The literature on GWR

Wesley Brooks

UW-Madison

April 12, 2013

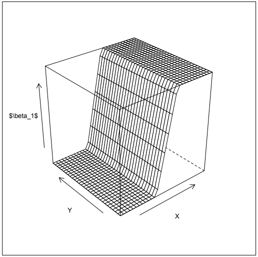
Outline

Introduction

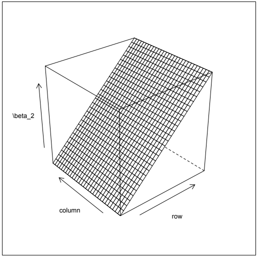
- A review of the development of the literature of geographically weighted regression (GWR)
- Some discussion of variable selection in varying-coefficient models

Varying coefficients regression

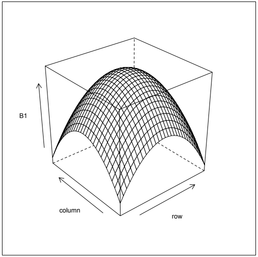
- Varying-coefficients regression (VCR) introduced in Hastie and Tibshirani (1993)
  - Linear regression: $y_i = x_i' \boldsymbol{\beta}$
  - VCR: $y_i = x_i' \boldsymbol{\beta}(s_i)$ (s is a location variable)



(a)



(b)



(c)

VCR estimation

- VCR is a special case of a generalized additive model (GAM) (Hastie and Tibshirani, 1986, 1990)
- Kernel-based methods: Hastie and Loader (1993); Loader (1999)
- Spline-based methods: R implementation is the gam package (Wood, 2006)

Kernel-based methods for estimation

- Local likelihood, given book-length treatment in Loader (1999)
- Most literature puts the smoothing kernel on the same variable as the coefficient
- GWR: kernel is on location, coefficient on other covariates.

Spline-based methods for estimation

- Smoothing splines: a method of estimating a smooth function through data (Wahba, 1990)
- The smooth function is pieced together, pieces centered on each observation, with the requirement that the first two derivatives match up at the joints
- Model can be parameterized to get coefficient functions (Hastie and Tibshirani, 1993)

Geographically weighted regression (GWR)

- Geographically-weighted regression (Brundson et al., 1998) is a kernel-based method of estimating spatial VCR models
- Main reference is the book Fotheringham et al. (2002)

Local regression

- Bias is known to be a problem in kernel smoothing (Hastie and Loader, 1993)

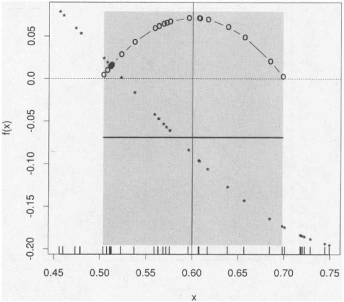


FIG. 1. *Effect of asymmetry on the Nadaraya–Watson estimator. Suppose we observe the data indicated by the asterisks; for clarity shown with no noise. We estimate $f(0.6)$ using the locally constant NW fit (thick line) using Epanechnikov's kernel $K(x/10) = (1 - x^2)I_{[-1,1]}(x)$, indicated by the circles. The asymmetry of observations causes substantial bias.*

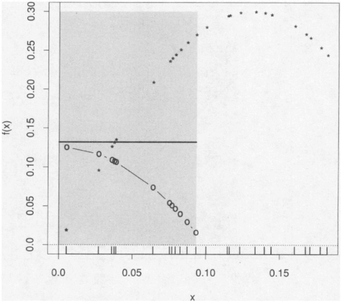


FIG. 3. *Nadaraya–Watson estimate, boundary effects. When the x_i are in the interval $[0, 1]$ and we attempt to estimate $f(0)$, the slope of the mean function induces particularly severe bias.*

Local regression

- Local regression (locally linear fits) reduce bias
- Local regression introduced in GWR context by Wang et al. (2008)

