Local Variable Selection and Parameter Estimation of Spatially Varying Coefficient Regression Models

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1. Simulation

Results from the simulation were summarized at five locations on the simulated grid (see Figure 2). The five key locations were chosen because they represent interesting regions of the β_1 coefficient surfaces. The results of variable selection and coefficient estimation are presented in the tables below.

1.1. Tables

1.1.1. Selection

1.1.2. Estimation

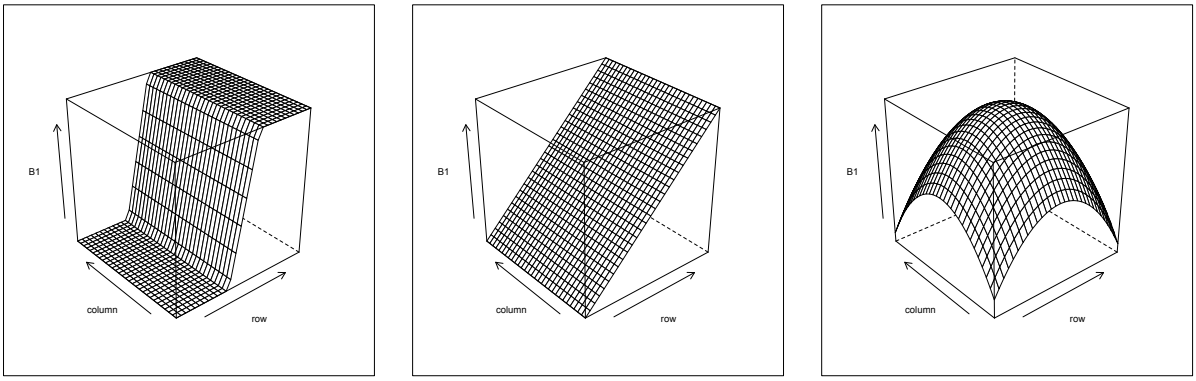

Figure 1: The actual β_1 coefficient surface used in the simulation.

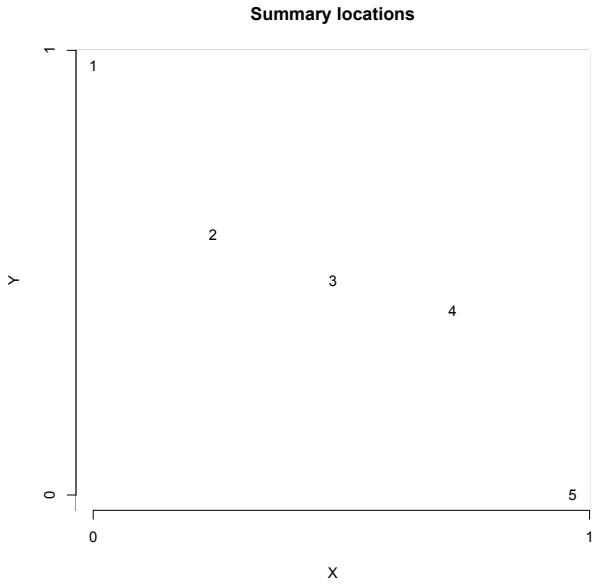

Figure 2: Locations where the variable selection and coefficient estimation of GWL were summarized.

Setting	function	ρ	σ^2
1	step	0	0.25
2	step	0	1
3	step	0.5	0.25
4	step	0.5	1
5	gradient	0	0.25
6	gradient	0	1
7	gradient	0.5	0.25
8	gradient	0.5	1
9	parabola	0	0.25
10	parabola	0	1
11	parabola	0.5	0.25
12	parabola	0.5	1

Table 1: Simulation parameters for each setting.

	step				gradient				parabola			
	`enet`		`glmnet`		`enet`		`glmnet`		`enet`		`glmnet`	
location	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5	β_1	β_2 - β_5
1	0.80	0.02	0.80	0.02	0.77	0.05	0.74	0.04	0.00	0.00	0.00	0.00
	0.80	0.04	0.80	0.04	0.69	0.07	0.69	0.07	0.03	0.01	0.03	0.01
2	0.97	0.00	0.98	0.00	0.80	0.01	0.80	0.00	0.51	0.01	0.49	0.01
	0.96	0.01	0.96	0.01	0.63	0.04	0.64	0.03	0.46	0.01	0.46	0.01
3	0.31	0.00	0.32	0.00	0.43	0.01	0.50	0.02	0.69	0.01	0.69	0.00
	0.36	0.02	0.36	0.02	0.35	0.04	0.36	0.04	0.53	0.03	0.54	0.03
4	0.00	0.00	0.00	0.00	0.24	0.01	0.26	0.01	0.56	0.00	0.59	0.00
	0.07	0.01	0.07	0.01	0.23	0.01	0.35	0.01	0.51	0.03	0.51	0.03
5	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01
	0.03	0.01	0.03	0.01	0.02	0.03	0.04	0.03	0.03	0.03	0.02	0.03

