Superconductors

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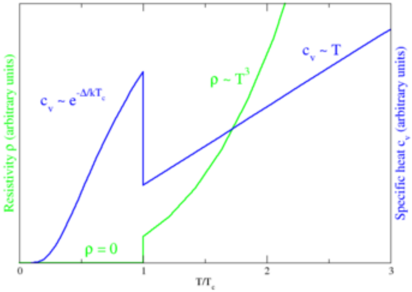
Introduction

Superconductivity is defined as a complete disappearance of electrical resistance in a substance especially at very low temperatures.



Why does this happen? The flaws and vibrations of the atoms in the materials should cause resistance in when the electrons flow through it, yet, the electric resistance is equal to zero although the flaws and vibrations still exist.

Electrons

Let's start with electrons, which are fermions. All fermions have half-integer spin. Now, because fermions have to have the half-integer spin, we get a constraint on a system with more than one fermion

This constraint is called the Pauli exclusion principle, which states that no two fermions can have the exact same set of quantum numbers.

This causes electrons that have the same energy to have different spins, one having spin up and the other spin down, and why we can only have two electrons in each energy level.

Fermi-Dirac Distribution

Electrons are fermions and so at temperature T and a state with energy E is occupied according to the Fermi-Dirac distribution:

$$f(\varepsilon) = \frac{1}{e^{\beta(E-\mu)} + 1} \tag{1}$$

where $f(\varepsilon)$ is the occupation probability of a state of energy ε, β is $\frac{1}{k_B T}$, and μ is the chemical potential.

At absolute zero the value of the chemical potential is defined as the Fermi energy. At room temperature, it turns out the chemical potential for metals is virtually the same as the Fermi energy, with a typical different being on the oder of 0.01%.

Superconductors

Things that make superconductors awesome and unique:

- Cooper Pairs
- Energy Gap
- Meissner Effect

All of which are gotten from the BCS theory.

Bardeen, Cooper, and Schrieffer

BCS is the first theory that explained superconductivity at a microscopic scale. It also correctly predicts the energy gap 2Δ, twice the energy gap, at the fermi level.

It turns out that the effective forces between electrons can sometimes be attractive, this is due to the phonon electron coupling. Cooper considered a single electron pair outside a occupied fermi surface and found that the fermions formed a stable pair bound state no matter how weak the attractive force.

Schreiffer constructed a many particle wave function in which all electrons near the fermi surface are paired up, this is a form of coherent states and the energy gap arises from it's analysis.

Cooper Pairs

One electron passes by positively charged ions in the lattice of the superconductor, which makes the lattice distort.

In distorting an area of increased positive charge, the electron attracts a closely following negative electron that will also pass along the relatively positive trough in the distorted lattice area.

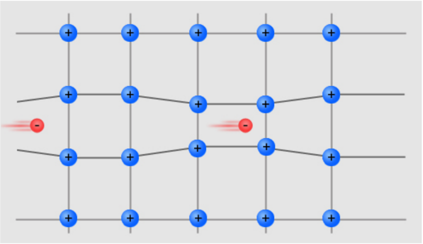
The electron pairs produced in this way are coherent (in phase) as they pass through the conductor in unison. The electrons are screened by the phonons and, although paired, are separated by some distance.

This means that the first electron of the Cooper pair has emitted a phonon and the second electron has absorbed that phonon.

Cooper Pairs

As long as collisions with the ionic lattice of the solid do not supply enough energy to break the Cooper pairs, the electron fluid will be able to flow without dissipation. As a result, it becomes a superfluid, and the material through which it flows a superconductor.

Cooper pairs make super conductors



At extremely low temperatures, an electron can draw the positive ions in a superconducting material towards it. This movement of the ions creates a more positive region that attracts another electron to the area.

Cooper Pairs

In a metal, electrons are waves. Each of these electrons is relatively independent and follows its own path independent of other electrons. In a superconductor, the majority of these electrons merge in order to form a large collective wave. In quantum physics, we call it macroscopic quantum wavefunction, or condensate. When the collective wave is formed, it requires each member to move at the same speed. In a metal, an individual electron is easily diverted by a flaw or an atom that is too big. In a superconductor however, this same electron can be diverted only if, at the same time, all the other electrons of the collective wave are diverted in the exact same manner. The flaw in a single atom surely cannot do that; the wave will not be diverted, and, thus, not slowed down.

Energy Gap

The Cooper pairs are able to occupy lower energy-levels than the electrons did and the spectrum of energy levels contains a gap between the highest occupied state and the next higher available state.