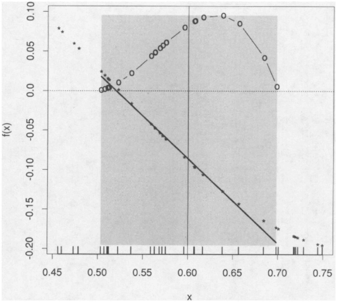


FIG. 4. *Local linear regression. We estimate f(0.6) using weighted least squares with weights assigned by Epanechnikov's kernel. This has substantially reduced the bias associated with the NW estimate. Moreover, the effective weights, shown by the circles, do not have the noisy behavior associated with the GM estimate.*

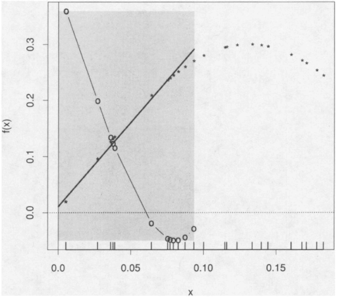


FIG. 5. *Local linear regression, boundary region. Fitting the weighted least squares has substantially reduced the bias of the Nadaraya–Watson estimate in the boundary region.*

Variable selection for spatial VCR models

- Global selection
- Local selection

Global variable selection

- Fan and Zhang (1999) for response variables that belong to an exponential-family distribution (as in the generalized linear model)
- Wang et al. (2008) for models with repeated measurements.

Local variable selection

- Antoniadas et al. (2012) estimates the coefficient functions with P-splines, and then uses the nonnegative garrote of Breiman (1995) to do local variable selection by selecting P-spline bases.
- Wheeler (2009) uses the LASSO with a jackknife criterion that limits selection to observation locations.

Antoniadas, A., I. Gijbels, and A. Verhasselt (2012). Variable selection in varying-coefficient models using p-splines. *Journal of Computational and Graphical Statistics 21*(3), 638–661.

Breiman, L. (1995). Better subset regression using the nonnegative garrote. *Technometrics 51*, 373–384.

Brundson, C., S. Fotheringham, and M. Charlton (1998). Geographically weighted regression - modeling spatial nonstationarity. *The Statistician 47*(3), 431–443.

Fan, J. and W. Zhang (1999). Statistical estimation in varying coefficient models. *The Annals of Statistics 27*(5), 1491–1518.

Fotheringham, A., C. Brunsdon, and M. Charlton (2002). *Geographically weighted regression: the analysis of spatially varying relationships*. Wiley.

Hastie, T. and C. Loader (1993). Local regression: automatic kernel carpentry. *Statistical Science 8*(2), 120–143.

Hastie, T. and R. Tibshirani (1986). Generalized additive models. *Statistical Science 1*(3), 297–310.

Hastie, T. and R. Tibshirani (1990). *Generalized Additive Models*. Taylor and Francis.

Hastie, T. and R. Tibshirani (1993). Varying-coefficient models. *Journal of the Royal Statistical Society. Series B (Methodological) 55*(4), pp. 757–796.

Loader, C. (1999). *Local regression and likelihood*. Springer New York.

Wahba, G. (1990). *Spline models for observational data*. Institute of Mathematical Sciences.

Wang, L., H. Li, and J. Z. Huang (2008). Variable selection in nonparametric varying-coefficient models for analysis of repeated measurements. *Journal of the American Statistical Association 103*(484), 1556–1569.

Wang, N., C.-L. Mei, and X.-D. Yan (2008). Local linear estimation of spatially varying coefficient models: an improvement on the geographically weighted regression technique. *Environment and Planning A 40*, 986–1005.

Wheeler, D. C. (2009). Simultaneous coefficient penalization and model selection in geographically weighted regression: the geographically weighted lasso. *Environment and Planning A 41*, 722–742.

Wood, S. (2006). *Generalized additive models: an introduction with R*. Texts in statistical science. Chapman & Hall/CRC.