Local Variable Selection and Parameter Estimation of Spatially Varying Coefficient Regression Models

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Abstract

intentionally empty

	step				gradient				parabola			
	GWEN		GWAL		GWEN		GWAL		GWEN		GWAL	
location	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5
	1.00	0.03	1.00	0.03	1.00	0.05	0.99	0.05	0.74	0.01	0.71	0.00
1	1.00	0.09	0.97	0.06	0.98	0.07	1.00	0.06	0.69	0.03	0.71	0.01
	0.98	0.35	0.89	0.06	1.00	0.35	0.97	0.07	0.68	0.21	0.40	0.06
	0.99	0.07	0.99	0.07	0.96	0.04	0.98	0.03	0.89	0.01	0.91	0.01
2	0.97	0.12	0.96	0.08	0.97	0.06	0.99	0.05	0.89	0.04	0.87	0.02
	0.94	0.38	0.81	0.11	0.98	0.37	0.91	0.07	0.86	0.29	0.58	0.09
	0.85	0.08	0.85	0.07	0.96	0.03	0.97	0.02	0.95	0.00	0.95	0.00
3	0.88	0.11	0.84	0.07	0.96	0.07	0.98	0.06	0.95	0.02	0.89	0.00
	0.85	0.33	0.71	0.10	0.97	0.34	0.86	0.07	0.96	0.30	0.69	0.07
	0.73	0.06	0.71	0.05	0.96	0.02	0.97	0.02	0.87	0.01	0.86	0.02
4	0.57	0.13	0.66	0.11	0.96	0.07	0.95	0.06	0.94	0.02	0.88	0.01
	0.61	0.24	0.52	0.08	0.96	0.35	0.88	0.07	0.90	0.30	0.66	0.07
	0.29	0.03	0.32	0.03	0.89	0.03	0.92	0.02	0.74	0.00	0.73	0.01
5	0.37	0.09	0.42	0.08	0.86	0.10	0.85	0.06	0.72	0.02	0.69	0.00
	0.30	0.12	0.28	0.05	0.82	0.32	0.81	0.09	0.75	0.25	0.53	0.04

Table 1: Selection frequency for the indicated variables.

function	location	GWEN	GWAL	GWEN-LLE	GWAL-LLE	oracle	GWR
step	1	**0.038**	*0.040*	0.072	0.078	0.082	0.056
		0.076	0.101	*0.055*	0.094	**0.048**	0.078
		0.140	0.183	0.218	0.240	**0.056**	0.233
	2	0.086	0.086	*0.071*	**0.070**	0.086	0.075
		0.131	0.127	0.098	0.102	*0.084*	**0.075**
		0.211	0.248	0.169	0.230	**0.086**	*0.100*
	3	0.050	0.050	0.036	0.035	*0.011*	**0.010**
		0.051	0.061	0.032	0.038	**0.016**	*0.017*
		0.073	0.092	*0.045*	0.065	**0.010**	0.048
	4	0.082	0.084	*0.060*	**0.059**	0.095	0.080
		0.095	0.104	**0.057**	*0.074*	0.098	0.084
		0.066	0.086	*0.053*	**0.053**	0.092	0.109
	5	0.045	0.046	*0.015*	0.016	**0.000**	0.073
		0.080	0.087	*0.069*	0.095	**0.000**	0.094
		0.048	0.049	0.023	*0.016*	**0.000**	0.265
gradient	1	0.156	0.158	0.036	0.037	**0.009**	*0.010*
		0.174	0.148	0.033	*0.020*	**0.011**	0.021
		0.237	0.195	0.090	*0.048*	**0.019**	0.108
	2	0.023	0.016	0.016	0.010	**0.002**	*0.002*
		0.026	0.014	0.027	0.007	**0.003**	*0.005*
		0.040	0.049	0.022	0.040	**0.004**	*0.021*
	3	0.018	0.017	0.011	0.009	*0.002*	**0.001**
		0.021	0.016	0.012	0.008	**0.002**	*0.003*
		0.031	0.044	0.021	0.037	**0.004**	*0.015*
	4	0.021	0.020	0.009	0.008	**0.002**	*0.002*
		0.022	0.024	0.010	0.012	**0.003**	*0.004*
		0.029	0.035	*0.017*	0.024	**0.003**	0.021
	5	0.186	0.189	*0.011*	0.012	**0.000**	0.013
		0.160	0.175	0.012	*0.012*	**0.000**	0.017
		0.132	0.170	0.034	*0.027*	**0.000**	0.092
parabola	1	0.057	0.056	0.039	*0.031*	**0.001**	0.046
		0.047	0.050	0.051	0.052	**0.001**	0.080
		0.035	0.030	0.046	*0.021*	**0.001**	0.187
	2	0.039	0.035	0.039	0.036	**0.018**	*0.019*
		0.045	0.048	0.042	0.049	**0.017**	*0.020*
		0.072	0.104	0.082	0.103	**0.015**	*0.061*
	3	0.038	0.037	0.033	0.034	*0.022*	**0.021**
		0.049	0.058	0.037	0.053	**0.023**	*0.023*
		0.074	0.110	0.069	0.107	**0.018**	*0.050*
	4	0.045	0.064	0.043	0.053	*0.016*	**0.015**
		0.037	0.043	0.029	0.043	**0.018**	*0.018*
		0.063	0.100	0.059	0.104	**0.016**	*0.038*
	5	0.059	0.058	0.054	*0.052*	**0.001**	0.060
		0.054	0.061	*0.043*	0.051	**0.001**	0.069
		0.037	0.039	0.048	*0.029*	**0.001**	0.142

