 2300$$

The Reynolds number is then greater than 2300, the condition for laminar flow. We can say then that the flow inside the tube is turbulent and fully developped*. As all properties here at calculated at T_m, where $T_m = \left(\frac{T_{fin} - T_{fout}}{2}\right)$, and a linear regression, like the one in the first question will be performed if this temperature doesen't align with the properties table given with the statement.

Following the flow conditions given by the Reynolds number, we use the Gnielinski (8.62) equation to calculate the Nusselt number:

$$Nu_i = \frac{h_i D_i}{k_{fluid}} = \frac{(f/8)(Re_i - 1000)Pr}{1 + 12{,}7 (f/8)^{1/2} (Pr^{2/3} - 1)}$$ where all properties

are evaluated at T_m, and $f = (0{,}790 \cdot \ln(Re_i) - 1{,}64)^{-2}$ (8.21)

* This is verified since $L/D = {}^{30}/_{6 \cdot 10^{-3}} > 10$

Calculating Nu_i while performing linear regressions for Pr yields

$$Nu_i = 102 = \frac{h_i D_i}{k_{fluid}} \Rightarrow h_i = 200 \text{ W/m}^2\text{K} \quad (Pr = 8{,}41,\ f = 0{,}029)$$

Similarly, h_o is calculated using the appropriate equations related to forced convection: $Re_e = \dfrac{V_\infty (D_i + 2e)}{\mu_{air}} = 13\,196$ where, like in the previous case, a linear regression was performed to evaluate μ_{air} at $T_{m\,air} = \left(\dfrac{T_{fin} + T_{fout}}{2} + T_\infty\right)\dfrac{1}{2}$, and $V_\infty = 3$ m/s.

(Table A.4)

⇓

Next, a correlation corresponding to our case is that of Hilpert (7.52):

$\overline{Nu}_e = C\,Re_e^m\,Pr_e^{1/3} = 60{,}203$ where, once again Pr_e was looked in the table A.4. and evaluated at $T_{m_{air}}$ thanks to a linear regression.

The m and C coefficients can be found at table 7.2 in function of $Pr_e = 0{,}6925 \simeq 0{,}7 \Rightarrow C = 0{,}193$, $m = 0{,}615$.

Then; $\dfrac{Nu_e k_{air}}{D_i+2e} = h_o = 20{,}055$ W/mK

Now, rewriting the equation from Q_2:

$$\frac{dT_f}{dx} = \frac{q''_{solar}.W}{\dot{m}_f c_p} - \frac{\pi D_i\,(T_f(x)-T_\infty)}{\left(\frac{1}{h_i} + \frac{D_i}{D_i+2e}\frac{1}{h_o} + \frac{D_i}{2\pi k_{glass}}\ln\left(\frac{D_i+2e}{D_i}\right)\right)\dot{m}_f c_p}$$

as: $\dfrac{dT_f(x)}{dx} = A - B\,(T_f - T_\infty)$ where $A = \dfrac{q''_{solar}W}{\dot{m}_f c_p}$, $B = \dfrac{U_i \pi D_i}{\dot{m}_f c_p}$

$\Leftrightarrow \dfrac{dT_f(x)}{dx} = A - BT_f + BT_\infty \Leftrightarrow \dfrac{dT_f(x)}{dx} + BT_f = A + BT_\infty$

where $B = 0{,}007256$ and $A + BT_\infty = 5{,}48039$

This is a differential equation is of the form $f' + Af = B$ and can be solved; yielding with a condition $T(0) = T_{fin}$:

$T(x) = 755{,}277 - 332{,}127\,e^{(-0{,}007256 \cdot x)}$

Now we can calculate T_{fout} since $T_{fout} = T_f(L) = \underline{488{,}120\text{ K}}$

The heat transported by the fluid is $q_{fluid} = q_{solar} - q_{loss}$

where $q_{loss} = U_i A_i\,(T_f(L) - T_\infty) = 23\,427$ W and

$q_{solar} = q''_{solar} \cdot W \cdot L = 56\,250$ W $\Rightarrow q_{fluid} = \underline{32\,822{,}7251\text{ W}}$

The energy efficiency is derived from the first question, which is an ideal case with no losses; hence:

$\eta = \dfrac{T_f(L) - T_{f,in}}{T_{fout} - T_{f,in}} = 0{,}654343$ where $T_{f,out} = 522$ K (Question 1).

So the energy efficiency is $\underline{65{,}43\,\%}$

Heat - transfer Homework 2 Emile Wertz

Q4. This question changes the nature of the external convection, which now will be free instead of forced. Furthermore, the assumption that $T_f(x)$ remains constant at $T_{f,in} = 150°C$ is given. * T_f $R_{conv,i}$ R_{cond} T_s $R_{conv,o}$ T_∞

An energy balance* over the entire tube, similar to that of question 3 can be performed: $q_{loss} = \frac{T_f - T_\infty}{R_{tot}}$ (1) where $T_f = T_{f,in}$ and $T_\infty = 15°C$

and $R_{tot} = \frac{1}{h_i A_i} + \ln\left(\frac{D_i + 2e}{D_i}\right)\frac{1}{2\pi L\, k_{glass}} + \frac{1}{h_o}\left(\frac{D_i}{D_i + 2e}\right)$

Here, h_i will be calculated in the same way a for question 3, with all properties evaluated at T_f, yielding Re_i (Reynolds) = 7380,74 $\Rightarrow h_i = 143{,}67$

For h_o, a different calculation is needed due to the nature of the flow: Free convection follows this equation**: $Ra_e = \frac{g\beta(T_s - T_\infty)}{Nu_e^2} \cdot Pr_e$ where Ra_e is the Rayleigh number needed for equation (9.34)

Churchill & Chu: $\overline{Nu}_D = \left[0{,}60 + \frac{0{,}387\, Ra_e^{1/6}}{[1 + (0{,}559/Pr)^{9/16}]^{8/27}}\right]^2$ for $Ra_e \leq 10^{12}$

In these equations, air properties are evaluated at $T_m = \frac{T_s + T_\infty}{2}$ and a linear regression will be performed based on the table A.4 in the same manner as described in Q1.

Since T_s, the tube's external surface temperature is unknown, a second equation will be needed to solve for T_s iteratively:

From * $\Rightarrow q_{loss} = \frac{T_s - T_\infty}{R_{conv,o}}$ (2). Combining (1) and (2):

$T_s = R_{conv,o} \cdot \left(\frac{T_\infty - T_f}{R_{tot}}\right) + T_\infty$ where $R_{conv,o} = \frac{1}{h_o} \frac{D_i}{D_i + 2e}$

** Note that β (thermal expansion coefficient) $= \frac{1}{(T_m)}$.

Solving iteratively yields $T_s = 418{,}3\,K$ and a heat rate $q_{loss} = 1524\,W$

Assuming $T_f(x)$ is constant is a bad approximation since q_{loss} is far from negligible and would definitely change the value of $T_{f,out}$ or $T_f(L)$.