Table 2: Selection frequency for the simulation experiment

function	location	enet	glmnet	u.enet	u.glmnet	oracular	gwr
step	1	0.241	*0.239*	0.520	0.520	0.514	**0.020**
		0.289	*0.287*	0.781	0.780	0.899	**0.047**
	2	0.062	0.054	0.048	0.040	*0.030*	**0.027**
		0.080	0.078	0.064	0.064	*0.049*	**0.031**
	3	0.122	0.121	0.125	0.124	*0.026*	**0.008**
		0.126	0.126	0.135	0.135	*0.063*	**0.017**
	4	**0.005**	**0.005**	**0.005**	**0.005**	0.037	0.026
		0.015	*0.015*	0.018	0.018	0.075	0.035
	5	*0.002*	0.002	0.007	0.007	**0.000**	0.026
		0.008	0.008	0.033	0.033	**0.000**	0.056
gradient	1	0.317	0.321	0.599	0.573	*0.291*	**0.051**
		0.376	*0.360*	0.550	0.558	0.615	**0.072**
	2	0.319	0.277	0.251	0.250	**0.115**	*0.116*
		0.438	0.396	0.383	0.365	**0.153**	*0.154*
	3	0.106	0.096	0.112	0.104	*0.027*	**0.016**
		0.125	0.129	0.138	0.138	*0.054*	**0.027**
	4	**0.038**	*0.056*	0.058	0.074	0.138	0.132
		0.053	*0.079*	0.083	0.122	0.164	0.150
	5	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.041
		0.006	0.022	0.024	0.113	**0.000**	0.061
parabola	1	*1.000*	*1.000*	*1.000*	*1.000*	1.373	**0.751**
		1.018	1.018	1.063	1.063	1.670	**0.726**
	2	0.179	0.184	0.173	0.180	*0.035*	**0.026**
		0.197	0.197	0.199	0.199	*0.056*	**0.024**
	3	0.077	0.077	0.083	0.082	*0.018*	**0.007**
		0.127	0.125	0.144	0.142	*0.047*	**0.012**
	4	0.084	0.078	0.095	0.089	*0.030*	**0.009**
		0.111	0.112	0.129	0.129	*0.062*	**0.017**
	5	0.006	0.006	**0.005**	**0.005**	0.261	0.046
		0.011	**0.008**	0.029	0.027	0.538	0.079

Table 3: Mean squared error of $\hat{\beta}_1$ (**minimum**, *next best*).

function	location	enet	glmnet	u.enet	u.glmnet	oracular	gwr
step	1	-0.221	-0.214	*-0.052*	-0.053	0.062	**0.014**
		-0.250	-0.242	*-0.017*	**-0.016**	0.139	0.024
	2	-0.129	-0.121	-0.063	*-0.057*	**-0.033**	-0.129
		-0.142	-0.137	*-0.042*	-0.042	**0.001**	-0.135
	3	-0.263	-0.257	-0.251	-0.247	**-0.011**	*0.016*
		-0.231	-0.231	-0.193	-0.192	**0.002**	*0.019*
	4	**-0.069**	**-0.069**	**-0.069**	**-0.069**	0.083	0.132
		-0.038	-0.037	*-0.034*	-0.034	**0.024**	0.130
	5	*0.005*	0.005	0.009	0.009	**0.000**	-0.013
		0.002	0.002	0.005	0.005	**0.000**	-0.018
gradient	1	-0.334	-0.286	-0.148	-0.169	**0.011**	*-0.054*
		-0.414	-0.351	-0.135	-0.135	**0.050**	*-0.054*
	2	-0.514	-0.464	-0.420	-0.418	**-0.317**	*-0.320*
		-0.597	-0.545	-0.514	-0.494	**-0.342**	*-0.364*
	3	-0.221	-0.166	-0.178	-0.131	**0.061**	*0.066*
		-0.238	-0.212	-0.197	-0.184	**0.033**	*0.051*
	4	**0.037**	*0.058*	0.060	0.077	0.335	0.348
		0.049	0.108	*0.079*	0.151	0.329	0.353
	5	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.018
		0.009	0.019	0.015	0.042	**0.000**	0.066
parabola	1	*-1.000*	*-1.000*	*-1.000*	*-1.000*	-1.066	**-0.851**
		-0.998	-0.998	-1.004	-1.004	*-0.974*	**-0.823**
	2	-0.331	-0.335	-0.298	-0.308	**-0.105**	*-0.132*
		-0.357	-0.354	-0.320	-0.320	*-0.136*	**-0.128**
	3	-0.129	-0.128	-0.087	-0.086	*0.041*	**0.031**
		-0.187	-0.181	-0.122	-0.115	*0.042*	**0.027**
	4	-0.137	-0.124	-0.092	*-0.079*	0.085	**0.052**
		-0.135	-0.136	-0.081	-0.082	*0.067*	**0.048**
	5	0.008	0.008	**0.007**	**0.007**	0.101	0.159
		0.018	**0.013**	0.024	0.019	0.102	0.171