Table 2: Mean squared error of $\hat{\beta}_1$ (**minimum**, *next best*).

function	location	GWEN	GWAL	GWEN-LLE	GWAL-LLE	oracle	GWR
step	1	-0.136	-0.133	*0.095*	**0.095**	0.141	0.148
		-0.188	-0.201	*0.049*	**0.021**	0.146	0.145
		-0.291	-0.265	**-0.027**	*-0.065*	0.171	0.173
	2	-0.225	-0.218	*-0.218*	**-0.215**	-0.276	-0.253
		-0.299	-0.269	-0.246	**-0.233**	-0.273	*-0.243*
		-0.380	-0.378	-0.341	-0.377	*-0.277*	**-0.251**
	3	*-0.003*	**0.003**	-0.004	-0.007	0.074	0.057
		0.008	0.024	*0.004*	**-0.003**	0.083	0.079
		-0.019	*-0.030*	-0.053	-0.063	0.070	0.074
	4	0.202	0.200	*0.181*	**0.174**	0.295	0.270
		0.181	0.216	**0.137**	0.183	0.299	0.270
		0.129	0.153	**0.118**	*0.121*	0.293	0.274
	5	0.106	0.111	*-0.038*	-0.043	**0.000**	-0.205
		0.152	0.161	*-0.052*	-0.061	**0.000**	-0.181
		0.106	0.106	-0.067	*-0.056*	**0.000**	-0.201
gradient	1	-0.369	-0.370	0.011	*-0.002*	**0.000**	0.014
		-0.384	-0.355	-0.026	-0.008	*0.001*	**0.001**
		-0.459	-0.401	-0.106	**-0.020**	*0.020*	-0.021
	2	-0.054	-0.036	-0.021	*-0.005*	0.005	**0.004**
		-0.048	-0.028	-0.010	-0.010	**-0.004**	*-0.006*
		-0.124	-0.101	-0.068	-0.067	*-0.009*	**-0.009**
	3	*0.003*	0.015	-0.016	-0.010	0.005	**0.003**
		-0.022	**-0.001**	-0.027	-0.019	*-0.006*	-0.012
		-0.086	-0.055	-0.072	-0.069	**0.000**	*-0.004*
	4	0.048	0.055	-0.012	-0.007	*0.007*	**0.005**
		0.020	0.041	-0.019	-0.018	**-0.007**	*-0.015*
		-0.043	*0.005*	-0.048	-0.051	**-0.003**	0.017
	5	0.389	0.398	*0.012*	0.013	**0.000**	0.013
		0.350	0.368	*0.006*	0.015	**0.000**	-0.021
		0.307	0.353	*-0.008*	-0.010	**0.000**	0.018
parabola	1	0.190	0.183	0.114	*0.107*	**-0.034**	0.160
		0.165	0.172	0.138	*0.133*	**-0.034**	0.217
		0.129	0.086	**0.027**	0.044	*-0.034*	0.103
	2	-0.143	-0.131	-0.136	-0.123	**-0.108**	*-0.121*
		-0.173	-0.155	-0.152	-0.140	**-0.105**	*-0.119*
		-0.236	-0.264	-0.240	-0.257	**-0.100**	*-0.145*
	3	-0.156	-0.146	-0.131	*-0.125*	**-0.121**	-0.130
		-0.198	-0.194	-0.143	-0.162	**-0.126**	*-0.140*
		-0.244	-0.265	-0.230	-0.248	**-0.107**	*-0.121*
	4	-0.150	-0.139	-0.127	-0.128	**-0.086**	*-0.105*
		-0.151	-0.151	-0.111	-0.133	**-0.099**	*-0.109*
		-0.220	-0.215	-0.199	-0.193	*-0.078*	**-0.073**
	5	0.194	0.192	0.155	*0.151*	**-0.034**	0.192
		0.182	0.188	*0.135*	0.136	**-0.034**	0.199
		0.147	0.124	*0.055*	0.072	**-0.034**	0.184

