1. Oracle property

Here we show that the estimation accuracy is just as good as if the relevant predictor groups were specified in advance.

Theorem 1.1. *If* $\sqrt{n}a_n \to 0$ *and* $\sqrt{n}b_n \to \infty$, *then* $\sqrt{nh^2 f(\boldsymbol{s})}\left(\hat{\boldsymbol{\beta}}_{(a)}(\boldsymbol{s}_i) - \boldsymbol{\beta}_{(a)}(\boldsymbol{s}_i) - \frac{\kappa_2 h^2}{2\kappa_0}\{\boldsymbol{\beta}_{uu}(\boldsymbol{s}_i) + \boldsymbol{\beta}_{vv}(\boldsymbol{s}_i)\}\right) \xrightarrow{d} N(0, \Sigma_{(a)}(\boldsymbol{s}_i))$.

Proof. The proof proceeds by showing that if the tuning parameter λ is chosen correctly, then the penalty term vanishes for the relevant predictor groups and becomes infinite for the irrelevant predictor groups.

$\square$

Since $\|\tilde{\boldsymbol{\beta}}(\boldsymbol{s}_i)\|^\gamma = O_p\left\{(nh^2)^{-\gamma/2}\right\}$ and $h = O(n^{-1/6})$, in order for $\sqrt{n}a_n \to 0$ and $\sqrt{n}b_n \to \infty$, we require that $\lambda = O(n^\alpha)$ where $\alpha \in \left(-\left\{1 + \gamma - \frac{\gamma}{6}\right\}/2, -1/2\right)$.