Table 4: Bias of $\hat{\beta}_1$ (**minimum**, *next best*).

function	location	enet	glmnet	u.enet	u.glmnet	oracular	gwr
step	1	*0.194*	0.196	0.523	0.523	0.515	**0.020**
		0.229	0.231	0.788	0.788	0.889	**0.046**
	2	0.045	0.039	0.044	0.037	*0.029*	**0.011**
		0.060	0.060	0.063	0.063	*0.049*	**0.013**
	2	0.054	0.055	0.063	0.063	*0.026*	**0.008**
		0.073	0.074	0.099	0.099	*0.064*	**0.017**
	2	**0.000**	**0.000**	**0.000**	**0.000**	0.030	0.009
		0.013	*0.013*	0.017	0.017	0.075	0.019
	2	*0.002*	0.002	0.007	0.007	**0.000**	0.027
		0.008	0.008	0.033	0.033	**0.000**	0.056
gradient	1	*0.207*	0.242	0.583	0.550	0.294	**0.049**
		0.206	0.239	0.537	0.546	0.618	**0.070**
	2	0.055	0.062	0.076	0.076	*0.015*	**0.014**
		0.083	0.100	0.120	0.123	*0.036*	**0.022**
	3	0.058	0.069	0.081	0.087	*0.023*	**0.012**
		0.069	0.084	0.100	0.105	*0.053*	**0.025**
	4	0.037	0.053	0.055	0.069	*0.026*	**0.011**
		0.051	0.068	0.078	0.101	0.056	**0.026**
	5	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	0.041
		0.006	0.022	0.024	0.112	**0.000**	0.057
parabola	1	**0.000**	**0.000**	**0.000**	**0.000**	0.238	0.027
		0.022	*0.022*	0.055	0.055	0.729	0.050
	2	0.070	0.073	0.086	0.086	*0.025*	**0.009**
		0.071	0.072	0.098	0.098	*0.037*	**0.008**
	3	0.061	0.061	0.076	0.076	*0.017*	**0.006**
		0.093	0.093	0.130	0.130	*0.045*	**0.011**
	4	0.066	0.064	0.088	0.084	*0.023*	**0.007**
		0.094	0.094	0.123	0.124	*0.058*	**0.015**
	5	0.006	0.006	**0.005**	**0.005**	0.253	0.021
		0.011	**0.008**	0.029	0.026	0.533	0.050

Table 5: Variance of $\hat{\beta}_1$ (**minimum**, *next best*).

function	location	enet	glmnet	u.enet	u.glmnet	oracular	gwr
step	1	*0.108*	0.108	0.108	0.108	**0.101**	0.223
		0.174	0.174	0.174	*0.174*	**0.146**	0.337
	2	0.340	0.341	*0.340*	0.341	**0.291**	0.462
		0.564	0.565	0.564	0.565	**0.498**	0.776
	3	0.276	0.274	0.276	*0.274*	**0.138**	0.281
		0.459	*0.458*	0.459	0.458	**0.333**	0.492
	4	0.256	*0.256*	0.256	0.256	**0.173**	0.307
		0.664	0.665	0.664	0.665	**0.518**	0.722
	5	0.236	0.236	0.236	0.236	**0.218**	*0.227*
		0.344	0.344	*0.344*	0.344	0.358	0.382
gradient	1	0.112	0.114	*0.112*	0.114	**0.096**	0.217
		0.278	*0.267*	0.278	**0.267**	0.320	0.558
	2	0.258	*0.246*	0.258	0.246	**0.216**	0.353
		0.379	0.386	**0.379**	0.386	0.413	0.570
	3	0.247	0.203	0.247	*0.203*	**0.181**	0.292
		0.455	*0.435*	0.455	*0.435*	**0.414**	0.574
	4	0.317	*0.285*	0.317	*0.285*	**0.227**	0.309
		0.490	*0.419*	0.490	0.419	**0.369**	0.532
	5	0.243	*0.238*	0.243	*0.238*	**0.237**	0.259
		0.327	**0.305**	0.327	**0.305**	0.414	0.424
parabola	1	0.298	*0.297*	0.298	*0.297*	**0.128**	0.304
		0.365	0.365	0.365	*0.365*	**0.189**	0.435
	2	0.326	0.303	0.326	0.303	**0.231**	*0.282*
		0.606	0.606	0.606	*0.606*	**0.560**	0.822
	3	0.292	0.290	0.292	*0.290*	**0.224**	0.332
		0.367	*0.365*	0.367	**0.365**	0.397	0.629
	4	*0.238*	0.243	0.238	0.243	**0.196**	0.295
		0.596	0.612	**0.596**	0.612	0.600	0.811
	5	0.219	*0.216*	0.219	*0.216*	**0.100**	0.306
		0.231	0.238	*0.231*	0.238	**0.150**	0.385

Table 6: Mean squared error of $\hat{Y}$ (**minimum**, *next best*).