Table 3: Bias of $\hat{\beta}_1$ (**minimum**, *next best*).

function	location	GWEN	GWAL	GWEN-LLE	GWAL-LLE	oracle	GWR
step	1	**0.020**	*0.022*	0.064	0.070	0.063	0.035
		0.041	0.061	0.053	0.095	**0.027**	0.058
		0.055	0.114	0.220	0.238	**0.027**	0.205
	2	0.036	0.039	0.024	0.024	**0.011**	*0.011*
		0.043	0.055	0.038	0.049	**0.010**	*0.017*
		0.068	0.107	0.054	0.088	**0.010**	*0.038*
	3	0.051	0.050	0.036	0.035	**0.006**	*0.007*
		0.052	0.061	0.032	0.039	**0.009**	*0.011*
		0.073	0.092	*0.042*	0.061	**0.006**	0.043
	4	0.042	0.044	0.028	0.030	*0.008*	**0.007**
		0.063	0.058	0.038	0.041	**0.008**	*0.011*
		0.050	0.063	0.040	0.038	**0.006**	*0.035*
	5	0.034	0.034	*0.013*	0.014	**0.000**	0.031
		0.057	0.062	0.067	0.092	**0.000**	0.062
		0.037	0.038	0.019	*0.013*	**0.000**	0.226
gradient	1	0.020	0.021	0.036	0.037	**0.009**	*0.010*
		0.027	0.022	0.033	*0.020*	**0.011**	0.021
		0.027	0.035	0.080	0.048	**0.019**	0.109
	2	0.020	0.015	0.015	0.010	**0.002**	*0.002*
		0.024	0.013	0.027	0.007	**0.003**	*0.005*
		0.025	0.039	*0.017*	0.036	**0.004**	0.021
	3	0.019	0.017	0.011	0.009	*0.002*	**0.001**
		0.021	0.016	0.011	0.008	**0.002**	*0.003*
		0.024	0.041	0.016	0.033	**0.004**	*0.015*
	4	0.019	0.018	0.009	0.008	**0.002**	*0.002*
		0.021	0.022	0.010	0.012	**0.003**	*0.004*
		0.027	0.036	*0.015*	0.022	**0.003**	0.021
	5	0.034	0.031	*0.011*	0.012	**0.000**	0.013
		0.038	0.040	0.012	*0.012*	**0.000**	0.016
		0.038	0.046	0.035	*0.027*	**0.000**	0.093
parabola	1	0.021	0.022	0.026	*0.020*	**0.000**	0.021
		0.020	0.021	0.033	0.035	**0.000**	0.034
		0.018	0.023	0.045	0.019	**0.000**	0.178
	2	0.019	0.018	0.021	0.022	*0.007*	**0.004**
		0.015	0.024	0.019	0.029	*0.006*	**0.006**
		0.016	0.034	0.025	0.038	**0.005**	0.040
	3	0.014	0.015	0.017	0.019	*0.008*	**0.004**
		0.010	0.021	0.017	0.027	*0.007*	**0.004**
		0.015	0.041	0.017	0.046	**0.007**	0.036
	4	0.022	0.046	0.027	0.037	*0.009*	**0.004**
		0.014	0.021	0.017	0.025	*0.008*	**0.007**
		0.015	0.054	0.020	0.067	**0.010**	0.033
	5	*0.021*	0.022	0.030	0.030	**0.000**	0.024
		0.020	0.026	0.025	0.033	**0.000**	0.029
		0.016	0.024	0.046	0.024	**0.000**	0.109