The pairs of electrons act more like bosons which can condense into the same energy level. The electron pairs have a slightly lower energy and leave an energy gap above them on the order of .001 eV which inhibits the kind of collision interactions which lead to ordinary resistivity.

The essential point is that below T_C the binding energy of a pair of electrons causes the opening of a gap in the energy spectrum at E_F, separating the pair states from the normal single electron states.

Energy Gap

Excitations within the solid are not sufficient to overcome this gap and so these paired electrons are able to travel through the solid without interacting (scattering) off anything else present in the solid.

The BCS theory gives the expression of the energy gap that depends on the Temperature T and the Critical Temperature T_C and is independent of the material:

$$E_G = 3.52 k_B T_C \left(1 - \frac{T}{T_C}\right)^{1/2} \tag{2}$$

Meissner Effect

When a material makes the transition from the normal to superconducting state, it actively excludes magnetic fields from its interior; this is called the Meissner effect.

It does this by setting up electric currents near its surface. The magnetic field of these surface currents cancels the applied magnetic field within the bulk of the superconductor. As the field expulsion, or cancellation, does not change with time, the currents producing this effect (called persistent currents) do not decay with time.

Meissner Effect

This constraint to zero magnetic field inside a superconductor is distinct from the perfect diamagnetism, which would arise from its zero electrical resistance. Zero resistance would imply that if you tried to magnetize a superconductor, current loops would be generated to exactly cancel the imposed field (Lenz's law). But if the material already had a steady magnetic field through it when it was cooled through the superconducting transition, the magnetic field would be expected to remain. If there were no change in the applied magnetic field, there would be no generated voltage (Faraday's law) to drive currents, even in a perfect conductor. Hence the active exclusion of magnetic field must be considered to be an effect distinct from just zero resistance.

$$\frac{d\phi}{dt} = 0 \tag{3}$$

The magnetic field remains constant as a function of time.

Type I or Type II

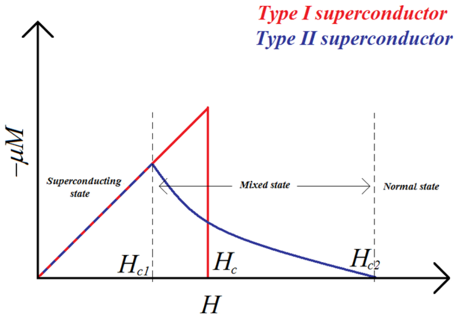
There are two types of superconductors, Type I and Type II.

When the magnetic field becomes too strong, the system becomes a metal again. Type I superconductors demonstrate this type of behaviour.

Type II superconductors act a bit differently. Under a small magnetic field, they react like type I superconductors and completely expel the magnetic field. But when the magnetic field is stronger, they prefer to adopt a compromise situation and allow some of the magnetic field to penetrate along vortices. Each vortex is a small region in a superconductoring metal that acts like a normal metal.

The material then becomes a sieve. In order to enable this magnetic field to go through the vortex, the material develops superconducting currents circulating around this pillar in a spiral motion justifying the name vortex.

Type I or Type II



Coherence Length and London Penetration Depth

Coherence length is defined as

$$\xi = \frac{2\hbar v_F}{\pi \Delta} \tag{4}$$

where v_F is the Fermi velocity, and Δ is the superconducting energy gap.

The London penetration depth is defined as

$$\lambda = \sqrt{\frac{mc^2}{4\pi n_s e^2}} \tag{5}$$

where m is the mass of the electon, e is the charge of an electon, and n_s is the number density of superconducting carriers.

Ginzburg-Landau Theory

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}\left(-i\hbar\nabla - 2eA\right)^2\psi = 0 \tag{6}$$

$$\nabla \times B = \mu_0 j; \qquad j = \frac{2e}{m}\Re\left\{\psi^*\left(-\hbar\nabla - 2eA\right)\psi\right\} \tag{7}$$

Ginzburg-Landau Equations

If we consider a homogeneous superconductor where there is no superconducting current, we can simplify ψ to

$$\alpha\psi + \beta|\psi|^2\psi = 0 \tag{8}$$

This gives us the trivial solution, which corresponds to temperatures above the critical temperature. Below T_C, ψ is expected to have non-trivial solutions:

$$|\psi|^2 = -\frac{\alpha}{\beta} \tag{9}$$

If we allow a temperature dependance on α, such that $\alpha(T) = \alpha_0(T - T_C)$, then $T > T_C$ means that only the trivial solution will be a solution, since the RHS would be negative, and the magnitude of a complex number must be a non-negative number.
When $T < T_C$, the RHS is positive, and we have non-trivial solutions

$$|\psi|^2 = -\frac{\alpha_0(T - T_C)}{\beta} \tag{10}$$

Ratio of the Two

Now, the penetration depth and the coherence length both diverge as the critical temperature, T_C.