Table 4: Variance of $\hat{\beta}_1$ (**minimum**, *next best*).

function	location	GWEN	GWAL	GWEN-LLE	GWAL-LLE	oracle	GWR
step	1	1.473	1.459	5.144	5.154	2.558	**1.253**
		0.729	0.695	3.072	2.997	**0.827**	0.677
		1.538	1.404	3.656	3.615	1.834	**1.149**
	2	2.969	2.883	20.687	20.605	4.232	**2.861**
		2.093	2.125	10.215	10.226	2.895	2.132
		1.016	1.048	4.133	4.276	1.176	**1.002**
	3	1.055	**1.015**	2.124	2.047	1.177	1.078
		1.577	1.562	2.953	2.683	1.642	**1.533**
		1.012	0.940	1.899	1.972	1.187	0.985
	4	**1.251**	1.266	2.480	2.370	1.330	1.275
		0.924	0.956	1.593	1.724	**1.003**	0.912
		0.944	0.931	1.837	1.507	1.053	**0.989**
	5	**0.959**	0.958	1.153	1.204	1.088	0.831
		1.008	0.983	1.585	1.776	1.203	0.883
		0.677	0.649	**1.127**	0.866	0.779	0.504
gradient	1	1.493	1.480	3.647	3.611	1.502	**1.396**
		1.530	1.554	7.603	7.678	1.604	**1.453**
		1.417	**1.207**	8.362	8.349	1.449	1.261
	2	**1.808**	1.873	5.178	5.213	1.876	1.886
		2.060	2.052	4.028	4.032	2.089	**2.037**
		1.282	1.268	2.989	2.880	1.272	**1.240**
	3	1.157	1.151	2.725	2.741	**1.138**	1.170
		1.245	1.220	2.503	2.492	1.213	**1.209**
		1.290	1.290	2.554	2.430	1.311	**1.189**
	4	1.164	1.156	2.221	2.226	1.164	**1.137**
		1.184	**1.069**	2.447	2.425	1.084	1.097
		0.722	0.725	2.024	2.013	**0.729**	0.725
	5	1.115	1.122	2.207	2.253	1.112	**1.034**
		1.059	1.085	2.350	2.498	**1.047**	0.950
		0.670	0.678	1.422	1.490	**0.759**	0.657
parabola	1	1.105	1.090	2.215	2.216	1.063	**0.981**
		0.892	0.844	1.495	1.410	0.875	0.761
		0.948	0.937	1.875	1.585	**1.009**	0.855
	2	1.032	1.052	1.963	2.106	1.045	**0.995**
		1.532	**1.441**	3.297	3.157	1.569	1.635
		1.099	1.094	2.293	2.251	1.125	**1.056**
	3	0.923	0.916	1.806	1.806	**0.932**	0.895
		1.388	1.348	2.700	2.493	1.408	**1.331**
		0.883	0.890	2.080	1.809	**0.910**	0.904
	4	1.145	**0.969**	2.447	2.455	1.240	1.138
		0.899	0.909	2.029	1.983	0.918	**0.936**
		1.384	**1.303**	2.799	2.656	1.367	1.335
	5	1.039	1.035	1.882	1.843	0.984	**1.008**
		0.837	0.842	1.107	**1.091**	0.774	0.788
		1.092	**0.983**	2.197	1.879	1.161	0.967

Table 5: Mean squared error of $\hat{y}$ (**minimum**, *next best*).