$$\lim_{T\to T_C} \xi(T) = \frac{\xi(0)}{\sqrt{1-(T/T_C)}} \qquad \lim_{T\to T_C} \lambda(T) = \frac{\lambda(0)}{\sqrt{1-(T/T_C)}} \tag{11}$$

But, if we take a ratio of the two, it will be independent of temperature.

$$\kappa \equiv \frac{\lambda}{\xi} \tag{12}$$

For a Type I superconductors,

$$0 < \kappa < 1/\sqrt{2} \tag{13}$$

For Type II superconductors,

$$\kappa > 1/\sqrt{2} \tag{14}$$

Most magnetic materials

For extreme Type II superconductors, or most magnetic materials,

$$\kappa \gg 1 \tag{15}$$

which means that

$$\lambda \gg \xi \tag{16}$$

The larger the ratio λ/ξ, the larger the ration of H_{C2} to H_{C1}.

$$H_{C1} \approx \frac{\Phi_0}{\pi\lambda^2} \tag{17}$$

$$H_{C2} \approx \frac{\Phi_0}{\pi\xi^2} \tag{18}$$

Vortices

Each vortex has a normal core – long thin normal cylinder along the field direction. The radius of the normal core is approximately ξ.

Around the normal core, there is a circulating supercurrent. The direction of circulation is such that the direction of magnetic field created by this current coincides with the direction of external magnetic field (along the normal core). The size of the region where the supercurrent circulates is approximately λ.

The penetration of the vortices in the superconductor takes place at $H < H_{C1}$. Each vortex carries one quantum of magnetic flux.

Magnetic Flux

This flux quantum has a value of

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\ \mathrm{Tm}^2. \tag{19}$$

This unit of flux is called a fluxoid.
This quantization is the result of the existence of the condensate combined with the formation of collective Cooper pairs. The exact value of Φ_0 is the experimental proof that electrons are paired in a superconductor.

Vortices

These quantized vortices are known as Abrikosov vortices.

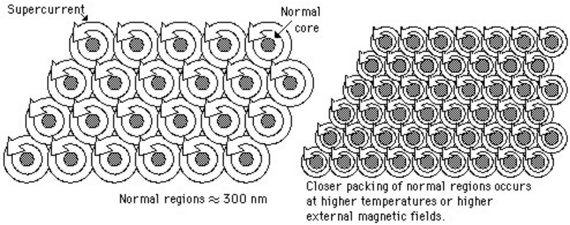
Vortices form a triangular lattice because vortices of the same sign (direction of circulation) repel each other

Feynman first calculated this triangular arrangement as the lowest energy approximation to solid-body rotation.

Vortices

As H increases from H_{C1} to H_{C2}, the density of the vortices increase.

At H_{C2}, the distance between the vortices becomes approximately ξ, and the second order phase transition to the normal state takes place.



Second-order Phase Transitions

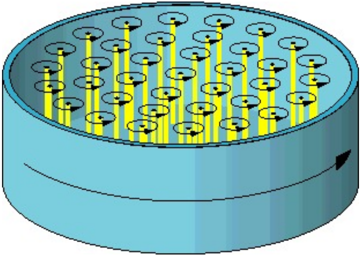
Second order phase transitions occur when a new state of reduced symmetry develops continuously from the disordered (high temperature) phase.

The ordered phase has a lower symmetry than the Hamiltonian – the phenomenon of spontaneously broken symmetry.

i.e. the difference between Type I and Type II superconductors.

Superfluids

Interestingly, superfluids have these same vortices, just under different conditions.



Superfluid Vortices

These quantized vortices align with the axis of rotation and form a triangular lattice.

The resulting course-grain flow closely approximates the analogous situation in classical fluid even though quantum mechanics imposes stringent constraints on vorticity.

Aside

The zero point energy of liquid helium is less if its atoms are less confined by their neighbors. Hence in liquid helium, its ground state energy can decrease by a naturally-occurring increase in its average interatomic distance.

The zero-point energy is the lowest possible energy that a quantum mechanical physical system may have.

Because of the very weak interatomic forces in helium, this element would remain a liquid at atmospheric pressure all the way from its liquefaction point down to absolute zero. Liquid helium solidifies only under very low temperatures and great pressures.

Conclusion

The key idea behind superconductors is that there is attraction between electron pairs, when normally we expect them to repulse one another. This gives us Cooper Pairs, which form the condensate (which can be thought of as a superfluid), that allows for an energy gap, and the zero resistivity.

Questions

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